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Technical Report No. 2
Contract AF 61 (052) - 506
Research on
Electrically Small Antennas
Prof. Dr. H. H. Meinke

Institut für Hochfrequenztechnik
der Technischen Hochschule München
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Technical Report No. 2

Research on electrically small antennas

Prof. Dr. H. H. Meinke, Institut für Hochfrequenztechnik
Technische Hochschule München, Germany
30. November 1962

"The research reported in this document has been sponsored by the Aeronautical Systems Division, AFSC through the European Office, Aerospace Research, United States Air Force".
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Abstract

This report describes the results obtained in the investigation of very short antennas which were surrounded with a dielectric medium especially ferrite. It has already been shown in Technical Report No. 1 that a performance improvement of the electrically small antenna can only be obtained by achieving a bandwidth enlargement of the antenna impedance. Also Technical Report No. 1 has shown that an enlargement of the bandwidth of a very short antenna was not obtained by inserting dielectric in the region between the feeding point and a field line \( z = z_0 \). This is caused by the fact that the dielectric in the immediate vicinity of the feeding point also causes an undesirable increase in the reactive power of the transmission line wave. Therefore an increase in the antenna bandwidth can not be obtained by inserting dielectric material in the immediate vicinity of the feeding point. In this report an improved shape as compared to that of Technical Report No. 1 is described for the dielectric shell surrounding the antenna. Here the medium begins at some distance away from the feeding point. (see Fig. 1)

In Section I the polystyrol body of improved shape according to Fig. 1 is discussed. The corresponding feeding point impedance (Fig. 2) and bandwidth (Fig. 3) is calculated and compared with measured values. An improvement in the bandwidth as compared to that of the antenna with polystyrol in Technical Report No. 1 was obtained. However the bandwidth of the antenna surrounded with the improved polystyrol shape is still smaller than the bandwidth of an antenna in air.

In Section II ferrite bodies according to Fig. 1 of two different material types were used and investigated. The corresponding feeding point impedance and antenna bandwidth
were calculated and compared with the measured values. The results show that compared to the radiator in air no improvement of the bandwidth of very small antennas ($\lambda_0$ larger than 1/20 antenna height) is obtained by surrounding the antenna with either a ferrite or polystyrol shell. If the height of the antenna is larger than $\lambda_0/20$ some improvement of the antenna bandwidth can be obtained by surrounding the antenna with a dielectric medium especially ferrite.
I. Very short Radiator in a Polystyrol Shell of improved Shape

Technical Report No. 1 described the investigations which were undertaken with a radiator surrounded with a dielectric medium which extended from the feeding point to a field line: \( z = z_0 \). In the following text TR1 will be used as an abbreviation for "Technical Report No. 1". In this Technical Report No.2 the behaviour of an improved geometric shape of the dielectric medium is described.

Figure 21 (TR1) shows that the additional radiation caused by the dielectric medium may first occur outside the field line \( z/h = 0.5 \). Between the field line \( z/h = 0.5 \) and the feeding point of the radiator, the dielectric medium only causes a bandwidth decreasing transformation effect. Therefore the dielectric material should only be placed in those regions of the antenna where it could aid the effect of additional radiation. Figure 1 shows the improved geometric shape of the dielectric material. This improved shape is constructed of the same polystyrol material as that of the shape in TR1 (\( \varepsilon_r = 2.5 \)).

The impedance caused by the outer space at the coordinate \( z = z_0 \) which is the border of the dielectric medium is always the same for very short antennas, independent of the dielectric medium. Curve \( II \) of Fig. 16 in TR1 shows this impedance \( Z_1 \) at \( z = z_0 \) which has been calculated from the impedance measurements of the antenna in air. The transformation of this known impedance \( Z_1 \) at the field line \( z = z_0 \) inwards towards the field line \( z = z_d \) of Figure 1 is obtained by using a computer in conjunction with the Runge-Kutta method for
solving eq. (1) below.

\[
\frac{dZ_1}{dz} = j \frac{2\pi}{\lambda \varepsilon} \frac{1}{Z_{lo} \sqrt{\varepsilon_r}} \left( \frac{Z_1^2 - F \frac{Z^2}{Z_{lo}}}{\varepsilon_r} \right)
\]  

(1)

this corresponds to equation (67) of TR\(\text{I}\).

\(Z_1\) is the impedance to be transformed.

\(Z_{lo}\) is the characteristic impedance of the feeding line.

\[
\lambda = \frac{Z_{lo}}{\varepsilon_r}
\]  

(2)

The transformation of the impedance from \(z = z_d\) inwards towards \(z = 0\) occurs according to the following equation:

\[
\frac{dZ_1}{dz} = j \frac{2\pi}{\lambda} \frac{1}{Z_{lo}} \left( \frac{Z_1^2 - F \frac{Z^2}{Z_{lo}}}{} \right)
\]  

(3)

This is eq. (68) of TR\(\text{I}\).

The dielectric and resistive losses are so small that they can be neglected in the calculation procedure as was done with the antenna in TR\(\text{I}\).

Curve I in the Smith-chart of Figure 2 shows the plot of the calculated feeding point impedance for the frequency range 200 - 1000 mc/s. Curve II gives the measured values.

If the antenna height \(h\) is smaller than \(\frac{\lambda_0}{20}\) then the measured and calculated impedance values are in good agree-
ment. If \( h \) is larger than \( \lambda_0/20 \) then the measured impedance values lie closer to the \( \text{SWR} = 1 \) point than the calculated values. This was also observed with the antenna described in TR1 and is caused by the additional radiation effect of the dielectric.

Figure 3 gives a plot of \( b_{TD}/b_{TA} \) which is the ratio of the relative bandwidth of the antenna with improved dielectric medium shape to that of an antenna in air.

\[
\frac{b_{TD}}{b_{TA}} = \frac{R_{SD}}{R_{SA}} \cdot \left| \frac{dZ_A}{dZ_D} \right|
\]

(4)

this is eq. (72) of TR1.
index D means antenna in dielectric medium.
index A means antenna in air.
\( R = \text{Re}(Z) \)

Curve I of Fig. 3 is a plot for this ratio using the calculated impedance values; curve II gives a plot for the same ratio using the measured impedance values. As a result of the additional radiation, the values for \( b_{TD}/b_{TA} \) obtained by using the measured impedance data is larger than that obtained by using the calculated impedance data.

If the curves of Fig. 3 are compared with the corresponding curves of TR1 (Fig. 20) then it is readily realized that the new form of the dielectric medium yields a noticeable improvement of the bandwidth. However the bandwidth of the antenna surrounded with the improved polystyrol shape is still smaller than the bandwidth of an antenna in air.
Fig. 2
II. Very short Radiator in Ferrite Shells of improved Shape

The medium which surrounds the spherical antenna shall now be constructed of Ferrite material. Ferrite powder mixed with melted paraffin (binding medium) was used. At warm temperatures this mixture could be formed to the desired shapes. Upon cooling the mixture has approximately the same rigidity as paraffin.

A test set-up was constructed for measuring the electric and magnetic characteristics of this material. (see Fig. 4). The latter consists of a variable short circuit tuning stub, a test-sample line and a slotted line arrangement. Figure 5 shows various ferrite samples which were inserted between the inner and outer conductor of the test-sample line. Figure 6 shows the test-sample line with inserted ferrite sample. The measuring equipment serves as a means of determining the following material constants:

\[
\varepsilon', \varepsilon'' \quad \text{with} \quad t \delta \varepsilon = \frac{\varepsilon''}{\varepsilon'},
\]

and

\[
\mu', \mu'' \quad \text{with} \quad t \delta \mu = \frac{\mu''}{\mu'},
\]

(5)

Figures 7, 8, 9, and 10 give the measured material constants of four different Ferrite materials obtained from Siemens & Halske Company. The constants \(\varepsilon'\) and \(\mu'\) remain approximately constant in the frequency range between 200 and 1,000 mc/s.
whereas $\varepsilon''$ increases slightly with increasing frequency and $\mu''$ increases rapidly with increasing frequency.

Ferrite of types U 60 (Fig. 7) and HFPi (Fig. 8) were used in the construction of the improved shapes (Fig. 1). Fig. 11a and b shows the plaster molds used for forming the ferrite shapes. Fig. 12 shows two finished ferrite shapes and one shape made out of polystyrol. Fig. 13 shows a bottom view of a Ferrite shape and Fig. 14 shows the latter with a spherical radiator in its center on the measuring plane.
Eine Achse logar. geteilt von 1 bis 10000, Einheit 0,5 mm, die andere in mm
mit Prozentmaßstab
Calculation of the Feeding Point Impedance taking into consideration the electric and magnetic field losses

The transformation of the known impedance at the field line \( z = z_0 \) (see TRI, Fig. 16 curve II) inwards towards the field line \( z = z_d \) occurs according to:

\[
\frac{dZ_1}{dz} = \frac{2\pi}{\lambda^*} \frac{1}{Z_L} \frac{\lambda}{\varepsilon_r} \left( Z_1^2 - F_0 \frac{Z_2^2 \mu_r}{\varepsilon_r} \right)
\]

with

\[
\lambda^* = \frac{\lambda_0}{\sqrt{\mu_r \varepsilon_r}}
\]

(6)

This corresponds to eq. (70) of TRI.

As was the case with the antenna in TRI, the resistive losses can be neglected since the outer surface of the radiator is silver plated. However, the electric and magnetic field losses of the ferrite body itself are so large in the frequency range between 200 and 1,000 mc/s that these must be included in the impedance calculation. Therefore a complex material constant is actually used.

\[
\varepsilon_r = \varepsilon_r' - j \varepsilon_r''
\]

\[
\mu_r = \mu_r' - j \mu_r''
\]

(7)

\( \varepsilon_r', \varepsilon_r'', \mu_r', \mu_r'' \) are obtained from Fig. 7 and 8. If eq. (7) is inserted into eq. (6) then a transformation equation for the inhomogeneous line with consideration of the electric and magnetic field losses is obtained. The impedance which is obtained in this manner is now transferred from the field
line \( z = z_1 \) inward towards the feeding point \( z = 0 \) by using eq. (3).

Curve I of Fig. 15 shows the calculated feeding point impedance of the antenna with ferrite U 60 (\( \varepsilon' = 4.9, \mu' = 1.48 \)). The latter is plotted on a Smith-chart for the frequency range between 200 and 1,000 mc/s. Curve II shows the measured values of the feeding point impedance of this antenna.

If the antenna height \( h \) is larger than \( \lambda_0/20 \) then the measured impedance values once again lie nearer the SWR = 1 circle than the calculated values, and can again be explained as being caused by the additional radiation due to the ferrite. However a second effect exists here: The measured impedance of curve II in Fig. 15 shows an indentation with a small loop in the frequency range about 600 mc/s. Here the impedance lies very close to the SWR = 1 point. This loop is caused by a cavity-resonance of the \( W_1 \)-wave within the ferrite body. The wavelength \( \lambda_{W1} \) of the \( W_1 \)-wave within the ferrite body is smaller than its equivalent in air by the factor:

\[
\frac{1}{\sqrt{\varepsilon' \mu'}}
\]

The first resonance of the \( W_1 \)-wave occurs in the frequency range about 600 mc/s where the electrical wavelength between \( z_1 \) and \( z_0 \) has the values \( \lambda_{W1}/2 \). The next resonance occurs at twice this frequency namely: 1,200 mc/s. In the impedance plot of curve II in Fig. 15 one can already notice the beginning of the second loop at 1,000 mc/s.
The exact transformation of the impedance $Z$ from the outer field line $z = z_0$ inwards towards the field line $z_d$ occurs according to the equation:

$$\frac{dZ}{dz} = \frac{2\pi}{\lambda^*} \left[ \frac{Z^2}{Z_{lo}^2} - Z_{lo} \sqrt{\frac{\mu_r}{\varepsilon_r}} \left( F_0 + \frac{1}{2} F_1 \frac{\omega_1}{\omega_0} + \cdots \right) \right] \quad (8)$$

with

$$\lambda^* = \frac{\lambda_0}{\sqrt{\varepsilon_r \mu_r}}$$

This corresponds to equation (62) of TR1.

In the vicinity of the cavity resonance of the $W_1$-wave, $W_1/W_0$ becomes very large and via the factor $F_1$ of eq. (8) influences the impedance transformation $dZ/dz$ of the $W_0$-wave. In the calculation of the impedance in accordance with eq. (6) the factor $1/2 F_1 \frac{\omega_1}{\omega_0}$ of eq. (8) was not taken into consideration.

Therefore the calculated impedance depicted by curve I of Fig. 15 does not contain a loop.

In the impedance calculation a consideration of the $W_1$-wave is not possible since the magnitude of $W_1/W_0$ is not known. Whereas the impedance $Z$ of the $W_0$-wave of a radiator in air can be determined via measurement and used for the impedance calculation of the $W_0$-wave of the ferrite antenna, this is
not possible for the \( W_1 \) cavity resonance since the antenna surrounded with air does not possess such a cavity resonance.

The resonance behaviour of the \( W_1 \)-wave in ferrite does not cause a change in the cosine shape of the directional pattern of the antenna as long as the geometric antenna height \( h \) is small compared to the wavelength. If a different antenna pattern is desired then the geometrical height \( h \) must be in the order of \( \lambda \). \[1\]

The cavity resonance can be considered as a transforming network in the inhomogeneous antenna line. Therefore it can be replaced by a suitably chosen four pole network which is inserted at the feeding point of the small antenna. As a result it can be realized that this phenomenon does not bring a real improvement of the antenna characteristics.

Figure 16 shows the ratio \( \frac{b_{RF}}{b_{RA}} \) of the relative bandwidth of an antenna imbedded in Ferrite U 60 to the relative bandwidth of the antenna in air. Curve I of Fig. 16 is that of the calculated impedance; curve II is a plot of the measured values. As a result of the additional radiation and the additional transformation effect of the cavity resonance at 600 mc/s, curve II lies above curve I. The definition of the bandwidth (4) is only plausible when the impedance plot does not contain loops. Therefore curve II is shown in broken line form above 450 mc/s.

Fig. 17 gives a plot of the feeding point impedance of an
antenna with Ferrite HFFi (\(\varepsilon_r' = 8.25, \mu_r' = 2.82\)) in the frequency range 200 to 1,000 mc/s. Curve I shows the calculated values, curve II the measured values of the feeding point impedance of this antenna.

Curve II once again lies closer to the point SWR = 1 of the Smith-chart than does curve I. As a result of the large values of \(\varepsilon_r'\) and \(\mu_r'\), the loop in the measured impedance plot already occurs at 350 mc/s. Curve I of Fig. 18 shows the ratio \(b_{RF}/b_{IA}\) using the calculated impedance values and curve II for the measured impedance values. Since the first loop already occurs at 350 mc/s, curve II is drawn in broken line form above 250 mc/s.
Conclusions

As was shown in TR1 the practical use of a very short antenna is dependent on its bandwidth. Therefore if one wants to improve the characteristics of a very short antenna, a means must be found to increase its bandwidth. This could be possible by surrounding the radiator with dielectric material especially ferrite. First the radiator was surrounded with polystyrol and the effect of various geometric forms of the dielectric material upon the antenna behaviour was investigated. Theory and measured data has shown that a larger bandwidth is obtained when the dielectric begins at some distance away from the feeding point of the antenna than in the case of having the dielectric already begin at the feeding point. That portion of the dielectric which is in the vicinity of the feeding point of very short antennas can not bring an improvement in the radiation behaviour and thus does not result with an increase in the bandwidth. Rather, this region of the dielectric only causes an increase in the reactive power of the transmission line wave. This reactive power is undesirable since it causes a decrease in the bandwidth.

These results seem to indicate that a sphere totally filled with dielectric material may not be the most suitable shape for surrounding a dipole.

According to [2] and [3] a fundamental limit is imposed on the bandwidth of antennas by the impedance of the spherical wave functions representing their radiated fields. A cosine
form of the directional antenna pattern is obtained with very short antennas whose geometric height $h$ is small compared to the wavelength regardless if the antenna is surrounded with ferrite or not. Therefore only the lowest order dipole mode of these spherical wavefunctions appears. Then the impedance of the latter mode is the same for all very short antennas. Therefore a ferrite shell surrounding a very short antenna does not bring a real improvement in antenna behaviour; rather only a different transformation form of the antenna impedance results. Our impedance measurements indicate a cavity resonance of the $W_1$-wave in the ferrite body. When a ferrite with very high $\mu_r$ is used, the cavity resonance is excited at very low frequencies. But this resonance does not contribute to radiation and thus has no bandwidth increasing effect because the air filled outer space region in immediate vicinity of the ferrite body can not support the $W_1$-wave in its propagating form. Only at very distant regions away from the antenna can $W_1$ again exist as a propagating wave in the air filled outer space. Thus this cavity resonance also has only a transforming effect. Since the antenna is very short all this transformations could also have been accomplished by inserting a suitable fourpole network in the feeding point of the antenna.

If the antenna is chosen to be larger than $\lambda_r/20$ then the ferrite material can also influence the impedance of the lowest order spherical wave function. This is especially true when the ferrite shell has approximately the same height
as that of the antenna itself. As the antenna height is increased the wave function already has propagating wave characteristics in the air-filled outer space region near the ferrite body. Then additional radiation may be transmitted from the ferrite body into space. This additional radiation causes some enlargement of the antenna bandwidth.
Bibliography

Contract No. DA - 36 - 039 SC - 73263

AFCRL - TN - 60 - 997 sect. VB

## Glossary of Symbols

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<th>Symbol</th>
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<tr>
<td>$b_{RA}$</td>
<td>Relative bandwidth of antenna in air</td>
</tr>
<tr>
<td>$b_{RD}$</td>
<td>Relative bandwidth of antenna in dielectric</td>
</tr>
<tr>
<td>$b_{RF}$</td>
<td>Relative bandwidth of antenna in ferrite</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>Relative dielectric constant</td>
</tr>
<tr>
<td>$\varepsilon_r'$</td>
<td>Real part of complex $\varepsilon_r$</td>
</tr>
<tr>
<td>$\varepsilon_r''$</td>
<td>Imaginary part of complex $\varepsilon_r$</td>
</tr>
<tr>
<td>$F_n$</td>
<td>Fourier coefficients</td>
</tr>
<tr>
<td>$h$</td>
<td>Geometrical height of the radiator</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>Wavelength in air</td>
</tr>
<tr>
<td>$\lambda_d$</td>
<td>Wavelength in dielectric medium</td>
</tr>
<tr>
<td>$\lambda_f$</td>
<td>Wavelength in ferrite</td>
</tr>
<tr>
<td>$\mu_r$</td>
<td>Relative permeability</td>
</tr>
<tr>
<td>$\mu_r'$</td>
<td>Real part of complex $\mu_r$</td>
</tr>
<tr>
<td>$\mu_r''$</td>
<td>Imaginary part of complex $\mu_r$</td>
</tr>
<tr>
<td>$R_{RA}$</td>
<td>Radiation resistance of antenna in air</td>
</tr>
<tr>
<td>$R_{RD}$</td>
<td>Radiation resistance of antenna in dielectric</td>
</tr>
<tr>
<td>$\beta_0$, $\beta_d$</td>
<td>Loss factor of the electric field</td>
</tr>
<tr>
<td>$\beta_0$, $\beta_f$</td>
<td>Loss factor of the magnetic field</td>
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<tr>
<td>$W_n$</td>
<td>Fourier coefficients of complex wave function</td>
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<tr>
<td>$Z_0$</td>
<td>Impedance</td>
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<tr>
<td>$Z_{Le}$</td>
<td>Characteristic impedance of feeding line</td>
</tr>
<tr>
<td>$Z_{LA}$</td>
<td>Impedance of antenna in air</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Impedance of antenna in dielectric</td>
</tr>
<tr>
<td>$z$</td>
<td>Coordinate in the inhomogeneous antenna line</td>
</tr>
<tr>
<td>$z_0$</td>
<td>Coordinate of the outer boundary of dielectric medium</td>
</tr>
<tr>
<td>$z_0$</td>
<td>Coordinate of the inner boundary of dielectric medium</td>
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broadening effect. Therefore a shape is used where the ferrite begins at some distance away from the feeding point of the antenna.