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Final Engineering Report

Digital Correlator Study

Navy Department Bureau of Ships

UNIVAC

DIVISION OF SPERRY RAND CORPORATION

December 1962
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December 1962  Contract N0b3 87424

M. F. Hodges

M. F. HODGES
By direction.
Final Engineering Report
for the
Digital Correlator Study

This report covers the period
1 May 1962 to 1 December 1962

UNIVAC
Division of Sperry Rand Corporation
St. Paul, Minnesota

Navy Department Bureau of Ships
Contract Number: 87424
Project Serial Number: SF-007-01-01, Task 7131

December 1962
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ABSTRACT

A systems design study of a digital correlator for waveform recognition purposes has produced a design that is practical for hardware implementation of waveform classification as outlined in the original contract. Two approaches to the correlation problem, referred to as the sum-of-products and the least-squared-difference methods, were investigated. With appropriate normalization the sum-of-products method and the least-squared-difference method can both be used for target classification. The latter method has the following advantages over the former:

a) more economical to implement
b) less time required to identify a given return
c) compensation for changes in the noise level and for certain a priori information
d) analytical justification

Comparisons are made of several designs based upon the different parameters.

Because of the uniqueness of the method proposed for classification, and the uncertainty of the data available in the sonar returns, a simulation study using recorded sonar returns is recommended. The simulation study will determine the validity and accuracy of the correlation process. It also will aid in the selection of various parameters that will lead to final systems design.
PART I

A. PURPOSE:

The study was to conduct a systems analysis and prepare a proposed design of a digital correlator for waveform classification purposes based on the following system parameters:

1. Maximum signal frequency of interest: 400 cps
2. Signal amplitude resolution: 32 levels
3. Range gate: target range ± 1000 yards
4. Pulse repetition intervals: 8-25 seconds
5. Pulse width: 10 milliseconds to 1 second

On-line classification was to be given priority over off-line procedures, and various designs were to be investigated with comparisons made for each system.

B. GENERAL FACTUAL DATA:

Personnel: The following personnel were employed in this study for the periods indicated.

R. B. Arndt, Principal Systems Design Engineer: 1.5 man months
G. F. Marette, Principal Systems Design Engineer: 4.0 man months
E. J. Farrell, Mathematician: 4.7 man months
R. R. Lachenmayer, Associate Systems Design Eng: 5.8 man months
C. DETAILED FACTUAL DATA:

1. Algorithm Determination. The initial approach to the correlation problem was based on the generation of a sum-of-products correlation between the range-gated unknown waveform and a library of standard waveforms. The correlator design was to be based on the availability of an AN/USQ-20 Unit Computer during the time required for the classification procedure.

Several methods were considered for implementing the sum-of-products computation, but because of the stringent time requirements of the on-line problem these methods were narrowed down to two: residue arithmetic and carry-store addition. The residue arithmetic was attractive because of the fast multiply and add times available in this system. These fast times result because there is no propagation of carries or borrows between residue groups. Disadvantages of the residue system are the conversions required between residue and binary representation at the input and output, and there is no practical method for comparing two residue numbers. This was particularly important since classification would be determined by the maximum correlation values computed.

The second method, the carry-store addition, was selected for use because it did not have the conversion and comparison limitations of the residue system, and could be operated at fast speeds for the size of the numbers involved. In this method the products are accumulated by a series of half adds, where the carries are stored, and are not allowed to propagate more than
one stage for each half add. At the end of the last product addition, the carries are allowed to propagate through the carry pyramid to obtain the final sum-of-products. The amount of extra time required for the final propagation of carries is not significant and therefore the total multiply-add time is essentially the time required to do the half adds.

In most digital multipliers, the multiplier is examined one bit at a time and the multiplicand or zero is added to the accumulated product, if the multiplier bit being sampled is a binary "1" or "0", respectively. The multiplicand must be scaled by powers of two depending upon the location of the "1" within the multiplier word. If this method were used, it would take five half add times to complete each sample by sample multiplication. This time would be excessive for the problem under study, so it was decided to examine the multiplier in two increments instead of five. The first sampling of the multiplier examines the lower 3 bits and the second sampling examines the upper two bits and the carries from low order bits. Monthly Status Report #3 presents a detailed description and example of this multiplication procedure. The end result of this scheme is that it allows the multiplications for 5-bit samples to be completed in two half add times instead of five.

2. **Timing Considerations.** With the multiplication scheme selected, the system parameters affecting speed were examined to determine a practical clock rate. The factors affecting this were: the maximum pulse repetition rate of 8 sec., the length
of the standard reference waveforms, the sampling frequency, and the number of stored reference waveforms. The first calculation assumed the number of stored reference waveforms to be 1000. The number of 5-bit multiplies and the time per multiply were plotted as a function of sampling frequency and standard waveform size. These results are shown in Figures 1 and 2, respectively. From Figure 1 it can be seen that for a sampling frequency of 800 cps and a standard waveform length of 150 yards, there would be $277 \times 10^6$ 5-bit multiplications in 8 seconds. Figure 2 shows that this would allow a maximum of 30 nanoseconds per multiply or 15 nanoseconds per half add which is impractical in terms of the present state of the computer art.

In view of the above, it was decided to utilize circuitry, developed for Project Lightning, operating with a four-phase, 4-megacycle clock that results in a 16-megacycle transfer rate. With the clock rate thus fixed, another series of calculations were run to determine the number of standard waveforms that could be referenced as a function of sampling frequency and standard waveform size. The results of these calculations are shown in Figure 3. From this graph one can see the various trade-offs possible at different sampling rates for the standard waveform sizes shown. For example, for a sampling frequency of 800 cps and standard waveform size of 150 yards, there is time to perform a sum-of-products correlation with 115 standard waveforms in 8 seconds. At a sampling frequency of 400 cps and the same
Figure 1

Graph of the number of 5 bit by 5 bit multiplies as a function of sampling frequency for different lengths, in yards, of reference waveforms, S. These calculations are based on a correlation of 1000 Reference Waveforms every 6 seconds.
Figure 2

Graph of allowed time for a 5 bit by 5 bit multiply as a function of sampling frequency for different lengths, in yards, of reference waveforms, S. These calculations are based on a correlation of 1000 Reference Waveforms every 8 seconds.
Graph of the maximum number of reference waveforms that may be correlated in an 8-second interval as a function of sampling frequency and reference waveform length, in yards, (S). These operations are based on a 5 bit by 5 bit multiply every 0.4 micro-second.
standard waveform size, the number of waveforms increases to 462. Also, as the size of the standard waveform is reduced, the number of standard waveforms increases.

3. Sum of Products System.

(a) System Description

Since these results indicated that the assumed clock rate could be used to correlate over a large number of standard waveforms for different trade-offs of sampling frequency and standard waveform lengths, a feasibility design utilizing the AN/USQ-20 Unit Computer was completed to determine the approximate number of circuits that would be required to implement a correlator. This design used the sum-of-products method of which a block diagram is shown in Figure 4.

In reference to Figure 4, the standard waveform is stored during correlation in registers $R_0$ thru $R_5$. Registers $R_6$ and $R_7$ are used to shift the standard waveform by the sample size in a circular manner in conjunction with the other $R$ registers. The quantized range gate samples are held in registers $G_0$ thru $G_5$. Registers $G_6$ and $G_7$ are used to shift the range gate information in much the same manner as registers $R_6$ and $R_7$ shift the standard waveform samples. Since the range gate information is correlated at one-sample offsets, another holding register $G_{in}$ is provided to supply the additional sample required for each offset.
The lower five bits of register $G_6$ act as the multiplier for each operation, and the corresponding bits of $R_6$ are the multiplicand. The multiplier is examined in two increments; the lower three bits first and then the upper two bits. This is accomplished by the sensors shown in the block diagram of Figure 4 as the sensor gates. The result of the first sensing operation will be to gate the appropriate multiple of the multiplicand through the $X_2$ gates to be half added with the quantities in the $S_2$ and $C_2$ registers. The second sensor will control the transmission of another multiple of the multiplicand to the $X_1$ register where it will be half added with the quantities in the $S_1$ and $C_1$ registers. The $S_1$, $C_1$, and $S_2$, $C_2$ registers contain the stored bit sums and carries resulting from the half adds.

In practice each standard waveform would contain a constant which would be a combination of normalizing, probability, and threshold constants. This constant would be initially entered into the $S_2$ register by the transmission path shown from the $R_0$ register to the $S_2$ register in Figure 4. At the completion of the multiply-add operation for each offset of the range gate information, the resulting sum is compared to the highest previous correlation value obtained for that specific standard waveform. If the resulting sum is greater, it is stored in the $H$ register, which holds the high correlation value.

(b) Control

A flow chart illustrating the control sequence is shown in Figures 5 and 6. For the purpose of this illustration, it will
be assumed that all registers of Figure 4 will be in the cleared state, and that the range gate and 1000 standard waveforms are stored in the memory of the AN/USQ-20 Unit Computer. The loading sequence, Figure 5, is initiated by the sonar operator. On the "start" command the loading of the first standard waveform is begun. As the loading sequence of the first standard waveform is completed, the loading of the range gate waveform is initiated. At its completion, the actual correlation process is begun as shown in Figure 6 and repeated through the 1000 standard waveforms, at which time the correlation value will be displayed and the loading and correlation sequences terminated.

4. Statistical Analysis. While the sum-of-products design was being completed, analysis of the general problem of waveform classification was continuing. Reasonable criteria for selecting a classification procedure lead to procedures that have the same general form and differ only in the value of certain parameters. A detailed discussion and derivation is presented in the attached report. Basically the procedures "correlate" the input return with all of the standard returns, and select the standard return with the highest "correlation". The "correlation" value used to compare the standard returns is the usual product-sum correlation corrected by an additive constant. If 

\[(u_{i1}, u_{i2}, ..., u_{in}) \text{ represents the } i\text{-th standard return, and} \]

\[(x_1, x_2, ..., x_n) \text{ represents the input return, the optimum procedure compares} \]

12.
Start

Set $E = 1$

Set $A = 1$

Initiate Read USQ-20

USQ-20 $\rightarrow R_0$

Yes

$A = 6$

Yes

$A = 5$

Yes

$A = 4$

Yes

$A = 3$

Yes

$A = 2$

Yes

$A = 1$

Yes

$A + 1 \rightarrow A$

$A + 1 \rightarrow A$

Set $A = 1$

Initiate Read USQ-20

USQ-20 $\rightarrow G_{in}$

Yes

$G_{in} \rightarrow G_0$

Yes

$G_0 \rightarrow G_1$

Yes

$G_1 \rightarrow G_2$

Yes

$G_2 \rightarrow G_3$

Yes

$G_3 \rightarrow G_4$

Yes

$G_4 \rightarrow G_5$

Loading Sequence
Figure 5
Correlation Sequence

1. To Figure 5

2. Figure 6

Set D = 1
Set C = 1
Set B = 1

R₆₋₋ → R₇₋₋
G₆₋₋ → G₇₋₋
R₇₋₋ → R₆₋₋
G₇₋₋ → G₆₋₋
Half Add S₁₋₋X₁ → S₂₋₋ (except when C = 34, 35 or 36)

When B = 1

G₇₋₋₋₋ → G₅₋₋₋₋
G₇₋₋₋₋ → G₃₋₋₋₋
G₇₋₋₋₋ → G₁₋₋₋₋ (when B = 1 and C ≠ 7)
Full Adder → S₂

H → C₂

Inhibit Sensors when C = 34, 35 or 36

Clear R₇
Clear G₇
Clear S₁₋₋X₁
K → S₂ (only when C = 1)
Clear H (if C = 35 and S₂ positive and Eₕ in Full Adder)

Clear R₆
Clear G₆
Clear R₅
Clear G₅
Clear S₂₋₋C₂ (except when C = 34)

Write H to USQ-20

E = 100? No E = 100? Yes
E + 1 → E

E + 1 → C

C = 37? Yes D = 40? No

No E + 1 → E

Yes

B = 6? No B + 1 → B

Yes

Stop
2( \eta^2 - \eta^2 ) \log(w_1 \lambda_1) - \sum_{r=1}^{n} (u_{1r} + \eta)^2 + 2 \sum_{r=1}^{n} (u_{1r} + \eta)x_r ,

2( \eta^2 - \eta^2 ) \log(w_2 \lambda_2) - \sum_{r=1}^{n} (u_{2r} + \eta)^2 + 2 \sum_{r=1}^{n} (u_{2r} + \eta)x_r ,

2( \eta^2 - \eta^2 ) \log(w_3 \lambda_3) - \sum_{r=1}^{n} (u_{3r} + \eta)^2 + 2 \sum_{r=1}^{n} (u_{3r} + \eta)x_r ,

etc., where \eta is the average (or d.c.) noise level and \overline{\eta^2} is the average squared noise level (or average noise power). The value of the parameters \lambda_i and w_i depend on criterion used in selecting a classification procedure.

For example, if the goal is to maximize the average probability of correct classification, \textstyle w_i = 1 \text{ for all values of } i \text{ and } \lambda_i \text{ is the relative frequency (or a priori probability) that the input return is from the } i\text{-th target. Introducing a priori probabilities has the effect of favorably weighting those standard returns that occur more often. The influence of this weighting increases with the noise power; if the noise level is high, the classification procedure relies more on a priori probabilities than on the input return. If the a priori probabilities change from one region of the ocean to another, one can adjust the classification procedure by changing the } \lambda_i \text{'s. Similarly, if}
the noise power increases due to changes in target range or position of the sonar, one can adjust the classification procedure by changing $\bar{\eta}^2$. Note that $u_{ir} + \bar{\eta}$ is obtained by averaging several returns from a known target.

Several assumptions were made to arrive at the above classification procedure: viz.,

i) The range of the target is known.

ii) The input to the digital correlator has been corrected for range attenuation; the input return and standard returns have comparable amplitudes. The "signal to noise" ratio may change with range.

iii) The quantized (in amplitude) samples from the input return can be approximated by continuous samples.

iv) The noise is a superposition of randomly positioned pulses with random amplitudes; i.e., the noise is Gaussian.

v) The noise characteristics are constant over the range gate: constant average, constant power, etc.

vi) The noise spectrum has a sharp high frequency cutoff; and the sampling frequency is twice this cutoff frequency.

In practice, the exact range of the target is unknown and the range gate must be searched. In this case, another term must be added to the correlation value: viz., $\sum_{r=1}^{n} x_r^2$. In our
analysis of classification with a known range, this term appears as a "constant" in comparing the standard returns and was dropped. Introducing this term, the classification procedure becomes a comparison of

\[ 2(\eta^2 - \bar{\eta}^2) \log(w_1 \lambda_1) - \sum_{r=1}^{n} (u_{1r} + \bar{\eta} - x_r)^2, \]

\[ 2(\eta^2 - \bar{\eta}^2) \log(w_2 \lambda_2) - \sum_{r=1}^{n} (u_{2r} + \bar{\eta} - x_r)^2, \]

\[ 2(\eta^2 - \bar{\eta}^2) \log(w_3 \lambda_3) - \sum_{r=1}^{n} (u_{3r} + \bar{\eta} - x_r)^2, \text{ etc.} \]

Basically, this classification procedure is a comparison of the least squared deviation between the input return and the various standard returns.

5. Least Squared Difference Method. Since the least squared difference method appeared attractive, a design utilizing this method was completed. The desired result of the least squared difference method is

\[ \sum_{r=1}^{n} (R_r - G_r)^2 = \text{correlation value} \]

where \( R \) represents the quantized amplitude of the stored reference
waveforms and \( G \) represents the quantized amplitude of the raw video waveform.

Figure 7 is a block diagram of the correlator implementing the above process. Comparing Figure 7 with Figure 4 shows the elimination of the 30-bit registers \( R_7 \) and \( G_7 \) plus associated circuitry.

The remaining \( R \) and \( G \) registers continue to perform the function of holding the standard waveform and a portion of the range gate return during the correlation process. The shifting formerly done between registers \( G_6 \) and \( G_7 \) and between \( R_6 \) and \( R_7 \) has been eliminated. The same effect is now obtained by gating directly out of the appropriate sections of \( G_6 \) and \( R_6 \). The two blocks labeled \((R-G)^2\) subtract one five-bit sample in \( G \) from one five-bit sample in \( R \), square the difference, and then transfer it to their respective \( X_1 \) or \( X_2 \) registers. The \( S \) and \( C \) registers and the half adders continue to perform the summation process. The \( H \) register is used to store the current correlation sum for comparison with the latest result.

This design along with the sum-of-products design had the disadvantage of being limited by the input/output speed of the AN/USQ-20 Unit Computer. The least-squared-difference system was capable of operating at twice the speed of the sum-of-products system, but since both systems were limited by the speed of the AN/USQ-20 Unit Computer, the main advantage of the least-squared-difference method was that slower, less expensive circuits could
Figure 7

Block Diagram of Correlator Utilizing $\sum (R-G)^2$ Process

Correlation High

To AN/USQ-20

REFERENCE

30 Bit Registers

(R-G)$^2$

16 Bit Registers

Half Add

Carry Pyramid

From AN/USQ-20

RAW VIDEO
be utilized in the design. Another disadvantage of both of the
aforementioned designs, was the need for holding registers for
both the standard waveform and a portion of the range gate wave-
form. These registers would have to be equal in length to the
size of a standard waveform. For large standard waveforms, high
sampling rates, and greater number of levels of quantitization,
the number of circuits required for these holding registers be-
comes many times greater than the number of circuits required for
the arithmetic operations. This can be seen in Table I which is
a tabulation of the approximate number of inverters required to
implement either of the two systems for various sampling fre-
quencies and standard waveform sizes. The numbers in the columns
labeled R & G represent the inverters required for the two holding
registers. The last column shows the percentage of the total num-
ber of inverters required for the holding registers.

6. Separate Memory Systems. In order to take advantage
of the speed of the least-squared-difference method and to partly
eliminate the need for the many holding registers, a system de-
sign utilizing a separate memory for the quantized range gate wave-
form was investigated. A block diagram of this system is shown in
Figure 8. Besides operating at twice the speed of the previous
least-squared-difference system, this design has eliminated most
of the holding registers required for the range gate waveform.
Two registers, \( G_0 \) and \( G_1 \), are required external to the memory for
shifting purposes. These registers are one sample size greater
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</table>

**Table 1**
Block Diagram of Correlator Employing A Range Gate Memory

Figure 8
than the word length of the memory. This was necessitated to allow for one sample offsets in the correlation process across the range gate samples.

In the initial phases of the study, it was assumed that the AN/USQ-20 Unit Computer would be available for use in the target classification process, and that memory space and input/output channels would also be available for these purposes. Later discussions with Bureau of Ships personnel have cast some doubt upon the validity of these assumptions. Because of this and the fact that the holding registers required for the standard reference waveforms can be very large at high sampling frequencies, another system was studied which included a separate memory for the reference waveforms. A block diagram of this system is shown in Figure 9.

This system is completely independent of the AN/USQ-20 Unit Computer. The high values of correlation are stored in the reference waveform memory and the control portion of the correlator updates these values after each reference waveform is completed. A disadvantage of this system is that extra circuitry must be added to enter reference waveform information from a peripheral storage device such as a magnetic tape unit. The reference waveform memory would also have to be faster than the AN/USQ-20 Unit Computer memory since it would have to keep pace with the speed of the correlation process. With a transfer rate of 16 megacycles, two samples from each waveform would be subtracted, squared, and summed
Figure 9
Block Diagram of Correlator Employing Range Gate and Reference Waveform Memories.
every 1/4 μsec. Therefore a memory word length of 6 samples requires a new word to be read every 3/4 μsec. If a larger word length were selected, the memory cycle time would be increased proportionally.

7. **Sonar Recording Observations.** A group of sonar returns were made available by the Underwater Sound Laboratories in the last phases of this study. Some of these returns were recorded for a cursory visual inspection. Figure 10 shows a submarine return at four different speeds plus a representative whale return. The filtering evidenced in Figures 10a, b, c and e is a result of mechanical filtering introduced by the strip recorder. Figure 10d shows the 800-cycle component from the AN/SQS-23 audio channel.

The initial examinations seem to indicate that each target has a representative waveform or class of waveforms that are not similar to the whale returns. Since none of the targets on the tapes were identified according to nature, size, or aspect, the results could not be extended to include any detailed analysis of the observed waveforms.

D. **CONCLUSIONS:**

The resulting designs and study have shown that a digital correlator for the classification of sonar waveforms is practical from both the mathematical and hardware viewpoints. For a sampling frequency of 800 cps, a standard waveform size of 150 samples, and the maximum pulse repetition rate of 8 seconds, it is possible to correlate over 200 reference waveforms. As these
Recorded Sonar Returns

(a) Submarine

(b) Submarine

(c) Submarine

(d) Submarine

(e) whale

Figure 10
parameters are relaxed, it is possible to correlate greater numbers of reference waveforms. The procedures developed also allow for changes in noise level and for differences in the frequency of occurrence of various targets.

PART II
RECOMMENDATIONS

Since the method proposed is relatively unique for the classification of sonar returns, there are several unknowns that will affect the selection of a specific design. Some of these are:

1) number of different waveforms needed to represent a specific target (or class of targets) at various aspects.
2) sampling frequency which is sufficient to identify different types of targets
3) length of standard reference waveforms necessary to uniquely identify a target or class of targets
4) number of levels of quantitization necessary for accurate identification
5) number of sample offsets in the range gate waveform to effect a correct identification with the standard reference waveforms
6) the availability of the AN/USQ-20 Unit Computer for the correlation process.
It is recommended that the above areas be studied further to determine the optimum parameters for a specific design. A simulation phase has been proposed that will simulate the mathematical procedures recommended with actual sonar returns. The simulation, besides determining the validity of the methods recommended, will be used with variations in the system parameters to obtain a final recommended system design.

For the extended study and simulation the recordings of sonar returns must contain explicit information regarding the targets. This will be necessary for the formulation of the standard reference waveforms and also to determine the accuracy of the simulated correlation process.
Waveform Identification Using
Digital Techniques

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October 2, 1962

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Waveform Identification Using Digital Techniques

I. Introduction

Many problems of waveform identification or recognition can be formulated in the following way. Several standard waveforms are specified. Given one of the standard waveforms that has been changed by "noise", the problem is to identify the standard waveform. In many situations waveform identification is difficult and accurate comparisons must be made. For this reason digital techniques are preferred over analog. The noisy waveform and standard waveforms are digitized; i.e., they are represented by uniformly space samples. The noisy waveform is identified using these digitized waveforms. In the following paragraphs an identification procedure is derived.
II. Selection of an Identification Procedure

By applying certain methods of decision theory, one can select an identification procedure without additional restrictions on the problem. There are several criterion for selecting an identification procedure. The following criterion will be discussed:

1) average probability of correct identification,
2) probability of correct identification for "least favorable" standard waveforms,
3) probabilities of correct identification which are in a specified ratio.

Assume there are \( N \) standard waveforms. Let \( p_i(I) \) be the probability of correct identification when the noisy waveform is the \( i \)-th standard waveform and identification procedure \( I \) is used. The average probability of correct identification is

\[
\sum_{i=1}^{N} \lambda_i p_i(I),
\]

where \( \lambda_i \) represents the a priori probability that the noisy waveform is the \( i \)-th standard waveform. The probability of correct identification for "least favorable" standard waveforms is \( \min\{p_1(I), p_2(I), \ldots, p_N(I)\} \). The third criterion is of a different nature. In certain situations it may be desirable to have the \( p_i(I) \)'s in a specified ratio; i.e., \( \lambda_1 p_1(I) = \lambda_2 p_2(I) = \ldots = \lambda_N p_N(I) \), where the \( \lambda_i \)'s are positive. The goal is to select an identification procedure \( I^* \) that satisfies the above ratio.
condition and for which there is no "uniformly better" procedure; i.e., there is no procedure \( I' \) such that \( p_i(I^*) \leq p_i(I') \) for all \( i \) and \( p_j(I^*) < p_j(I'') \) for some \( j \).

These criteria lead to identification procedures that have the same form and differ only in the value of certain parameters. Let \( x = (x_1, x_2, x_3, \ldots, x_n) \) represent \( n \) uniformly-spaced samples of the noisy waveform. Let \( f_i(x) \) represent the multivariate density function of \( x \) given the noisy waveform is the \( i \)-th standard waveform. The optimum identification procedure for all three criteria is of the form

\[
I_\lambda = \begin{cases} 
  w_1 \lambda_1 f_1(x) = \max\{ w_1 \lambda_1 f_1(x), \ldots, w_N \lambda_N f_N(x) \}, \\
  \text{then select the } i \text{-th standard waveform.}
\end{cases}
\]

If \( w_1 = 1 \) for all \( i \) and \( \lambda_1 \) is the a priori probability that the noisy waveform is the \( i \)-th standard waveform, identification procedure \( I_\lambda \) maximizes the average probability of correct identification. If \( w_1 = 1 \) for all \( i \) and if the \( \lambda_i \)'s are selected so that

\[
p_1(I_\lambda) = p_2(I_\lambda) = \cdots = p_N(I_\lambda),
\]

then \( I_\lambda \) maximizes the probability of correct identification for "least favorable" standard waveforms. For specified \( w_1 \)'s, if the \( \lambda_i \)'s are selected so that

\[
w_1 p_1(I_\lambda) = w_2 p_2(I_\lambda) = \cdots = w_N p_N(I_\lambda),
\]

then \( I_\lambda \) satisfies the third criterion, provided the \( \lambda_i \)'s are positive. These statements are proven in the following paragraph.
An identification procedure I can be characterized by a collection of N sets in Euclidian n-space; i.e., if x is in the first set, select the first standard waveform, if x is in the second set, select the second standard waveform, etc.

Let $X_i(I, x)$ be a function of x which is one when x is in the $i$-th set and zero otherwise. Note that

$$\sum_{i=1}^{N} X_i(I, x) = 1$$

for all I and x. Also

$$p_i(I) = \int X_i(I, x) f_1(x) \, dx,$$

where the integration is over all values of x. Using $X_i(I, x)$, it is easy to show that $I^*_\lambda$ maximizes

$$\sum_{i=1}^{n} \lambda_i w_i p_i(I).$$

First

$$\sum_{i=1}^{n} \lambda_i w_i p_i(I) = \sum_{i=1}^{n} \lambda_i w_i \int X_i(I, x) f_1(x) \, dx$$

$$= \int \sum_{i=1}^{n} \lambda_i w_i X_i(I, x) f_1(x) \, dx.$$
From the definition of $I_\lambda$ it follows that
\[
\sum_{i=1}^{n} \lambda_i w_i x_i(I_\lambda,x)f_1(x) \geq \sum_{i=1}^{n} \lambda_i w_i x_i(I_\lambda) f_1(x)
\]
for each $x$, and hence
\[
\sum_{i=1}^{n} \lambda_i w_i p_1(I_\lambda) \geq \sum_{i=1}^{n} \lambda_i w_i p_1(I) .
\]

If the $\lambda_i$'s are selected so that $w_1 p_1(I_\lambda) = w_2 p_2(I_\lambda) = \cdots = w_n p_n(I_\lambda)$, then
\[
\min \{ w_1 p_1(I_\lambda), w_2 p_2(I_\lambda), \cdots \} = w_1 p_1(I_\lambda) = \sum_{i=1}^{n} \lambda_i w_i p_1(I_\lambda) \geq \sum_{i=1}^{n} \lambda_i w_i p_1(I) \geq \min \{ w_1 p_1(I), w_2 p_2(I), \cdots \} .
\]

If the $\lambda_i$'s are positive, there is no uniformly better procedure. Assume there is a $I'$ such that
\[
p_1(I_\lambda) \leq p_1(I') \text{ all } i
\]
and
\[
p_j(I_\lambda) < p_j(I') \text{ for some } j.
\]
Then \( \sum \lambda_i p(I_{\lambda}) < \sum \lambda_i p(I') \), provided the \( \lambda_i \)'s are positive; but this is a contradiction. It follows that \( I_{\lambda} \) is optimum for all three criteria; the proof is completed. In the following section, it will be shown that \( I_{\lambda} \) is relatively simple with Gaussian noise.
III. Identification with Gaussian Noise

Assume the density function \( f_j(x) \) is from the exponential family; i.e., \( f_j(x) \) can be expressed as

\[
f_j(x) = a_j h(x) \exp \left( \sum_{r=1}^{n} b_{jr} x_r \right),
\]

where \( h(x) \) is independent of \( j \), and where \( a_j \) and \( b_{jr} \) are independent of \( x \). Note that

\[
a_j = \left[ \int h(x) \exp \left( \sum_{r=1}^{n} b_{jr} x_r \right) \, dx \right]^{-1}
\]

since the integral of \( f_j(x) \) is one. The identification procedure \( I_\lambda \) is a comparison of \( w \lambda_1 f_1(x) \), \( w \lambda_2 f_2(x) \), \( w \lambda_3 f_3(x) \), ..., \( w \lambda_N f_N(x) \) for a given \( x \). Since \( h(x) \) appears as a constant factor in each expression, \( I_\lambda \) reduces to a comparison of \( w \lambda_1 a_1 \exp \left( \sum_{r=1}^{n} b_{1r} x_r \right) \), \( w \lambda_2 a_2 \exp \left( \sum_{r=1}^{n} b_{2r} x_r \right) \), etc., or equivalently a comparison of

\[
c_1 + \sum_{r=1}^{n} b_{1r} x_r, \quad c_2 + \sum_{r=1}^{n} b_{2r} x_r, \quad \text{etc.}, \quad \text{where} \quad c_i = \log w \lambda_i a_i.
\]

If the noise is a superposition of randomly positioned pulses with random amplitudes, it can be approximated by Gaussian noise; \( f_j(x) \) will be a multivariate normal density

\[
f_j(x) = \left[ \frac{\det(m_{rs})}{(2\pi)^n} \right]^{\frac{1}{2}} \exp \left[ -\frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} (x_r - u_{jr} - \eta) m_{rs} (x_s - u_{js} - \eta) \right]
\]
where \((m_{rs})\) is the inverse of the covariance matrix of the noise, where \(\bar{\eta}\) is the average (or d.c.) noise level, and where \((u_{j1}, u_{j2}, \ldots, u_{jn})\) represents the \(j\)-th standard waveform. The average noise level is assumed to be constant over the sampling interval. A Gaussian approximation is reasonable when the average density of the noise pulses is "high" and the variance of the amplitudes is "small". The normal density function is in the exponential family; the constants \(a_j\) and \(b_{jr}\) are

\[
a_j = \exp \left[ -\frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} (u_{jr} + \bar{\eta}) m_{rs} (u_{js} + \bar{\eta}) \right]
\]

\[
b_{jr} = \sum_{s=1}^{n} (u_{js} + \bar{\eta}) m_{sr}.
\]

The procedure \(I_\lambda\) reduces to a comparison of

\[
\log(w_{x_1}) = -\frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} (u_{1r} + \bar{\eta}) m_{rs} (u_{1s} + \bar{\eta}) + \sum_{r=1}^{n} b_{1r} x_r,
\]

\[
\log(w_{x_2}) = -\frac{1}{2} \sum_{r=1}^{n} \sum_{s=1}^{n} (u_{2r} + \bar{\eta}) m_{rs} (u_{2s} + \bar{\eta}) + \sum_{r=1}^{n} b_{2r} x_r, \text{ etc.}
\]

The noise is assumed to be Gaussian in the remainder of this paper.

Assume the statistical characteristics of the noise are independent of time (i.e., stationary noise) over the sampling
interval, and that the autocorrelation of the noise $\varphi(t)$ can be evaluated. Then $(\mathbf{m}_r^2)$ is the inverse of

$$
\begin{pmatrix}
\varphi(\tau) - \bar{\eta}^2 & \varphi(2\tau) - \bar{\eta}^2 & \cdots \\
\varphi(\tau) - \bar{\eta}^2 & \varphi(\tau) - \bar{\eta}^2 & \cdots \\
\varphi(2\tau) - \bar{\eta}^2 & \varphi(\tau) - \bar{\eta}^2 & \cdots \\
\vdots & \vdots & \ddots \\
\end{pmatrix},
$$

where $\tau$ is the time between samples and $\bar{\eta}^2$ is second moment (or average power) of the noise.

If the noise spectrum has a sharp high-frequency cutoff, and if $1/\tau$ is twice this cutoff frequency, the procedure $\mathbf{H}_r$ takes on a simple form. Consider the noise shifted by its average $\bar{\eta}$. If the noise has a spectrum of the form

$$
G(\omega) = \begin{cases} 
G_0 & 0 \leq \omega \leq \omega_0 \\
0 & \omega > \omega_0 
\end{cases},
$$

where $G_0$ is a constant, then the correlation function is

$$
\varphi(t) = \int_0^\infty G(\omega) \cos \omega t \, d\omega = G_0 \int_0^\infty \cos \omega t \, d\omega = G_0 \frac{\sin \omega_0 t}{t},
$$

for $t > 0$ and $\varphi(0) = G_0 \omega_0$. If $1/\tau = 2(\omega_0/2\pi)$, $\varphi(k\tau) = G_0 \omega_0 \frac{\sin k\pi}{k\tau} = 0$, for $k \in \mathbb{Z}$.
for \( k = 1, 2, 3, \ldots \). In other words, the covariance matrix of the noise is diagonal and \( m_{rs} = \left[ \eta^2 - \eta^2 \right]^{-1} \delta_{rs} \). Hence \( I_X \) reduces to a comparison of

\[
\left[ \eta^2 - \eta^2 \right] \log(w_1 \lambda_1) - \frac{1}{2} \sum_{r=1}^{n} (u_{1r} + \bar{\eta})^2 + \sum_{r=1}^{n} (u_{1r} + \bar{\eta}) \chi_r,
\]

\[
\left[ \eta^2 - \eta^2 \right] \log(w_2 \lambda_2) - \frac{1}{2} \sum_{r=1}^{n} (u_{2r} + \bar{\eta})^2 + \sum_{r=1}^{n} (u_{2r} + \bar{\eta}) \chi_r, \text{ etc.}
\]
IV. **Probability of Correct Identification**

Let $E_{ij}$ be the set of $x_i$'s such that

$$\left\{ c_i - c_j + \sum_{r=1}^{n} (b_{ir} - b_{jr}) x_r \leq 0 \right\}.$$

Then $p_j(I_j)$ is the probability of $E_{1j} \cap E_{2j} \cap E_{3j} \cap \cdots \cap E_{nj}$ given the noisy waveform is the $j$-th standard waveform. Theoretically one can evaluate $p_j(I_j)$ since the $x_i$'s have a known multivariate normal density function. In practice it may be very difficult to evaluate $p_j(I_j)$ for large $N$ and $n$. If it is not practical to evaluate $p_j(I_j)$ directly, one can use upper and lower bounds for $p_j(I_j)$.

In analyzing a system the parameters of the system are given and $p_j(I_j)$ must be determined. With a lower bound for $p_j(I_j)$ one can make statements like--for the given parameter values the average probability of correct identification is at least .93. An upper bound for $p_j(I_j)$ would be of little help in analyzing a given system. An upper bound is more helpful in design than in analysis. In design, $p_j(I_j)$ is specified and the system parameters must be determined. With an upperbound for $p_j(I_j)$ one can make statements like--to achieve the specified value of $p_j(I_j)$ one must take at least 31 samples from each input waveform. On the other hand, with a lower bound for $p_j(I_j)$ one can make statements like--to achieve the specified value of $p_j(I_j)$ it is not necessary to take more than 43 samples from each input waveform. A lower bound will result in over designing and an upper bound will result in under designing the system.
The probability of $E_{ij}$ given the noisy waveform is the $j$-th standard waveform is an upper bound for $p_j(I_x)$ for all values of $i$. One can select $i$ to minimize this bound; in other words, the "best" upper bound is

$$\min\{ Pr(E_{ij} \mid j\text{-th stand.}), Pr(E_{2j} \mid j\text{-th stand.}), \ldots, Pr(E_{nj} \mid j\text{-th stand.}) \}.$$ 

This bound is easy to evaluate. The probability of $E_{ij}$ given the noisy waveform is the $j$-th standard waveform is equal to the probability that

$$\sum_{r=1}^{n} (b_{1r} - b_{jr})(x_r - u_{jr} - \eta) \leq c_j - c_i - \sum_{r=1}^{n} (b_{ir} - b_{jr})(u_{jr} + \eta).$$

The left-hand side of this inequality is a normal random variable with mean zero and variance

$$\sum_{r=1}^{n} \sum_{s=1}^{n} (b_{1r} - b_{jr})(b_{is} - b_{js}) \left[ \phi(\mid r-s \mid \tau) - \eta^2 \right]$$

Hence $Pr(E_{ij} \mid j\text{-th stand.})$ equals the probability a normal random variable with zero mean and unit variance is less than

$$\left[ c_j - c_i - \sum_{r=1}^{n} (b_{ir} - b_{jr})(u_{jr} + \eta) \right]$$

$$\cdot \left[ \sum_{r=1}^{n} \sum_{s=1}^{n} (b_{1r} - b_{jr})(b_{is} - b_{js}) \left[ \phi(\mid r-s \mid \tau) - \eta^2 \right] \right]^{-\frac{1}{2}}.$$
Let $K_{ij}$ represent this constant. Since $K_{ij}$ gives an upper bound for all values of $i$, the "best" upper bound is obtained when $i$ minimizes $K_{ij}$. If the noise spectrum has a sharp high-frequency cutoff, and if $1/\tau$ is twice this cutoff frequency,

$$K_{ij} = \frac{1}{\sqrt{n\rho_{ij}}} \log \left( \frac{w_i \lambda_i}{w_j \lambda_j} \right) + \frac{1}{2} \sqrt{n\rho_{ij}},$$

where $\rho_{ij} = \frac{1}{n} \sum_{r=1}^{n} \frac{(u_{jr} - u_{ir})^2}{\bar{\eta}^2 - \eta^2}$.

Note that $\rho_{ij}$ is the ratio of the mean square deviation between $u_{jr}$ and $u_{ir}$ to the expected square deviation of the noise from its mean. In Figure 1, $Pr(E_{ij}|j\text{-th stand.})$ is graphed against $n$ and $\rho_{ij}$ for $w_i \lambda_i = w_j \lambda_j$. If $w_i \lambda_i = w_j \lambda_j$ for all $i$, then the value of $i$ that minimizes $\rho_{ij}$ will minimize $K_{ij}$.

The probability of $E_{ij} \cap E_{kj}$ given the noisy waveform is the $j$-th standard waveform is also an upper bound for $p_j(I_\lambda)$. This bound is smaller than the bound derived above, but it is also more difficult to evaluate. The probability of $E_{ij} \cap E_{kj}$ can be evaluated by transforming the random variables to a standard form and using tabulated values of the bivariate normal distribution.
Figure 1. Graph of \( \Pr(E_{ij} \mid j\text{-th stand.}) \)
V. Preliminary Rejection

If the number of standard waveforms \( N \) is large, it is awkward to select the value of \( i \) that maximizes \( c_i + \sum_{r=1}^{n} b_{ir} x_r \).

The selection can be simplified with preliminary rejection of non-contenders. If \( c_i + \sum_{r=1}^{n} b_{ir} x_r \) is less than a fixed threshold \( t_i \), it is not compared to \( c_2 + \sum_{r=1}^{n} b_{2r} x_r, c_3 + \sum_{r=1}^{n} b_{3r} x_r \), etc.

It is easy to evaluate the probability that the \( j \)-th waveform will be rejected erroneously, i.e.,

\[
\Pr \left\{ c_j + \sum_{r=1}^{n} b_{jr} x_r < t_j \bigg| j \text{-th type} \right\}.
\]

The threshold \( t_j \) is selected so that probability of erroneous rejection is "sufficiently" small.