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CHEMICAL DEFENCE EXPERIMENTAL ESTABLISHMENT

LIGHT TRANSMISSION THROUGH A FOG-OIL SMOKE LAYER WITH AN ABSORBING OR PARTIALLY REFLECTING BOUNDARY

BY

M.W. WOOTTON AND W.R. LANE

PORTON TECHNICAL PAPER No. (R) 34
SUMMARY

Comparison is made between measurements of the light flux at the boundary of a layer of polydisperse fog-oil smoke, due to radiation from a point source outside the layer, and predicted values based on the "Six Flux" method of calculating light scattering by small particles. Agreement is found when the boundary is absorbing, but with a partially reflecting boundary (albedo 0.5) the experiments show a greater increase in flux, due to reflection, than the theory predicts. Again contrary to the theory the flux increase becomes greater as the angle of incidence is reduced.

(Sgd.) H.L. Green,
Supt., Physics Research Division.

(Sgd.) A.S.G. Hill,
Deputy Director.
LIGHT TRANSMISSION THROUGH A FOG-OIL SMOKE LAYER WITH AN
ABSORBING OR PARTIALLY REFLECTING BOUNDARY

By

N.W. Wootten and V.R. Lane

INTRODUCTION

The development of a method of producing a stable well-defined shallow layer of fog-oil smoke near the floor of a chamber (1) has made it possible to compare measurements of the transmission of light through such a cloud with those predicted from the theoretical analysis of radiation transport in such a system made by Chu, Clarko and Churchill (2), and with some results of small-scale experiments with dense dispersions of latex particles undertaken by Scott and Churchill (3).

Chu, Clarko and Churchill based their mathematical treatment of transmission of electromagnetic radiation through a multiple scattering medium on a model in which the angular distribution of radiation scattered by a single particle is represented by six components - one forward, one backward and four perpendicular to the direction of propagation. They have used this "six-flux" representation to obtain analytical solutions for certain cases including that of a parallel plane dispersion of particles, bounded by one absorbing or partially reflecting surface and illuminated by a point source outside it; for this configuration they have computed the flux incident on the boundary surface as a function of the optical thickness of the dispersion, the angle of incidence of the radiation and the boundary reflectivity. A brief outline of the six flux model of multiple scattering is given in the Appendix to this paper.

Scott and Churchill (3) later published measurements of the flux received from a point source by a 2π receiver located in the partially reflecting boundary. Their experimental dispersion consisted of very uniform (0.814 μm diam) polystyrene latex particles suspended in an aqueous solution of glycol. Heptane was introduced above the dispersion
to avoid optical discontinuity at the interface, and the source, a neon lamp ($\lambda = 0.427 \mu$) located in the heptane, was moved horizontally and vertically relative to the receiver. The ratio of the flux striking the receiver in the presence and absence of the dispersion was measured for a number of dispersion depths, boundary reflectivities, and source positions. The results were correlated graphically and found to be in reasonable agreement with the theoretical solution.

The work described in the present paper was undertaken to ascertain to what extent the theoretical computations are also supported by measurements made in a polydisperse oil-smoke layer.

**SMOKE CHAMBER EXPERIMENTS**

A layer of fog-oil smoke, 0.5 m deep, was produced on the floor of a chamber (approx. 11m x 6m x 3m) by passing the output of a thermal generator through boxes containing crushed solid carbon dioxide as described in the appendix to reference (1). A light source consisting of a 24 volt, 150 watt lamp in a water-cooled jacket could be placed at any height above the floor and 2$\pi$-cosine receivers (consisting of photovoltaic cells, or photomultipliers, covered by ground opal glass diffusers) were placed facing upwards at known distances from the ground zero of the source to measure the flux falling on a horizontal surface. The ratio of the flux in the presence of smoke to that in the absence of smoke was found from these measurements. The floor was normally black (albedo 0.05) but could be overlaid with white lining paper (albedo 0.50).

Other parameters determined during these experiments were (a) the optical density of the smoke, measured by the transmission of a collimated-beam densitometer, (b) the mass concentration of the smoke, determined by aspiration through filters of "Millipore" material, and (c) the droplet size distribution, determined with the aid of a conifuge fitted with glass slides coated with a thin oleophobic film. These measurements were made at intervals during each experiment and were plotted against time. The values of all the parameters at any given time could be obtained from these graphs and could, therefore, be related to each other.

**RESULTS AND CALCULATIONS**

The most convenient means of comparing the results of our experiments with the published theoretical values is in the form of a graph of the flux ratio plotted against the optical thickness of the dispersion for various values of the angle between the direction of the incident beam and the normal to the receiving surface. Our experimental data were reduced to this form by means of the procedure outlined below. The symbols used are mainly those given in (3) to make for ease of comparison.
The angle \( \theta \) between the direction of the incident beam and the normal (Fig. 1) is determined by the height of the source and the distance of the receiver from ground zero. The flux ratio \( P \) is the ratio between the flux falling on a horizontal surface in the presence of a dispersion to that in the absence of the dispersion. The optical thickness \( T \) is the product of the dispersion depth \( t \), the number \( N \) of particles per unit volume and the effective scattering area \( q_0 \) of the droplets. The latter is determined by the droplet size and the scattering coefficient \( K \), which is the ratio of the total flux scattered by a droplet in all directions to the flux incident on the geometrical cross section of the droplet. \( K \) in turn depends on the parameter \( \alpha = \pi d/\lambda \) (where \( d \) is the droplet diameter and \( \lambda \) is the wavelength of the light used). The values of \( K \) were obtained from Penndorf's tabulations (5) for a substance of the same refractive index as the for-oil, namely 1.50. From consideration of the spectral distribution of the radiation and the spectral response of the photocells and photomultipliers used it was calculated that the value of \( \lambda \) appropriate to these experiments was 0.60 \( \mu \).

All the chamber experiments were started with as nearly as possible the same smoke concentration. A complete analysis of drop-size distribution as determined by the conifuge measurements, and mass concentration, from the millipore samples, was made for three of the eight experiments and these were related to the attenuation coefficient, \( \sigma \), determined by the densitometer. The value of \( \sigma \), which is given by the expression

\[
\tau = e^{-\sigma x}
\]

(whence \( \tau \) is the transmission of a collimated beam over a distance \( x \)), has been used in all previous chamber experiments to characterize the smoke cloud.

The change in the cumulative percentage number distribution was determined as the cloud aged and this showed that the mean droplet diameter increased with time (Fig. 2). The variation of mass concentration and of \( \sigma \) with time was also found and in Fig. 3, the relation between the two is given. By taking the mean value of the drop size in each successive 20% range of the number versus size distribution the relative masses in each range could be determined. These are given in Table 1.
Table I

Droplet size and mass distributions in fog-oil smoke

<table>
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<tr>
<th>Time from commencement of expt (min)</th>
<th>0</th>
<th>30</th>
<th>60</th>
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<tr>
<td>% No Range</td>
<td>Mean diam. (μ)</td>
<td>Mean diam. (μ)</td>
<td>Mean diam. (μ)</td>
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<tr>
<td>0 - 20</td>
<td>0.50</td>
<td>0.62</td>
<td>0.69</td>
</tr>
<tr>
<td>20 - 40</td>
<td>0.63</td>
<td>0.82</td>
<td>0.88</td>
</tr>
<tr>
<td>40 - 60</td>
<td>0.75</td>
<td>0.97</td>
<td>1.02</td>
</tr>
<tr>
<td>60 - 80</td>
<td>0.88</td>
<td>1.13</td>
<td>1.22</td>
</tr>
<tr>
<td>80 - 100</td>
<td>1.12</td>
<td>1.40</td>
<td>1.50</td>
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(\text{It is interesting to note here that although the size distribution changes as the cloud ages the relative mass in the 20\% ranges remains almost constant}).

From the total measured mass and the relative estimated mass, the number of droplets of a given size range per unit volume of the cloud could be determined. The mean size in each range was also used to determine \( a \).

The effective value of the optical thickness \( T \) was taken as \( 2 \pi \sigma T \) for the five ranges into which the drop sizes were divided. The effective value of \( a \) was taken as the mean value found for these ranges. The relationships between \( T, \sigma \) and \( a \) are shown in Fig. 4.

The close agreement of the various relationships in the three experiments (well within the limits that could be expected from the nature of the experiment and the methods of sampling the fog-oil smoke) and the nearly linear relationship between the calculated optical thickness and the measured attenuation coefficient gave confidence in the use of \( \sigma \) as the characterizing parameter. This was taken as the reference for the comparison of the flux ratios for the different boundary conditions and geometrical configurations in the whole series of experiments.

The flux ratio \( P \) measured in the experiments when the source was above the smoke layer were plotted for given values of \( \sigma \) against \( \tan \theta \) and the best curve was fitted to them (Fig. 5). From this curve the appropriate values were plotted against \( T \) (Fig. 6) giving the final curve on which the comparison with the theoretical predictions can be made.

DISCUSSION OF RESULTS

For comparison with a previous series of smoke-chamber experiments (1) \( P \) was plotted against ground distance from the source. The agreement was
very good for all readings when the source was above the smoke layer but
for a source height of 0.5m, which is the approximate height of the smoke
layer, the values of P were sometimes above those previously obtained
and sometimes below. Although the smoke layer is stabilised at about
0.5m its height may well vary by a few centimetres so that the source
may sometimes be immersed in the smoke and sometimes be outside it; this
would affect the fluxes received by distant receivers. Such slight
variations in the smoke layer height will have very little effect on
the flux received when the source is located well above the layer.

When plotted as a function of tan θ the curves of P for different
source heights show little deviation from a constant value. This is in
accordance with the findings of Churchill and his collaborators (3).

**COMPARISON WITH THEORY**

Only a few of the data given in (3) are suitable for a direct
comparison but in (4) additional values of the flux ratio as a function
of tan θ, T and boundary reflectivity are given. The flux ratio E,
quoted there, however, is defined as the ratio of the flux on a
horizontal surface in a dispersion to that falling normally onto a
surface facing the source in the absence of the dispersion. P is there-
fore obtained by dividing E by the appropriate value of cos θ.

In Fig. 6 interpolated and extrapolated values of P, for a reflecting
boundary of albedo 0.50 and at given values of T and tan θ, obtained from the
data (4) are plotted together with the results obtained in our experiments.

Comparison of the theoretical values of P given in (2) and (3) for
different values of α (in the former α = 10.0 and in the latter α = 6.0)
indicate that P is a function of α. The value of α in the smoke chamber
experiments varies slightly with c but lies between 4.5 and 5.5
The theoretical values quoted are for α = 6.0 so this will account for
their being slightly higher than the experimental values with an absorbing
boundary. Apart from this the smoke chamber results and theoretical
predictions for an absorbing boundary show good agreement.

For a partially reflecting boundary (albedo 0.50), however, the smoke
chamber results are somewhat higher than the theory predicts. This is
contrary to the trend in the latex dispersion experiments (3) where Scott
and Churchill conclude that "the effect of reflectivity on the transmission
is essentially independent of the angle of incidence but increases signifi-
cantly with the optical thickness of the dispersion". Whilst the smoke
chamber experiments support the latter part of this conclusion comparison
of the curves in Fig. 6 shows that the increase of flux due to the presence
of a reflecting boundary is greater at low values of tan θ than at higher
values.
CONCLUSIONS

It has been shown by chamber experiments that the attenuation of light flux in a layer of polydisperse fog-oil smoke with absorbing boundaries agrees with the values predicted by the "Six Flux" theoretical method of calculating light scattering in aerosols. It appears, however, that the theory underestimates the increase of flux due to a reflecting boundary, particularly at narrow angles of incidence.

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REFERENCES


Appendix to P.T.P.(R)4

Outline of Six-Flux Model of Multiple Scattering by Dispersions

For thin dispersions of uniform particles or limited paths of transmission such that re-scattering is negligible, the transmission and angular distribution of scattered radiation can be described by a simple exponential attenuation law and application of the Mie theory of scattering of electromagnetic radiation by dielectric spheres. In dense or extensive dispersions such that the mean free path for radiation is comparable with or less than the least dimension of the system, multiple scattering becomes significant and the classical expressions for scattering are inadequate.

Multiple scattering by a dispersion of non-absorbing particles is described mathematically by the transport equation (1, 2) obtained by an energy balance over a differential volume of unit cross section and length ds along the direction \( \hat{\Omega} \).

\[
\frac{di(r, \hat{\Omega})}{ds_{\hat{\Omega}}} = - N \sigma_s^{-1} i(r, \hat{\Omega}) + N \sigma_s \int_{\Omega} f(\hat{\Omega}, \hat{\Omega}') d\Omega' \tag{1}
\]

in which

- \( i(r, \hat{\Omega}) \) is the specific intensity, defined as the energy intercepted per unit time by a unit area at \( r \), per unit solid angle from the direction \( \hat{\Omega} \) normal to the area, where \( r \) is the radius vector from the origin of the co-ordinate system and \( \hat{\Omega} \) is a unit vector representing direction. In general \( i(r, \hat{\Omega}) \) is a function of five variables, three for position and two for direction.

- \( f(\hat{\Omega}, \hat{\Omega}') \) is the angular distribution function for single scattering, which gives the fraction of energy scattered from a beam in the direction \( \hat{\Omega} \) into a unit solid angle in the direction \( \hat{\Omega}' \). For spherical particles the angular distribution is symmetrical about the incident direction and \( f(\hat{\Omega}, \hat{\Omega}') \) reduces to \( f(\theta) \) as illustrated in fig.A.1.

\( N \) = Number of particles per unit volume of the dispersion.

\( \sigma_s \) = effective cross section of a particle for scattering.

The scattering cross section and the density of particles can be combined to give the mean free path for scattering \( (1/N \sigma_s) \). The cross section, the mean free path and the angular distribution function are functions of the dimensions and refractive index of the particles and of the wavelength of the radiation (Mie theory).

Exact solutions of the transport equation have been worked out for only a few cases and for highly idealized conditions - isotropic scattering.
in dispersions of infinite thickness — but useful approximate solutions can be obtained if the angular distribution from a single scattering is represented by a number of discrete components. In the six flux model of Chu and Churchill (3), illustrated in Fig.A1, the angular distribution function for single scattering by spherical particles, \( f(\theta) \), is resolved into a forward-scattering component \( f \), a backward scattering component \( b \), and four equal sideways-scattering components \( s \), defined as:

\[ f = 2\pi \int_{\pi/2}^{\pi} f(\theta) \cos^2 \theta \sin \theta \, d\theta \]
\[ b = 2\pi \int_{\pi/2}^{\pi} f(\theta) \cos^2 \theta \sin \theta \, d\theta \]

and \( s = \frac{1}{4} (1 - f - b) \)

\( f(\theta) \) is the fraction of radiation scattered into a unit solid angle in the direction \( \theta \), see Fig.A1).

The specific intensity within a dispersion can thus be represented by six discrete components, \( E_1 \ldots E_6 \), in the positive and negative direction in three orthogonal directions. The representation for the case of obliquely incident radiation from a source outside a plane-parallel dispersion is shown in Fig.A2. Use of this six flux model permits reduction of the transport equation to a set of six simultaneous differential equations, whose solution yields the six components of the flux at any point.

The solutions obtained by Chu and Churchill for the case of obliquely incident radiation from a source outside a plane-parallel dispersion having a partially-reflecting boundary can be expressed functionally as:

\[ E_i = \phi_i (Z, \theta, bT, sT, R) \]

where \( Z = N_s \cdot z \) = normal distance \((z)\) into the dispersion expressed in mean free paths,

\( T = N_s \cdot t \) = thickness \((t)\) of the dispersion expressed in mean free paths.

\( \theta \) = azimuth angle of the incident radiation.

\( R \) = reflectivity of the boundary.

The analytical solution is extremely complex and computations are feasible only with the aid of an automatic computer. Chu, Clarke and Churchill (4) have undertaken such computations for a number of particular cases and have published graphical correlations illustrating the effects of the various variables on the ratio of the flux at any point in the presence and absence of a dispersion of particles of given optical properties.

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APPENDIX REFERENCES


GEOMETRICAL REPRESENTATION OF TRANSMISSION THROUGH A SCATTERING LAYER.

FIG. I.
FIG. 2. SIZE DISTRIBUTION OF DROPLETS IN FOG-OIL SMOKE.

ATTENUATION COEFFICIENT AS A FUNCTION OF MASS CONCENTRATION.

FIG. 3.
RELATIONSHIPS BETWEEN T, \( \sigma \) AND \( \infty \)

FIG. 4.
ESTIMATION OF MEAN VALUES OF P IN TERMS OF $\sigma^-$ AND TAN $\theta$. 

FIG. 5
FIG. 6. FLUX RATIOS IN TERMS OF T, R AND TAN θ
FIG. A1. ANGULAR DISTRIBUTION OF RADIATION FROM A SINGLE SCATTERING PARTICLE DIAMETER COMPARABLE WITH WAVE-LENGTH; (a) CONTINUOUS DISTRIBUTION, (b) SIX FLUX REPRESENTATION.

FIG. A2. SIX-FLUX REPRESENTATION OF THE INTENSITY IN A PLANE-PARALLEL DISPERSION HAVING A REFLECTING BOUNDARY WITH OBLIQUELY INCIDENT RADIATION (E5 AND E6 ARE PERPENDICULAR TO THE PLANE OF THE PAPER.)

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