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Solar Flux Measurements, April 1962, and Antenna Considerations

JOHN P. CASTELLI
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Research Note

Solar Flux Measurements, April 1962, and Antenna Considerations

JOHN P. CASTELLI
Abstract

For ten days in April 1962, solar power flux density was measured simultaneously at four frequencies using the 84-ft diameter radio telescope at Hamilton, Mass. The frequencies were 2965 Mc, 1200 Mc, 400 Mc, and 225 Mc. In addition, data were available for several days at 94 kMc through the courtesy of C. W. Tolbert of the University of Texas. Data at 9180 Mc were taken at Hamilton, Mass. with an 8-ft antenna. Apart from the solar data presented, various antenna parameters of the 84-ft parabola are reviewed and presented along with the method of determination. Finally, the method is presented by which the recorded data were corrected for those in determining true flux.
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The author is indebted to Dr. H. C. Ko of Ohio State University for his kind help in the basic antenna problem. He is grateful also to Carl P. Ferioli of AFCRL and Donald Knight of Lowell Technological Institute for their assistance in computations and measurements.
1. INTRODUCTION

During two weeks in April 1962, solar flux measurements were made simultaneously at 225 Mc, 400 Mc, 1200 Mc, and 2965 Mc at Hamilton, Mass., using the 84-ft antenna. In addition, data on selected days were available at 9180 Mc, taken also at Hamilton, using an 8-ft diameter parabola. For two days in the period, data were furnished on the 94,000 Mc solar flux from the University of Texas.

Normally, a large antenna with a narrow beam is unsatisfactory for making solar flux measurements where absolute accuracy is desired. It was used here because it was available and instrumented for four frequencies and to demonstrate that it can be used with appropriate correction factors. A discussion of these factors is included.

2. THE ANTENNA

The basic method of measuring solar antenna temperature $T_A$ is by drift technique. Here the recorded signal deflection of five or six drifts above a constant temperature sky before and after each drift is measured by noise injection into the system through a directional coupler using calibrated noise generators and precision attenuators. This peak calibration method gives merely an apparent solar $T_A$. True solar $T_A$ is found from consideration of radio-frequency losses between the
antenna feed system and receiver calibration point. In all cases, these losses were measured using noise generators and the actual solar receivers. In some cases, the losses were also measured by signal generators, bolometers, bolometer amplifiers, and so forth. Agreement between both methods was good; however, the former seems more suited to radio astronomy systems.

Uniform drift patterns for the calculation of $T_A$ sun were available in all cases except at 2965 Mc where the antenna beam width was approximately one-half the solar diameter. Hence, these drifts contained some solar profile data. At this frequency and from the records, an effective solar $T_A$ was determined from inspection of the drifts. The method seems reasonably valid because final flux values agree well with 2800 Mc data from the National Research Council of Canada at Ottawa which are reported in various publications.

An inspection of the equation used for determining solar flux indicates which other factors must be considered. Basically,

$$\text{Flux} = \frac{2kT_A}{A_{em}}$$

(1)

where $k$ = Boltzmann's constant $1.38 \times 10^{-23}$, $T_A$ is solar antenna temperature discussed above, and $A_{em}$ is the maximum effective area of the antenna in square meters ($\text{m}^2$). It is contrasted with $A_{phy}$ which is the physical aperture of the dish ($\pi r^2$ for an antenna such as a parabola). $A_{phy}$ for the 84-ft antenna is 514 $\text{m}^2$. Therefore, the aperture efficiency is

$$\eta_A = \frac{A_{em}}{A_{phy}}$$

(2)

The problem of determining $A_{em}$ for a large antenna is difficult to solve directly. For small parabolic antennas, perhaps up to 10-ft diameter - a size which can be easily taken to an antenna test range - and where side lobes and ground effects are no problem, the gain of the antenna can be measured relative to a standard gain horn at the frequency desired. Then $A_{em}$ can be calculated from the expression,

$$A_{em} = \frac{\lambda^2}{4\pi \text{ gain}}$$

(3)

No such routine is practical with a large antenna because of the following conditions: ground interference effects, distance which is quite great to the far field for a test transmitter for a large antenna at short centimeter wavelengths, large
difference between the gain of a standard horn and a large dish, and so forth. At
times, it is possible to determine the gain of a very large antenna by comparing the
flux from a star source measured with the large antenna with the flux from the same
star source measured with an intermediate size or small size antenna whose gain
has already been measured and whose $A_{em}$ has been computed by using the standard
gain horn technique or some other method. This method seems most useful.

A series of measurements of the flux of the source Cassiopeia at certain fre-
quencies with large horns, where the gain is related to the horn's geometry, has fur-
nished radio astronomers with a norm and a spectral index of Cassiopeia A and engi-
eers with a tool for measuring the gain and $A_{em}$ of their large antenna. The engineer
has simply to make a flux measurement of Cassiopeia A and compare it with the
norm for the frequency in question. If it is assumed that Cassiopeia has a Gaussian
distribution, then the $A_{em}$ of the large dish can be found by solving for $A_{em}$:

$$G = \frac{2kTA}{A_{em}} \left[ 1 + \left( \frac{\frac{\theta_d}{2}}{\theta_H} \right)^2 \right]$$

where

- $G$ = flux of the norm
- $\theta_d$ = the Gaussian source half-intensity width
- $\theta_H$ = the antenna half-power beam width.

For such a source as Cassiopeia and even at $\lambda = 10$ cm, the correction factor in
brackets above for an 84-ft antenna is quite small.

Measurements made at the National Radio Astronomy Observatory (NRAO) at
Greenbank, West Virginia by Findlay in 1961, using a large horn, indicate the 1440
Me flux of Cassiopeia A at $2560 \times 10^{-26}$ watts/m$^2$/cps. This is information of para-
mount importance. With this 'norm' and a spectral index of -0.8 for Cassiopeia
gained for a plot of all known flux measurements of this source, as made by Whitfield
and others, and omitting only the early observations, the gain of the large antenna
can be calibrated. Since flux is a frequency$^x$, a plot of flux values on log-log paper
yields the spectral index from determination of slope of the line. (See Figure 1.)

Or the spectral index ($x$) can be determined from the expression:

$$\left( \frac{I_2}{I_1} \right)^{x_1} = \frac{S_1}{S_2}$$
where $S_1$ and $S_2$ are fluxes at frequencies $f_1$ and $f_2$.

Other reliable measurements of Cassiopeia A flux have been made over the spectrum. Most useful is that at 3200 Mc by Broten and Medd.\(^5\)

Now the picture becomes complicated for it is believed from repeated measurements that the flux density of Cassiopeia is decreasing at the rate of 1 percent per year. This must be included in antenna gain calculations where the Cassiopeia comparison flux method is used.

We have used this method and information rigorously to determine the $A_{em}$ of the Hamilton 84-ft antenna and feed system at 1200 and 2965 Mc, and only roughly at 225 and 400 Mc. Figure 2 lists the $A_{em}$ for the frequencies under consideration.

In Eq. (4), the term $\theta_H$ appeared. This is the antenna half-power beam width. It is necessary to know this quantity accurately for the above and for other correction-factor determinations. The values used in Eq. (4) and other equations in this paper where $\theta_H$ appears are the averages of the half-power beam width in the E and H planes. These were found by making hour-angle drifts of Cassiopeia at all frequencies under consideration; first at one orientation of the feed, and later with the feed rotated 90°. Half-power levels were located on the drift curves and related to actual time duration and finally angle $D$ where 1 min of time = 15 min of arc (cosine declination of star). Actually,

$$D^2 = S^2 + B^2,$$  

where $B$ is the antenna half-power beam width and $S$ is the Gaussian source half-intensity width. Figure 2 lists this information.

It has been pointed out by Ko\(^7,8\) that for large antennas, such as parabolas and extended circular sources of uniform brightness distribution, the flux in Eq. (1) is only an apparent one. The formula is accurate for sources very small in diameter in relation to the antenna beam width. In fact, Ko has indicated that with a Gaussian antenna main lobe and for $\theta_d/\theta_H$ under 0.2, the error between true flux and apparent flux is under 1 percent. (See Figure 3.) As the ratio $\theta_d/\theta_H$ increases above 0.2, a correction factor $K$ should be used to determine true flux. For small ratios, $K \approx 1$. As the source begins to approach the antenna beam width or exceed it, as is the case with the sun or moon when using the 84-ft parabola at 'L' or 'S' bands, a correction $K$ is definitely required. Ko\(^7\) has shown further that not only is a correction required based on the $\theta_d/\theta_H$ ratio, but also that the shape of the antenna power pattern is a further consideration in determining the correction factor $K$. He has considered three typical antenna pattern shapes and has shown that $K$ may vary for different antenna response curves.

Accordingly, at 2965 Mc we have plotted the normalized antenna drift pattern of the source Cassiopeia and compared this with a true Gaussian and Bessel function curve. At 2965 Mc, our antenna pattern closely resembles the Bessel function curve.
Therefore, we have used this correction which appears in Eq. (7). At 1200 Mc, trial calculations for Gaussian and Bessel function responses yielded approximately the same numerical value for K. Hence, actual antenna response was not determined at 1200 Mc. The same holds true for 225 and 400 Mc. Actually the correction factors for the two lower frequencies can be taken from Figure 3 or calculated from the following expressions for greater accuracy.7

\[
\text{Gaussian } [K] = \frac{\Omega_d}{\iint_{\text{source}} f(\phi, \theta) d\Omega} = \frac{(1n 2) \left( \frac{\theta_d}{\theta_H} \right)^2}{1 - e^{-\left(1n 2\right) \left( \frac{\theta_d}{\theta_H} \right)^2}} \tag{7}
\]

\[
\text{Bessel function } [K] = \frac{\Omega_d}{\iint_{\text{source}} f(\phi, \theta) d\Omega} = \frac{(1.616 \frac{\theta_d}{\theta_H})^2}{4 \left[ 1 - J_1^2 \left( 1.616 \frac{\theta_d}{\theta_H} \right) - J_0^2 \left( 1.616 \frac{\theta_d}{\theta_H} \right) \right]} \tag{8}
\]

where \(\theta_d\) = source diameter = 32 min of arc for the sun, the universally used optical value

and \(\theta_H\) = average antenna half-power beam width at observed frequency.

At times it is desirable to know the average brightness temperature of the sun \(T_B\), sometimes called average disk temperature. Having determined solar flux where \(\text{flux} = \frac{2kT}{A_{em}} [K]\), one can solve the expression below for \(T_B\).9

\[
\text{flux} = \frac{2kT_B \omega}{3280 \lambda^2} \tag{9}
\]

where \(\omega\) = sun extent = 0.224 \(\text{deg}^2\)

\(\lambda^2\) = wavelength in \(\text{m}^2\)

3280 = conversion factor - 1 \(\text{sq radian} = 3280 \text{ sq deg}\)

or one may measure temperature directly from7

\[
T_D = \frac{T_A \theta_A}{\eta_r \theta_d} [K] \tag{10}
\]
where $T_D =$ average disk temperature in degrees K

$T_A =$ sun's antenna temperature

$\eta_r =$ antenna radiation efficiency 98-100 percent

$\Omega_A =$ total antenna solid angle in steradians

$$= \frac{\pi^2}{\Omega_{\text{em}}}$$ (11)

and $\Omega_d =$ source solid angle in steradians

$$= \frac{\pi D^2}{4} = \frac{\pi}{4} (32)^2 \left( \frac{1}{60} \right)^2 \times \frac{1}{3280} = 6.8112 \times 10^{-5} \text{ steradians.}$$

If smaller antennas are used, $A_{\text{em}}$ presents no problem. No correction factor K is necessary and the antenna pattern shape becomes unimportant.

Reference has been made to 3.2 mm observations from the University of Texas and 3.27 cm observations made at Hamilton. No antenna parameters other than those listed in Figure 2 are available for the Texas instrumentation. The various parameters for the 3.27 cm instrumentation are contained in Figure 2. Although the antenna in the latter case is quite small (8-ft diameter), a correction factor for Eq. (1) is required. It was found by the method outlined earlier. Antenna gain and $A_{\text{em}}$ could not be found using the Cassiopeia A comparison method because of low flux from Cassiopeia, lack of antenna gain and relatively low receiver sensitivity. However, measurements of the gain of the antenna and feed system by the standard gain horn comparison method are listed in Figure 2. Since routine data taking was made before and after gain evaluation, no attempt was undertaken to improve the efficiency.

3. SOLAR DATA

Calculated values of solar flux for observed days in April 1962 are presented in Figure 4. In all cases except the furnished millimeter data, values listed reflect the method of calculation outlined in Section 2. In the case of the millimeter data, flux was determined by the Texas group.

It is a well-known fact that solar flux varies at a 26-27 day rate. This is most pronounced in the 10-25 cm wavelength region, especially during sunspot maximum. During sunspot minimum, the periodic variation may be scarcely apparent. During sunspot maximum, the correlation between even 3 cm and 10-25 cm wavelengths is close. Approaching sunspot minimum, the correlation has been observed to fall...
off. For the period in April 1962, the correlation remained good at some wavelengths. Admittedly, insufficient days were recorded to establish an average flux for the period in relation to the eleven-year cycle. However, the average of the days observed is plotted in Figure 5.

The disadvantages of large antennas for absolute solar flux measurements were indicated earlier. Despite this, the agreement between the average of nine days at 'S' band at Hamilton and the average of the same nine days reported by the National Research Council of Canada for the 2800 Mc solar flux is good. The Hamilton average was $78 \times 10^{-22}$ watts/m$^2$/cps, while the Ottawa average was $81 \times 10^{-22}$ watts/m$^2$/cps.

Observations were possible on only two days during the period at 3.2 mm at Austin, Texas because of weather problems. It is significant to note that even at this wavelength, the solar emission appears to have a variable component. (See Figures 4 and 6 where observations of all days and frequencies observed are found.) The period in question may be located in relation to the daily 10.7 cm solar flux shown for April, May, and June 1962 in Figure 7, as reported by the National Research Council of Canada at Ottawa.

Factors for converting solar flux to average disk temperature have been found by solving for $T_B$ in the expression

$$\text{Flux} = \frac{2kT_B^\omega}{3280A^2}.$$  

At 225 Mc, the $T_B = (\text{Flux})(9.35 \times 10^{26})$ and $T_B$ average from Figure 5

$$= (1.0775 \times 10^{-21})(9.35 \times 10^{26}) = 1,007,460\text{K}.$$  

At 400 Mc, $T_B = (\text{Flux})(2.98 \times 10^{26})$ and $T_B$ average from Figure 5

$$= (1.271 \times 10^{-21})(2.98 \times 10^{26}) = 378,758\text{K}.$$  

At 1200 Mc, $T_B = (\text{Flux})(0.332 \times 10^{26})$ and $T_B$ average from Figure 5

$$= (5.81 \times 10^{-21})(0.332 \times 10^{26}) = 192,982\text{K}.$$  

At 2965 Mc, $T_B = (\text{Flux})(54.2 \times 10^{23})$

$$T_B = (7.829 \times 10^{-21})(54.2 \times 10^{23}) = 42,433\text{K}.$$  

At 2800 Mc, $T_B = (\text{Flux})(60.8 \times 10^{23})$

$$T_B = (8.16 \times 10^{-21})(60.8 \times 10^{23}) = 49,612\text{K}.$$
At 9180 Mc, $T_B = (\text{Flux})(5.7 \times 10^{23})$

$$T_B = (4.62 \times 10^{-20})(5.7 \times 10^{23}) = 26,400^\circ \text{K}.$$  

4. CONCLUSION

Large diameter parabolic antennas may be used at the centimeter wavelengths for determining solar flux with reasonable accuracy by giving careful attention to antenna correction factors. At the long centimeter and meter wavelengths, correction factors are no problem, although the gain determination requirement is always present. It is suggested that the utilization of the Cassiopeia A flux norm compared with measurements made with the large antenna of unknown aperture efficiency is an easy way to determine the antenna gain parameter.
Figure 1. The Spectrum of Cassiopeia A, 23N5A, from Absolute Measurements of Flux Density Reported by Various Observers. Sources of information are represented by the same letters as in Whitfield.4

<table>
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<tr>
<th>ANT DIAMETER FEET</th>
<th>FREQUENCY mc</th>
<th>λ cm</th>
<th>λ² METERS</th>
<th>NORMAL R-FLOSS Db</th>
<th>LOSS FACTOR</th>
<th>ANT HPBW E PLANE</th>
<th>θH-ANT HPBW AVERAGE</th>
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<td>9'</td>
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Figure 2. Parameters
Figure 3. Correction Factor for Converting the Apparent Flux Density to the True Flux Density of Circular Radio Source With Uniform Brightness Distribution. Courtesy H. C. Ko, Ohio State Univ.

### Table

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<td>78.29</td>
<td>3.94</td>
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**FLUX UNIT** WATT/m² /CPS

- UNIVERSITY OF TEXAS
- NATIONAL RESEARCH COUNCIL OF CANADA

Figure 4. Solar Flux, April 1962
Figure 5. Solar Flux, April 1962. Average of days observed
Figure 6. Solar Flux, Selected Days. Hamilton, Mass. (0.32 cm Univ. Texas)
Figure 7. Solar Flux 2800 Mc Recorded at National Research Council of Canada at Ottawa, 1962
References


10. Tolbert, C. W., University of Texas, private communication.
For ten days in April 1962, solar power flux density was measured simultaneously at four frequencies using the 84-ft diameter radio telescope at Hamilton, Mass. Frequencies were 2965 Mc, 1200 Mc, 400 Mc, and 225 Mc. In addition data were available for several days at 94 kMc from the University of Texas. Data at 9180 Mc were taken at Hamilton, Mass., with an 8-ft antenna. Apart from the solar data, various antenna parameters of the 84-ft parabola are reviewed and presented along with the method of determination. Finally, the method is presented by which the recorded data were corrected for these in determining true flux.