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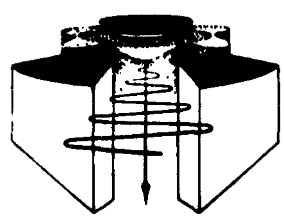
AN ANALOG METHOD OF DETERMINING ELECTRODE SHAPES OF ELECTRON GUNS HAVING CURVED TRAJECTORIES

TECHNICAL REPORT NO. 54

ELECTRON PHYSICS LABORATORY
Department of Electrical Engineering



By: R. J. Lomax

Approved by: J. E. Rowe

November, 1962

CONTRACT WITH:

ELECTRONIC TECHNOLOGY LABORATORY, AERONAUTICAL SYSTEMS DIVISION, WRIGHT-PATTERSON AIR FORCE BASE, OHIO. DEPARTMENT OF THE AIR FORCE, PROJECT NO. 4156, TASK NO. 415603, CONTRACT NO. AF-33(657)-8050

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Ronald J. Lomax

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J. E. Rowe, Director
Electron Physics Laboratory

Project 05000

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DEPARTMENT OF THE AIR FORCE
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PLACED BY: ELECTRONIC TECHNOLOGY LABORATORY
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ABSTRACT

In the resistance analog method of determining Pierce electrode shapes to maintain a known electron beam, the solution of Laplace's equation is sought for values of the electric potential and normal field specified on the curved beam surface. The difficulty of satisfying two conditions simultaneously can be eased by dividing the problem into two parts. The potential distribution when there is the specified potential but zero normal field on the beam surface is determined, then when there is the specified normal field but zero potential. The linearity of Laplace's equation permits the two solutions to be superimposed to give the required equipotentials, two of which can be taken as electrodes.

Experimental results obtained by this method are presented and compared with known analytical results.

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AN ANALOG METHOD OF DETERMINING ELECTRODE SHAPES OF ELECTRON GUNS HAVING CURVED TRAJECTORIES

INTRODUCTION

This report describes a method by which Pierce-type electrodes may be designed for electron beams with curvilinear trajectories using a Poisson cell or electrolytic tank analog. The main difficulty which arises in the analog approach is the simulation of the boundary conditions along the edge trajectory. The non-zero curvature of the electron paths implies that the normal as well as the tangential component of the electric field is non-zero and the necessity arises of simulating two boundary conditions simultaneously. The simplest conditions which can be represented in a resistive medium are zero normal electric field and zero tangential electric field (constant potential). These conditions exist at an insulating and a conducting boundary respectively. If both field components are to be non-zero along a boundary, either current must be injected along the boundary to simulate space charge, which is not very accurate near the cathode, or the bottom of the resistive medium must be contoured, which is not very convenient.

An alternative approach described below, in which the two boundary conditions are satisfied separately, makes use of the simplicity with which zero normal and tangential fields can be simulated.

SEPARATION OF THE TWO BOUNDARY CONDITIONS

Suppose the boundary conditions on the potential and normal field along the edge of the electron beam are

$$V = V_o \, , \tag{1}$$

$$\frac{\partial V}{\partial n} = \left(\frac{\partial V}{\partial n}\right)_o \, , \tag{2}$$

where V_o and $(\partial V/\partial n)_o$ vary along the beam edge. Define two potential distributions V_1 and V_2 which satisfy

$$\nabla^2 V_1 = \nabla^2 V_2 = 0 \tag{3}$$

in the region outside the beam and

$$V_1 = V_o \, , \quad \frac{\partial V_1}{\partial n} = 0 \; ; \tag{4}$$

$$V_2 = 0 \, , \quad \frac{\partial V_2}{\partial n} = \left(\frac{\partial V}{\partial n}\right)_o \tag{5}$$

on the beam edge. Clearly V_1+V_2 satisfies Laplace's equation and in addition it also satisfies the boundary conditions as originally specified. Consequently it is the required potential distribution.

SATISFYING THE POTENTIAL CONDITION

In order to determine the V_1 potential distribution, the beam edge is simulated by an insulator. Two (or more) electrodes at different potentials are adjusted until the potential distribution along the beam edge is matched to a satisfactory accuracy. When adequate electrode shapes have been obtained in this way, the potential is read off at a series of points which are most conveniently situated at the mesh points of a square net.

SATISFYING THE NORMAL FIELD CONDITION

Direct Method

To satisfy the V_2 boundary conditions, the beam edge must be replaced by a conductor at zero potential. Electrodes are adjusted until the field normal to the conductor is sufficiently close to the required value at a number of points. Subsequently the potential is read off at the same set of points as the V_1 solution.

From the two sets of potentials, V_1+V_2 can be determined and the equipotentials constructed by interpolation to give the possible electrode shapes.

Indirect Method

An accurate measurement of the field near the beam edge is not easily obtained because the field is measured by taking the difference of the potential at two closely spaced points whose separation can only be determined to within a relatively large error. Small imperfections of the beam edge simulating electrode can also produce large local errors in the field. In order to circumvent some of these difficulties, an alternative method of finding the V_2 solution can be used for planar beams. This makes use of the properties of functions of a complex variable in the following way.

Because V_2 satisfies Laplace's equation and can be written as a function of x and y, it can be expressed as the real part of a function

W_2 of x+jy. Let U_2 be the function conjugate to V_2, which must also satisfy Laplace's equation. Thus

$$W_2(x+jy) = V_2(x,y) + j\,U_2(x,y) \quad . \tag{6}$$

The Cauchy-Riemann equations require that

$$\frac{\partial V_2}{\partial s} = \frac{\partial U_2}{\partial n} \;, \quad \frac{\partial V_2}{\partial n} = -\frac{\partial U_2}{\partial s} \;, \tag{7}$$

where s refers to the derivative along the direction of the tangent to the beam edge, and n refers to the derivative along the normal to the beam edge. The boundary conditions

$$V_2 = 0 \;, \quad \frac{\partial V_2}{\partial n} = \left(\frac{\partial V}{\partial n}\right)_0 \tag{8}$$

on the beam edge are therefore equivalent to the following conditions on U_2:

$$\frac{\partial U_2}{\partial n} = 0 \;, \quad \frac{\partial U_2}{\partial s} = -\left(\frac{\partial V}{\partial n}\right)_0 \;, \tag{9}$$

which can be written as

$$U_2 = -\int_{\text{beam edge}} \left(\frac{\partial V}{\partial n}\right)_0 ds \;,$$

$$\frac{\partial U_2}{\partial n} = 0 \quad . \tag{10}$$

These conditions are similar to those imposed on V_1 and so a similar technique can be used to obtain the potential distribution U_2.

U_2 and V_2 are simply related: equipotentials of U_2 are field lines of V_2 and vice versa. This allows the V_2 distribution to be deduced from the U_2 distribution.

EXPERIMENTAL INVESTIGATION

In order to test the feasibility of these methods, experiments were performed on electrode configurations whose exact forms are known.

Meltzer Flow

The first flow pattern investigated was that of Meltzer[1] in which the electron trajectories are arcs of concentric circles. The electrons are emitted from a space-charge-limited cathode, then accelerate to a maximum velocity after rotating through 60 degrees, and decelerate to zero velocity when they have rotated 120 degrees. The normalized potential on the beam edge is

$$\frac{\left(\cos \frac{3}{2}\theta\right)^{4/3}}{r^2} , \qquad (11)$$

where θ is the polar angle measured from the axis of symmetry (position of maximum velocity), and r is the radius of the trajectory. The electrode shapes can be calculated by the method of analytical continuation and have been given elsewhere[2]. The beam and the analytically calculated electrode shapes are shown in Fig. 1.

The potential distributions corresponding to the two sets of boundary conditions described above were determined using the analysis of reference 2. Figure 2 shows V_1, the potential satisfying

$$V = \left(\cos \frac{3}{2}\theta\right)^{4/3} , \quad \frac{\partial V}{\partial r} = 0 \qquad (12)$$

on the beam edge r = 1, and Fig. 3 shows V_2, which satisfies

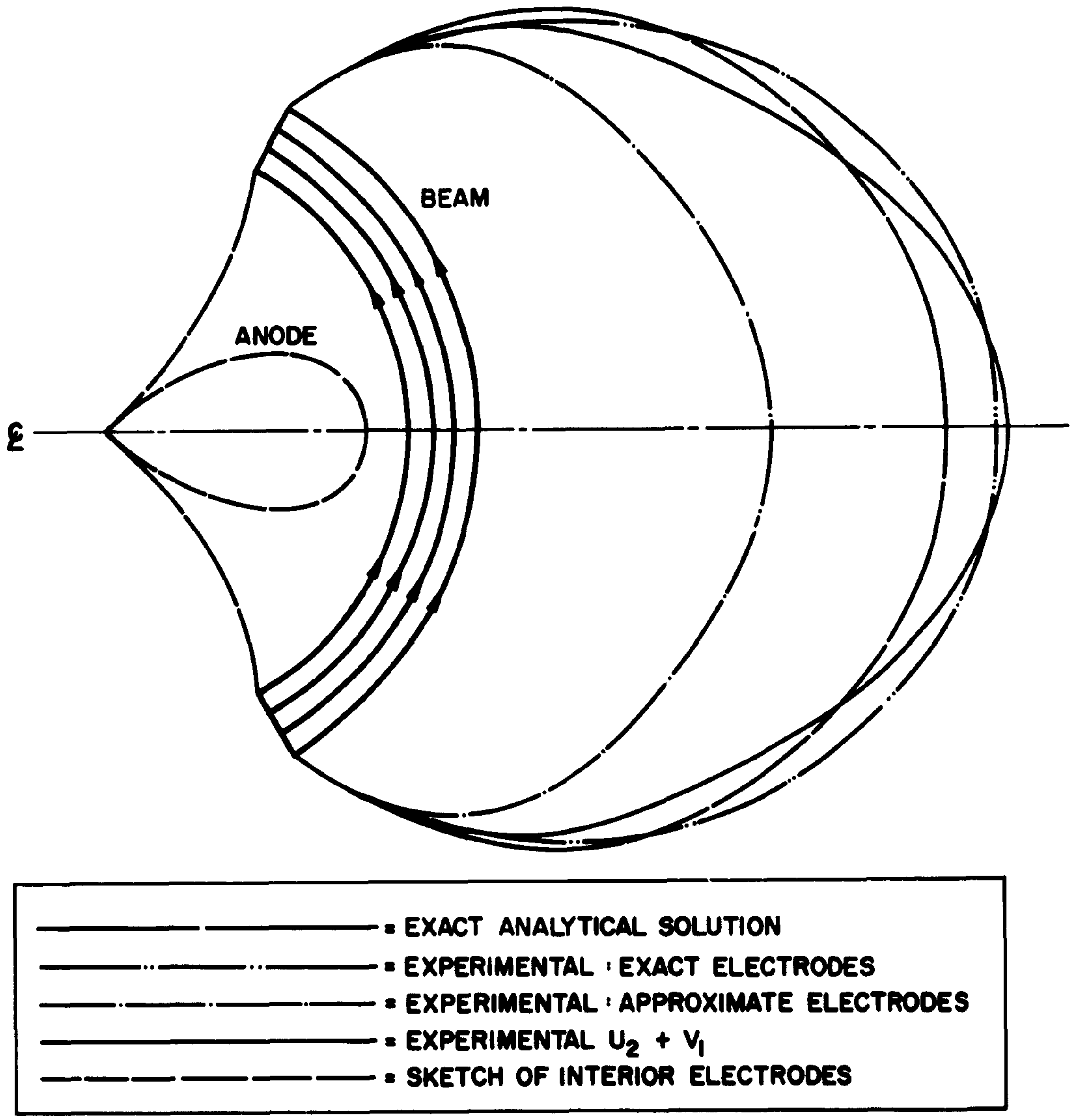

FIG. 1 ELECTRODE AND BEAM SHAPES (MELTZER FLOW).

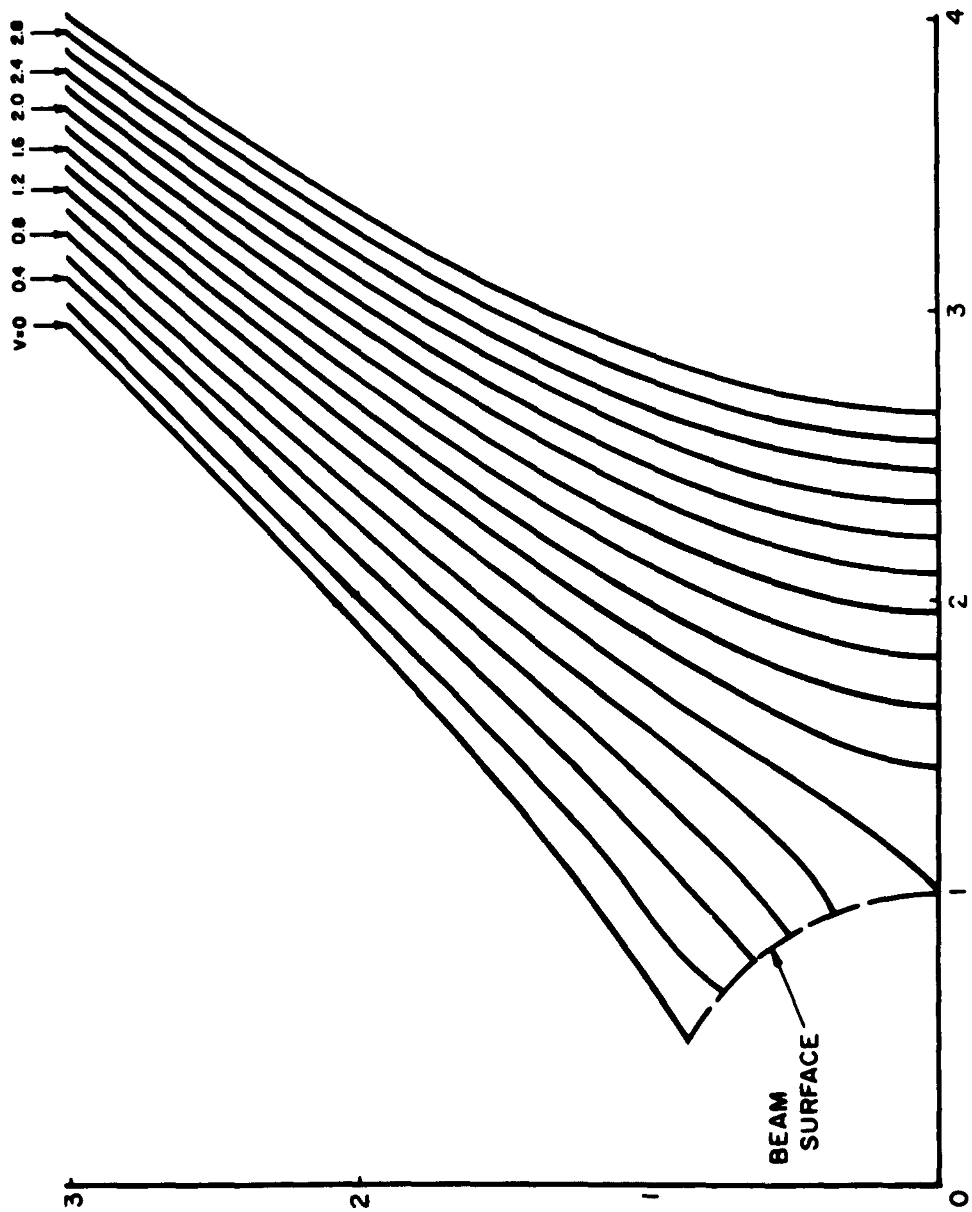

FIG. 2 POTENTIAL DISTRIBUTION V_1 (MELTZER FLOW).

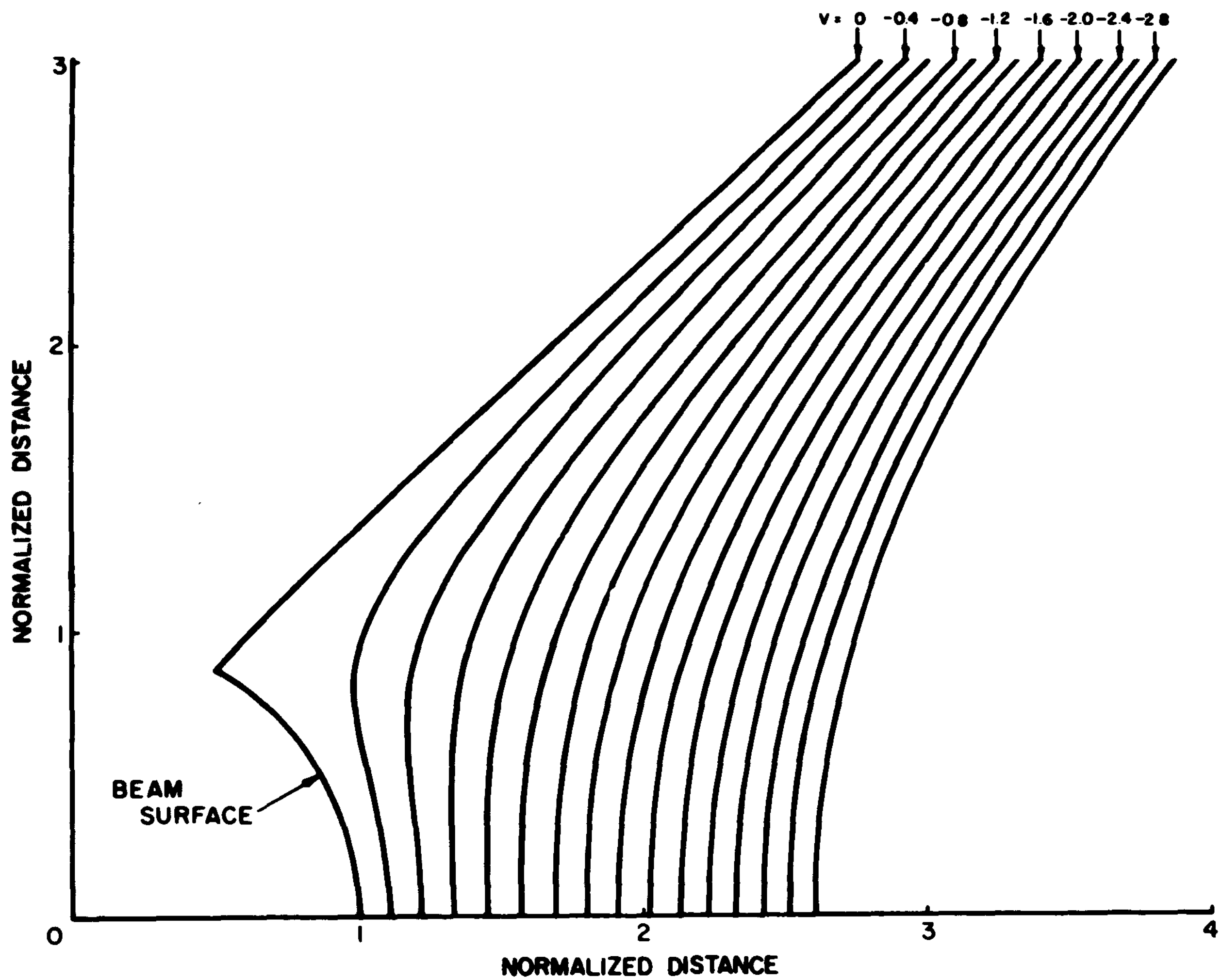

FIG. 3 POTENTIAL DISTRIBUTION V_2 (MELTZER FLOW).

$$V = 0 \ , \ \frac{\partial V}{\partial r} = -2\left(\cos\frac{3}{2}\theta\right)^{4/3} \tag{13}$$

on r = 1. Only the half region above the axis of symmetry is shown in both figures.

The known electrode shapes for the potential distributions V_1 and V_2 were then set up on a Poisson cell consisting of a thin film of conducting material deposited on a plastic sheet. Potential readings were taken at mesh points on a net parallel to the axes and at a spacing of 0.2 in the units of Figs. 2 and 3. The potential V_1 was measured every 5 degrees along the beam edge, which was simulated by scraping off the conductor along the beam contour. The V_2 field was also measured at corresponding points. The beam edge in this case was simulated by painting the resistive material with conducting silver paint, and the axis of symmetry was represented by an insulated edge. In Table I these experimentally determined values are compared with their exact values. V_1 (second line of the table) agrees to within about 2 percent, V_2 (fifth line) agrees to between 8 and 10 percent. When the two potential distributions were added, the zero potential line labelled "Exact Electrodes" in Fig. 1 was obtained.

A second set of readings was taken in which the electrode shapes were changed in order to produce a deliberate error in the potential distribution and to evaluate its effect on the potential and field values along the beam edge and on the final electrode shape. (In practice the situation would be reversed; unavoidable errors in matching the boundary conditions would lead to errors in the electrode positions.) The modified electrode shapes are shown as dashed lines in Fig. 4 for both V_1 and V_2. Apart from this change, the details of the procedure remain the same. The

Table I

Comparison of Experimental and Analytical Values of the Boundary Conditions
(Meltzer Flow)

Degrees of Arc Along Cathode from Axis of Symmetry		0	5	10	15	20	25	30	35	40	45	50	55	60
V	Analytical Solution	1.00	0.99	0.95	0.90	0.83	0.73	0.63	0.52	0.40	0.28	0.17	0.07	0.00
	Experimental Values for Exact Electrodes	1.02	1.02	0.97	0.92	0.83	0.74	0.63	0.52	0.40	0.28	0.17	0.07	0.00
	Experimental Values for Modified Electrodes	0.92	0.91	0.87	0.82	0.75	0.67	0.57	0.47	0.36	0.26	0.16	0.06	0.00
$-\frac{1}{2}\frac{\partial V}{\partial n}$	Analytical Solution	1.00	0.99	0.95	0.90	0.83	0.73	0.63	0.52	0.40	0.28	0.17	0.07	0.00
	Experimental Values for Exact Electrodes	1.00	1.00	0.92	0.93	0.88	0.79	0.70	0.49	0.38	C.29	0.15	---	---
	Experimental Values for Modified Electrodes	1.06	1.09	1.03	0.95	0.88	0.83	0.77	0.56	0.44	0.31	0.20	---	---
	Experimental Values from U_2 Solution	0.90	0.93	0.96	0.90	0.80	0.73	0.63	0.58	0.42	0.30	0.16	0.15	---

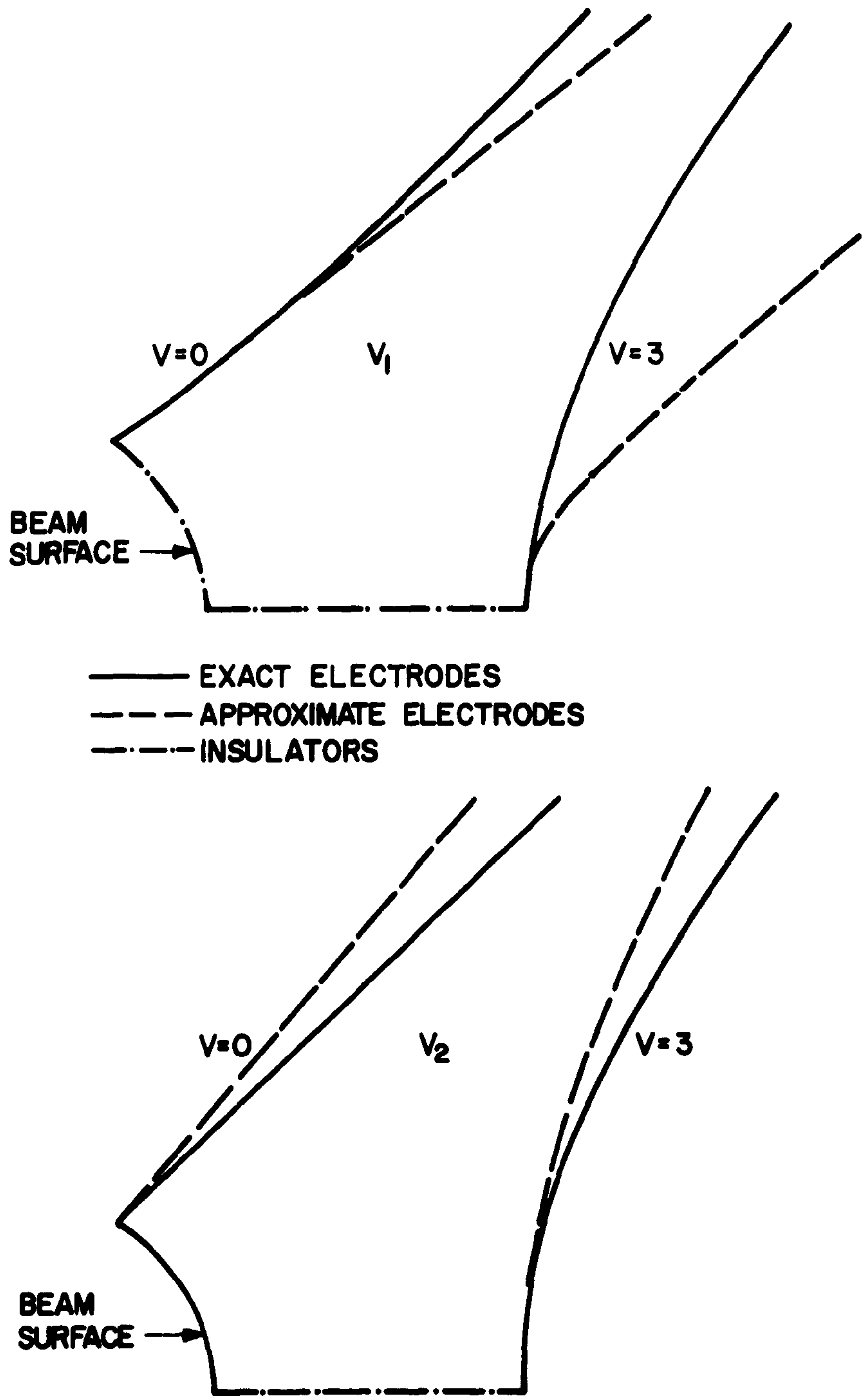

FIG. 4 MODIFIED ELECTRODE SHAPES.

potential values along the beam edge for the V_1 solution and the normal field values for V_2 are given in the third and sixth lines of the table. The considerable departure of the electrodes from their exact shape produces changes of about 8 percent in the potential, and up to 15 percent in the normal field. The zero equipotential obtained from adding these values of V_1 and V_2 is shown in Fig. 1 "Approximate Electrodes".

Finally, the indirect method of determining V_2 via U_2 was used. U_2 satisfies the conditions

$$U_2 = 2 \int_0^{\theta} \left(\cos \frac{3}{2} \theta'\right)^{4/3} d\theta' \quad , \tag{14}$$

$$\frac{\partial U_2}{\partial r} = 0 \tag{15}$$

on r = 1. Again the electrode shapes were adjusted until adequate agreement at 5 degree intervals along the beam edge was obtained. This time it was necessary to use three electrodes at potentials 0, 64.5 and 100 volts (subsequently scaled). The values 0 and 100 were fixed, but the choice of 64.5 was made during adjustment of the electrode positions to give optimum agreement with the boundary values. Field lines were then traced out; this could be done automatically on the trajectory plotter used for these experiments by programming the associated analog computer appropriately. Although the field lines and thus the equipotentials of V_2 can be determined, the potential to be assigned to each one is not known. The simplest way of determining these potentials is to revert to a representation of V_2 using as electrodes the equipotentials now known. This can be done rapidly since no further adjustments of the electrodes are needed. The beam edge is once more simulated by a conductor and the field is measured at a number of points along the beam edge. The ratios of these values should already be correct; their absolute values may be determined by scaling all potential and field values by a factor which yields the closest agreement on the average with the prescribed field values. The potential distribution can then be read off directly. Figure 5 shows a plot of the U_2 field lines. This can be compared with Fig. 3; the field lines of Fig. 5 leave the axis of symmetry (horizontal axis) at the same points as the equipotentials of Fig. 3. The zero equipotential obtained from these results is shown in Fig. 1.

Discussion of Results

The departures of the three experimental results from the analytical solution observed in Fig. 1 deserve some comment. V_1 and V_2 have cancelling singularities which means that, when V_1 and V_2 are added, the final result is the sum of two almost equal but opposite values. This leads to the possibility of a large relative error and explains why the "Exact Electrodes" curve does not coincide with the analytical solution. The "Approximate Electrodes" curve is very far from the analytical value. This is a manifestation of the well known instability of the Cauchy problem for an elliptic partial differential equation. Fortunately, there is the compensating property that electrodes which are only approximately correct far from the beam will give potentials close to those desired along the beam itself. The electrode derived by the use of U_2 is reasonably close to the analytical solution.

The Kino Gun

The second investigation was for the more complex geometry of the Kino gun[3]. The beam trajectories are described parametrically in terms of

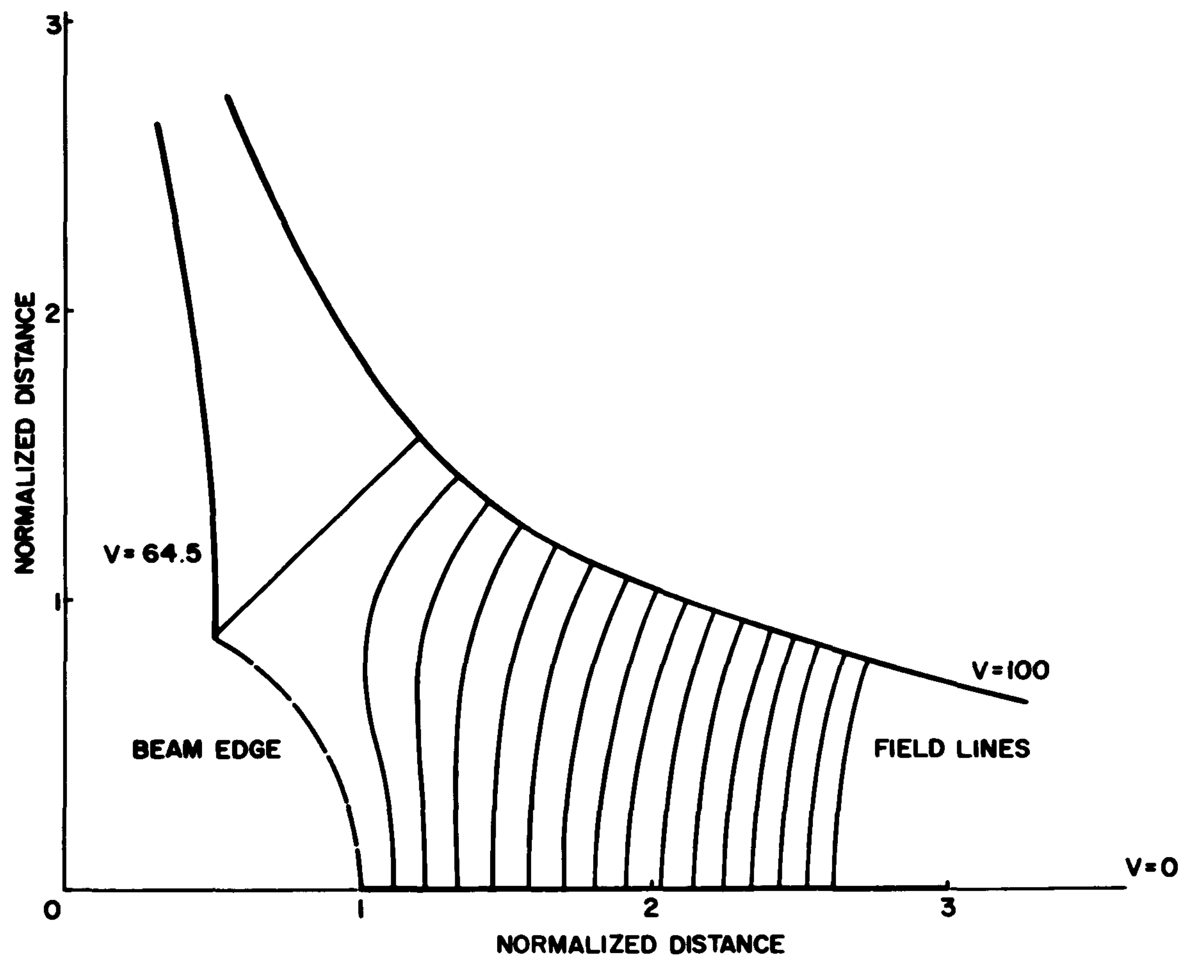

FIG. 5 POTENTIAL DISTRIBUTION U_2 FOR THE MELTZER FLOW SHOWING FIELD LINES.

u, the electron transit time normalized with respect to the cyclotron angular frequency, by

$$x = x_o + \frac{1}{2}u^2 + \cos u - 1 \quad , \tag{16}$$

$$y = u - \sin u \quad . \tag{17}$$

Each trajectory is geometrically similar, merely being translated in the x-direction. In the beam region, the potential at the point x(u), y(u) is given by

$$V = \frac{1}{2}u^2 - u \sin u + 1 - \cos u \quad . \tag{18}$$

The analytically continued potential is singular at the points corresponding to $u = 2n\pi$; $n = 0, 1, 2, \ldots$ where three equipotentials leave the beam edge simultaneously. The location of the singular point corresponding to $u = 2\pi$ is shown in Fig. 6 and the behavior of the potential in the immediate vicinity of this point is shown on a much larger scale in Fig. 7.

Associated with each singular point is a branch point of the potential distribution function where the potential becomes multiple valued. One of these is shown in Fig. 6. The presence of the branch points makes it impossible to satisfy the boundary conditions with only two electrodes when these electrodes meet the beam edge at points which include one of the singular points between them. Figure 6 shows some of the V_1 equipotentials determined analytically; the wide divergence of lines of neighboring potential is clearly seen. In fact the lines to the left and right of the singular point become members of different branches of the solution. Consequently, the region to the left of the singular point was investigated.

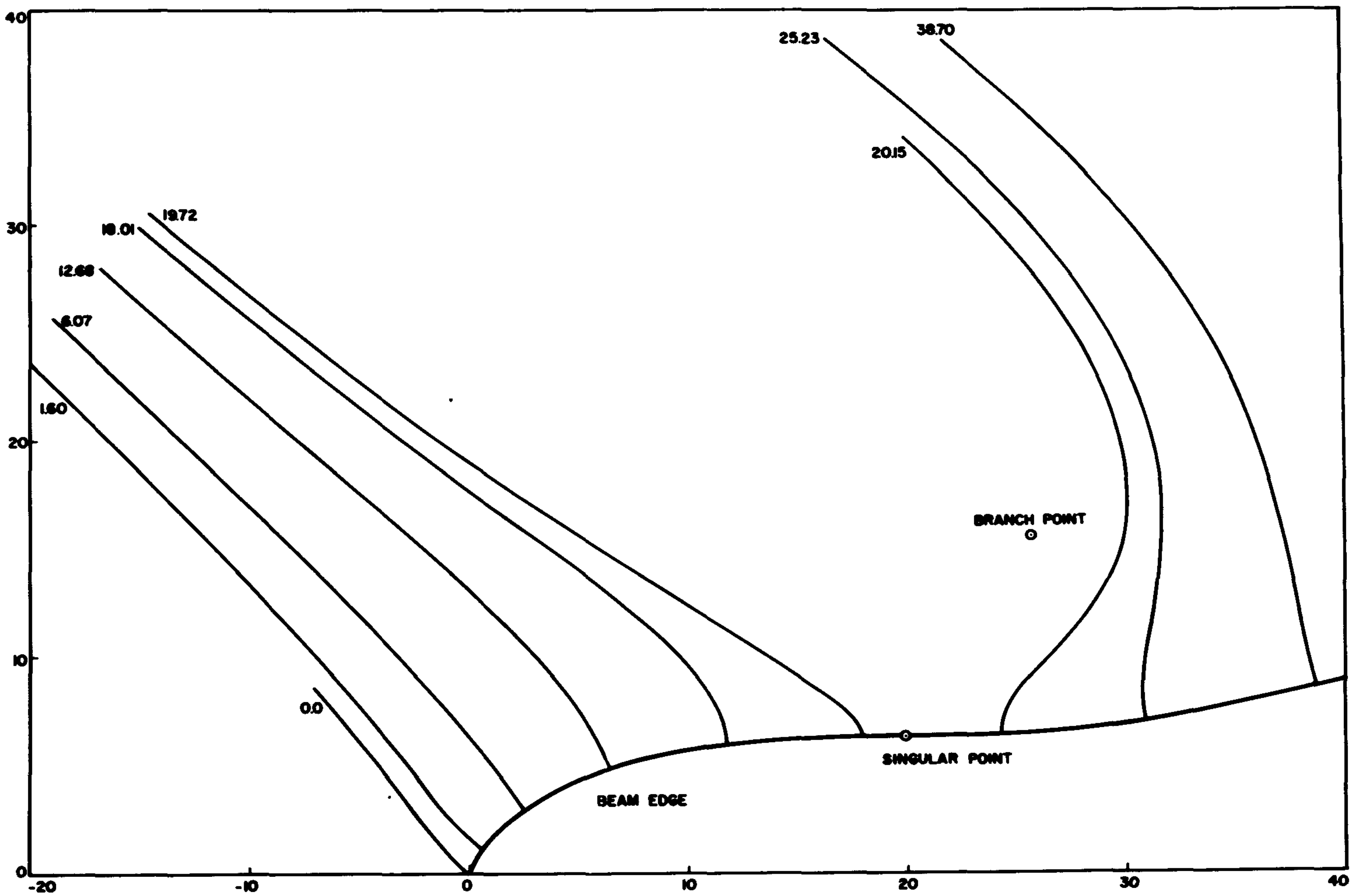

FIG. 6 POTENTIAL DISTRIBUTION V_1 (KINO GUN).

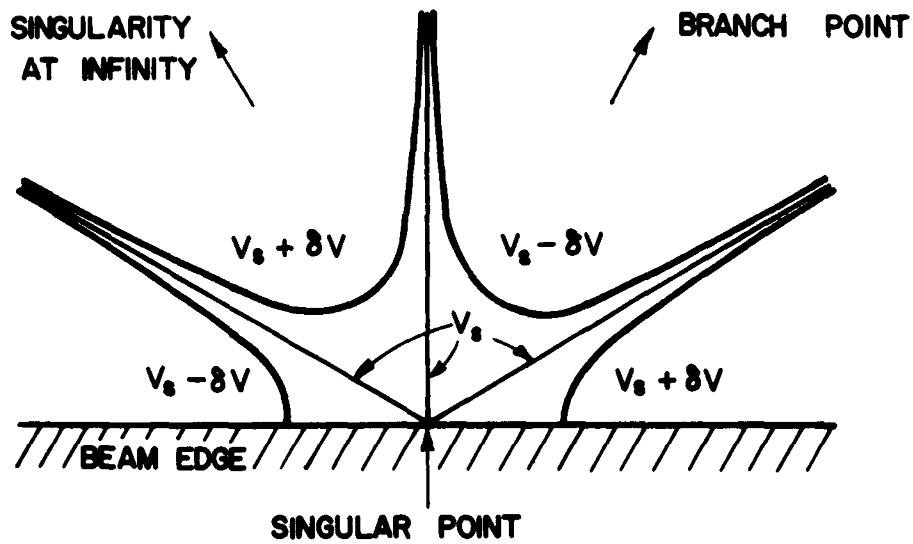

FIG. 7 POTENTIAL V_1 IN THE VICINITY OF THE SINGULAR POINT ON THE EDGE OF THE BEAM.

The V_1 solution shown in Fig. 6 was also obtained by the analog method between the 0 and 19·72 equipotentials. The V_2 solution was first obtained by means of the dual solution U_2, but it was found that for electrodes starting at the same points on the beam edge as the V_1 solution, the overlap of the two solutions was very small. (The final solution can only be obtained in the region common to both of the individual solutions.)

The V_2 solution was then obtained directly using a positive, zero and negative potential electrode in order to obtain a greater overlap of V_1 and V_2. The equipotentials which result from the combination of V_1 and V_2 are shown in Fig. 8 and are reasonably close to the analytical solution.

In order to find the electrode shapes over a larger region, it would be necessary to divide the whole region external to the beam into a number of smaller ones in which the potential distributions would be found separately. The reason for this is that in order to avoid the singularity, the electrodes would have to be truncated only a short distance from the beam, and to define the potential adequately the separation of the electrodes must be somewhat less than their length. A considerable amount of work would then be involved, which makes the technique seem unattractive for this and similar configurations in which singularities occur close to the beam surface.

CONCLUSIONS

The analog technique of determining electrode shapes that has been described should have practical utility for certain types of problems. The type of electron gun which is not suitable for this method is one

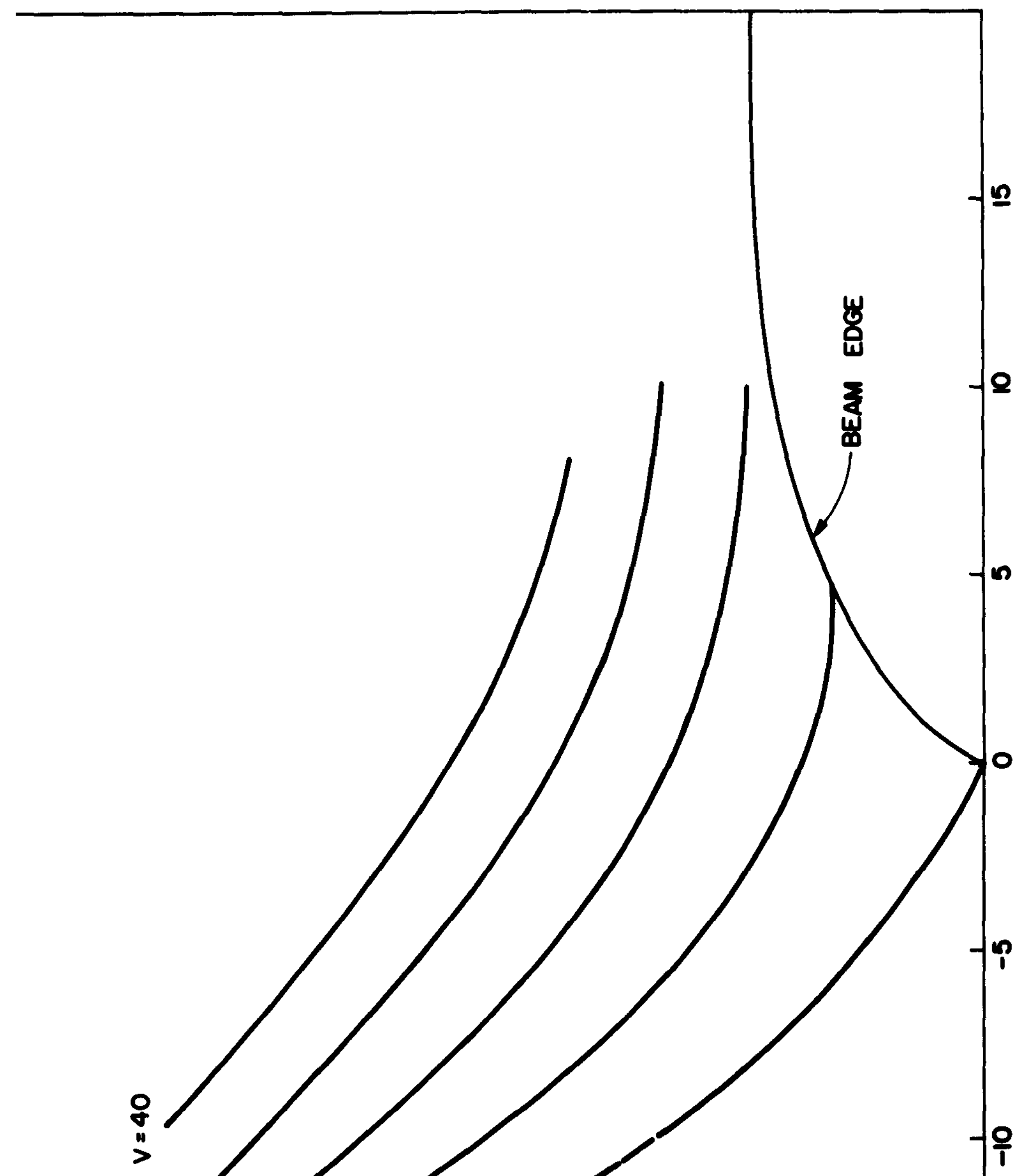

FIG. 8 EXPERIMENTALLY DETERMINED EQUIPOTENTIALS EXTERNAL TO THE BEAM (KINO GUN).

which has a singularity in the potential distribution close to the beam surface. This frequently means that in any case a practical two potential gun will not be possible whatever the means of design. In cases where the singularities can be excluded by taking a finite section in which the solution is well behaved, as in the Meltzer solution described above, or when they are far from the region of interest, the process described is a feasible one.

ACKNOWLEDGMENT

Some of the ideas upon which this work is based were obtained in conversation with Dr. Mark Barber, then at the Department of Electrical Engineering, University of Cambridge.

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In the resistance analog method of determining Pierce electrode shapes to maintain a known electron beam, the solution of Laplace's equations is sought for values of the electric potential and normal field specified on the curved beam surface. The difficulty of satisfying two conditions simultaneously can be eased by dividing the problem into two parts. The potential distribution when there is the specified potential but zero normal field on the beam surface is determined, then when there is the specified normal field but zero potential. The linearity of Laplace's equation permits the two solutions to be superimposed to give the required equipotentials, two of which can be taken as electrodes. Experimental results obtained by this method are presented and compared with known analytical results.

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