AERODYNAMIC COMPRESSION OF A GAS STREAM

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AERODYNAMIC COMPRESSION OF A GAS STREAM

Ye. D. Nesterov

(Kazan)

Let us examine the interaction of two streams of compressible fluid directed at an angle to one another (Figs. 1 and 10). The main stream of gas is compressed by an active stream, which is supplied through an annular slit b, located in the region of minimum cross section of the main stream.

Ratios of aerodynamic compression are derived for two cases: a) a convergent channel (Fig. 1) and b) a Laval nozzle (Fig. 10). The difference between these two cases is that in a divergent channel the separation zone is in communication with the atmosphere, and in a Laval nozzle it is a closed region. This gives unique conditions for solution of the problem.

Basic Premises

1. Let us consider the interaction of streams at ratios of total pressure corresponding to the velocity coefficients of the active stream $\lambda_3 < 1.0$, and at relative flow rates of the active stream $\Delta\overline{G}_3 = \frac{\Delta G_3}{G_1} < 0.06$.

2. Let us derive the fundamental ratios in the absence of external heat transfer within the framework of one-dimensional theory.

3. As in Martin's work [3], let us assume instantaneous mixing of the streams.

4. The active stream expands up to the static pressure of the main stream at a jet inlet angle $\frac{\pi}{2} \leqslant \varphi \leqslant \pi$ and up to the pressure of the partially retarded stream $P_3 = \dfrac{P_1}{n(\lambda_1 \cdot \cos\varphi)}$ when $0 \leqslant \varphi \leqslant \frac{\pi}{2}$.

A Divergent Channel

The diagram is shown in Fig. 1.

Additional Conditions

A. Between the wall and the surface of the stream after mixing there exists a stagnant zone with pressure equal to atmospheric.

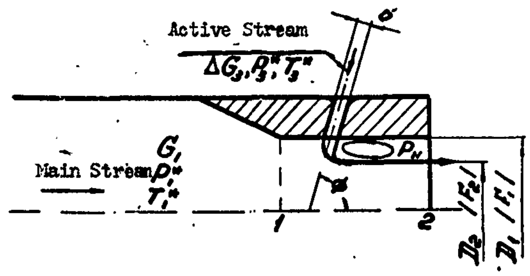

Fig. 1.

B. In cross section 2-2, static pressure is $p_2 = p_H$ at a subcritical flow regime of the main stream, i.e., at the set drops: $\dfrac{p_1^*}{p_H} = \dfrac{1}{n(\lambda_{1d})} < \left[\dfrac{k+1}{2}\right]^{\frac{k}{k-1}}$, where $k = \dfrac{c_p}{c_v}$ is the ratio of specific heats, and $p_2 = p_2^* \left[\dfrac{2}{k+1}\right]^{\frac{k}{k-1}}$ at a supercritical flow regime, when

$$\lambda_2 = 1{,}0, \quad n(\lambda_{1d}) \leqslant \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}.$$

Let us set up a system of equations which describe the process.

1. The condition of cylindricalness

$$F_1=F_2+\Delta F_H \quad \text{or} \; \overline{F}_2+\Delta\overline{F}_H=1. \qquad (1)$$

2. From the condition of expansion of the active stream, the ratio of total pressures of the streams

$$\pi_3^*=\frac{\Pi(\lambda_1)}{\Pi(\lambda_3)\cdot\Pi(\lambda_3\cdot\cos\varphi)}. \qquad (2)$$

3. The continuity equation: $G_1 + \Delta F_3 = G_2$, where the subscripts 1, 3, and 2 denote the main stream, active stream, and the stream after mixing, respectively.

Using gasdynamic functions [1], we obtain

$$1+\Delta\overline{G}_3=\frac{m_2}{m_1}\cdot\frac{p_2^*}{p_1^*}\cdot\frac{g(\lambda_2)}{g(\lambda_1)}\cdot\frac{\overline{F}_2}{\vartheta_2}, \qquad (3)$$

where $\vartheta_2 = \frac{T_2^*}{T_1^*}$ is the temperature ratio,

$$m=\sqrt{k\left[\frac{2}{k+1}\right]^{\frac{k+1}{k-1}}}\cdot\sqrt{\frac{g}{R}},$$

g the acceleration due to gravity (9.81 m/sec^2), and R the gas constant (kg-m/kg·deg).

In Martin's work [3], the hypothesis is taken that the active stream expands up to pressure p_1^*. In this case, when the ratios of total pressures of the streams $\pi_3^* \leqslant 1.0$, calculation cannot be applicable, since according to this hypothesis the flow rate of the active stream is zero, but on the basis of experimental data [2,3] it can be said that interaction between the streams exists, i.e., the active stream compresses the main stream.

4. The momentum equation for the volume of fluid between cross section 1-1 and 2-2:

$$I_1+\frac{\Delta G_3}{g}\cdot v_3\cdot\cos(\pi-\varphi)=I_2+p_H\Delta F_H;$$

where $I = \frac{G}{g} v + pF$ is the total momentum [kg], and $\underline{v}$ is the velocity of the stream [m/sec]. Using the gasdynamics functions, we obtain

$$f(\lambda_2) + k_1\left[\frac{2}{k_1+1}\right]^{\frac{k_1}{k_1-1}} \cdot \lambda_3 \cdot g(\lambda_1) \cdot \Delta\bar{G}_3 \frac{a_{cr.3}}{a_{cr.1}} \cdot \cos(\pi - \varphi) = \Pi(\lambda_{1d}) \cdot \Delta\bar{F}_n + \frac{P_2^*}{P_1^*} \cdot \bar{F}_2 \cdot f(\lambda_2), \qquad (4)$$

where $a_{cr} = \sqrt{\frac{2k}{k+1} gRT^*}$ is the critical velocity [m/sec].

5. The energy equation

$$\frac{A_3}{A_1} \cdot \Delta\bar{G} \cdot \vartheta_3^2 + 1 = (1 + \Delta\bar{G}_3)\frac{A_2}{A_1} \cdot \vartheta_2^2, \qquad (5)$$

where $A = \frac{k+1}{2(k-1)} \cdot \sqrt{\frac{2k}{k+1}} \cdot R$.

6. The condition for relationship between total pressures.

A. Flow without losses: total pressure after mixing should be determined from the condition of constancy of entropy S = const, which leads to the following equation at $R_1 = R_3$:

$$\frac{1 + \Delta\bar{G}_3}{\Delta\bar{G}_3} \cdot \ln \frac{T_2^{*\frac{k_2}{k_2-1}} / T_1^{*\frac{k_1}{k_1-1}}}{P_2^*/P_1^*} = \ln \frac{T_3^{*\frac{k_3}{k_3-1}} / T_1^{*\frac{k_1}{k_1-1}}}{\pi_3^*}$$

From calculation made for $k_3 = k_1$, $\vartheta_3 = 1.0$, $\Delta\bar{G}_3 = 0.05$, $\pi_3^* = 0.75$ to 1.4, we have a difference of $\mp 1.5\%$ between p_2^* and p_1^*. Therefore, let us take $p_2^* = p_1^*$ for calculation.

B. Flow with losses in the main stream. In this case $\frac{p_2^*}{p_1^*} = \sigma$, where σ is determined by experimental data as a function of $\Delta\bar{G}_3$, π_3^*, φ.

7. The ratio of flow rates of the streams

$$\Delta\bar{G}_3 = \frac{m_3}{m_1} \cdot \frac{\pi_3^* \cdot \mu_3 \cdot q(\lambda_3)}{\vartheta_3 \cdot q(\lambda_1)} \cdot \Delta\bar{F}_3, \qquad (6)$$

where μ_3 is the flow-rate factor of the active stream, selected from experimental data. It is a function of the geometry of the slit (inlet conditions) and the Reynolds number $Re = \frac{v \cdot 2b}{\nu}$, where ν is the kinematic coefficient of viscosity [m²/sec] and b is the width of the slit. With smooth input to the slit, μ_3 is close to 1.0.

As a result, we have a system of six equations with eight unknowns: λ_1, λ_3, $\Delta\overline{F}_3$, ϑ_3, ϑ_2, $\overline{F}_2$, $\Delta\overline{F}_H$, $\Pi(\lambda_{1d})$.

The gas constant and the ratio of the specific heats of the mixture are determined by the formulas

$$R_2 = \frac{R_1 + \Delta\overline{G}_3 \cdot R_3}{1 + \Delta\overline{G}_3}, \quad k_2 = \frac{\frac{k_1}{k_1 - 1} \cdot R_1 + \frac{k_3}{k_3 - 1}\Delta\overline{G}_3 \cdot R_3}{\frac{R_1}{k_1 - 1} + \frac{R_3 \cdot \Delta\overline{G}_3}{k_3 - 1}}.$$

The two additional equations are functions of the energy aggregate of the main stream. Let us consider the following schemes.

1. Apparatus operating by suction. Let $\Pi(\lambda_{1d}) =$ const $T_1^* =$ const.

The ratio of the flow rates of the main stream in operation with compression and without it, at a subcritical flow regime of the main stream

$$\Pi(\lambda_{1d}) > \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \rightarrow \overline{G} = \frac{G_1}{G_0} = \frac{q(\lambda_1)}{q(\lambda_{1d})},$$

at a supercritical flow regime

$$\Pi(\lambda_{1d}) < \left[\frac{2}{k+1}\right]^{\frac{k}{k-1}} \rightarrow \overline{G} = \frac{q(\lambda_1)}{q(1)},$$

where the subscript ($_0$) pertains to the parameters of the main stream in the absence of aerodynamic control.

2. The main stream is supplied by a centrifugal compressor. Two versions are possible here.

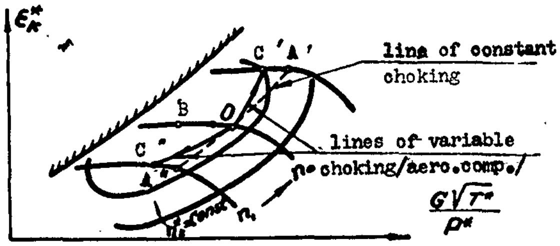

Fig. 2.

A. The number of revolutions is held constant; compression is accomplished by the interaction of the two streams. On the characteristic of the compressor, the point of joint operation is shifted from position O to position B. Assuming that the characteristic of the compressor is horizontal, we obtain for the degree of compression the condition

$$\pi_k^* = \frac{p_1^*}{p_{bx}^*} = \text{const},$$

where p_{bx}^* is the total pressure of the stream at the input to the compressor.

Under the condition of adiabatic efficiency of the compressor η_k^* = const, we have T_1^* = const, i.e., the problem reduces to the previous case.

B. The number of revolutions is varied. Consequently, on the characteristic of the compressor, the point of joint operation is shifted forward (the number of revolutions increases) from position O to C, and backwards (the number of revolutions is decreased) to C", instead of OA' and OA", respectively, with choking unchanged.

Assuming efficient operation of the compressor proportional to the square of the number of revolutions, and η_k^* = const, we have

$$\frac{\pi_{k_1}^{*\frac{k-1}{k}}-1}{\pi_{k_0}^{*\frac{k-1}{k}}-1}=\left[\frac{n_1}{n_0}\right]^2, \quad \frac{T_1^*}{T_0^*}=\frac{\eta_k^*+\left[\frac{n_2}{n_0}\right]^2\cdot\left(\pi_{k_1}^{*\frac{k-1}{k}}-1\right)}{\eta_k^*+\left(\pi_{k_0}^{*\frac{k-1}{k}}-1\right)}.$$

3. The main stream is supplied by an axial compressor. In this case let us examine a variation.

The number of revolutions is held constant; compression is accomplished by the interaction of the two streams. On the characteristic of the compressor, the point of joint operation of the system is shifted from position 0 to B. Assuming that the characteristic of the compressor is vertical, for η_k^* = const we can write

$$\pi_{k_1}^*=\pi_{k_0}^*\frac{q(\lambda_{1d0})}{q(\lambda_1)}\cdot\sqrt{\frac{T_1^*}{T_0^*}} \quad \text{for} \quad \Pi(\lambda_{1d})\geqslant\left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}.$$

$$\pi_{k_1}^*=\pi_{k_0}^*\frac{q(1)}{q(\lambda_1)}\cdot\sqrt{\frac{T_1^*}{T_0^*}} \quad \text{for} \quad \Pi(\lambda_{1d})\leqslant\left[\frac{2}{k+1}\right]^{\frac{k}{k-1}}.$$

$$\frac{T_1^*}{T_2^*}=\frac{\pi_{k_1}^{*\frac{k-1}{k}}-(1-\eta_k^*)}{\pi_{k_0}^{*\frac{k-1}{k}}-(1-\eta_k^*)}.$$

In reality, the efficiency of a compressor is usually somewhat lower. In the presence of such compressor characteristics, the calculation for both cases is carried out more strictly.

By analogy with the examples examined, we can obtain additional conditions for the main stream for the use of aerodynamic control on direct-flow or rocket engines. As an example, let us examine the solution for apparatus operating by suction (applicable to experiments [3]).

The ratio of specific heats $\frac{c_p}{c_v} = k_1 = k_3 = 1.4$.

The temperature ratio $\vartheta_3^2 = 1.0$.

The drop in the main stream $p_1^*/p_H = 1.89$.

In this case the problem reduces to the solution of a system of two equations with two unknowns, λ_1 and λ_3:

$$\Pi(\lambda_{1d}) + \frac{q(\lambda_1) + \pi_3^* \Delta\bar{F}_3 \cdot q(\lambda_3)}{q(\lambda_2)} \left[f(\lambda_2) - \frac{\Pi(\lambda_{1d})}{\sigma} \right] =$$

$$= f(\lambda_1) + k \left[\frac{2}{k+1} \right]^{\frac{k}{k-1}} \cdot \Delta\bar{F}_3 \cdot q(\lambda_3) \cdot \lambda_3 \cdot \pi_3^* \cdot \cos(\pi - \varphi),$$

$$\pi_3^* = \frac{\Pi(\lambda_1)}{\Pi(\lambda_1 \cdot \cos\varphi) \cdot \Pi(\lambda_3)} \quad \begin{matrix} \text{at} \quad \sigma = 1{,}0, \quad \Pi(\lambda_2) = \Pi(\lambda_{1d}). \\ \text{at} \quad \sigma \neq 1{,}0, \quad \sigma = \dfrac{\Pi(\lambda_{1d})}{\Pi(\lambda_2)}. \end{matrix}$$

The solution is carried out by the method of successive approximations. Letting $\lambda_1' < 1.0$, we find λ_1'' of the second approximation. Let us check the convergence etc. up to the required accuracy.

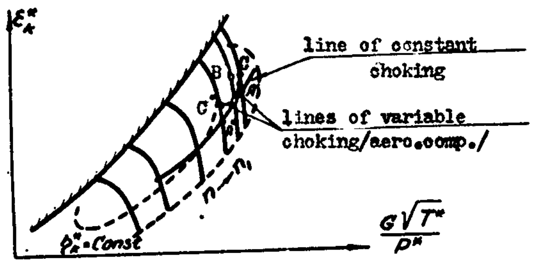

Fig. 3.

The efficiency of aerodynamic compression is conveniently evaluated by the specific area of compression

$$\delta = \frac{1 - \bar{F}_2}{\Delta\sigma_3} \cdot 100\%.$$

The calculation was made for relative magnitudes of the slits $\Delta\bar{F}_3 =$ $= 0.053$, 0.04, 0.027, and 0.007. The results are given in Figs. 4, 5 and 6.

In Fig. 4 it can be seen that at low $\Delta\bar{G}_3 \leqslant 0.05$ for $\Delta\bar{F}_3 = 2.7\%$; at $\varphi = 70°$, the calculation is in good agreement with experimental data [3].

In order to determine the effect of the isentropic exponent and the ratio of the stream temperatures, calculations were made for

$$\Delta\bar{F}_3 = 0{,}027, \quad \varphi = 90^\circ, \quad \frac{p_1^*}{p_н} = 1{,}89.$$
$$k = 1{,}4, \quad k = 1{,}33, \quad k = 1{,}25.$$
$$\vartheta_3^2 = 1{,}5, \quad 1{,}0, \quad 0{,}5.$$

The results are shown in Fig. 7.

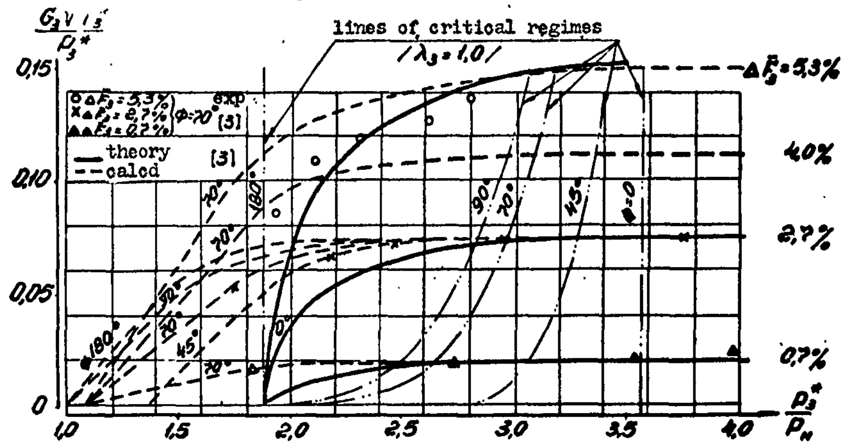

Fig. 4. Variation in the parameter $\frac{\Delta G_3 \sqrt{T_3^*}}{p_3^*} = f(p_3^*/p_н)$ for various values of φ and $\Delta\bar{F}_3$.

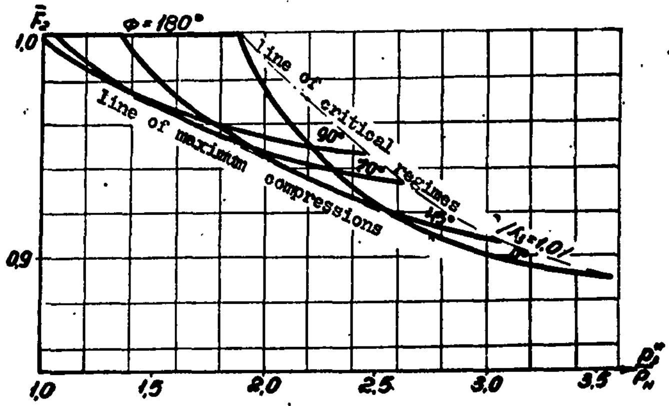

Fig. 5. The function $\bar{F}_2 = f\left(\frac{p_3^*}{p_н}\right)$ for slit $\Delta\bar{F}_3 = 0.7\%$.

Using the experimental dependence $\frac{G_1}{G_0} = f(\Delta\overline{G}_3)$ (Fig. 6), let us calculate the losses in total pressure of the main stream (Fig. 9). For aerodynamic control of the engine nozzle, the inlet angle of the active stream must be, from the condition of allowable losses in total pressure, 90 to 70°.

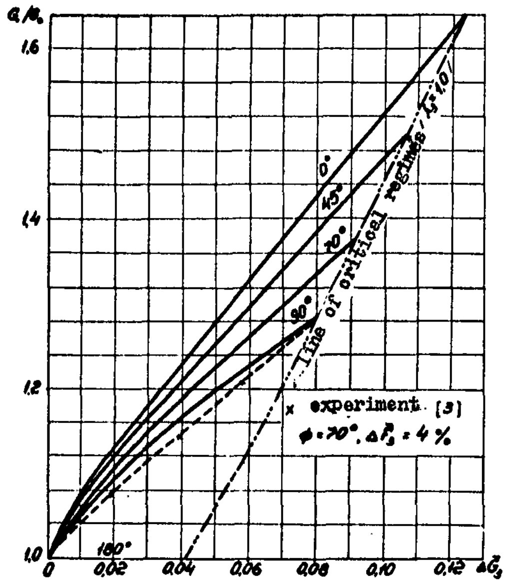

Fig. 6. Variation in $\frac{G_1}{G_0} = f(\Delta\overline{G}_3)$ for a slit $\Delta\overline{F}_3$ = 4.0%.

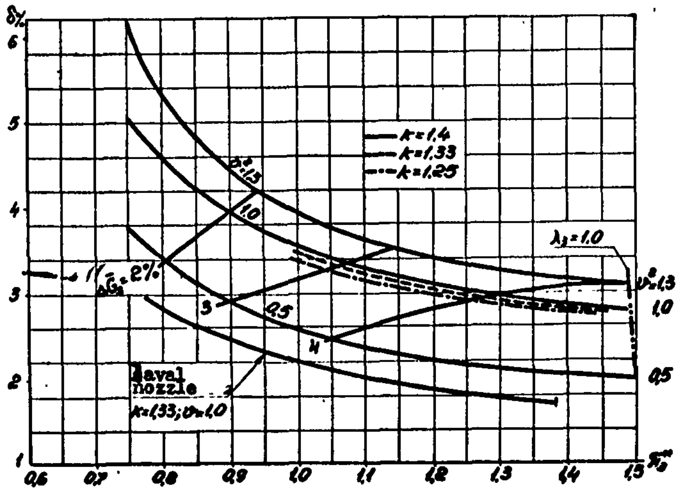

Fig. 7. Variation in specific area of π_3^* for divergent nozzle and Laval nozzle at $\Delta\overline{F}_3$ = 2.7%, φ = 90°.

Let us compare the calculation with experimental data [2] for a slit $\Delta\overline{F}_3 = 5.0\%$, $\varphi = 90°$. The depth of penetration of the active stream in Zenukov's work [2] was estimated by the magnitude of the aperture ring h_9 (Fig. 9), which exerts on the main stream an effect which is equivalent to aerodynamic compression, i.e., the same flow rate of air, with the same total pressure and temperature. Transition from the depth of the aperture ring h_9 to the effective depth h_{ef} is by the formula

$$\overline{h}_{ef} = \frac{h_{ef}}{R_1} = 1 - \sqrt{\mu}(1 - \overline{h}_9),$$

where μ is the flow-rate factor for the aperture*.

Calculation of the effective depth of compression of the main stream is by the formula

$$\overline{h}_{ef} = 1 - \sqrt{\frac{\overline{F}_2}{1 + \Delta\overline{G}_3}},$$

at $k_1 = k_3 = 1.4$; $\vartheta_3 = 1.0$.

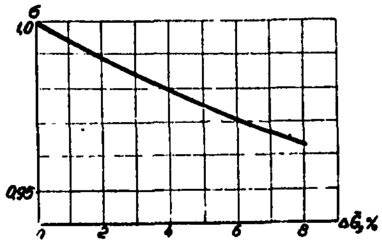

Fig. 8. Variation in total-pressure recovery factor as a function of $\Delta\overline{G}_3$ for $\Delta\overline{F}_3 = 2.7\%$, $\varphi = 70°$.

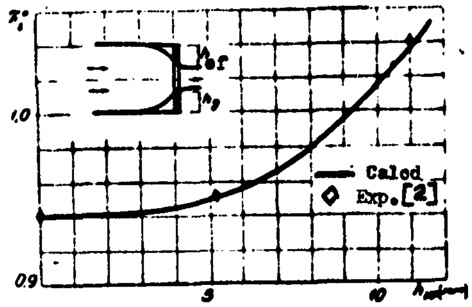

Fig. 9. The dependence $h_{ef} = f(\pi_3^*)$ for $\Delta\overline{F}_3 = 5.0\%$, $\varphi = 90°$.

*Rules 27 to 54 on the use and checking of flow-rate meters with normal apertures, nozzles and Venturi tubes. Committee on Standards, Measures and Measuring Instruments, Under the Council of Ministers, USSR, 1956.

The result of the calculation is shown in Fig. 9. The agreement of the calculation with the experiment is satisfactory.

Laval Nozzle

The diagram is shown in Fig. 10.

Additional premises: a) the stagnation zone is not extended beyond the cylindrical section, and b) the pressure in the stagnation zone is assumed equal to the static pressure in cross section 2-2. For a Laval nozzle without a cylindrical section, it is necessary to have, from experiments, the pressure in the stagnation zone for various values of $\Delta\bar{G}_3$, φ, and π_3^*.

Let us set up a system of equations for this case, assuming $k_1 = k_3$.

1. The equation of continuity between cross sections

a) $1 - \min$ $q(\lambda_1) = \frac{\bar{F}_2 \cdot \sigma}{(1+\Delta\bar{G}_3)}\sqrt{\frac{T_1^*}{T_2^*}}$,

where $\sigma = \frac{P^*_{min}}{P^*_1}$ – is the total-pressure recovery factor;

b) $\min - 2$ $\sigma_2 = \frac{P_2^*}{P^*_{min}} = \frac{\bar{F}_{min}}{q(\lambda_2)}$,

where $1 - \sigma_2$ are the losses in total pressure in the section min – 2.

2. The momentum equation for the volume of fluid between the cross sections

a) $1 - \min$ $I_1 + \frac{\Delta G_3}{g} v_3 \cdot \cos(\pi - \varphi) = I_{min} + p_{3.3} \cdot \Delta F_{3.3}$.

Using gasdynamic functions, we obtain

$$z(\lambda_0) + \frac{k}{k+1} \cdot \Delta\bar{G}_3\lambda_3 \cdot \theta_3 \cdot \cos(\pi - \varphi) = (1 + \Delta\bar{G}_3)\theta_2 \cdot z(\lambda_2);$$

b) $\min - 2$ $I_2 = I_{min} + p_{3.3} \cdot \Delta F_{3.3}$.

Taking point 1 (b) into account, we obtain

$$\Delta\bar{F}_{3.3} = 1 - \bar{F}_{min} = 2\left[\frac{2}{k+1}\right]^{\frac{1}{k-1}} \cdot y(\lambda_2)[z(\lambda_2) - z(1)].$$

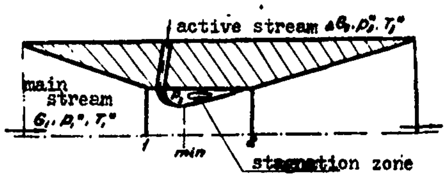

Fig. 10

3. The energy equation, the ratio of the flow rates of the streams, and the condition of expansion of the active stream are analogous to formulas (5), (6), and (3), respectively.

As a result we have a system of seven equations with seven unknowns: λ_1, λ_3, λ_2, $\Delta\overline{F}_{3.3}$, $\overline{F}_{min}$, σ_2, and ϑ_2. The parameters $\Delta\overline{G}_3$, π_3^*, φ, and ϑ_3 are given by the conditions of the problem, and σ is determined from experimental data.

Therefore, all parameters are determined uniquely. Calculation is by successive approximations, similarly to the problem examined above.

The graph in Fig. 7 gives the results of the calculation of the specific area of compression δ as a function of the ratio of total pressures for $\Delta\overline{F}_3 = 0.027$, $\varphi = 90°$, $k_1 = k_3 = k = 1.33$, $\vartheta_3 = 1.0$.

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