UNDERWATER SOUND TRANSMISSION

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Preface

The subject "Underwater Sound" covers phenomena as complex and diverse as the imagination and patience of the investigator will allow. An important part of the subject deals with the sonar parameters, in the form of loss or strength coefficients. During the twenty years that appreciable effort has been expended in experimentation and analysis, the great emphasis has been in two areas.

a) Quantitative investigation of the sonar parameters.

b) Quantitative investigation of the ocean environment.

Knowledge of the environment has been applied to the estimation of the parameters, and to the description of other aspects of underwater sound.

The principle effects of the environment which determine losses in a propagating sound wave were identified by the early part of World War II. The work since that time has served to provide an increased understanding of the physical processes involved, and to make possible more detailed and reliable calculations of transmission loss.

The first concern of the user or the designer of underwater sound equipment is with energy, or power. These things establish the general operational features, size and weight of equipment, and class of components. The state of present knowledge permits the evaluation with reasonable accuracy, of energy/power levels under prescribed conditions. Unfortunately, conditions prescribed must be quite exact, if definite energy calculations are required. This is true because present descriptions of the sound field are synthetic, in the sense that the whole picture is made up of many parts, each part contributing an important element to the calculation. Each part also contributes its share to the uncertainties of the reliability of the calculation.

Table I outlines the conditions which must be specified for a calculation of transmission loss, as an example. It is clear that a specific calculation will apply only to the specific conditions chosen. Now, of course, no one has more than passing interest in such a specific situation. Therefore, in practice one either makes many calculations, corresponding to many conditions, and then averages them in some sense, or else chooses a "representative" condition, and regards it as typical.

The process of research and application as discussed above amounts to measurement of the sound field, its analysis into many constituents, and then a synthesis of the predicted field by a sort of reverse process. But the reverse does not lead back to the original measurement, even in principle. All of the detail, and most of the features of the sound field have been irreversibly discarded. Therefore, no answers can be provided to questions concerning other features of the sound field, and if the questions must be answered, the process of research and application must be renewed and redirected.

Today, the questioners are pounding on the door, seeking information vital to the problems of present operational performance and future design. They are concerned for the most part with features of the sound field which are simply not contained in the sonar parameters. There are two ways in which answers to such questions can be sought at this time. One is the direct way, going to sea again, with new kinds of instrumentation, making new analyses and building up a new picture of the kind that seems important. The second way is synthetic, using a physical description of the ocean as the basis for the necessary equations to describe the required features of the sound field. Both ways are useful; both are being employed today.

Unfortunately, the kind of experiments being conducted today cannot be regarded as satisfactory. On the "synthetic" side, considering the very large expenditures of effort in the past, future results cannot be expected to provide more than marginal increases in knowledge, except in special cases. On the "direct" side, there is the inescapable dilemma that vastly increased detail is required, on a subject which is already almost unmanageable because of detail. The fact is, new ideas and concepts for characterizing the sound field are sorely needed. What are the essential features of the sound field, which both <u>must</u> and <u>can</u> be determined ?

Research is presented on a
~~This report presents a~~ physical representation of the ocean, together with constants and equations sufficient to permit a reasonable calculation of the average sound field. In addition, it discusses other features of the sound field, and hints at some new methods of calculation using the physical picture.

is given
~~Chapter 1 presents~~ A ~~brief~~ review of the state of knowledge concerning underwater sound transmission from its status in 1946, to its present state. A discussion of recent advances and studies in several important areas is made. An oceanographic acoustic description of the physical environment of underwater sound is proposed in outline form.

In Chapter 2, the proposed description is developed quantitatively, and detailed methods are given for computing the average sound field.

Chapter 3 discusses aspects of the sound field which are not covered by the quantitative model, either in principle or because of practical difficulties. These include mean variability of the sound field, fluctuations in the sound field, turbulence effects and other measures of the sound field.

In Chapter 4, recommendations for the direction of future research in underwater sound propagation are given.

To the reader not concerned with detail, it is suggested that Chapters 1 and 4 be perused.

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Chapter 1

INTRODUCTION

1.1 Orientation

Since the end of the war, only two status reports on the subject of underwater sound transmission have been published. These include the NDRC[1] reports of 1946, and the Geological Memoir No. 27[2] by Ewing, Worzel, and Pekeris of 1948. The present report reviews the progress made in the field since that time.

Rapid strides have been made in the understanding and prediction of transmission loss. The previous summaries consisted of collections of articles by various authors with diverse points of view. Unfortunately, gaps existed in available data and in the general unity of the subject matter. While much remains to be done in both experiment and concept, we have strived to achieve unity and completeness in treatment and to establish an acoustical description of the ocean into which most of the present and future data can be placed without undue strain on the structure.

The present state of agreement between the "oceanographer's ocean" and the "acoustician's ocean" has allowed the formulation of an acoustic description of the ocean as a framework for additional experimental design and operational applications. The cells of the descriptive model must be fine-grained enough to admit data from general research and yet large enough to permit easy assembly and integration for specific operational applications which exist now and those which will be required in the future. The proposed description of this model is shown in Table I.

Table I

PHYSICAL ENVIRONMENT OF UNDERWATER SOUND

Outline

1.0	Regions
1.1	Atmosphere
1.2	Ocean Surface
1.2.1	Ice cover
1.3	Hydrosphere
1.4	Ocean floor
1.5	Ocean sub-bottom
2.0	Geophysical Properties - (point functions of space-time in each region)
2.1	Physical properties
2.1.1	Depth
2.1.2	Velocity (wind or current)
2.1.3	Temperature
2.1.4	Pressure
2.2	Chemical properties (composition)
2.3	Biological properties
2.3.1	Presence of species
3.0	Acoustic Properties - (point functions of the geophysical properties and the acoustic parameters)
3.1	Phase velocity
3.2	Absorption coefficient
3.3	Boundary reflection and scattering coefficients
4.0	Parameters (independent variables)
4.1	Acoustic
4.1.1	Frequency
4.1.2	Wave amplitude
4.2	Sonar
4.2.1	Source location
4.2.2	Receiver location
5.0	Sound Properties - (point functions of the acoustic properties and sonar parameters)
5.1	Sound pressure
5.1.1	Mean value
5.1.2	Mean variability
5.2	Transmission Time

Note: The model may be extended to other sound properties such as reverberation, etc., by adding to items 4.0 (possibly) and 5.0, above.

1.2 Review of Present Status

1.2.1 General Comments

The chief advances since 1946 consist of the probing of the sound field as a function of near-surface depth for both source and receiver; a better recognition of the role of surface scattering in the attenuation of sound in surface sound channels; establishment of the form of the frequency and temperature dependence of absorption and its relation to the $MgSO_4$ ions; the establishment of preliminary curves for estimating bottom loss as a function of grazing angle and frequency; some estimate of the gains and geometry of convergence zone fields; a summary of shallow water propagation results in terms of acoustic frequency, water depth, and bottom type; studies concerning the relationship between thermal microstructure and sound fluctuations; and the direct measurement in the laboratory of the speed of sound in sea water, correcting Kuwahara's surface values.

A great deal of measurement work continues to be done, but the emphasis is still on the specific needs of operational equipment. Information about variability in both time and space has become even more important and the three-dimensional representation of the oceans is needed to describe it as a medium for practical acoustic transmission.

1.2.2 Speed of Sound and Absorption Measurements

With the development of the Greenspan-Tschiegg[3] sound velocity meter into a field instrument, increasing effort is being given to the actual measurement of the speed of sound and its fluctuations. It has been used in connection with propagation measurements at sea. While controversy still exists regarding the exact nature of the sound speed which this device measures, the formulas of W. Wilson[4-6] have been accepted for the conversion of oceanographic measurements of temperature and salinity as a function of pressure. These expressions were determined from laboratory measurements, but have been fairly well verified in the field. Accurate velocity measurements are important for determining range to convergence zones, in long-range communications and in fathometry. Mackenzie[7] has furnished a gravity correction for the density of sea water in converting pressure to ocean depth. Maunsell and Scrimger[8] have shown that Kuwahara's formula yields ranges too short for the observed convergence zones in the North-East Pacific.

Since absorption is an important part of attenuation in sea water, a more precise determination of that quantity will aid in understanding the residual term, or the attenuation due to other factors. Much work has been done relating the absorption properties of sea water to those of magnesium sulphate in solution. While there is reasonable agreement with regard to the data on relaxation frequencies and the form of the expression and consequent temperature dependence, the value for the temperature independent coefficient appears to be in dispute. There is a 35% difference in determinations of this coefficient which should be resolved. There are only two large sets of measurements at sea from which absorption characteristics have been determined. The first set, that of Marsh and Schulkin,[9, 10] is in moderate agreement with that of Del Grosso,[11] and of Kurtze and Tamm.[12, 13] Measurements for the second set, that of Murphy and Garrison,[14] are at higher frequencies, and were made in the low salinity water of Puget Sound. Some suspicion exists here, because even though they agree with Beyer's nomogram, Beyer[15] acknowledges that the consistency of the different salts in sea water fails in water of low salinity, and absorption has been found to be proportional to salinity. Thus a study of the temperature independent term in sea water as opposed to magnesium sulphate solution can turn out to be quite important.

Del Grosso[11] has computed a maximum dispersion of 0.024 m/sec from an expression by Kneser that the percentage dispersion is proportional to the maximum absorption at the relaxation frequency. The relationship between the temperature pressure, and salinity dependence of sound speed and absorption has not been worked out yet. Nor has the pressure dependence of absorption in sea water been measured, although this has been done for pure water.

1. 2. 2. 1 Attenuation Coefficient

One of the earliest techniques of describing propagation loss over a path was to fit data to an equation which had a term considering spreading loss and a term in which the loss was proportional to range. The coefficient of this latter term is called the attenuation coefficient and is frequency dependent. Two expressions for the attenuation coefficient which are in fairly widespread use are those of Horton[16] and Sheehy and Halley[17]. The Horton formula fits empirical data above 5 kc/s to the same frequency dependence as is given by the absorption formula. Sheehy and Halley found a $^3/_2$ power law dependence on frequency from 40 cps to 100 kc/s. Recently E. L. Peterson[18] developed an equivalent network for sea water using the Horton formula. He showed that the impulse response for the network agreed with the properties of explosion impulses in the sea.

1.2.3 Ocean Boundaries

1.2.3.1 Surface Scattering

Surface scattering leads to a large attenuation term when coupled with the predominant occurrence of surface sound channels. Marsh et al[19-21] have presented a solution to these problems, including application of the Neumann-Pierson expression for ocean wave spectra. The formulation also allows the determination of the sound intensity fluctuation distribution. Marsh[22,23] has also applied his analysis to surface reverberation and has shown how to deduce ocean wave spectra from shot spectra. There is evidence pointing to the existence of other factors which must be considered to obtain a better understanding of the nature of the propagation mechanism. For example, Urick[24] has observed scattering returns emanating from a point well below the depth of the waves on the sea surface. In addition, Urick and Hoover[25] have observed an independence of back-scattering intensity below a 10° grazing angle in relatively rough water which they attribute possibly to bubbles. Garrison, Murphy and Potter[26] observed an extremely sharp falloff of back-scattering at these low angles when the water surface was comparatively undisturbed.

1.2.3.2 Under Ice Propagation

Several reports on transmission experiments under ice have appeared recently, and the subject is receiving accelerated attention. Work of the group at Lamont Geological Observatory[27] and Naval Research Establishment (Canada)[28] has served to demonstrate several important features of the under ice situation, foremost among them being the sharp cutoff in propagation above 50 or 100 cps. This has been attributed to the scattering effects of the rough underside of the ice. Coupled with this effect is the loss of high order modes, when the refraction is upward, as it usually is. The result is a situation which can be characterized by a few modes giving rise to the effects of dispersion and modal interference.

On the quantitative side, the scattering losses under ice can be computed from reasonable assumptions about the under-ice roughness, using formulas developed for sea surface scattering. Mellen[29] has made extensive calculations interpreting the mode patterns.

1.2.3.3 Acoustic Properties of Ocean Bottom

The effects of the bottom in its influence on the sound field variability are being studied by various groups. An understanding of the acoustic properties of the ocean bottom has become quite important. Attempts are being made to separate the various factors affecting apparent bottom losses such as sound velocity structure, pulse widths, and scattering and absorption, in addition to the varying nature of the bottom itself. Fry and Raitt[30] have started a classification of bottom types into "fast" and "slow". A dependable estimate of average bottom loss as a function of frequency and grazing angle is required, as well as a measure of the dispersive and delay properties.

Hamilton[31-34] et al at NEL have made laboratory measurements of bottom properties by bringing back samples in a state as undisturbed as possible. Comparing these values with in situ measurements using probes, good agreement has been obtained. Mackenzie[35-37] has related Hamilton's measurements to observed bottom loss. Despite the great importance of quantitative knowledge of the ocean bottom, comparatively little effort has been devoted to this subject. Much improvement is hoped for in the next year or two. For further material, references 30-48 in the bibliography may be consulted.

1.2.4 Shallow Water Propagation

Mackenzie[49-51] and Marsh and Schulkin[52] have made extensive analyses of the sound field and its fluctuations in shallow water.

Mackenzie has developed a theory of shallow water propagation based on Hamilton and Shumway's description of the bottom in terms of density, sound speed, and absorption. In addition, a scattering property is superposed using a $\cos\theta$ or $\cos^2\theta$ law of scattering. Taken together with the empirical $f^{3/2}$ dependence of absorption, these expressions are used to account for the attenuation. Sea surface scattering is neglected. A frequency shift or dispersion attributed to the bottom has also been observed by Mackenzie in studies of fluctuation statistics in shallow water propagation.

Marsh and Schulkin have described propagation in shallow water through a semi-empirical bottom loss factor which is also dependent on sea state. The coupling between the bottom and the sea surface arises because the average angle of incidence of the sound on the bottom is a function of sea state. This description has the happy faculty of being consistent with deep water surface scattering results as a function of sea state and the deep water bottom loss results. In the first limiting ray zone, both the Mackenzie and the Marsh and

Schulkin descriptions are identical, despite the use of the different bottom loss curves, and the different absorption laws. There is also further theoretical and experimental evidence[53,54] for a spreading law of r^{-1} (cylindrical) beyond a certain range, preceded by a law of $r^{-3/2}$ at intermediate ranges, while the spherical law of r^{-2} prevails at short ranges. Williams[55-57] et al have made extensive studies of low frequency propagation in shallow water, also finding a sea state dependence. They are now studying the loss of energy into the bottom through the establishment of transverse waves. Tolstoy[58] has extended the normal mode treatment of Pekeris[2].

1.2.5 Internal Fluctuations in the Sound Field

The sound field may be changed by the phase interference of existing rays, focusing due to the widening or narrowing of beamlets, or the addition or subtraction of new rays. Phase interference usually occurs when a few rays following slightly different paths recombine. If the phases occur randomly over the interval of 0 to 2π, then the statistics of amplitudes will follow a Rayleigh distribution[1]. If the signal is very much larger than the others, then a "Rice" distribution[59] (sine wave plus random background) is obtained. Changes in focusing lead to normally-distributed amplitudes.

The addition or subtraction of rays may be brought about by relative motion of transmitter or receiver beams; the motion of reflecting surfaces such as the ocean surface, due to waves or tides; or the change in the sound speed structure (refraction) due to internal waves, surface waves, turbulence, currents, or heating and cooling.

Such processes may also cause phase changes, amplitude changes or even frequency shifts along single rays.

Understanding fluctuations in the sound field[60-63] thus involves an understanding of the process of scattering (and reverberation) in relation to the size, degree of discontinuity, distribution, and time behavior of ocean inhomogeneities. In the following paragraphs we summarize the situation with regard to various types of scattering.

1.2.5.1 Volume Scattering

Volume scattering contributes second-order effects to fluctuations in field intensity and direction of propagation. Although a small scattering attenuation factor does exist, this should become more important at great ranges and low frequencies.

The basic investigations of volume scattering in the ray-theory range were performed by P. G. Bergmann[64] who derived expressions for the fluctuations of the acoustic path and the intensity. The wave-theory treatment of scattering was essentially formulated by C. L. Pekeris[65] and E. L. Carstensen and L. L. Foldy,[66] who investigated the effect of bubbles in water. D. Mintzer[67] extended the work of Pekeris and succeeded in deriving an expression for the first-order space-average value of the scattered intensity. A. M. Obukhov[68] obtained an expression for the fluctuation of phase and amplitude, which in the low frequency range is equivalent to Mintzer's result and, in the ray-theory range, is equivalent to Bergmann's solution. Chernov[69] investigated the correlation distance for two sound receivers and also wrote a book on the propagation of energy through random media. Other Russian workers active in the field are Krasilnikov, and Karauainikou.[70, 71]

D. S. Potter and S. R. Murphy[72] who studies the effect of the correlation functions $e^{-\rho/a}$ and $e^{-(\rho/a)^2}$ in scattering, derived an expression for the maximum phase delay of the space average contributions of the scattering patches. They also derived an expression for the coefficient of variation in the whole frequency range.

Lighthill,[73] Kraichnan,[74] Lyon[75] and Tatarski[76] have studied the propagation of sound waves through turbulent media. Skudrzyk et al[77,78] made an important contribution to this subject through their unified theoretical treatment, the introduction of the Kolmogorov "patch"-size distribution of temperature in the ocean and the application of these concepts to actual thermal and acoustic experimental results. Skudrzyk has also contributed to the bearing-error problem by giving an expression for the deviation in direction due to focusing and interference fluctuations. This analysis is also applicable to volume reverberation.

Volume reverberation, as related to the statistical distribution of the scattering mechanisms, has been treated by Carleton,[79] and also Cron and Schumacher.[80] Lafond[81,82] has been making detailed studies of the thermal microstructure. The original work in this area was carried out by Lieberman,[83] and Urick and Searfoss.[84]

The bibliography of Skudrzyk (reference #77) may be consulted for references to the material cited in this section.

1.2.5.2 Internal Waves

Lee[85] has made computations of the effects of internal waves on sound field intensity. In addition, some study has been made of the propagation effects of the magnitude of the sound speed gradients in the thermocline.

1.2.5.3 Bearing Error Studies

A satisfactory theory of scattering should enable one to study the effects of fluctuations in horizontal homogeneity both in bearing error studies, and phase front fluctuations and correlations. In this connection, important work has been done by S. J. Gershman and G. I. Tuzhilkin.[86]

1.2.5.4 Bubbles

The existence and persistence of layers of bubbles arising from the wakes of ships, and wind and wave action have been known to furnish an important propagation effect. During World War II, it was found that bubble screens absorb and scatter non-resonant sound waves according to an $f^{1/2}$ law, a dependence which has been observed for sea surface scattering at low grazing angles. The original treatments by Carstensen and Foldy,[66] and by Meyer and Skudrzyk[87] have been applied by Laird and Kendig,[88] and Fox et al[89] to measurements of absorption and phase velocity. Surface reverberation measurements by Urick[24,25] indicate that a layer of bubbles may explain the observed phenomena of:

a) Reverberation becomes independent of grazing angle at about 10°

b) Pulses start returning to the receiver at a depth of about 40 ft. from the surface.

The subject of characteristics of bubbles in water continues to receive increased attention.

1.2.5.5 Intensity Computations

The use of high-speed, high-storage computing machines has made ray-tracing a practical and useful tool. There are still some questions raised by Pederson,[90] concerning the propriety and necessity of rounding off discontinuities in sound velocity as applied to linear fitting of computed sound speeds versus depth. Intensity computations require the knowledge of all rays which go through two points in addition to divergence computations and diffraction corrections. Pederson has also programmed the mode solution of the wave equation as a boundary value problem.

Chapter II

QUANTITATIVE DESCRIPTION OF THE SOUND FIELD

2.1 Introduction

This chapter presents a quantitative method for the calculation of the average sound field. It is based on a simple representation of the physical environment, tables and equations relating acoustic parameters to the environment, and approximate equations for the sound field.

The number of parameters which must be stated in order to describe the average sound field is rather large; this fact precludes the feasibility of preparing precomputed tables or nomographs of the sound field itself, except possibly in special cases. It is, therefore, necessary to resort to a synthetic method for calculating the sound field. We have chosen to use ray geometry as the main basis for this calculation, modified and supplemented in various special situations.

Section 2.2 contains a brief physical representation of the environment sufficient to permit calculations of the sound field. Equations for the acoustic properties of this representation are presented in Section 2.3 as functions of the conventional oceanographic measures. In Section 2.3, detailed equations are given for the sound field, and in Section 2.4 simple methods are given for estimating the sound field in the special cases of shallow water, and of sound channels.

2.2 Physical Environment of Underwater Sound

2.2.1 General Considerations

The physical laws describing the propagation of sound contain several functions which are properties of the environment on which the sound is impressed, and others characteristic of the generation and measurement of the field. Table I presents these functions in outline form. According to this outline, the sound field, as represented by Item 5, is determined as a function of Items 1 through 4, which in turn are interrelated. The outline states that there are more or less well defined regions of space which potentially influence the sound field. In each region, the complete time dependent equations of state must be known; these state equations are divided into physical, chemical, and biological groups.

It is assumed that the equations of state are available. This is, of course, not literally true. Qualitatively, we know very little about suitable representations of the ocean floor and its underlying materials. Likewise, a similar statement can be made about regions possessing appreciable ice covers. Quantitatively, we know little concerning the dynamic character of sound speed in the hydrosphere. In addition, the number of data points necessary to specify a given situation can be very large, and we are forced, for reasons of economy, to adopt simple, approximate representations.

Despite the foregoing, the results of practical application have shown that even a gross representation of the environment permits a useful and reasonable calculation of the sound field. At this time, it appears that the important basic mechanisms are understood and, with the exception of the ocean bottom, the necessary physical variables have been fairly well determined, and the connections between them have been established.

Given the necessary data corresponding to Item 2 of the outline, the related acoustic properties (Item 3) may be determined. These are functions of both the environment and the generators of the sound field (Item 4.1). The important acoustic parameter is, of course, the frequency. We have admitted a dependence on wave amplitude, partly for generality, but also as a reminder that the usual brand of acoustics is an approximate linearized aspect of the larger subject of hydrodynamics. Having made this observation, any consideration of finite amplitude effects will be ignored in what follows. The acoustic properties; sound speed; absorption; reflection and scattering coefficients are presented below in quantitative form.

Having obtained the acoustic properties, and stated the sonar parameters, the sound field is determined by the equations given in Section 4 of this chapter.

2.2.2 Geophysical Properties of the Environment

2.2.2.1 Physical Properties

These properties are time dependent point functions in each region. The physical properties of velocity, temperature, and pressure need no elaboration. By depth, we mean the function describing the geometrical interface between the regions. As such, it refers to instantaneous wave elevation at a geographical point; to the projection of ice into the water; to the bathymetric variable characterizing the ocean floor; or to substrata in the ocean bottom. Of these, only the wave elevation enters explicitly in this treatment of the subject.

For the calculation of the mean sound field, the statistical properties of the sea surface appear to be sufficient, because of both theoretical and experimental reasons. A descriptive account of the sea surface has been given recently[20] in connection with theories of sound scattering. The single variable of significance is the mean wave height h, which is derivable from the local wind speed for the fully developed sea, or from the wind, fetch, and duration in case of the developing sea. In the latter case, the methods of reference 91, may be employed for determining both wave height and the associated spectral distribution.

The influence of the sea surface upon the average sound field is represented in terms of the wave spectrum $A^2(\omega)$, which gives the distribution of wave amplitude among the spectral components at angular frequency ω. This influence is described in reference 20. That discussion is limited to the fully developed sea, and we so limit the discussion here. However, the same methodology can be applied to the co-cumulative spectra for the non-equilibrium sea.[91]

According to Neumann and Pierson,

$$A^2(\omega) = \frac{c}{\omega^6} \exp\left(\frac{2 g^2}{- \omega^2 s^2}\right) \text{cm}^2 \text{ sec} \qquad (1)$$

where $c = 4.8 \times 10^4$ cm^2 sec^{-5}, g is the acceleration of gravity (980 cm/sec^2), s is the wind speed in cm/sec and ω the angular frequency in rad/sec. In addition, the relation

$$h^2 = 2.42 \times 10^{-6} s^5 \text{ cm}^2 \qquad (2)$$

is employed for the mean square wave height,

$$H = 1.77 h \qquad (3)$$

for the average trough to crest wave height.

The sea is presumed to be isotropic, or rather, the average scattering over all azimuths is considered in developing the sound field.

2.2.2.2 Chemical Properties

For most practical purposes, the "salinity" of sea water can be employed as the single variable controlling the sound field. No chemical properties of the atmosphere or ocean bottom are known to be significant.

The salinity, along with temperature and pressure, determines both the acoustic phase velocity and absorption coefficient, although the various salts contributing to total salinity do not contribute equally in their affect upon these acoustic properties. The individual contributions are not known, and as a practical matter, the total salinity is employed in determining the acoustic properties. This is reasonably accurate, since the ratios of the various salt contents are remarkably independent of total salinity, except in very brackish water.

The presence of air or vapor, either dissolved or suspended as bubbles, is known to have an influence upon sound propagation. Measurements of attenuation in water, "quenched water" and other related effects were reported during World War II. More recently, reflections from apparent bubble layers beneath the sea surface have been reported by several observers. The subject has not received enough attention, however, to warrant including methods either or predicting effects upon the sound field, or for predicting the presence of scattering or absorbing regions of this type in the water. A similar statement can be made concerning biological effects. In considering other sonar properties, such as reverberation, the existence of biological scatterers cannot, of course, be ignored.

2. 3 Acoustic Properties

2. 3. 1 Speed of Sound in Sea Water

2. 3. 1. 1 Introduction

In recent years, with the development of an operational speed of sound meter, a re-examination has been made of the formulas developed over the years by less direct methods.

R. J. Urick[92] was the first to use an interferometer to make sound speed measurements in the ocean during the World War II. The basic instrument which has come to be accepted for this purpose is a 5 mc pulse-type apparatus. It measures the time for a pulse to traverse a fixed path between transmitting and receiving crystals. This instrument, called the "sing-around" velocimeter, was designed at the National Bureau of Standards by Greenspan and Tschiegg.[3]

W. Wilson[4] of NOL has used this technique to make the most extensive laboratory measurements of the speed of sound in sea water as a function of temperature, pressure and salinity. Del Grosso[93] of NRL made the first set of laboratory measurements at atmospheric pressure uncovering an error of -3 meters/sec in the semi-theoretical tables of Kuwahara, and also in the tables of Matthews. Del Grosso used an interferometer technique, however.

K. V. Mackenzie[94,7] of NEL and E. E. Hays[95] of WHOI have made measurements with the NBS-type velocimeter at sea as a function of depth and have verified Wilson's laboratory measurements and resultant tables with respect to depth.

These measurements point to a new pressure effect term in isothermal water of about 0.0165 m/sec/meter near the surface instead of 0.0181. Figures supplied by Mackenzie show the sound speed corrections as a function of depth, between the results of Wilson and Kuwahara. They show that these are not linear and reach a minimum at about 4000 meters in depth.

Table II, taken from Wilson, summarizes the results at atmospheric pressure, for various workers in the field.

Table II

COMPARISON OF THE SOUND FIELDS OBTAINED
BY DIFFERENT AUTHORS

$T = 30°C$
$P = 1.033\ kg/cm^2$

Author and Reference	Date of Determination	Speed of Sound (meters/sec) Distilled Water	Sea Water S = 35 $^0/_{00}$
Matthews[97]	1939	1504.4	1543.1
Kuwahara[98]	1939	------	1543.2
Del Grosso[93]	1952	1509.6	1546.2
Greenspan[3]	1958	1509.44	------
Wilson[4]	1959	1509.66	1546.16

Also in the present section, we reproduce Wilson's equation of sound speed in sea water. We prefer this equation to the equation subsequently published by Wilson[96], since it represents the situation in actual sea water better. Wilson's first equation, valid for salinity in the range between 33 and 37 $^0/_{00}$, was deduced from measurements made on actual sea water samples. The latter equation considered measurements made by diluting the samples with distilled water. The first equation has a standard error of about 0.22 m/sec and the second a value of 0.30 m/sec.

Equation 1, $$C = 1449.22 + C_T + C_P + C_S + C_{STP} \qquad (4)$$

where:

$$C_T = 4.6233T - 5.4585 \times 10^{-2} T^2 + 2.822 \times 10^{-4} T^3 - 5.07 \times 10^{-7} T^4,$$

$$C_P = 0.60518 \times 10^{-1} P + 1.0279 \times 10^{-5} P^2 + 3.451 \times 10^{-9} P^3 - 3.503 \times 10^{-12} P^4$$

$$C_S = 1.391 (S-35) - 7.8 \times 10^{-2} (S-35)^2, \text{ and}$$

$$\begin{aligned} C_{STP} = {} & (S-35)(-1.197 \times 10^{-2} T + 2.61 \times 10^{-4} P - 1.96 \\ & \times 10^{-7} P^2 - 2.09 \times 10^{-6} PT) + P(-2.796 \times 10^{-4} T \\ & + 1.3302 \times 10^{-5} T^2 - 6.644 \times 10^{-8} T^3) + P(-2.391 \\ & \times 10^{-7} T + 9.286 \times 10^{-10} T^2) - 1.745 \times 10^{-10} P^3 T. \end{aligned}$$

In Equation (4), P is expressed in kg/cm^2, T is in °C, S is in parts per thousand, and C is in m/sec. The coefficients have been rounded to give the nearest 0.001 m/sec in each term. The rounding of coefficients will not affect the computation of sound speed by more than 0.005 m/sec, if the computation is made within the range of variables considered here. To convert meters/sec to kiloyards/sec, multiply by 1.0936×10^3; to convert meters/sec to feet/sec multiply by 3.2808.

2.3.1.2 Review of Sound Speed Computations from Specific Volume Data

The differences between the measured sound speeds and the predicted sound speeds at atmospheric pressure were discussed by Del Grosso[93] and Beyer[15]. To account for the differences between the computed and measured sound speeds at other pressures, a review was made of the method used for computing sound speeds from specific volume data. Kuwahara and Matthews computed sound speeds in sea water from Newton's formula.

$$C^2 = \frac{\gamma}{\rho\beta} \qquad (5)$$

where γ is the ratio of specific heats, ρ is the density, and β is the isothermal compressibility. The density was computed from the formula

$$\rho = \frac{\rho_0}{(1 - \mu P)} \qquad (6)$$

where μ is the mean compressibility as defined in Equation (8) below. The true compressibility, β, was found from the mean compressibility by the relation,

$$\beta = \frac{1}{V}\frac{dv}{dP} = \frac{\mu + P(d\mu/d\rho)}{1 - P\mu} \qquad (7)$$

It may be seen from this equation that the mean compressibility is defined by

$$\mu = -\frac{1}{V_0}\frac{(v - v_0)}{(P - P_0)} \qquad (8)$$

where P = 0. Substituting Eqs. (6) and (7) into Eq. (5), we obtain

$$C^2 = \frac{\nu}{\rho_0\,\mu}\frac{(1 - P\mu)^2}{P(d\mu/dP)} \qquad (9)$$

The object in writing the equation for sound speed in terms of μ instead of β was to allow the use of an empirical equation obtained by V. Ekman. In Ekman's equation, μ is written as a function of temperature, pressure, and salinity. This empirical equation was based principally on Amagat's specific volume tables for distilled water and a few representative measurements of the specific volume of sea water made by Ekman. The pressure dependence of μ in Ekman's equation relies upon Amagat's pressure measurements. Therefore, the effect of pressure on the speed of sound in water should have the same characteristics regardless of whether Ekman's or Amagat's data is used. In Equation (9), μ can be obtained either from Equation (8) and Amagat's data, or from Ekman's empirical equation. If Equation (8) and Amagat's data are used, the second derivative of sound velocity in distilled water with respect to pressure is positive. If Ekman's equation for μ is used, the second derivative is negative. It is apparent, therefore, that the negative curvature of sound speed as a function of pressure has been introduced by the inclusion of Ekman's work. Since Ekman's equation is estimated to be accurate to about three parts per thousand, sound speeds computed from this equation cannot be more accurate than 4.5 m/sec. In actuality, the differences observed between the measured and the computed sound speeds are less than this amount. Comparison of the effect of pressure on the speed of sound is not possible in other work because of the large pressure increments taken and the small amount of data obtained in the pressure range considered here.

2.3.1.3 Note on Operational Experience With Sound Velocimeter

The "sing-around" velocimeter is essentially an ultrasonic delay line which synchronizes a relaxation oscillator. The repetition frequency is controlled by the transit time in the water for a pulse traveling between two transducers. The transit time in turn depends on the speed of sound. High stability and sensitivity are feasible in the system, permitting absolute measurement of speed to be made with an uncertainty of less than 1 ft/sec. Readings taken in the field are reproducible to within about 0.5 ft/sec., taking into account battery aging and temperature effects on the circuit elements. Lowering of the meter at the rate of a fathom per second is feasible but tends to eliminate detail in the velocity structure. Calibration tests after a three month cruise have shown velocity measurements made with the instrument to be still correct within 1 part in 10,000. The only maintenance required was flushing of transducers with fresh water and an occasional battery change.

Comparisons of velocimeter measurements with Wilson's tables have been reported by Mackenzie[94] and Hays[95]. These indicate comparatively large deviations near the surface of the ocean, and offer additional confirmation to earlier evidence[99] which has attributed velocity anomalies to the presence of entrapped air bubbles. If this is the case, it is still not clear whether the observed effect would be dispersive, and possibly not applicable to most sonar frequencies. Very strong evidence of dispersion has been reported by Fox et al[89] in water containing bubble concentrations of 2×10^{-4} parts by volume. The effect has also been studied by Laird and Kendig[88], on the basis of calculations of phase velocity made from measured absorption.

2.3.2 Sound Absorption in Sea Water

The recommended expression[9] for absorption, α, in sea water as a function of temperature, pressure and salinity is:

$$\alpha = \left(\frac{SA f_T f^2}{f_T{}^2 + f^2} + \frac{B f^2}{f_T} \right) (1 - 6.54 \times 10^{-4} P) \text{ nepers/meter} \qquad (10)$$

where:

S is the salinity in parts per thousand

f_T is the temperature dependent relaxation frequency in kc at atmospheric pressure = $21.9 \times 10^{\left[6 - 1520/(T + 273)\right]}$

f is the acoustic frequency in kc

A is a constant (2.34×10^{-6}) for the ionic relaxation process in sea water

B is nearly constant (3.38×10^{-6}) for the pure water viscosity mechanism.

P is pressure in Kg/cm^2 or atmospheres

T is the temperature in °C

To convert α to db/kyd, multiply by the factor 7.943×10^3.

2.3.2.1 Discussion

Apparently there is now comparatively little disagreement with the temperature dependence of f_T. The approximate form which it follows, and which is plotted in Figure 1, is $f_T = Ce^{-D/T}$, although the more accurate dependence is $f_T = ETe^{-D/T}$. $1/f_T$ represents the temperature dependence of the pure water viscosity term. However, it must be emphasized that $1/2\pi f_T$ is not the time constant associated with the viscous process. Figure 1 compares measured values for the temperature dependence of the pure water relaxation with $1/f_T$. It may

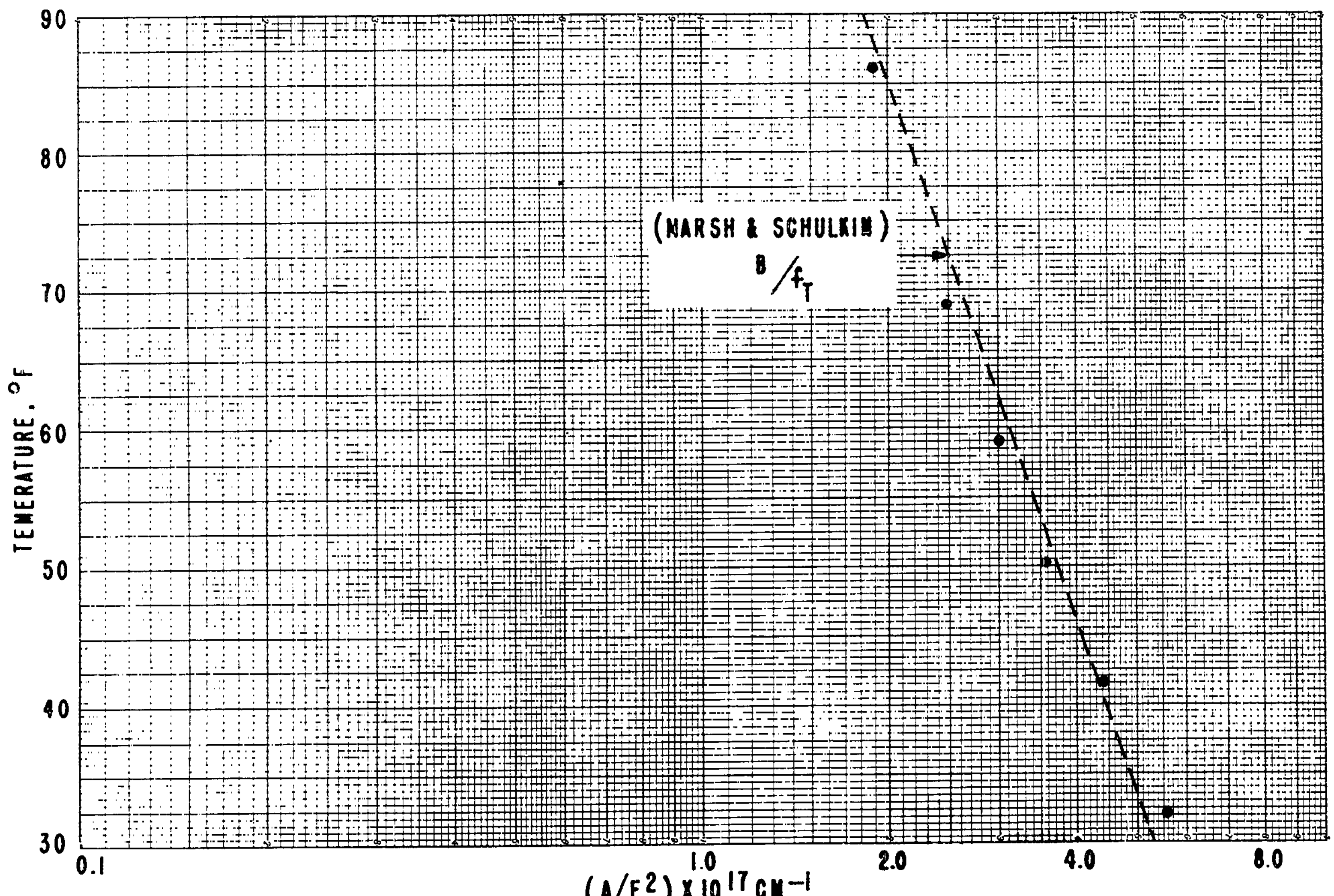

FIGURE I. TEMPERATURE DEPENDENCE OF RELAXATION ABSORPTION IN SEA WATER, PLOTTED DATA TAKE FROM PINKERTON be seen that, except for temperatures above 75° F, the error in this term is less than 10%, the term itself usually being considerably less than the first terms for frequencies normally encountered in underwater applications.

Murphy and Garrison[14] have made measurements over transmission paths in Puget Sound at frequencies of 60 kc, 142 kc, 272 kc and 467 kc. They conclude: "Although the frequency dependence of the absorption coefficient given by these measurements follow the normal relaxation law, the magnitudes are lower than those given by Del Grosso by 4 to 9 db/kyd".

2.3.2.2 Concentration Dependence

For $MgSO_4$ solutions with concentrations n in range 0.05 to 0.10 mole/liter, Kurtze and Tamm[12] found α/n to be substantially constant. Their data, taken from Del Grosso[11] is plotted in Figure 2. They also noted that sea water of salinity 35 ‰ was equivalent to $MgSO_4$ solutions with n = 0.014. Leonard and Wilson[100-102] investigated the concentration dependence for values of n between 0.003 and 0.020. Their data shows α/n to be substantially constant also, although it is about 35% lower.

Previously, Beyer[103] indicated that the excess absorption in $MgSO_4$ solutions varied as the square root of the concentration at lower frequencies and had a "more complicated dependence" at higher frequencies. A theory by Leontovich[104] calls for the absorption to be proportional to the square root of the concentration at low frequencies and proportional to the square of the concentration at high frequencies.

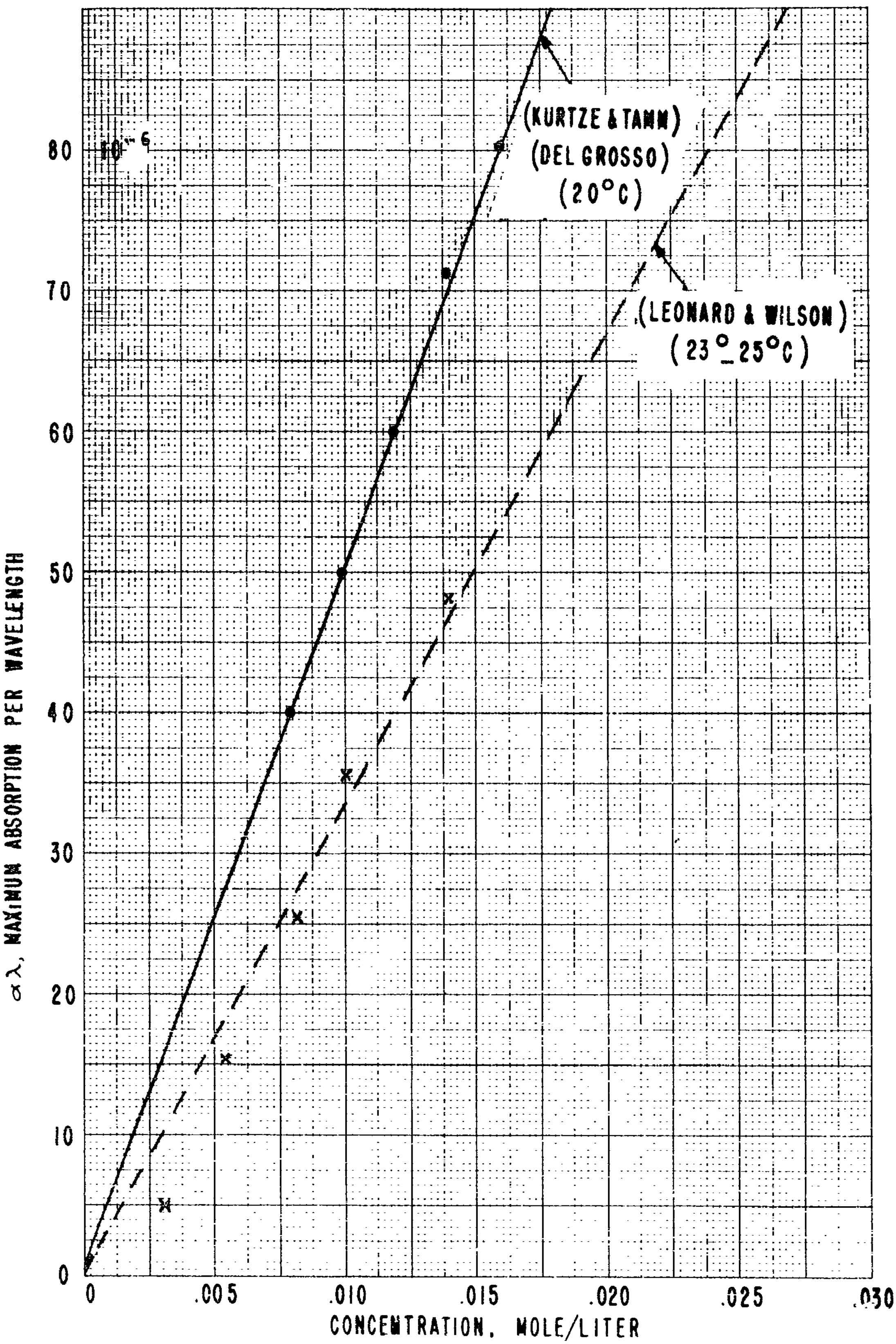

FIGURE 2. DEPENDENCE OF ABSORPTION ON CONCENTRATION OF $MgSO_4$ SOLUTIONS

2.3.2.3 Discrepancy in A

Since the linear dependence of absorption in concentration n is in agreement for these low concentrations it seems that the major problem can be traced to a disagreement in the determination of A.

Thus, it appears that Leonard and Wilson's laboratory value for A is lower than the laboratory value obtained by Kurtze and Tamm for $MgSO_4$ (see Figure 2). Likewise the field values for A found by Murphy and Garrison[14] are definitely lower than those obtained by Marsh and Schulkin[7] at lower frequencies in about the same ratio. It is felt that the low saline waters of Puget Sound which Murphy and Garrison used could have a lower percentage of $MgSO_4$ than sea water.

2.3.2.4 Absorption Dependence on Pressure

Curiously enough the dependence of absorption on pressure <u>in sea water</u> has not been studied. However, studies have been made of the dependence of absorption in pure water and also, associated and non associated liquids[105-107]. Measurements <u>on water</u> have been made up to <u>pressures</u> of 2000 kg/cm^2. Table III, reproduced from reference 108, shows that the absorption loss decreases as the pressure is raised. One would expect similar behavior in sea water.

Table III

DEPENDENCE OF ABSORPTION ON PRESSURE FOR WATER, 30°C

p (atm)	0	500	1000	1500	2000
$10^{17}\ \alpha\ f^2$	18.5	15.4	12.7	11.1	9.9

2.3.2.5 Dispersion

The relaxational absorption in sea water gives rise to a change of sound speed with frequency or dispersion. This effect is treated in great detail by Hertzfeld and Litovitz[108]. Del Grosso[11] has computed that the maximum dispersion in a 0.014 molar solution of $MgSO_4$ (equivalent to sea water) is about 2-4 cm/sec. He used an expression of Kneser

$$\frac{C - C_0}{C} = \frac{(\alpha\lambda)_{max}}{\pi} \qquad (11)$$

Such an extremely small value of dispersion would hardly cause trouble, even for systems with long range communication applications. However, it should be pointed out that the dispersion problem is much more severe than indicated above. For any attenuation curve that is a function of frequency, a dispersion curve for any four terminal network may be deduced. Since the attenuation curve will include scattering losses at the surface, losses at the bottom and into the bottom, scattering losses in the volume of the ocean leakage losses from sound channels, as well as absorption, the problem can be quite complex and important. Theoretically there is a relation between the temperature, pressure and salinity dependence of sound speed and absorption in sea water. If we know these relationships for one with great precision, then the effect of these variables on the other should be determined in principle.

2.3.3 Acoustic Properties of the Ocean Floor

At the time of this writing, major attention is being given to this subject, and it is not practicable to give a useful account of advances in the general state of knowledge. It is, therefore, recommended that the curves reproduced in Figure 3 be employed until such a time as they can be replaced by data giving dependence of the properties upon environment and possibly other factors.

However, T. Bell[109] has pointed out that the curves do not take explicit account of multipath effects in bottom reflection, and that differences of as much as six db can exist depending upon the number of ray paths involved in a particular situation.

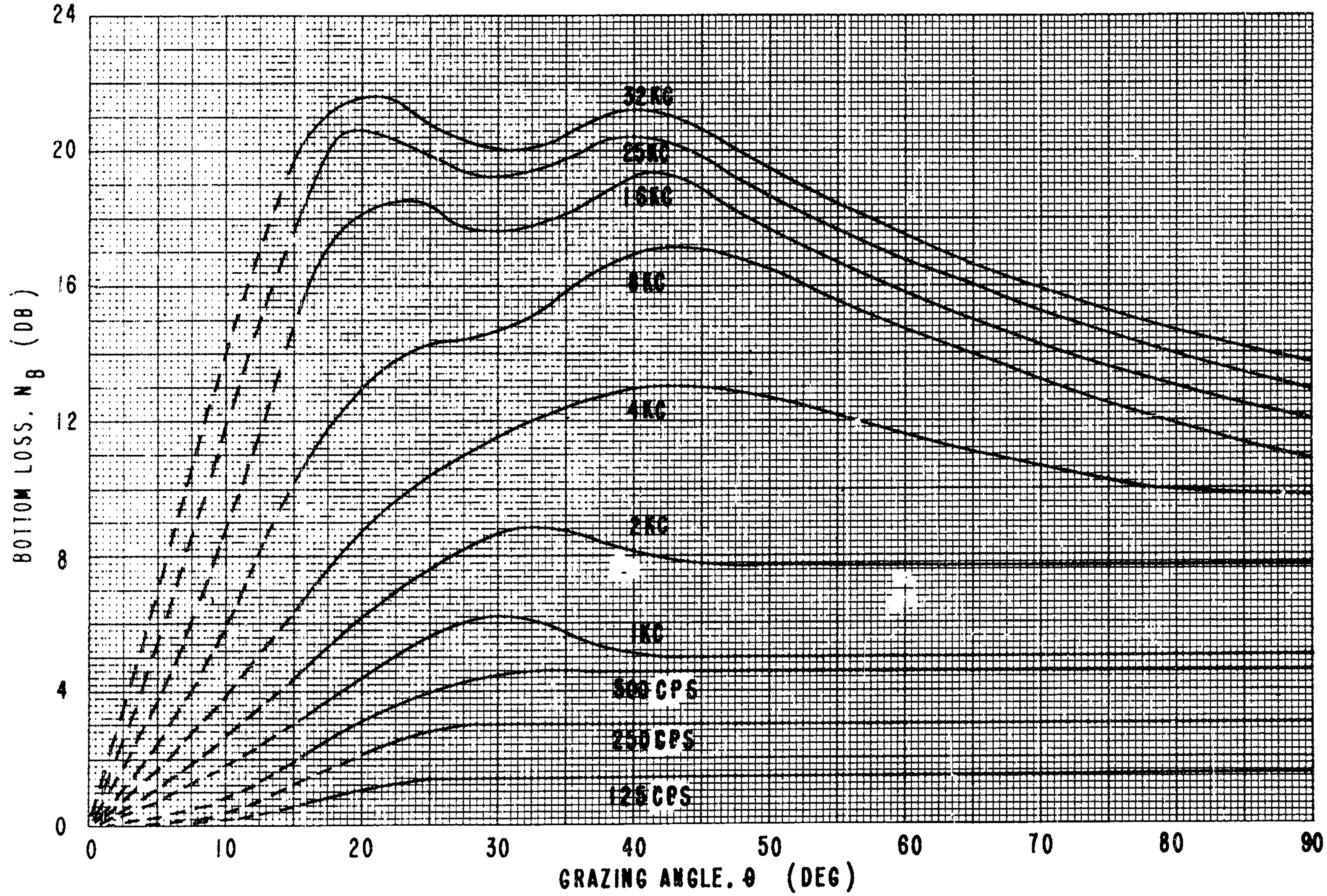

FIGURE 3 BOTTOM VS. GRAZING ANGLE

2.4 Calculation of the Sound Field

2.4.1 General Equations and Parameters

In this section, the equations connecting the sound field with the other variables in the model will be listed and developed where necessary.

The excess pressure P is the variable to be used in describing the sound field, and the fundamental equation governing P is the wave equation

$$\nabla^2 P = \frac{\partial^2 P}{\partial t^2 c^2} \tag{12}$$

In Equation (12), ∇^2 is the Laplacian operator and c the speed of sound. For a unique determination of P, boundary conditions and initial conditions must be stated. The boundary conditions require that P and the particle velocity $\bar{\nu}$ be everywhere continuous. The initial conditions prescribe the distribution of some linear combination of P and $\bar{\nu}$ over a closed surface (or set of closed surfaces) representing the source of the sound field.

In a fluid, the P and $\bar{\nu}$ are connected by the relation including the density ρ:

$$\nabla P + \frac{\rho \, \partial \bar{\nu}}{\partial t} = 0 \tag{13}$$

Because of this, the pressure and velocity generally cannot be stated independently for initial conditions.

In the following developments, the source will be taken as an isotropic, point source, with a simple harmonic angular frequency of ω.

In this case, Equation (12) becomes

$$\nabla^2 p + k^2 p = 0 \tag{14}$$

where $k = \frac{\omega}{c}$ and $p = Pe^{i\omega t}$. Near the source, if R is the distance to the source,

$$P = \frac{e^{i(kR - \omega t)}}{R} \tag{15}$$

For other initial conditions, results may be obtained by Fourier synthesis, (i. e., complex superposition).

The solution for the sound field may be obtained with the aid of ray geometry. Corresponding to Equation (12), there is a set of surfaces determined by the equation

$$\left|\nabla\theta\right|^2 = n^2 \tag{16}$$

where $n = \frac{c_0}{c}$, and is the index of refraction relative to the reference speed c_0.

The surfaces, where θ is constant, constitute a set of wave fronts. Orthogonal to these wave fronts are skew curves, which are the rays of the sound field. If s is the arc length measured along a ray, the equation of the ray is

$$\frac{d}{ds}\left(\frac{n\,dx}{ds}\right) = \frac{\partial n}{\partial x} \tag{17}$$

and similarly for y and z.

From the manifold of rays, those rays connecting specific points may be selected by integrating Equation (17) and applying boundary conditions. The selected set of rays provide the geometry for determining the sound field at the points of interest. The equations for this determination are presented in the sequel. There appears to be no point in developing the subject in complete spatial generality, because of the lack of detailed knowledge of the instantaneous oceanographic variables, and because of the enormous complexity of any calculations which might be attempted. Thus, while special situations such as that discussed by Lee[85] will continue to be of interest, the detailed description of the sound field presented below will be confined to the case of depth dependent sound speed.

Equation (12) is derivable from the hydrodynamical equations together with the equations of state of the materials in the environment, under certain simplifying assumptions[110]. It is basically valid for static conditions, in which case the parameters of the field are constant in time. This is more particularly true of Equation (14). However, if the parameters vary slowly, compared to the harmonic variation of the source, then time varying fields can be constructed from a sequence of static fields. This point will be developed subsequently. Likewise, spatial variations in the parameters, other than in depth, can be treated by connecting local, one-dimensional environments, after the manner employed by Gardner.[111]

The wave number k is complex valued and is generally not a linear function of the frequency as might be implied by Equation (14). The situation in sea water is presented in Section 2. 3. 1.

The boundaries are represented by the Rayleigh reflection coefficients, and in the case of the ice free sea surface, by a scattering coefficient. Sufficient information is not available at this time to permit the use of more than the reflection coefficient. The bottom is to be treated as though it were a fluid, and if velocity and attenuation data are available for a particular situation, effects of propagation into the bottom may be included by treating it as an extension of the hydrosphere.

2.4.2 Ray Geometry

There are two methods of displaying the ray geometry pictorially. The first is called the ray diagram and is illustrated in Figure 4. The ray diagram shows the paths taken by selected rays through the field. Many computer methods and programs have been developed for dealing numerically with ray diagrams.[112-115] A convenient program at USNUSL, based on the V_x method of Cole, is discussed in reference 117.

For the purpose of computing the sound field, a second display, which we called the field diagram, is illustrated in Figure 5. Although this class of diagram is mentioned by Horton[16] and is widely used in radio propagation, its application to underwater sound has been limited. The field diagram is employed to identify the rays passing from the source through a fixed point in the field. When the rays have been identified, the total field may be determined by combining the contributions of each ray. Figure 6 shows, for the parameters indicated below, the number of rays arriving at the receiver as a function of range. It is clear that the job of calculating the sound field by this method is not trivial. It should be equally clear, however, that all the rays contributing to a point should be considered, since the relative contributions of different classes will vary from point to point. Although many rays may

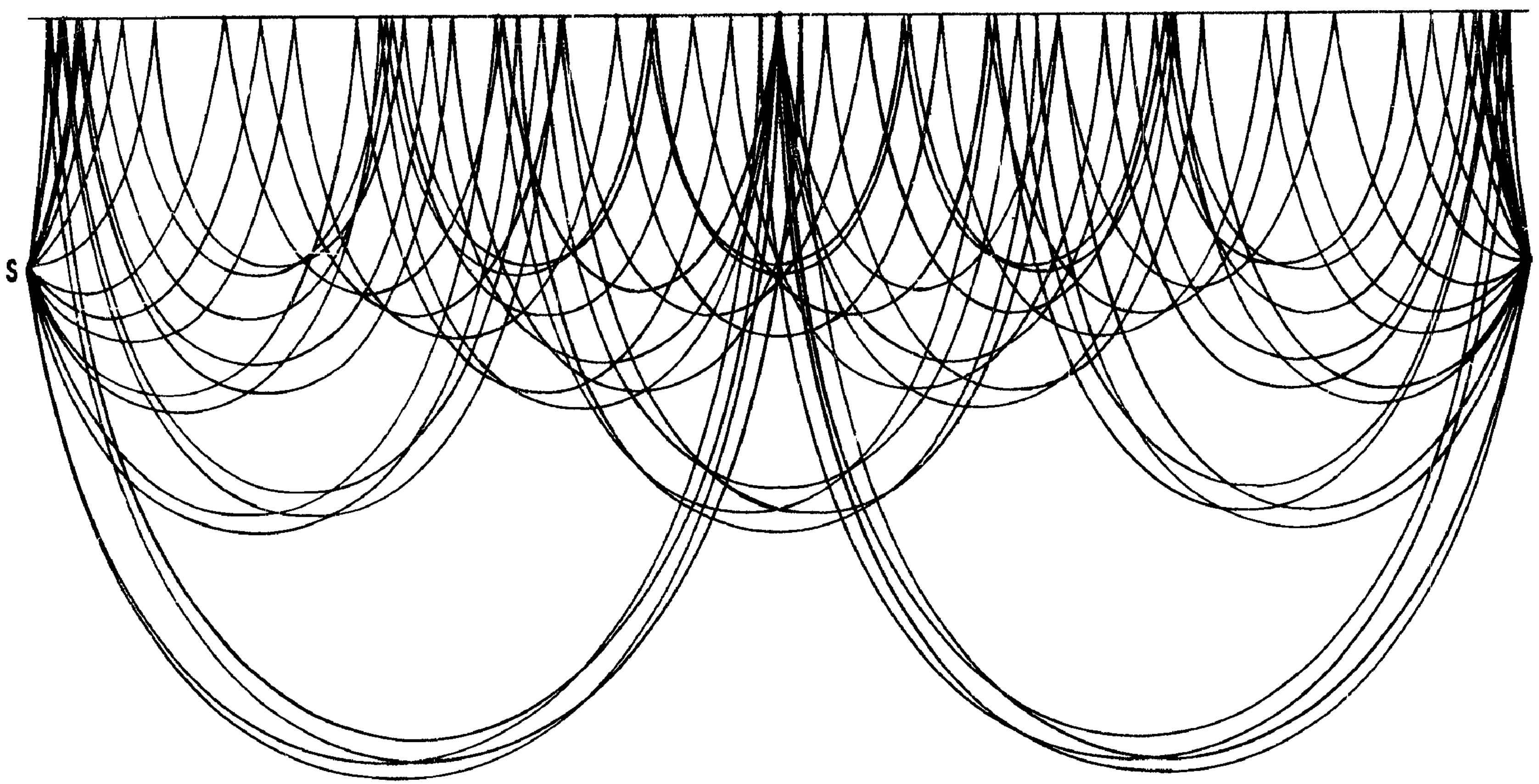

FIGURE 4 NON DIMENSIONAL RAY DIAGRAM

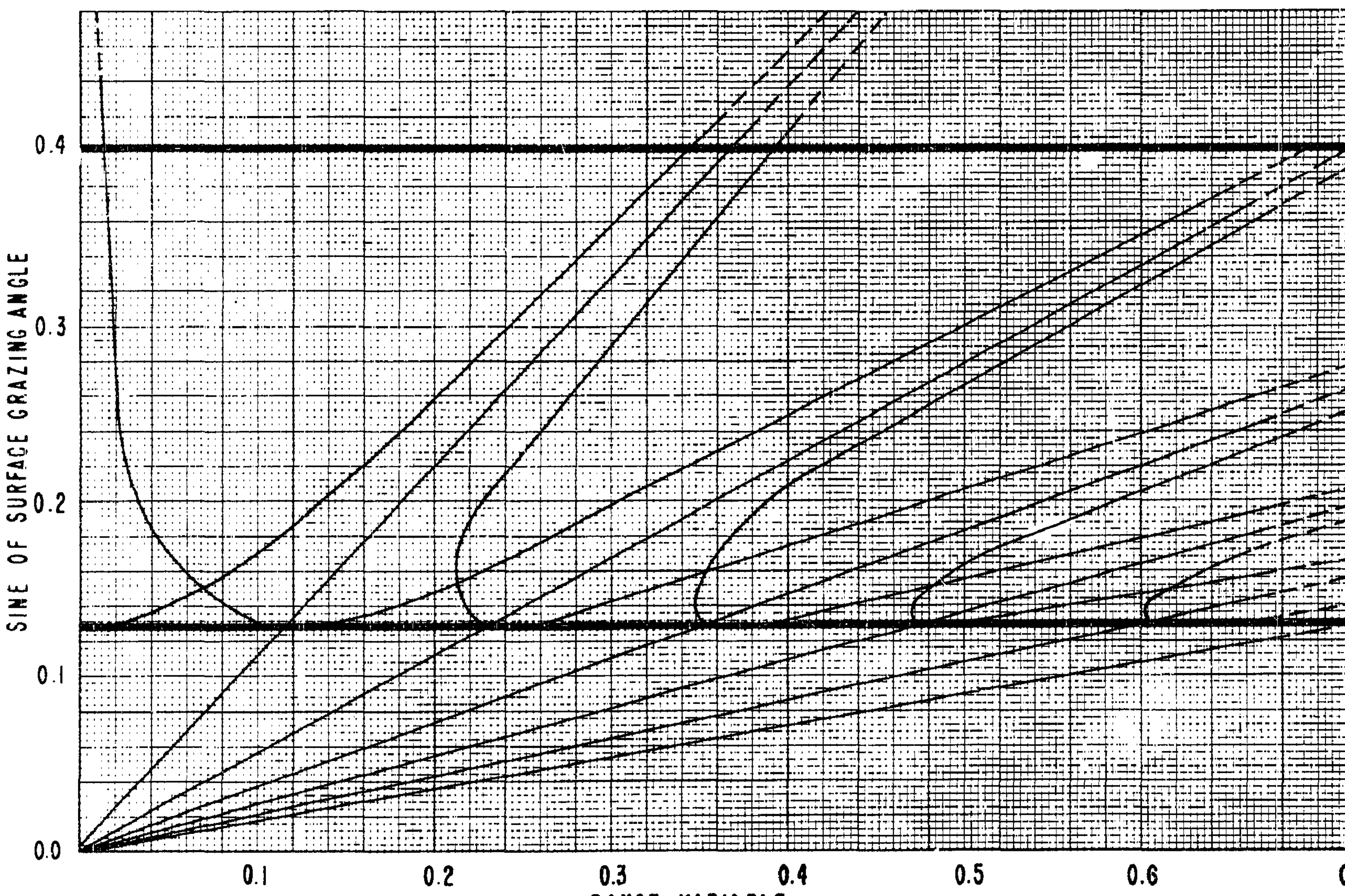

FIGURE 5. FIELD DIAGRAM

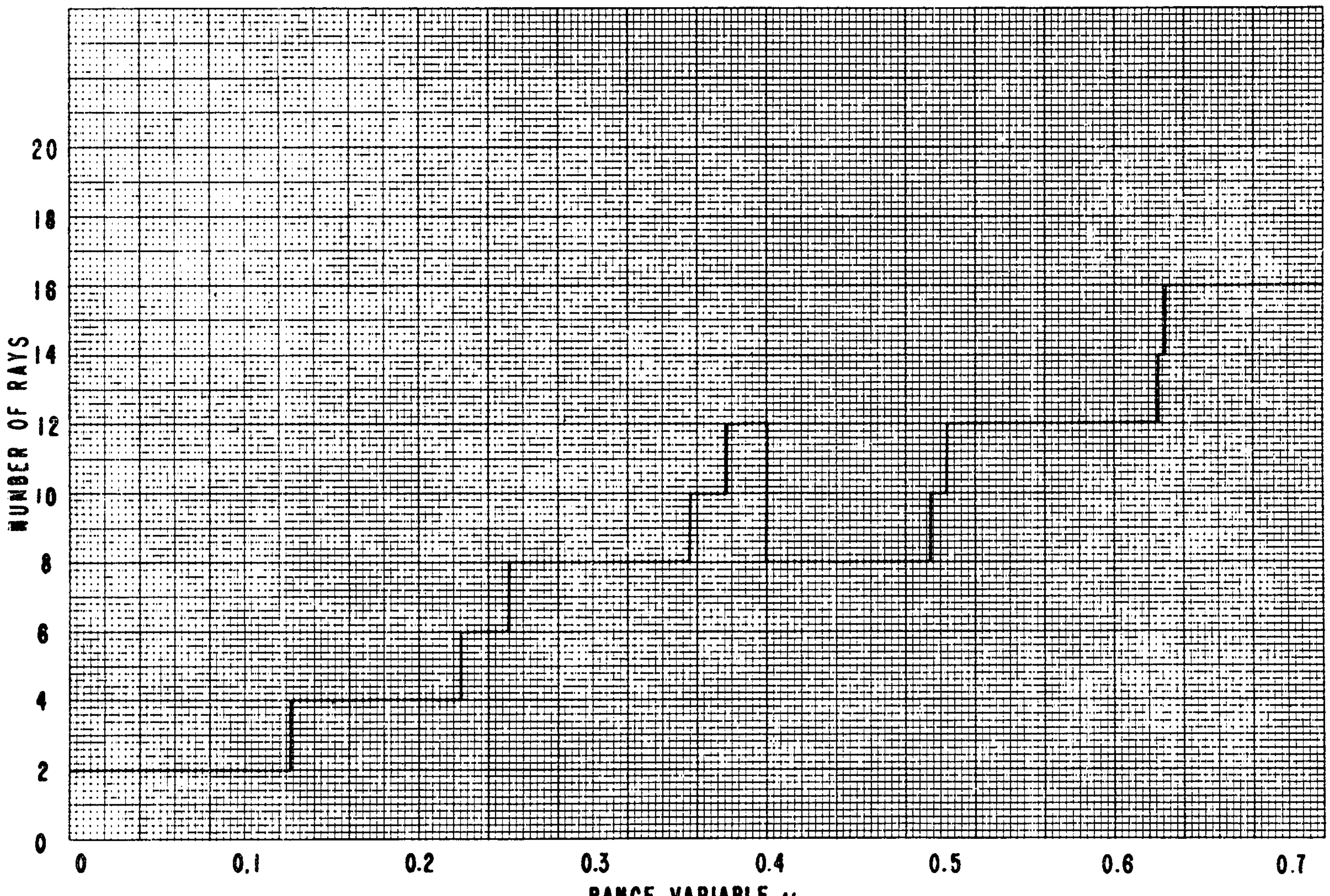

FIGURE 6. NUMBER OF RAYS CONNECTING SOURCE AND RECEIVER

be required per point, the method is completely tractable in comparison to other formulations.

In Figure 5, the range between the source, S, and the receiver, R, is plotted against the ray inclination at the surface, for all rays with the exception of rays reflected from the bottom at short ranges. A vertical line drawn at any range of interest will intersect the given curves at several points. Each point of intersection identifies a particular ray, providing the ray inclination exceeds the minimum value corresponding to the horizontal tangency at the source.

In order to clarify Figures 4 through 6, the following explanation of the parameters used is given.

In Figure 4, the non-dimensional ray diagram, the horizontal scale is the range variable, u, which is equal to gr/2. The vertical scale is the depth variable, v, equal to $\sqrt{2gz}$. In both cases, g is the fractional increase in sound velocity, per unit depth. The source is located at $v = 4/10^{3/2}$. Six cycles of the ray tangent to the source are shown between the source and the symmetrical receiver point.

In Figure 5, the field diagram, the scales are equal to those in Figure 4. The vertical scale also indicates the sine of the grazing angle at the surface, $(v = o)$. The heavy lines represent the source depth $(v = 4/10^{3/2})$ and an assumed bottom $(v = 0.4)$. Equal source and receiver depths are assumed.

Figure 6, number of rays connecting source and receiver, is plotted with the same parameters as Figures 4 and 5. Bottom reflected rays are not considered here.

Temporarily, take the origin of cylindrical coordinates (r, z) to be at the source, and the source inclination to be θ_0. Then the equation of a point on the ray may be written

$$r = \int_0^z \cot \theta \, d\zeta \tag{18}$$

θ is the inclination of the ray at the point ζ.

In Equation (18), $\cot \theta$ must be finite and continuous throughout the interval of integration. In general, there will be points at which one of these conditions is not satisfied. Furthermore, the general ray is periodic in nature, and there will be a set of such points in each period, or cycle of the ray.

Cot θ will be infinite, and the tangent to the ray will be parallel to the r axis at points where the sound speed is a relative maximum. These points are called turning points. The relative maxima can occur at fixed depths, or at depths determined by the parameters of the ray in question.

Cot θ will be discontinuous where a ray is reflected at a boundary. Here, the depths of the points of discontinuity will be fixed. If the boundaries are not perpendicular to the depth axis, special consideration must be given to the relationship between the angles of incidence, reflection and transmission. Several computer programs have been developed to account for rays reflected from sloping bottoms (see reference 111, for example). Williams[116] gives an approximate method of treating sloping bottoms in shallow water. The field diagram is particularly useful in the ray solution, since one need only draw in the bottom profile and make the proper adjustments to the rays affected.

A point of discontinuity or infinity will be called a fiducial point. Regardless of the ray parameters, there will be exactly two depths where fiducial points occur for a given ray, and the ray will cycle between these depths. Since the origin of coordinates is taken at the source of rays, the origin will lie between these two depths, including the special case where the origin is at one of the depths. These depths will be denoted Z_+ and Z_-. It is obvious, then, that $Z_+ \leq Z \leq Z_-$.

Let

$$F_\epsilon(z) = \int_z^{z_\epsilon} \cot\theta \, d\zeta \tag{19}$$

Here ϵ may be + or -.

$$r = 2F_+(0) + 2F_-(0) \tag{20}$$

has been termed the "cycle range" by Cole,[117] and is twice the skip-distance defined by Marsh and Schulkin[21].

Equation (18) can now be written

$$r = \eta_+ \left[F_+(0) - F_+(z)\right] + \eta_- \left[F_-(0) - F_-(z)\right] + 2mF_+(0) + 2nF_-(0) \tag{21}$$

The integers m and n are the numbers of contacts with the fiducial points Z_+ and Z_- respectively, and the integers η_ϵ take values 1, 0, -1 according to the following table.

		$\theta \geq 0$	$\theta \leq 0$
$Z \geq 0$	η_+	1	-1
	η_-	0	0
$Z \leq 0$	η_+	0	0
	η_-	-1	1

The smallest of the integers m, n is the number of complete cycles in the ray. If $\theta_0 > 0$, $m \geq n$ and vice versa. In any case, $|m - n| \leq 1$.

In addition to the purely geometrical properties of the rays the eikonal K is important. We have

$$\frac{K}{\omega} = \int_0^z \frac{ds}{c(1 - i\alpha)} \sim \int_0^z \frac{ds\,(1 + i\alpha)}{c} = T + iA \qquad (22)$$

T is the "Travel time," and A represents absorption. Corresponding to the quantity F_ϵ, define

$$G = \int_z^{z_\epsilon} \frac{d\zeta}{\sin\theta\cos\theta}, \quad H = \int_z^{z_\epsilon} \frac{\alpha\, d\zeta}{\sin\theta\cos\theta} \qquad (23)$$

Then,

$$\frac{c_0 T}{\cos\theta_0} = \eta_+ \Big[G_+(0) - G_+(z) \Big] + \eta_- \Big[G_-(0) - G_-(z) \Big]$$
$$+ 2mG_+(0) + 2\eta\, G_-(0)$$

$$\frac{c_0 A}{\cos\theta_0} = \eta_+ \Big[H_+(0) - H_+(z) \Big] + \eta_- \Big[H_-(0) - H_-(z) \Big]$$
$$+ 2m\, H_+(0) + 2\eta\, H_-(0) \qquad (24)$$

It is of interest to note that, by virtue of the mean value theorem of the calculus,

$$T = \frac{r}{c \cdot \cos\theta} \tag{25}$$

where c and cos θ correspond to some point along the ray. This equation shows the extreme possible departure of the travel time from that which would be associated with the corresponding refraction free situation.

2.4.3 Primary Field

Any point in the sound field reached by one or more rays will be called a primary point, and the field produced at these points by the rays will be called the primary field. The total field is the sum of the primary field and the field due to scattering and diffraction. The primary field will almost always be dominant at primary points, as discussed in Section 2.4.6. At other points, the secondary effects must be considered, and are treated below in Sections 2.4.4 and 2.4.5.

Take the origin of coordinates at the mean level of the sea surface, with the source on the polar axis, and let the cylindrical coordinates of the source be $(0, 0, Z_0)$; that of a field point be (r, ϕ, Z) as in Figure 7.

Let Γ_i be any ray connecting the two points and θ_i the inclination of the ray at the source. The pressure associated with Γ_i will be designated p_i.

The total primary field is the complex sum

$$p = \Sigma_i \; p_i$$

over all rays reaching the field point. The equation for p_i will be stated,

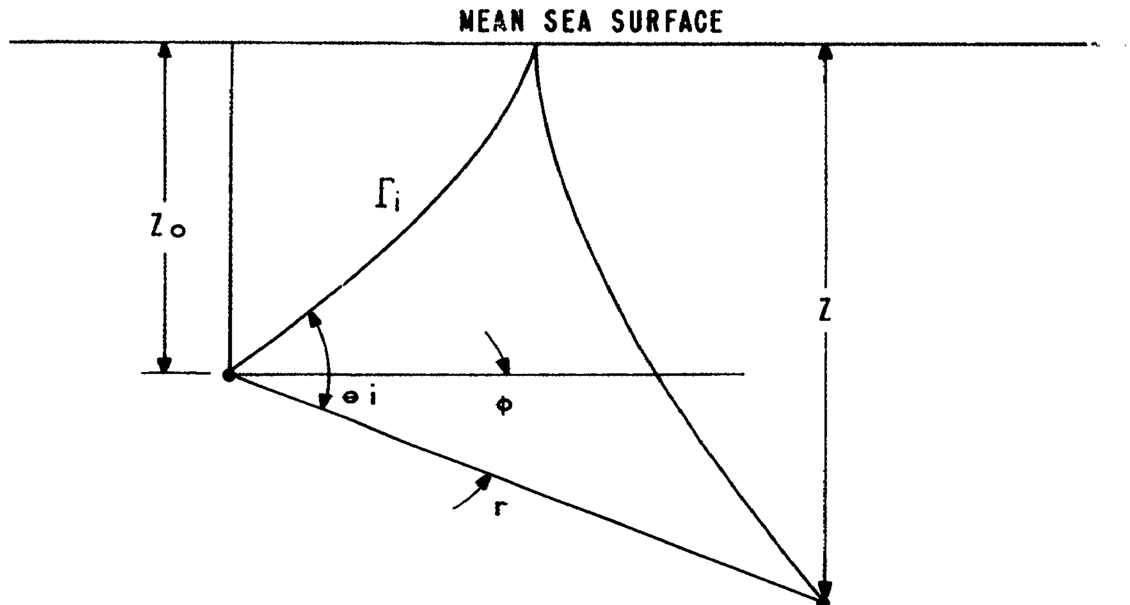

FIGURE 7. GEOMETRY OF THE SOUND FIELD

and then the individual terms discussed.

$$p_i = qJ(\Re)\, S^m\, B^n\, \Omega^m\, e^{-A} \tag{26}$$

Each term in this equation is a function of the ray parameters and so a subscript (i) is implicit.

The quantity q represents the optical approximation to the pressure, and is equivalent to the pressure reduction due to geometrical divergence of the ray bundle as discussed by Horton[16], among many others.

Explicitly,

$$q = e^{-i\omega(t - T)}(-rn \tan\theta \; dr/d\theta_0)^{-\frac{1}{2}} \tag{27}$$

Again, T is the "Travel time".

$$T = \int \frac{ds}{c} \tag{28}$$

The quantity $qJ(\Re)$ has been discussed by Marsh[118], who developed a correction to the optical limit, taking into account the finite wave length.

For $J(\Re)$, we have

$$J(\Re) = e^{i(2\Re/3 - 5\pi/12)}\, \pi^{1/2}\, \Re^{1/6}\, 2^{1/3}\, 3^{1/6}\, h_2\left(\Re^{2/3}\right) \tag{29}$$

in which

$$\Re = \frac{k_0 (dr/d\theta_0)^3}{2 \sin\theta_0 \,(d^2 r/d\theta_0^2)^2} \tag{30}$$

and h_2 is the modified Hankel function as defined in reference 119.

The quantity $qJ(\Re)$ is not applicable when both $d^2 r/d\theta_0^2$ and $dr/d\theta_0$ are zero. However, this can happen only at singular points (cusps).

The quantities S and B are the Rayleigh reflection coefficients for the ocean surface and floor, respectively, m the number of surface contacts and η the number of bottom contacts. If the ray penetrates either surface and returns by refraction or further reflection, the reflection coefficients must be replaced by the proper combination of reflection and transmission coefficients in an obvious manner.

The quantity S is sufficiently close to -1 for nearly all purposes, and in the sequel this will be assumed. The state of knowledge concerning B has been presented in Section 2. 3. 3. B is a function of the grazing angle of the ray at the bottom, the acoustic frequency, and possibly the environment. The functional dependence is largely empirical at this time, although Mackenzie[35] has shown that the acoustic properties of sediments may be reduced to first principles when sufficiently detailed knowledge of the material is available. We write

$$B = B(\omega, \theta_B, \epsilon), \tag{31}$$

θ_B being the grazing angle at the bottom and ϵ indicating potential environmental dependency.

Ω is the specular scattering coefficient of the sea surface[20]. For small scattering,

$$\Omega = 1 - 0.485\, b^{3/2}\, H^{1/10} \sin\theta_s \tag{32}$$

where H is wave height in feet, b is equal to $\omega H/2\pi$, and θ_s is the grazing angle at the sea surface. We take the same parametric dependence to hold for large scattering, so that, in general,

$$\Omega = \Omega\left(b^{3/2}\, H^{1/10} \sin\theta_s\right) \tag{33}$$

Figure 8 shows measured values from many sources,[22] with Equation (32) plotted as a solid curve in the form, surface loss = $-10 \log \Omega$.

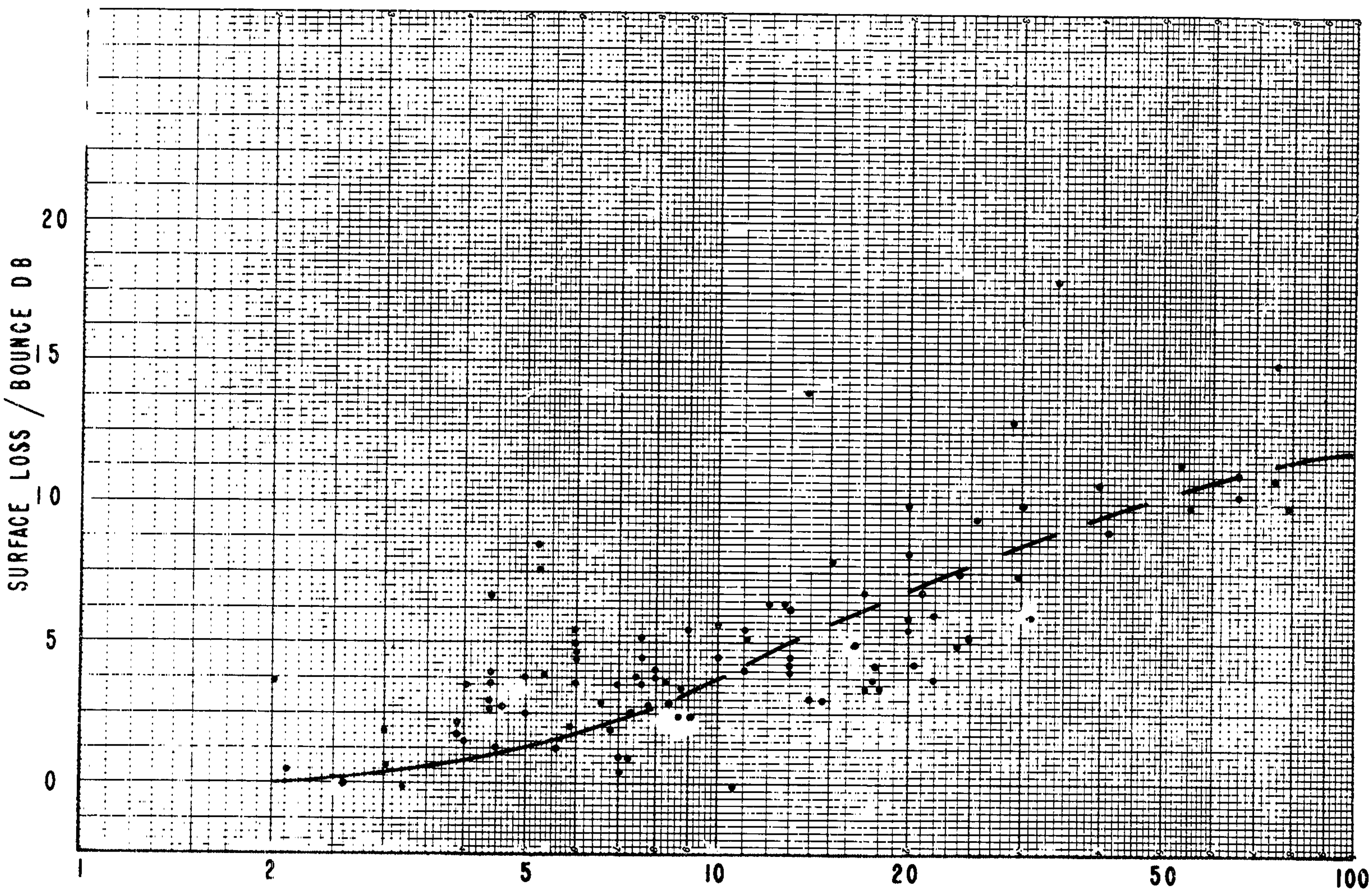

FIGURE 8. SEA SURFACE SCATTERING LOSS

The dotted curve is the recommended extension of Equation (33) to use.

This curve has been drawn, taking into account proper weighting, and the additional fact that the scattering should level off at high frequencies.

The irreversible conversion of acoustic to thermal energy, or absorption, is represented by the term e^{-A}, in which

$$A = \int \alpha ds \qquad (34)$$

2.4.4 Scattered Field

Sound scattering, as a propagation effect, produces three basic effects. These include:

a) contribution to the primary field

b) creation of sound energy in shadows

c) contribution to fluctuation

The latter effect is discussed in Chapter 3. The other effects are primarily caused by the ocean surface and bottom. Although the bottom has scattering properties, they are not sufficiently understood at this time to permit quantitative calculations. The scattering properties of the sea surface have been discussed by Skudrzyk[77], among others, and the model employed by Marsh[120] et al will be employed here for estimating the scattered field.

The scattered field is analyzed into its plane wave components, and for an incident plane wave. In terms of the geometry of Figure 9, Marsh gives, for small scattering,

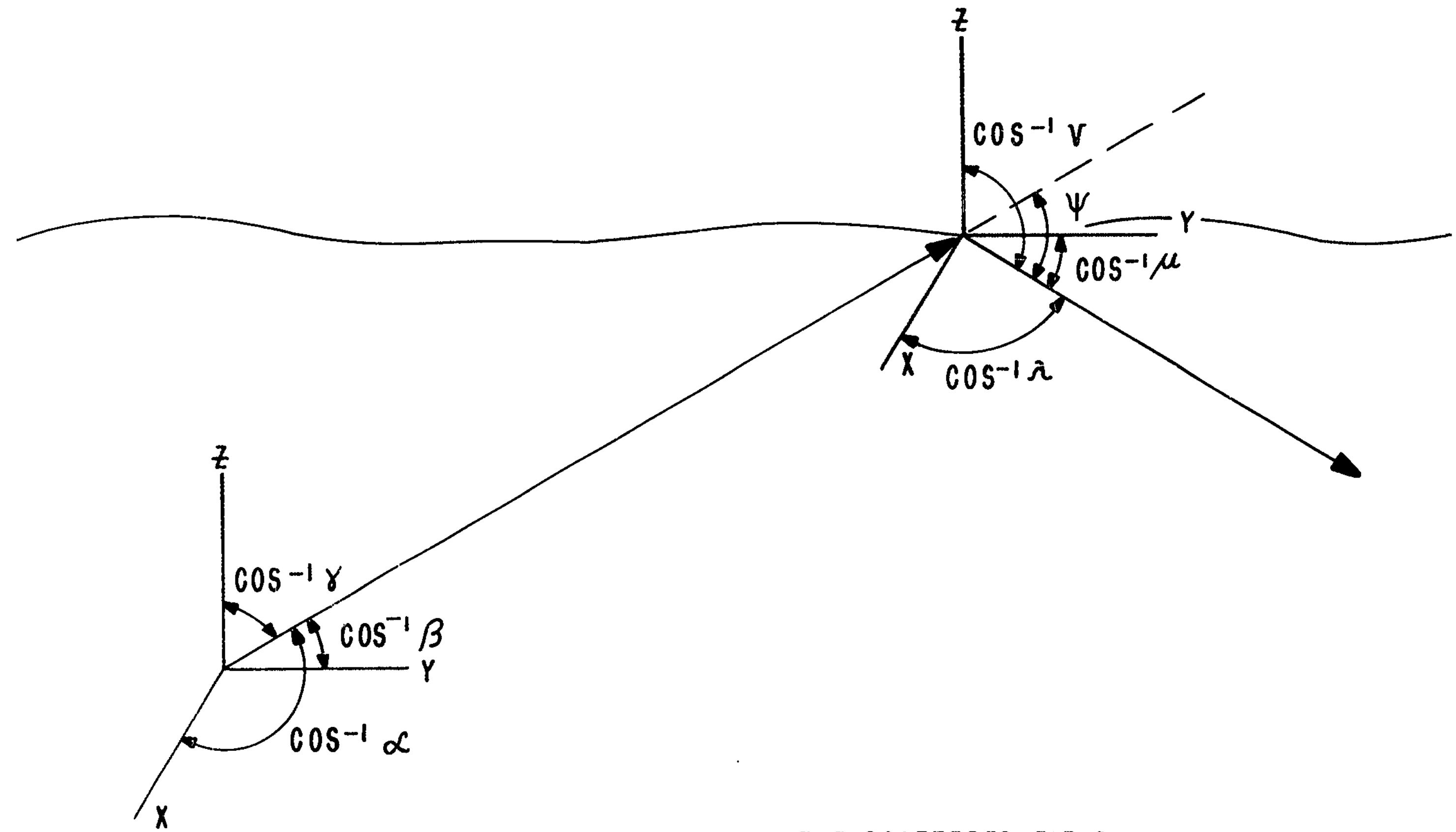

FIGURE 9. GEOMETRY OF THE SCATTERED FIELD

$$\Gamma = 0.098\, H^{2/5} (1 - \nu^{1^2}) \exp(-1/a^1)\, a^{1^{-\frac{1}{2}}} \qquad (35)$$

where:

$$a^1 = 6.33\, H^{-1/5}\, b\,(1 - \nu^{1^2})^{1/2}$$

$$(1 - \nu^{1^2}) = 2 - (\nu - \gamma)^2 - 2 \cos \psi$$

$$I = \Gamma\, d\lambda\, d\mu$$

as the ratio of scattered intensity in the direction (λ, μ) to incident intensity. For application to a definite calculation, integration over (λ, μ) must be effected. This is prohibitively difficult. It would appear, however, that the case of greatest interest is that in which propagation is supported by repeated surface reflection. In that case, the calculation can be materially simplified by some approximations. We neglect multiple scattering and joint refracted-scattered processes. Consider Figure 10.

We calculate the scattered field at the mid-point of the cycle and use it as an average over the entire cycle. Since multiple scattering is neglected, only primary rays need be considered.

The scattering process entails no loss of energy, but rather a redistribution in direction of propagation. Thus, the total scattered intensity will equal that lost from the specularly reflected ray. Accordingly, the scattered field due to the primary ray in question is approximately

$$p^2 = \frac{C\, \Gamma e^{-aR}}{R^2} \qquad (36)$$

where

$$C \int_0^{\pi/2} I \sin\beta \, d\beta = \Omega$$

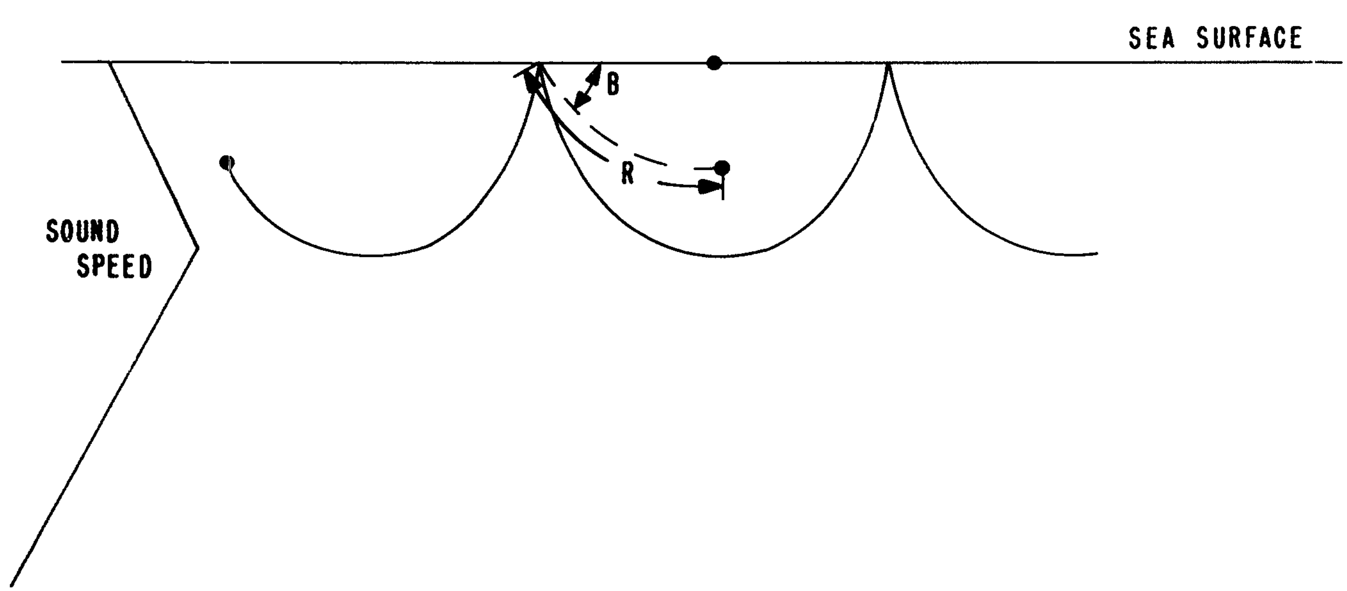

FIGURE 10 MODEL FOR THE SCATTERED FIELD

The results[7] give empirical expressions for the sound field below the isothermal layer, which should be equivalent on the average to those obtained from Equation (36), where they can be compared. The data is limited to frequencies above two kilocycles per second. Below this, the scattered field will be rather small. Furthermore, at about this frequency, the effects of diffraction start to become important.

2.4.5 Diffraction Field

The diffraction features of the sound field at points reached by rays has been considered above. At other points (shadows) the diffraction field can be predominant.

Shadows always have a boundary which is a limiting ray having a turning point as a sound speed maximum. In the shadow, the diffraction field will be nearly constant along lines parallel to the limiting ray, and will diminish exponentially in proceeding away from the limiting ray. In Figure 11 there are two limiting rays, and there will be a diffraction field associated with each.

The development leading to Equation (29) is applicable in principle to the present situation. However, no useful results can be obtained. In case the sound speed is a linear function of depth, and if

$$\gamma = \left| \frac{1}{c} \frac{dc}{dZ} \right| \tag{37}$$

then the diffraction field pressure is nearly equal to

$$p_l \, e^{-3 \left(\frac{\omega}{2\pi}\right)^{1/3} \left(H + 2\sqrt{\frac{V}{\gamma}}\right)} \tag{38}$$

H and V are the horizontal and vertical distances shown in Figure 11 and p_l is the pressure at the limiting ray.

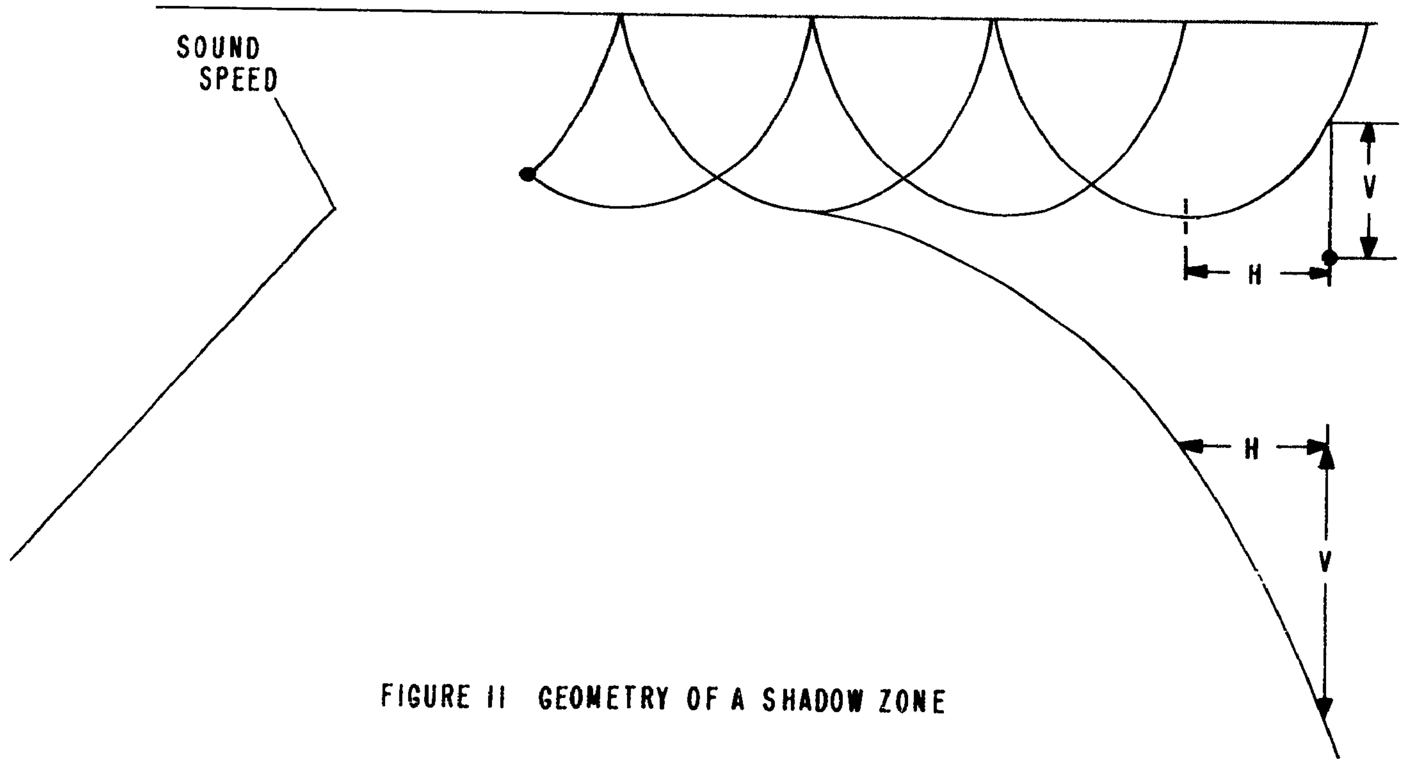

FIGURE II GEOMETRY OF A SHADOW ZONE

There appear to be few instances that Equation (38) can be applied with reliable results. Reference 117 indicates that the shadow zone field is much larger than that calculated from the above equation. On the other hand, the equation is surely applicable if the prevailing assumptions are justified. Two definite possibilities exist:

1) There are other sources of sound in the shadow zone,
2) The sound speed structure is not accurately known.

These can be discussed for the cases shown in Figure 11. When the turning point is at the surface, and if bottom reflections do not contribute appreciably, the diffraction field should dominate. Morse[121] has shown the qualitative effect to be expected from small changes in the speed gradient near the surface. This, coupled with recent evidence in near surface sound speed [99], indicates that a reliable calculation of the field can be anticipated if the speed structure is definitely known.

When the turning point is at depth, as under a surface sound channel, the scattered field can be expected to predominate at the higher frequencies where measurements exist. At the lower frequencies, a reasonable calculation of the diffracted field can be expected using Equation (38).

2.4.6 Normal modes

At sufficiently low frequencies, the methods of calculation based on ray geometry are no longer valid. The use of normal modes, or solutions based upon separation of the wave equation can be utilized in such cases. As a practical matter, the normal mode solution will be particularly useful, and will differ appreciably from the ray solution, when there is a single mode which is propagated without substantial diffraction losses.

The basic theory and many references to the normal mode theory are presented by Officer in reference 122. Since the appearance of that publication, some work has been published[27, 123, 124] indicating generally good agreement between theoretical and experimental results.

Other than the shallow water mode field, the only application of practical interest appears to be to surface sound channels. However, at low frequencies, surface sound channels are seldom important in determining the total field. On this account, there does not seem to be sufficient interest to develop general equations and procedures for the application of normal mode solutions.

As an aid, however, the results of Voorhis[125] are reproduced in Figure 12. This shows the net attenuation due to diffraction of the first mode for various values of the parameters. When this attenuation is appreciable compared with other attenuations (scattering, reflection and absorption), then consideration should be given to the use of normal mode calculations. For that purpose, Pedersen[126] has made certain corrections and improvements to the work of Marsh. His results are quoted below, since they represent the only known complete results necessary for the calculation of the sound field itself, rather than the features of it, such as are presented in the other works cited.

The computation is based on four physical parameters and three parameters determined by the experimental set-up. The physical

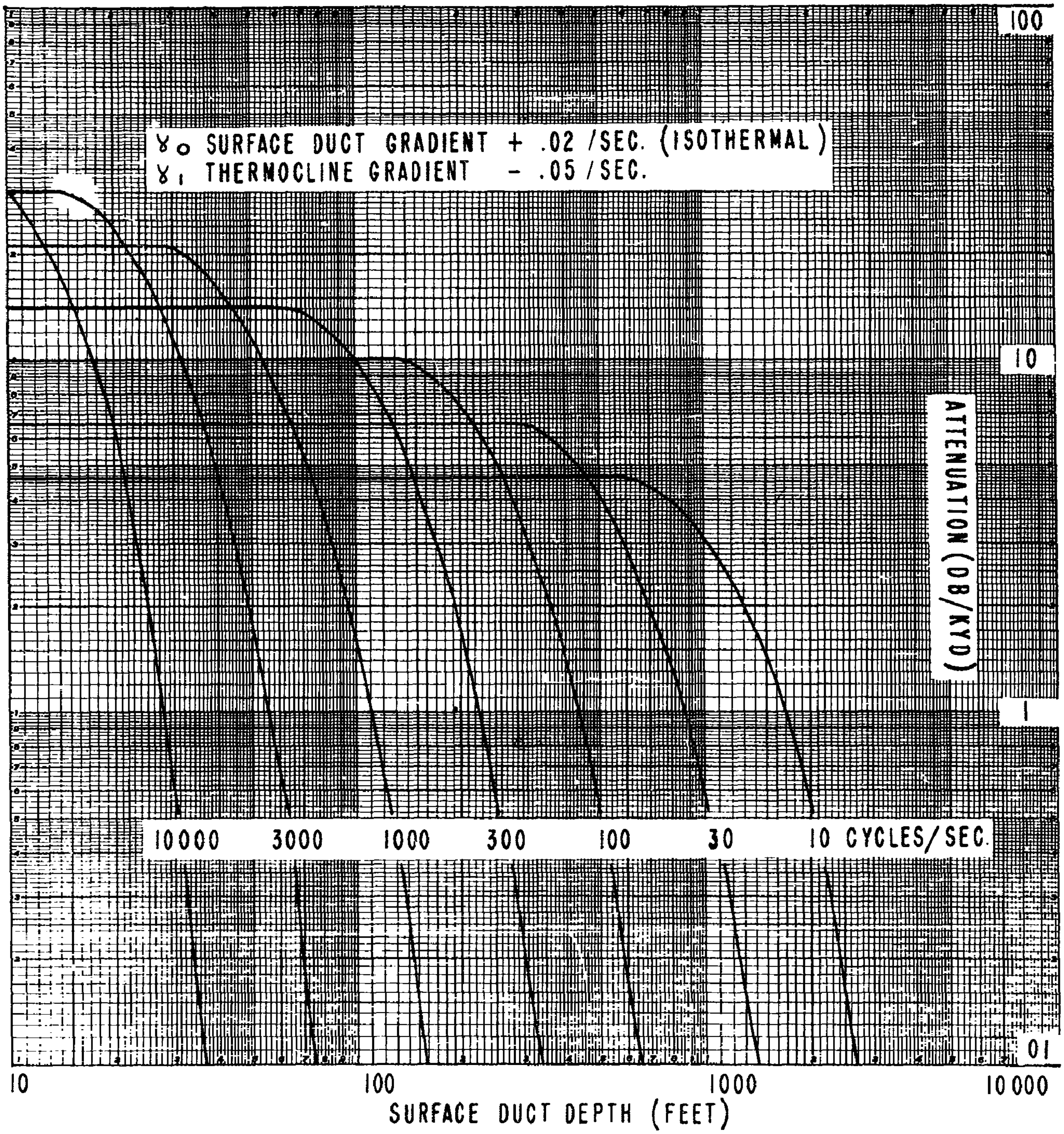

FIGURE 12. ATTENUATION DUE TO DIFFRACTION OF FIRST MODE

parameters are: C_0, the sound velocity at the sea surface; Z_1, the depth of the surface channel; and γ_0 and γ_1, the gradients in and below the channel respectively. In this study γ_0 is positive and γ_1 is negative. The other three parameters are the frequency, f, and the source and receiver depths. In this theory the source and receiver depths are interchangeable — i. e. reciprocity holds. The shallower of these two depths is designated by Z_a, and the deeper by Z_b.

Certain useful combinations of these parameters are:

$$t_a = Z_a/Z_1, \; t_b = Z_b/Z_1, \; \rho^3 = \gamma_0/\gamma_1$$

$$\Gamma = (\pi f \gamma_0{}^2)^{1/3}/c_0, \; M = 2c_0 \; Z_1 \Gamma^2/\gamma_0$$

where N = greatest integer $\leq (2M^{3/2}/3\pi) + .25$, the number of trapped modes.

When Marsh's equations[110] are evaluated in detail the following form results for the product of the depth functions:

$$U_\eta(t_a) \cdot U(t_b) = (M/Z_1) \cdot F(t_a, t_b)/D \qquad (39)$$

D will be discussed later. $F(t_a, t_b)$ takes one of three possible forms as follows:

Case I - when both source and receiver are in the channel; i. e. $t_a \leq t_b \leq 1$

$$A(t_a) \cdot A(t_b) \qquad (40)$$

Case II - when both source and receiver are below the channel; i. e. $1 \leq t_a \leq t_b$

$$\left[\frac{A(1)}{h_2(B_1)}\right]^2 h_2(Bt_a)\, h_2(Bt_b) \qquad (41)$$

Case III - when one is in and one below the channel; i. e. $t_a \leq 1 \leq t_b$

$$\left[\frac{A(1)}{h_2(B_1)}\right] A(t_a)\, h_2(Bt_b) \qquad (42)$$

where

$$A(t) = h_1(MX_\eta)\, h_2(MX_\eta - Mt) - h_2(MX_\eta)\, h_1(MX_\eta - Mt) \qquad (43)$$

The h_1 and h_2 functions are solutions of Stokes' equation and are discussed at length and tabulated in reference 119. First derivatives of these functions are designated as h_1' and h_2'. MX_η is the complex eigen value and will be given presently.

$$B_t = \rho^2 (MX_\eta - M) + \frac{(M - Mt)}{\rho} \qquad (44)$$

$A(1)$ and B_1 are obtained by setting $t = 1$ in the above expressions.

$$D = (\rho^3 - 1) \left[B^2 + (MX_\eta - M)\, A^2(1) \right] - 2.1242930 \qquad (45)$$

where the negative constant is the square of the Wronskian,

$W(h_1 h_2)$ and B is defined by $B = h_1(MX\eta) h_2'(MX_\eta - M) - h_2(MX_\eta) h_1'(MX_\eta - M)$

(46)

$MX\eta$ is a complex root of the characteristic equation

$$\rho h_2(B_1) \cdot B - h_2'(B_1) \cdot A(1) = 0 \qquad (47)$$

In the case of strong trapping the following approximation to the roots may be useful.

$$\mathrm{Re}(MX_\eta) = \left[\frac{3\pi}{2} (\eta - \tfrac{1}{4}) \right]^{2/3}$$

$$\mathrm{Im}(MX_\eta) = \frac{1}{4} \left[\mathrm{Re}(MX_\eta) \right]^{-1/2} \exp \left\{ -\frac{4}{3} (1 - \rho^3)(M - \mathrm{Re}MX_\eta)^{3/2} \right\}$$

(48)

The velocity potential ϕ is given by

$$\phi = -\pi i e^{i\omega T} \sum_1^N H_0^2 (\lambda_\eta r)\, U_\eta(t_a) \cdot U_\eta(t_b) \qquad (49)$$

$$\lambda_\eta = \sigma_\eta - i \tau_\eta$$

$$\sigma_\eta \sim K - \Gamma\ \mathrm{Re}(MX_\eta)$$

c

$$\tau_\eta \sim \Gamma \, \mathrm{Im}\,(MX_\eta)$$

$$k = \frac{2\pi f}{c_0}$$

The damping coefficient is given by τ_η while σ_η is needed in the phase determination.

For sufficiently large values of r

$$H_0^2(\lambda_\eta r) \sim \left(\frac{2}{\pi k r}\right)^{1/2} \exp\left\{-i\left(\sigma_\eta r - \frac{\pi}{4}\right) - \tau_\eta r\right\} \quad (50)$$

The transmission loss H is given by

$$H = -10 \log |\phi|^2$$

From Equations (49) and (50)

$$|\phi|^2 = \frac{c_0}{f}\left\{\left[\sum_1^N A_\eta \cos\theta_\eta\right]^2 + \left[\sum_1^N A_\eta \sin\theta_\eta\right]^2\right\}/r \quad (51)$$

where

$$A_\eta = |U_\eta(t_a) \;\; U_\eta(t_b)| \, e^{-\tau_\eta r} \quad (52)$$

$$\theta_\eta = \psi_\eta - \sigma_\eta \cdot r$$

$$\psi_\eta = \arg \cdot \left\{U_\eta(t_a) \cdot U_\eta(t_b)\right\}$$

2.5 Practical Calculation of the Sound Field

2.5.1 General Considerations

For detailed and precise calculation, the methods of Section 2.4, or others of equivalent scope, appear to be necessary. There are two practical difficulties, however, which demand that simpler methods be employed when circumstances warrant. In the first place, since the environmental model is idealized, there is often a lack of sufficient environmental data to permit complete employment of the model. Secondly, one is often more interested in the average sound field, or in general aspects of the field which can be stated approximately. This is especially so in systems engineering, and in strategic problems, where only the gross structure of the sound field can possibly be considered.

Two situations exist which have great practical importance, and which can be characterized rather simply with fair approximation. These are sound channels and shallow water. Both are defined below, and methods given for the approximate determination of the sound field.

2.5.2 Sound Channels

A sound channel is defined to be the region of space in which the cyclic character of the rays is predominant, and in which the deepest fiducial point is a turning point. In other words, the range is several multiples of the cycle ranges, and the influence of the ocean bottom is ignored.

Equation (21) shows that, when the source and receiver depths are equal, the range is

$$\begin{aligned} r &= 2mF_+(o) + 2nF_-(o) \\ &= mr_0 + 2(n-m)F_-(o) \end{aligned} \tag{53}$$

Hence when the range is an integral multiple of the cycle range,

$$\begin{gathered} r = mr_0 \\ \frac{dr}{d\theta_0} = m\frac{dr_0}{d\theta_0} \end{gathered} \tag{54}$$

and so the ray divergence q between any such points is exactly spherical. This has been pointed out by Cole[117]. In the case of a symmetrical channel, with source located on the axis, the ray divergence is also spherical between points which are a half integral multiple of the cycle range. In the latter case, if the field point is at the depth of a fiducial point then the ray divergence is determined by

$$|q|^2 = \frac{16}{m^2 r_0 n \tan\theta \, dr_0/d\theta_0}$$

$$\text{provided} \qquad r = \frac{mr_0}{4} \tag{55}$$

If θ is zero, $qJ(\Re)$ must be evaluated for the limit $\Re \to 0$, and we have $\Re = m\Re_0$, $\Re_0$ being the value of $\Re$ corresponding to $r = r_0/4$; $z = o$. This gives

$$|J(\Re)\,q|^2 = \frac{|J(\Re_0)q_0|^2}{m^{5/3}} \; \frac{\tan\theta'}{\tan\theta} \tag{56}$$

θ' being the ray angle at the point $r = r_0/4$. It can be seen that either θ' or θ is zero, and hence a further limit must be determined. If θ is zero we have the case of a surface bounded sound channel with source at the surface and field point at a turning point; if θ' is zero, we have the case of an internal (depressed) channel. The two are exactly equivalent with respect to the present analysis.

Let Z be the depth of a turning point, so that

$$n^2\,(Z) = \cos^2\theta_0 \tag{57}$$

In the vicinity of Z, when z is slightly less than Z, we may always write

$$n(z) = \cos\theta_0 + a\,(Z - z)\,, \quad a \geq o \tag{58}$$

If $a = o$ we have a relative maximum in the sound speed; if $a = \infty$ we have a cusp in the speed profile. The case $o < a < \infty$ can include relative maxima if the slope of the speed-depth curve is discontinuous at the turning point. The case $a = \infty$ is of no particular interest and will be disregarded.

We are concerned with the behavior of the derivatives of F near a turning point. Using higher order terms as $z \rightarrow Z$, we may derive the relationship

$$\sin\theta \frac{dF}{d\theta} \rightarrow \frac{-\sin\theta}{a} \tag{59}$$

as $\mathfrak{g} \rightarrow o$, which yields

$$q \rightarrow e^{i(\omega t - kr)} \left(\frac{na}{2\, r\, \sin\theta_0} \right)^{1/2} \tag{60}$$

From Equation (60) we obtain

$$qJ(\mathfrak{g}) \rightarrow \frac{(2\pi)^{1/2}\, e^{i(\omega t - kr + \pi/12)}}{2/3\,(3)^{2/3}} \left(\frac{k_0 a^2}{r^2 \sin\theta_0} \right)^{1/6} \cos^2\theta_0$$

We note in passing that if $N^2 = 1 - \gamma Z$, the case discussed by Marsh[127] when $2a = \gamma$,

$$r_0 = \frac{2\sin\theta_0\cos\theta_0}{\gamma} \tag{61}$$

and

$$|q|^2 = \frac{\cos^2\theta_0}{r^2} \tag{62}$$

which is a well known result[16].

The preceding discussion applies to isolated points in the sound channel. The general situation can be estimated as follows. Firstly, we observe that both of the quantities q and $J(\mathfrak{g})$ tend toward values

o

representing spherical spreading along any given ray, as the distance along the ray increases. This is trivially true for non-cyclic rays, or for reflected, non-refracted rays. For cyclic rays the result is true because, as r increases,

$$\begin{aligned} r &= m r_0 + \epsilon \\ \frac{dr}{d\theta_0} &= m \frac{dr_0}{d\theta_0} + \epsilon_1 \\ \frac{d^2 r}{d\theta_0^2} &= m \frac{d^2 r_0}{d\theta_0^2} + \epsilon_2 \end{aligned} \tag{63}$$

in which ϵ, ϵ_1, and ϵ_2 have maximum values, independent of r. Hence for points chosen at random along a ray, the probability that $\left| r - mr_0 \right| < E$ tends to 1 as r increases, where E is any preassigned number. Equivalent statements hold for $dr/d\theta_0$ and $d^2r/d\theta_0^2$. We can thus say qualitatively that every ray is ultimately spherical. Now, in a sound channel, the average number of rays reaching a field point increases as the range increases. Hence the total field ultimately follows a cylindrical spreading law. More precisely, if N is the number of rays, then

$$|p|^2 \sim \frac{N}{R^2} \tag{64}$$

The quantity N can be determined by constructing a field diagram (Figure 5), or can be estimated as follows. Let r_0' and r_0'' be the maximum and minimum cycle ranges for all possible angles. Then

$$1 + \frac{R}{r_0'} \leq \frac{N}{2} \leq 1 + \frac{R}{r_0''} \tag{65}$$

2.5.3 Shallow-Water Transmission[52]

Accounts of shallow-water transmission have recently appeared in the literature. In particular, references 2, 55, 58, 122, 128 discuss the boundary-valueproblems associated with the normal mode solutions, while references 49, 50, 51, 129 discuss solutions at higher frequencies involving ray approximations.

The principal problem in calculating the sound field in shallow water is the lack of detailed knowledge of the shallow water environment. However, highly variable sound speed is the rule, and this has an important bearing on high-frequency transmission. The properties of the bottom, important at any frequency, are controlling at the lower frequencies.

Even given the detailed parameters of a particular environment, the variability to be expected is such that calculation will be very com plex and tedious. There is thus the need for comparatively simple equations representing the average sound field, while retaining dependence upon the principal features of the environment.

A normal mode treatment is necessary if the ratio of water depth to wavelength is not large. In this case, the accounts of Williams[55, 128] cited above appear to represent the state of knowledge rather well, and hence this specific aspect of the problem will not be treated further here.

If the water depth exceeds about four wavelengths, then Mackenzie's, along with other results (not published) may be expressed in a relatively compact semiempirical way compatible with a general theory of underwater sound propagation. Reasonable assumptions are made about the mean character of the shallow-water environment.

Both the results of Mackenzie and others indicate no systematic dependence upon the depth of source or receiver. Propagation at extended range is supported by repeated bottom and surface reflection, regardless of the thermal conditions. Thus, there is a strong surface-bottom coupling such that the propagation losses are controlled by the number of contacts of rays with both surface. The surface scatters the rays, whereas the bottom absorbs them. The thermal structure of the water affects propagation through its influence on skip distance and the number of surface and bottom contacts. The propagation loss is thus represented in terms of sea state (wave height) bottom type (or bottom loss, if known) water depth, and isothermal-layer depth.

A skip distance, $H = \left[\frac{1}{8}(D + L)\right]^{\frac{1}{2}}$ kyd is defined for water depth D and layer depth L in feet. If the range R between source and receiver is less than H, then the propagation loss is

$$N = 20 \log R + \alpha R + 60 - k_L \text{ db;} \qquad (66)$$

k_L, the near field anomaly, represents the mean contribution to the field of the multiple bottom and surface reflections. It is a function of bottom loss and surface loss such that

$$k_L = 10 \log (1 + 2r_a r_b + r_a + r_b) \text{ db,} \qquad (67)$$

r_a and r_b being the surface and bottom intensity-reflection coefficients, respectively. The absorption coefficient in sea water (in kyd) is α and is discussed in references 9 and 14.

$$N = 15 \log R + \alpha R + \alpha_T (R/H - 1) + 5 \log H + 60 - k_L \text{ db.} \qquad (68)$$

For long ranges, $R \geq 8$ H,

$$N = 10 \log R + \alpha R + \alpha_T (R/H - 1) + 10 \log H + 64.5 - k_L \text{db.} \qquad (69)$$

These equations represent the gradual transition from spherical spreading in the near field to cylindrical spreading in the far field.

The loss coefficient α_t (db per cycle of bottom and surface reflection) is represented as follows:

Table IV. Shallow-water attenuation α_t (db/bounce)

Sea state / f kcps	0		1		2		3		4		5	
	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud
0.1	1.0	1.3	1.0	1.3	1.0	1.3	1.0	1.3	1.0	1.3	1.0	1.3
0.2	1.3	1.7	1.3	1.7	1.3	1.7	1.3	1.7	1.3	1.7	1.4	1.7
0.4	1.6	2.2	1.6	2.2	1.6	2.2	1.6	2.2	1.7	2.4	2.2	3.0
0.8	1.8	2.5	1.8	2.5	1.9	2.6	2.2	3.0	2.4	3.8	2.9	4.0
1.0	1.8	2.7	1.9	2.7	2.1	2.9	2.6	3.7	2.9	4.1	3.1	4.3
2.0	2.0	3.0	2.4	3.5	3.1	4.4	3.3	4.7	3.5	5.0	3.7	5.2
4.0	2.3	3.6	3.5	5.2	3.7	5.5	3.9	5.8	4.1	6.2	4.3	6.4
9.0	3.6	5.3	4.3	6.3	4.5	6.7	4.7	6.9	5.0	7.3	5.1	7.5
10.0	4.0	5.9	4.5	6.8	4.8	7.2	5.0	7.5	5.2	7.8	5.3	8.0

Table V. Near-field anomaly k_L (db)

Sea State / f kcps	0		1		2		3		4		5	
	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud	Sand	Mud
0.1	7.0	6.2	7.0	6.2	7.0	6.2	7.0	6.2	7.0	6.2	7.0	6.2
0.2	6.2	6.1	6.2	6.1	6.2	6.1	6.2	6.1	6.2	6.0	6.2	6.0
0.4	6.1	5.8	6.1	5.8	6.1	5.8	6.1	5.8	6.1	5.8	4.7	4.5
0.8	6.0	5.7	6.0	5.6	5.9	5.6	5.3	5.0	4.3	3.9	3.9	3.6
1.0	6.0	5.6	5.9	5.5	5.7	5.3	4.6	4.2	4.1	3.7	3.8	3.4
2.0	5.8	5.4	5.3	4.9	4.2	3.8	3.8	3.4	3.5	3.1	3.1	2.8
4.0	5.7	5.1	3.9	3.5	3.6	3.1	3.2	2.8	2.9	2.4	2.6	2.2
8.0	4.3	3.8	3.3	2.8	2.9	2.5	2.6	2.2	2.3	1.9	2.1	1.7
10.0	3.9	3.4	3.1	2.6	2.7	2.2	2.4	2.0	2.2	1.7	2.0	1.6

Table VI. Probable error of propagation loss (db)
(semi-inter quartile range)

Range, kyd	Frequency, cps 112	446	1120	2820
3	2	4	4	4
9	2	4	5	6
30	4	9	11	11
60	5	9	1[illegible]	12
90	6	9	11	12

In the case where upward refraction occurs, it is expected that the sea state would have a strong influence on the propagation. Actually, the loss per contact looks like the bottom loss, rather than like the sea-surface loss in that the surface-scattering loss has a much larger frequency dependence. This behavior with sea state might be expected if most of the energy scattered from the surface is reflected by the bottom and returned for propagation down the channel. In deep-water, α_s measures the energy which is scattered out of the surface channel and which is either lost in absorption on the way to the bottom or absorbed in the bottom through high grazing-angle incidence.

The problem then is to determine the extent of the coupling between the surface and the bottom. If r_s is the surface-reflection coefficient, then $(1 - r_s)$ is the surface-scattering coefficient and $\alpha_s = - 10 \log_{10} r_s$ is the surface loss in db/bounce. The surface components r_s and $(1 - r_s)$ must suffer different interactions with the bottom if any sea-state dependence is exhibited. We would expect scattered rays to suffer greater loss because they are steeper. The simplest expression satisfying this requirement and the observed data was

$$r_t = r_a \, r_b + (1 - r_s) \, r_b^2 \qquad (70)$$

where r_t is the fraction of energy transmitted down the channel when a bottom event is coupled with each surface reflection, $\alpha_t = - 10 \log_{10} r_t$ (shallow-water attenuation constant, db/bounce), r_b is the bottom reflection coefficient for the angles of incidence occurring in shallow water and is given in Table IV for sand and for mud bottoms, ($\alpha_b = - 10 \log_{10} r_b$).

The observed behavior for α_t has profound significance in the interpretation of the shallow-water propagation mechanism. The fact that the grazing-angle rays represented by r_s must be multiplied by r_b indicates that there is a bottom loss suffered by near grazing rays. The fact that the scattered rays must be multiplied by r_b^2 means that the angular dependence of bottom reflection loss is such that these steeper rays suffer twice the loss of the near grazing rays.

The observed bottom loss at grazing angles seems to imply a mode-changing process that takes energy for a refracted-ray system and converts it to a simple bottom and surface bounce mode at great ranges. This is equivalent to the normal mode treatment of propagation in shallow water. This mode-changing process seems to occur for any sound-speed structure. The sound-speed structure determines the skip-distance zones. It would appear that after the first few zones, the skip-distance has a purely local significance representing a limiting free path for rays which happened to be scattered into grazing angles at that range. If, as is shown in the data, grazing rays must suffer one bottom contact loss per skip zone, then they cannot be continuous from one zone to another. The possibility that horizontal rays suffer a bottom loss is hardly conceivable. The transition from one propagation mode to the other probably occurs over several skip zones and we have adopted the convenient model that it occurs over the first eight skip zones. When the foregoing expressions are used to compute propagation loss for all the shallow water data, representing about 100,000 observations, then Table VI gives the probable errors as a function of range and frequency.

Chapter III

OTHER EFFECTS

3.1 Introduction

The requirements of modern and probable future sonar systems are such that the variability of the oceanographic parameters in time and space can become an integral part of the sound field problem. Fluctuations of the sound field in the ocean are intimately related to the changing structure of the transmission path or paths between the transmitter and receiver. How we treat these fluctuations in intensity, phase, direction resolution, frequency, or pulse width depends entirely on what result is to be accomplished.

If, for example, we are trying to communicate over long range paths, in the design of an optimum system, we must consider (a) the time and space variability of signal levels for the modes of propagation which can exist, (b) multiple arrivals and multipath propagation which arises from the physical nature of the problem, and (c) frequency sensitive effects such as dispersion, absorption, and time and space correlation of signal levels.

3.2 Mean Variability of Transmission Loss

The practical application of the formulas for transmission loss in Chapter 2 usually requires an estimate of the associated variability. This is necessary in specifying performance measures of equipment, and in optimizing system design, especially with regard to the selection of the best operating frequency.

While work is in progress, notably that of Skudrzyk[77], Mintzer[67], and Tatarski[76] in the ocean medium, with the fluctuations in the acoustic parameters, we are forced to accept the environment as it occurs. Thus, specification of the variability in transmission loss can be expressed using empirical data taken from experiments made with the type of system to be studied.

For example, in previous attempts to correlate acoustic behavior with the oceanographic variables, the variables were considered to be quasistatic over the acoustic paths. The propagation behavior was described in terms of mean values. The propagation modes considered were those which minimized the number of classes necessary to account for the modes occurring most of the time in most locations. However, there remained a residual variability in propagation loss within each class. No particular study was made relating the variability within a class with the variability of the oceanographic parameters. This is a perfectly feasible approach for the prediction of design information for the conditions of the experiments.

If the sound field is computed by the methods of Chapter 2, the mean variability can be determined from probable errors of the acoustic properties. Figure 13, shows the probable error in the bottom loss. The probable error of the surface loss may be determined from Figure 8.

In addition to boundary variability, there is variation due to the body of water itself. This is evident even at short range, and can be estimated using the methods of Skudrzyk. As a practical matter, the inherent volume variability is about 3 db in signal level.

If the joint statistics of the multiple reflection processes were known, the variance of the composite sound field could be calculated.

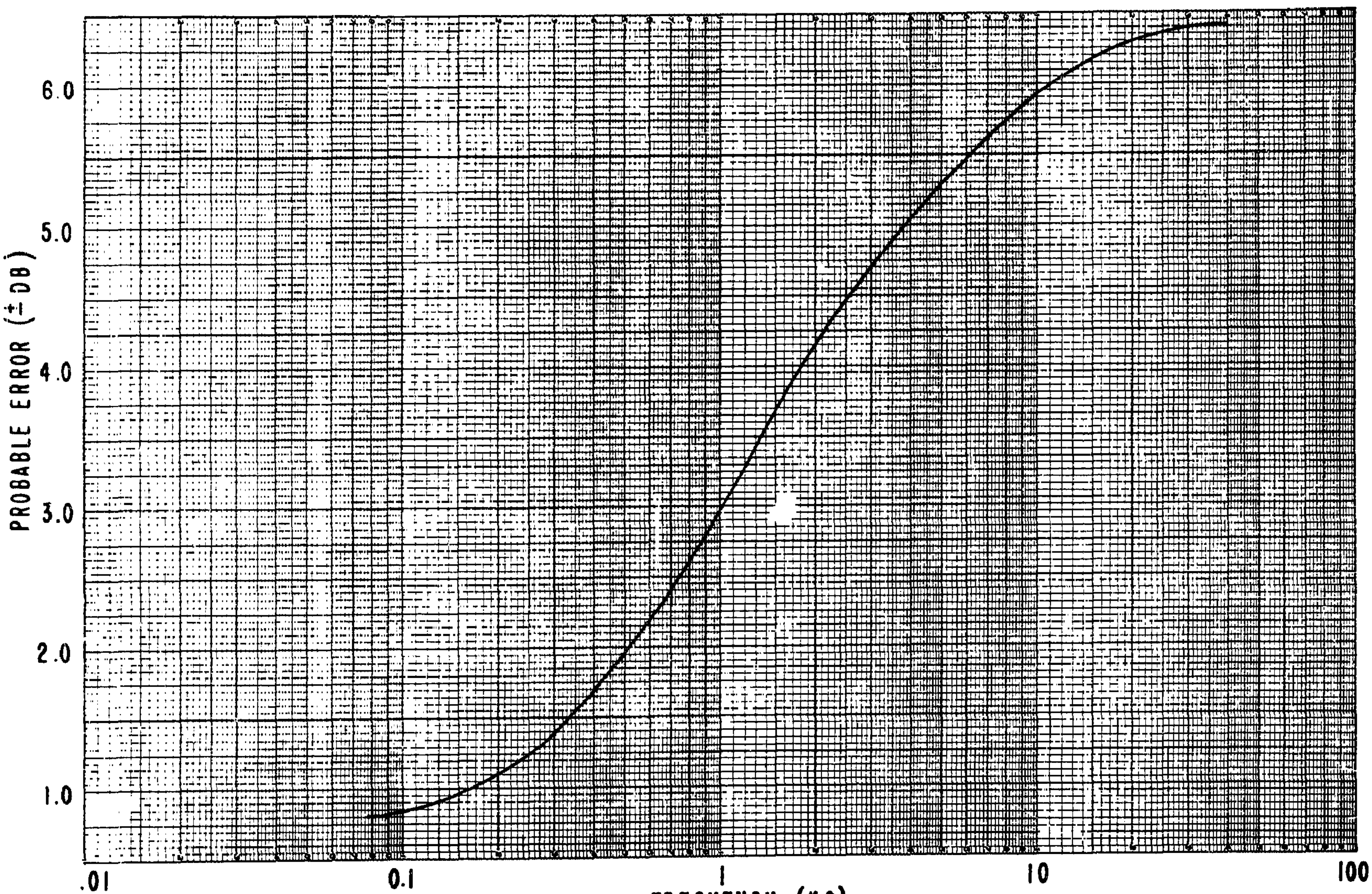

FIGURE 13 PROBABLE ERROR VS. FREQUENCY

In the absence of this knowledge, two extreme calculations can be made. One assumes independence from, and the other assumes complete dependence upon the probable errors associated with each reflection process. If the variances computed in this manner are σ_-^2 and σ_+^2 respectively, then the actual variance σ^2 should be bounded $\sigma_-^2 \leq \sigma^2 \leq \sigma_+^2$.

An expression for the variance of the propagation loss in db is:

$$\sigma = 10 \log \left[2^2 + (\text{ratio equivalent of boundary loss})^2\right]^{\frac{1}{2}} \text{db} \quad (71)$$

This overestimates the situation, but is a proper estimate for conservative calculations.

At the other extreme, we have simply

$$\sigma = \left[3^2 + \sigma_S^2 + \sigma_B^2\right]^{\frac{1}{2}} \quad (72)$$

as an example of the minimum variance of the propagation loss for a field dominated by surface and bottom reflected paths, and σ_S^2, σ_B^2 being the variances associated with single refractions at surface and bottom, respectively. Some tables giving the gross variability of the sound field have been published. Table VI gives the probable error of propagation loss for the average shallow water environment along the east coast of the United States. Mackenzie[50] has reported results for shallow water, and has commented on the nature of the distribution function.

3.3 Factors Influencing Variability

If the parameters of the sound field were constant in time, there would be no variability. Changes in the geometry of the field, such as motion of source or receiver, or changes in the environment are responsible for the changes in the sound field from its mean value.

Calculations of the fluctuation to be expected from thermal variations range from microscopic effects, through intermediate effects of internal waves to the seasonal influences which are readily observed. There is a continuum of periods of environmental changes ranging from seconds, or fractions of seconds through seasons or even years. These changes are entirely in the body of the hydrosphere and its surface.

The effect of the environmental changes is imposed upon the sound field through changes in the elementary constituents of the field. In general, the composite field is a superposition of contributions from a number of energy paths, which pass through different volumes of water and interact separately with the sea surface. It is the changing interaction of these paths which causes variability.

An obvious example is that of a ship suspended hydrophone, moving about as the ship moves in the seaway. Such motion can cause enormous variation in the sound field. Even with source and receiver fixed in space, there is still variability.[130] Measurements show a mean variation of some 2db in the rms pressure, and indicate a more or less well defined period on the order of 7 seconds. This variability is most likely attributable to surface waves.

In shallow water, with source and receiver fixed above the bottom, tidal periods have been observed.

The static sound field in the presence of an instantaneous internal wave pattern has been discussed by Lee.[85] A potential basis now exists for estimating the variability of the sound field to be expected because of internal waves.

Both the characterization and the measurement of variability are very difficult matters to deal with. Still, it is only because of variability that the modern interest in space-time coherence of the sound field has developed. It appears possible to calculate variations in the sound field from a sequence of static field calculations, in which the environment, as represented by the model, is varied in accordance with the physical processes giving rise to the environmental changes. These processes are of two basic types: a) flux of energy into the environment, and b) relaxation of energy concentrations within the environment (increase of entropy). In the first case, energy flows into the sea through the surface, because of wind action and solar heating, and through the bottom because of thermal gradients in the structure of the earth. In addition, energy is supplied by body forces (gravitational), represented in the tides and in Coriolis effects represented in current patterns. Secondly, the relaxation effects represent a redistribution of energy stored from earlier eras, and are, on a time scale, very long compared with the periods of variation due to energy flux.

It would appear that the predominant effects giving rise to variability are wind, the sea surface heating (cooling) and tides. Steady currents do not produce variability. The time scale of environmental changes associated with these effects range from seconds to hours to days to seasons. When a periodic change in the environment is impressed upon the sound field, there generally will be many periods produced in the sound field variation, and these will be sub-multiples of the driving period.

Two predominant effects are present. One may be the changing phase relationships between different paths contributing to the sound field. If so, the fundamental period of fluctuation can be a very small fraction of the driving period when many complete cycles of phase are varied during the environmental cycle. If the phase change is one cycle or less, the fundamental period will equal the driving period. The other type of effect occurs when changes in the shape or slope of the reflecting or refracting surface introduces a varying convergence or divergence. In this case, the driving period usually will equal the period of the acoustic field.

3.4 Other Measures of the Sound Field

There are many properties of the sound field which are not conveniently or practically related to the mean field and its variability. Some of these are discussed below. The subject will be more generally covered in Chapter IV, under new research.

3.4.1 Impulse Response Function

In some communications applications, we are interested in matching the transmitted and received signal. Dispersion properties of the medium could seriously affect the performance of such a correlation system. One way of obtaining this information is to interpret the impulse response of the propagation path as a four-terminal network, or filter. This will furnish phase and attenuation information as a function of frequency for the path. The change of the impulse response with time and oceanographic parameters, would then be studied and charted for the oceans much like the acoustic properties of the bottom. Peterson[18] has shown that Horton's attenuation formula can be used in this fashion, to account for the impulse response of the bottom to bomb shots.

3.4.2 Space and Time Correlation Functions

For some transmission problems, the size of the receiving array is limited by the space-correlation function of the wave front, i.e. how far apart can we put hydrophone elements and still have the signals received in phase ? Time correlation of signals at the same receiver is also important in signal processing.

3.4.3 Concurrent vs Sequential Statistics

In some systems, a single pulse may be transmitted involving several discrete frequencies. In this case, we would be interested in the correlation of signals from one frequency to another at the same instant (concurrent statistics) and from one instant to another at the same frequency (sequential statistics).

3.4.4 Multipath Interference

The tendency of the ocean to confuse signals by combination of the images being reflected from the surface and bottom will require study also. Thorp and Powers[82] have shown the nature of the problem including the effect of mixed modes, or leakage arrivals. There are two effects to be considered here:

3.4.4.1 Effective Noise Level Due to Multipaths Arrivals (Coherent)

If four equal reflected and/or refracted rays arrive simultaneously with the direct signal, as might happen for low frequencies over long paths, then the signal interference could be as much as 6 db above the signal.

3.4.4.2 Effective Pulse Lengthening (Incoherent)

The effective pulse lengthening can be taken as the time difference between the first arrival and the last arrival which is less than 20 db below the signal.

Chapter IV

REQUIREMENTS FOR FUTURE RESEARCH

4.1 General Comments

One way to appraise requirements for future research is to examine the material presented in the earlier sections of this document. We shall do this, employing the outline offered in Table I, as a method of organizing the subject. In so doing, we shall try to present priorities, based both on need in terms of the ultimate consumers, and on the scientific adequacy of present information.

One may ask first, "Is Table I an acceptable outline ?" We believe that it is, at the present time, and that future developments will at most add items to the outline, without disturbing its structure. This does not imply that certain sections might be completely reorganized. For example, the geophysical properties might in the long run be described statistically rather than deterministically. We will, in any event, regard the outline as acceptable, and base research requirements on the need for providing information to fill up the data points implicit in the outline.

4.2 Regions

This item of the outline is simply a list. As written, volumes and boundaries are listed as semantically equivalent. There is some logical objection to this; a more serious objective is that the sea surface, for example, is not a mathematical boundary, but a region, more or less well defined, which may include subsidiary regions such as the boundary layer topside, and turbulent air-water regions bottomside.

Equivalent statements can be made for the ocean floor, and for ice cover.

Another aspect of the regional division concerns turbulence paths within the hydrosphere, as well as "scattering layers". Are these to be regarded as identifiable regions, with peculiar properties, or are they automatically represented if the state variable of the ocean are given ?

Questions of this type are not entirely a matter of taste, but do in fact depend upon the physical measures employed in describing the regions. Furthermore, commonness of nomenclature among the various disciplines which concern themselves with the different regions is important in developing the unified work necessary for understanding the cross discipline of underwater sound.

We feel, therefore, that thought devoted to the organization of physical data on the earth environment can be fruitful, and that it is an interdisciplinary problem.

4.3 Geophysical Properties

This section of the outline refers to the "meat" of the data. Herein falls the quantitative description of the environment. The following appear to be the most important measurements needed:

Depth	Detailed and descriptive profiles of the ocean bottom, and of ice cover.
Velocity	Current data at depth.
Temperature	Precision measurements in thermal microstructures.
Chemical Properties	The chemical (and related physical) properties of the water just below the sea surface. A redetermination of the salt contents of various sea waters.
Biological Properties	Habits, distribution and physical properties of all forms of marine biology.

4.4 Acoustic Properties

Given the geophysical properties, we compute the acoustic properties. The overwhelmingly important lack today is concerned with the effects of the ocean bottom upon sound propagation. We can say that the bottom reflection and scattering coefficient should be determined, but what we mean is that ways must be found to describe the bottom sufficiently to predict its effects.

The second requirement is to determine the dependence of sound speed upon factors other than temperature, pressure and salinity. This should include near surface effects as well as possible associated dispersion. In connection with such a study, it would be highly desirable to relate the coefficients (temperature, etc.) of sound speed and absorption.

4.5 Parameters

Again, this is simply a list. Possibly, however, there are other parameters more suitable than these. Perhaps the impulse function is more suitable than the frequency function. We will consider this type of question below.

4.6 Sonar Properties

The great emphasis today is on detail. The words heard most often are "space-time correlation", "multiple order probability distribution", and "fluctuation", among others. Very often, one gathers that the need is for much more "data". It is important to recognize that there is a definite trend in progress, from the circumstances where average propagation loss was a decisive factor in acoustic problem, to the situation where much more must be known about the sound field. It is even more important to recognize that this trend is qualitative, in the sense that one wishes to know new kinds of things, more than it is quantitative, in the sense that more quantitative data of familiar form is required.

If more detailed understanding of the sound field is required, the measures employed are going to have to conform to some kind of conservation laws, which can provide powerful relationships vastly simplifying the description and interpretation of the field.

Conservation of energy is perhaps the most important single principle in physics, and it is curious that it has found so little application in underwater acoustics. Thus, although energy units are frequently employed in the reduction of transient phenomena, this is but a formal convenience. On the other hand, Urick[131] has shown how conservation of energy leads to a natural extension of the sonar equations to cover transient signals. One difficulty is, of course, that the acoustic energy or intensity do not obey simple differential equations. However, there may be related quantities, such as discussed by Wolf[132] which are conserved and obey a wave-like equation.

In addition to conservation, it would be highly desirable for measures to be subdivided as little as possible. A considerable loss of valuable interpretation can arise because of artificial division of the field into "reverberation", "scattering", "transmission", "target strength", and the like, even though these terms have a certain qualitative value. The single function "Impulse Response" has a unifying significance in describing the total result of applying a driving force to the ocean.

4.7 Summary

In summary, the following research, in order of importance, is recommended.

(1) Find new and pragmatic measures of the sound field.

(2) Find suitable methods for describing the acoustic effects of the ocean bottom and establish these effects.

(3) Obtain detailed profiles of the ocean bottom and of ice cover.

(4) Investigate the chemical (and related physical) properties of water just below the sea surface.

(5) Determine the dependence of sound speed upon factors other than temperature, pressure and salinity.

(6) Make a redetermination of the salt contents of various sea waters.

(7) Establish the habits, distribution and physical properties of marine biological forms.

(8) Study the organization and methodology associated with geophysical measurement.

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