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Carnegie Institute of Technology

Pittsburgh 13, Pennsylvania

GRADUATE SCHOOL of INDUSTRIAL ADMINISTRATION

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CRITICAL PATH METHOD --
A NEW TOOL FOR MANAGEMENT*

by

F. K. Levy
G. L. Thompson
J. D. Wiest

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Graduate School of Industrial Administration
Carnegie Institute of Technology
Pittsburgh 13, Pennsylvania

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Recently added to the growing assortment of quantitative tools for business decision-making is the Critical Path Method -- a powerful but basically simple technique for analyzing, planning, and scheduling large, complex projects. In essence, the tool provides a means of determining 1) which jobs or activities, of the many which comprise a project, are "critical" in their effect upon total project time, and 2) how best to schedule all jobs in the project in order to meet a target date at minimum cost. Widely diverse kinds of projects lend themselves to analysis by CPM, as is suggested in the following list of applications:

The construction of a building (or a highway, or a new plant)

Planning and launching a new product

A turn-around in an oil refinery (or other maintenance projects)

Installing and debugging a computer system

Research and engineering design projects

Scheduling ship construction and repairs

The manufacture and assembly of a large generator (or other job-lot operations)

Missile countdown procedures

Required Characteristics for Critical Path Analysis

Each of these projects has several characteristics that are essential for analysis by the Critical Path Method:

1) The project consists of a well-defined collection of jobs (or activities) which, when completed, mark the end of the project.

2) The jobs may be started and stopped independently of each other, within a given sequence. (This requirement eliminates continuous-

flow process activities, such as oil-refining, where "jobs" or operations necessarily follow one after another with essentially no slack.)

3) The jobs are ordered -- that is, they must be performed in a given sequence. (For example, the foundation of a house must be constructed before the walls are erected.)

The Method

The concept of CPM is quite simple and may best be illustrated in terms of a Project Graph. The Graph is not an essential part of CPM; computer programs have been written which permit necessary calculations to be made without reference to a graph. Nevertheless, the Project Graph is valuable as a means of depicting, visually and clearly, the complex of jobs in a project and their inter-relations. First of all, each job necessary for the completion of a project is listed with a unique identification (usually a number), the time required to complete the job, and its immediate prerequisite jobs.[1/] Then each job is drawn on the graph as a circle, with its number and time appearing within the circle. Sequence relationships are indicated by arrows connecting each circle (job) with its immediate successors, with the arrows pointing to the latter. For convenience, all circles with no predecessors are connected to a circle marked "start;" likewise, all circles with no successors are connected to a circle marked "finish."[2/] Typically, the graph then depicts a number of different "arrow paths" from start to finish. The time required to traverse each path is the sum of the times associated with all jobs on the path. The critical path (or paths) is the longest path (in time) from start to finish; it indicates the minimum time necessary to complete the entire project.[3]

In essence, the critical path is the bottleneck route. Only by finding ways to shorten jobs along the critical path can the overall project time be reduced; the time required to perform non-critical jobs is irrelevant from the viewpoint of total project time. The frequent (and costly) practice of "crashing" all jobs in a project in order to reduce total project time is thus unnecessary. Typically, about 10% of the jobs in large projects are critical. (This figure will naturally vary from project to project.) Of course, if some way is found to shorten one or more of the critical jobs, then not only will the project time be shortened but the critical path itself may shift and some previously non-critical jobs may become critical.

An Example: Building a House

A simple and familiar example should help to clarify the notion of critical path scheduling and the process of constructing a graph. The project of building a house is readily analyzed by the CPM technique and is typical of a large class of similar applications. While a contractor might want a more detailed analysis, we will be satisfied with the following list of major jobs (together with the estimated time and the immediate predecessors for each job).[4]

Job No.	Description	Immediate Predecessors	Normal Time (Days)
a)	Start	--	0
b)	Excavate and pour footers	a	4
c)	Pour concrete foundation	b	2
d)	Erect wooden frame including rough roof	c	4
e)	Lay brickwork	d	6
f)	Install basement drains and plumbing	c	1

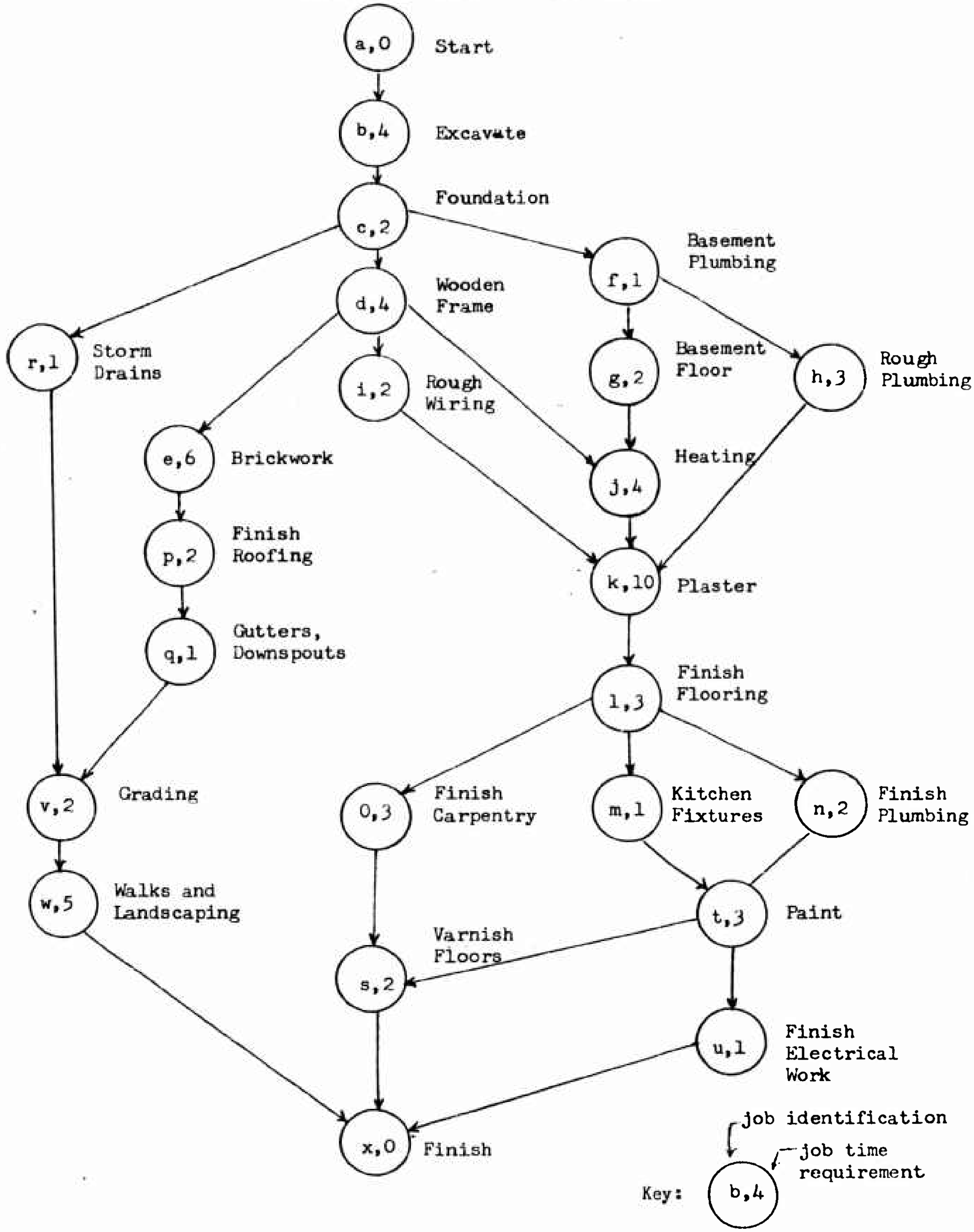
Job No.	Description	Immediate Predecessors	Normal Time (Days)
g)	Pour basement floor	f	2
h)	Install rough plumbing	f	3
i)	Install rough wiring	d	2
j)	Install heating and ventilating	d, g	4
k)	Fasten plaster board and plaster (including drying)	i, j, h	10
l)	Lay finish flooring	k	3
m)	Install kitchen fixtures	l	1
n)	Install finish plumbing	l	2
o)	Finish carpentry	l	3
p)	Finish roofing and flashing	e	2
q)	Fasten gutters and downspouts	p	1
r)	Lay storm drains for rain water	c	1
s)	Sand and varnish flooring	o, t	2
t)	Paint	m, n	3
u)	Finish electrical work	t	1
v)	Finish grading	q, r	2
w)	Pour walks and complete landscaping	v	5
x)	Finish	s, u, w	0

The column "Immediate Predecessors" determines the sequence relationships of the jobs and enables us to draw the project graph (Figure 1). In each circle the job identification (letter) appears before the comma and the job time appears after the comma.

Following the rule that a "legal" path must always move in the direction of the arrows, we could enumerate twenty-two unique paths from start to finish, with associated times ranging from a minimum of 14 days (path a-b-c-r-v-w-x) to a maximum of 34 days (path a-b-c-d-j-k-l-n-t-s-x). The latter is the critical path; it determines the overall project time and tells us which jobs are critical in their effect on this time. If the contractor wishes to complete the house in less than 34 days, it would be useless to shorten jobs not on the critical path. It may seem to him, for example,

Figure 1

Building a House: Project Graph



that the brickwork (e) delays progress, as work on a whole series of jobs (p-q-v-w) must wait until it is completed. But it would be fruitless to rush the completion of the brickwork, as it is not on the critical path and therefore is irrelevant in determining total project time.

Shortening the Critical Path

If the contractor were to use CPM techniques, he would examine the critical path for possible improvements. Perhaps he could assign more carpenters to job d (framing), reducing it from four to two days. Then the critical path would change slightly, passing through jobs f and g instead of d. Notice that total project time would be reduced only one day, even though two days had been shaved off job d. Thus the contractor must watch for possible shifting of the critical path as he affects changes in critical jobs.

Shortening the critical path requires a consideration of both engineering problems and economic questions. Is it physically possible to shorten the time required by critical jobs (by assigning more men to the job, working overtime, using different equipment, etc.)? If so, would the costs of speed-up be less than the savings resulting from the reduction in overall project time? CPM is a useful tool because it quickly focuses attention on those jobs that are critical to the project time, it provides an easy way to determine the effects of shortening various jobs in the project, and it enables the user to evaluate the costs of a "crash" program.

Two important applications of these features come to mind:

- DuPont, a pioneer in the application of CPM to construction and maintenance projects, was concerned with the amount of downtime for maintenance at its Louisville works, which produces an intermediate product in the neoprene process. Analyzing the maintenance schedule by the critical path method, DuPont engineers were able to cut downtime for maintenance from 125 to 93 hours. CPM pointed to further refinements that were expected to reduce total time to 78 hours. As a result, performance of the plant improved by about a million pounds in 1959, and the intermediate was no longer a bottleneck in the neoprene process.

- PERT, a technique closely related to the critical path method, is widely credited with helping to shorten by two years the time originally estimated for completion of the engineering and development program for the Navy's Polaris missile. By pinpointing the longest paths through the vast maze of jobs necessary for completion of the missile design, PERT enabled the program managers to concentrate their efforts on those activities that vitally affected total project time.

Even with our small house-building project, however, the process of enumerating and measuring the length of every path through the maze of jobs is tedious. A simple method of finding the critical path and, at the same time, developing useful information about each job is next described.

The Critical Path Algorithm

If the start time or date for the project is given (we denote it by S), then there exists for each job an earliest starting time (ES) which is the earliest possible time that a job can begin, given that all its predecessors are also started at their ES. And if the time to complete the job is t we can define, analogously, its early finish time (EF) to be ES + t, that is, the early start time plus the time it takes to do the job.

There is a simple way of computing ES and EF times using the project graph. It proceeds as follows: (1) mark the value of S to the left and to the right of Start; (2) consider any new unmarked job <u>all of whose predecessors have been marked</u>, and mark to the left of the new job the <u>largest</u> number marked to the right of any of its immediate predecessors (this is its early start time); (3) add to this number the job time and mark the result to the right of the job (early finish time); (4) continue until Finish has been reached, then stop. Thus at the conclusion of this calculation the ES time for each job will appear to the left of the circle which identifies it, and the EF time will appear to the right of the circle. The number which appears to the right of the last job, Finish, is the early finish time (F) for the entire project.

<u>An Illustration</u>

To illustrate these calculations we consider the following simple production process. An assembly is to be made from two parts A and B. Both parts must be turned on the lathe and B must be polished while A need not be. The list of jobs to be performed together with the predecessors of each job and the time in minutes to perform each job is given in Figure 2.

Job No.	Description	Immediate Predecessors	Time
a	Start		0
b	Get materials for A	a	10
c	Get materials for B	a	20
d	Turn A on lathe	b, c	30
e	Turn B on lathe	b, c	20
f	Polish B	e	40
g	Assemble A and B	d, f	20
h	Finish	g	0

Figure 2

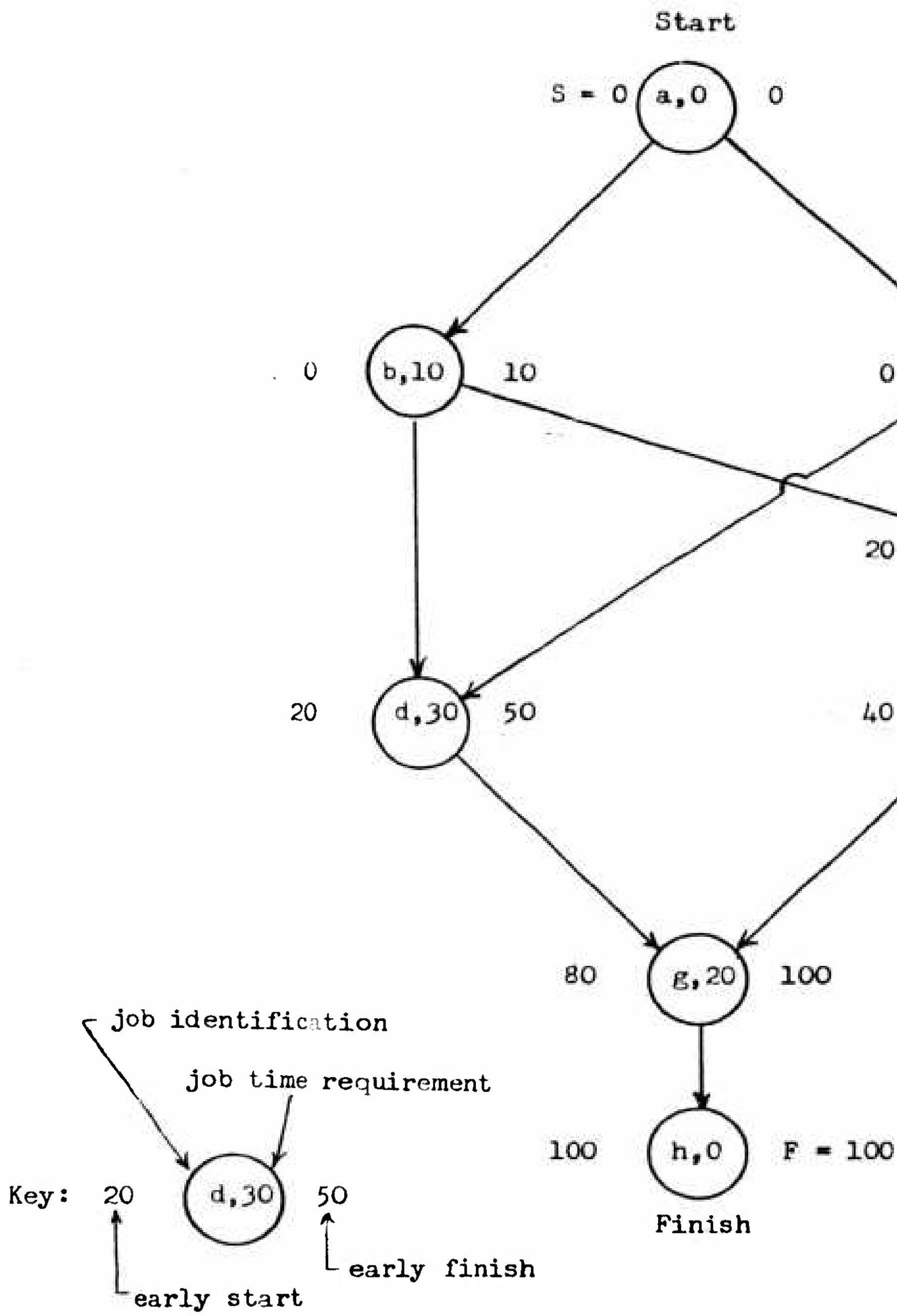
The graph for this project is shown in Figure 3. As before, the number of each job appears before the comma and its job time after the comma. Also shown on the graph are the ES and EF times for each job, assuming that the start time, S, is 0. The ES time appears to the left of the circle representing a job and the EF time appears to the right of the circle. Note that F = 100. The reader may wish to duplicate the diagram without these times and carry out the calculations for himself as a check on his understanding of the computation process described above.

Latest Start and Finish Times

Suppose now that we have a target time (T) for completing the project. In order to be feasible it is clear that T must be greater than or equal to F, the early finish time for the project. Assuming T is greater than or equal to F, we can define the concept of late finish (LF), or the latest time that a job can be finished without delaying the total project beyond its target time (T). Similarly,

Figure 3

Calculation of Early Start and Early Finish Times for Each Job



late start (LS) is defined to be LF - t, where t is the job time.

These numbers are determined for each job in a manner similar to the previous calculations except that we work from the end of the project to its beginning. We proceed as follows: (1) mark the value of T to the right and left of Finish; (2) consider any new unmarked job all of whose successors have been marked, and mark to the right of the new job the smallest number marked to the left of any of its immediate successors; (3) subtract from this number the job time and mark the result to the left of the job; (4) continue until Start has been reached, then stop. At the conclusion of this calculation the LF time for a job will appear to the right of the circle which identifies it, and the LS time for the job will appear to the left of the circle. The number which appears to the right of Start is the latest time that the entire project can be started and still finish at the target time T.

In Figure 4 we carry out these calculations for the example of Figure 2. Here T = F = 100, and we separate early start and finish and late start and finish times by semicolons so that ES; LS appears to the left of the job and EF; LF to the right. Again the reader may wish to independently check these calculations.

The Concept of Slack

Examination of Figure 4 shows that some jobs have their early start equal to late start, while others do not. The difference between a job's early start and its late start (or between early finish and late finish) is called total slack (TS). Total slack represents the maximum amount of time a job may be delayed beyond its early start without necessarily delaying the project completion time.

We earlier defined critical jobs as those on the longest path through the project. That is, critical jobs directly affect the total project time. We can now relate the critical path to the concept of slack.

Finding the Critical Path

If the target date (T) equals the early finish date for the whole project (F), then all critical jobs will have zero total slack. There will be at least one path going from Start to Finish that includes critical jobs only, i.e. the critical path.

If T is greater than F, then the critical jobs will have total slack equal to (T - F). This is a minimum value; all non-critical jobs will have greater total slack. The critical path, once again, includes only critical jobs, and hence those with minimum TS.

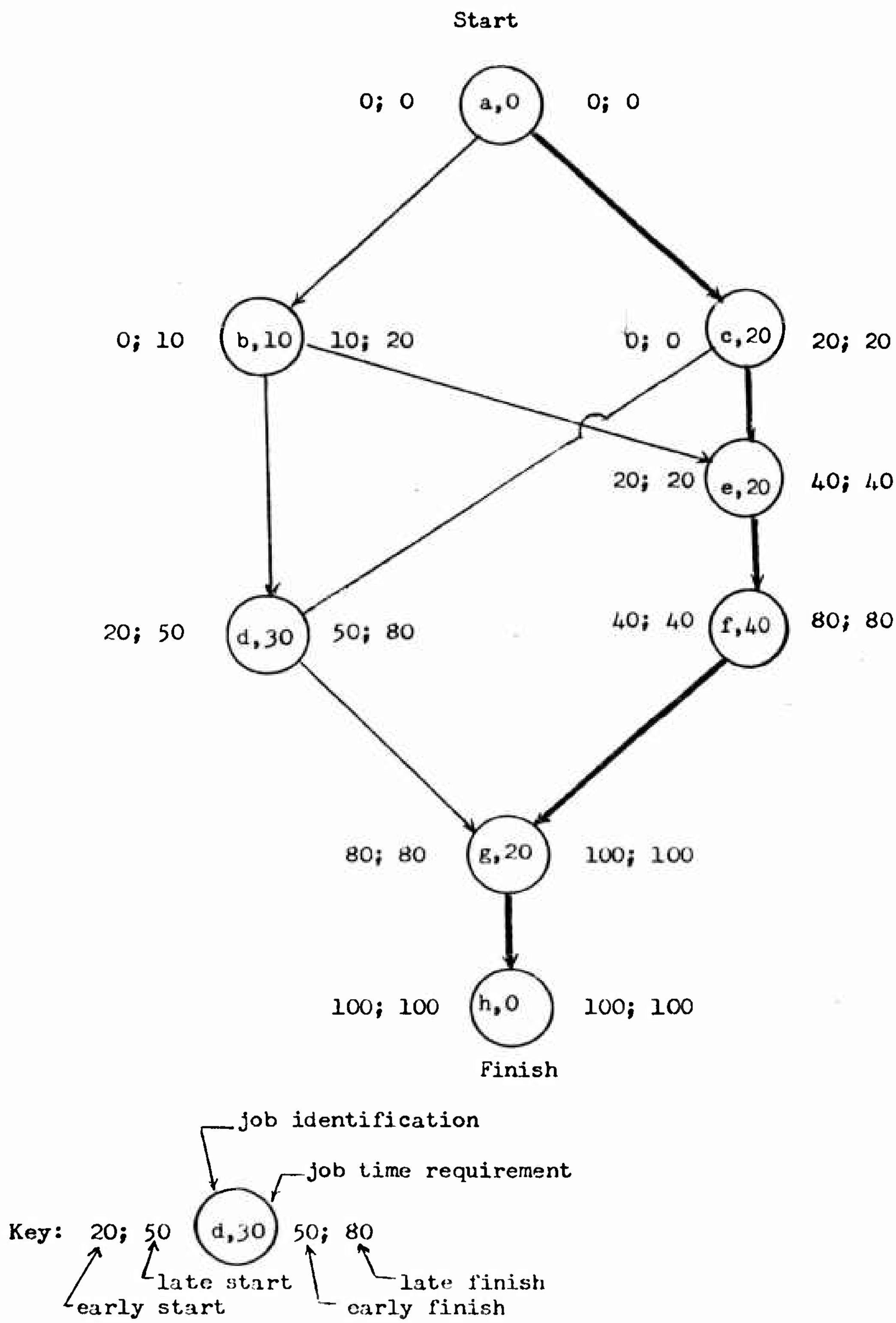
In Figure 4, the critical path is shown by darkening the arrows connecting critical jobs. In this case there is just one critical path, and all critical jobs lie on it. (In other cases there may be more than one critical path.) Note that T = F; thus the critical jobs have zero total slack. Job b has TS = 10, and job d has TS = 30; either or both of these jobs could be delayed by these amounts without delaying the project.

Free Slack

Another kind of slack is worth mentioning. Free Slack (FS) is the amount a job can be delayed without delaying the early start of any other job. A job with positive total slack may or may not also have free slack, but the latter never exceeds the former. For purposes of

Figure 4

Calculation at Late Start and Late Finish Times for Each Job



computation, the free slack of a job is defined as the difference between the job's early finish time and the earliest of the early start times of all its immediate successors. Thus in Figure 4, job b has FS = 10, and job d has FS = 30. All other jobs in Figure 4 have 0 free slack.

The Significance of Slack

When a job has zero total slack, its scheduled start time is automatically fixed (early start equals late start); to delay the calculated start time is to delay the whole project. Jobs with slack, however, allow the scheduler some discretion in setting their start times. This flexibility can usefully be applied to smoothing work schedules. Peak loads that develop in a particular shop (or on a machine, or within an engineering design group, to cite other examples) may be relieved by shifting jobs on the peak days to their late starts. Slack allows this kind of juggling without affecting project time.[5/]

Free slack can be used effectively at the operating level. For example, if a job has free slack, the foreman may be given some flexibility in deciding when to start the job. Even if he delays the start by an amount equal to (or less than) the free slack, the delay will not affect the start times or slack of succeeding jobs (which is not true of jobs that have no free slack). For an illustration of these notions, we return to our house-building example.

Back to the Contractor

In Figure 7 we reproduce the diagram of house building jobs, marking ES; LS to the left and EF; LF to the right of each job.

We assume that construction begins on day zero and must be completed by day 37. Total slack for each job is not marked, since it is evident as the difference between the pairs of numbers ES-LF or EF-LF. However, jobs that have positive free slack are so marked. There is one critical path, which is shown darkened in the diagram. All critical jobs on this path have three days total slack.

Several observations can be drawn immediately from the diagram:

1) The contractor could delay starting the house three days and still complete it on schedule (barring unforeseen difficulties). This would reduce the total slack of all jobs by three days, and hence reduce TS for critical jobs to zero.

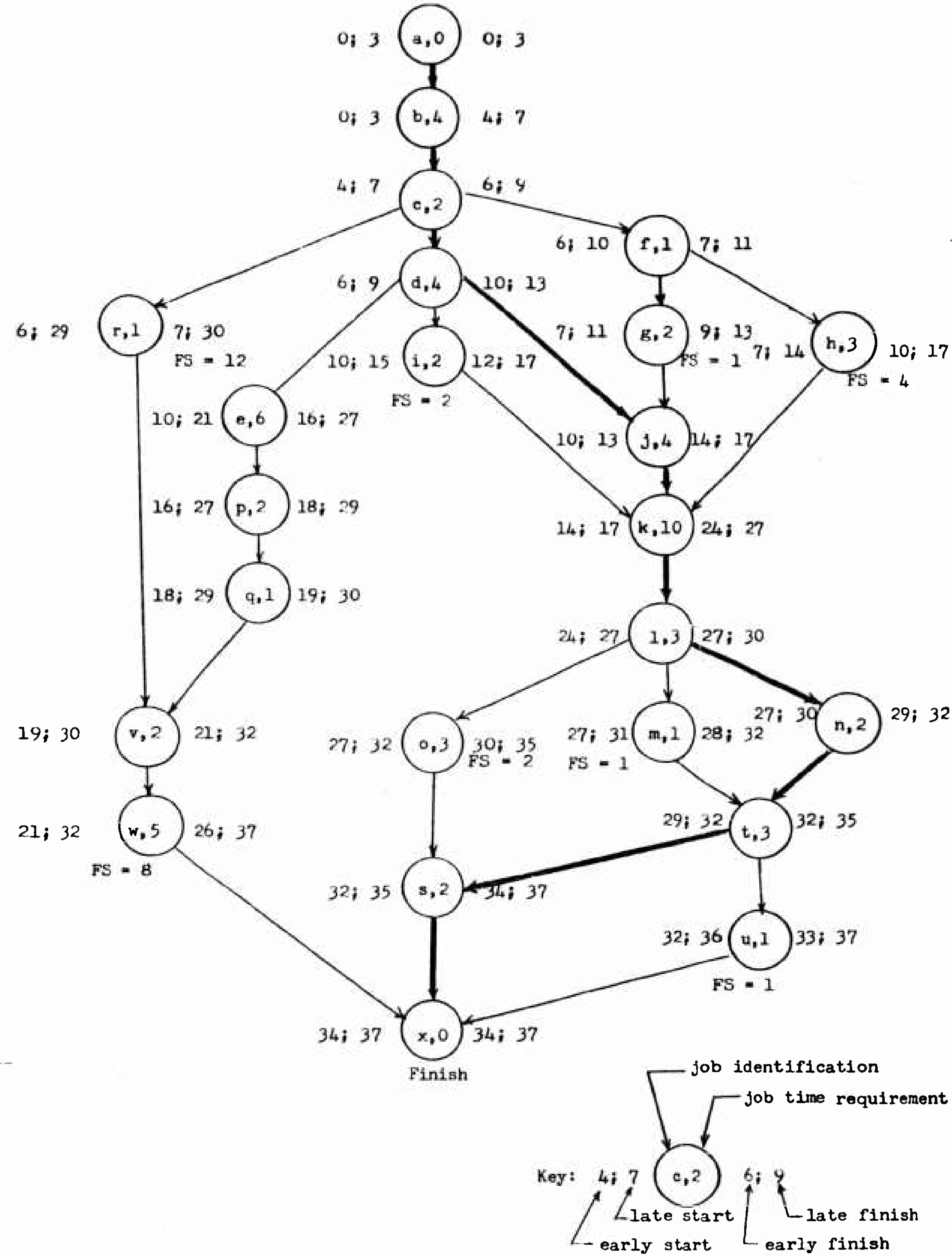
2) Several jobs have free slack. Thus the contractor could delay the completion of rough wiring by two days, the basement floor by one day, rough plumbing by four days, the storm drains by 12 days, and so on -- without affecting succeeding jobs.

3) The series of jobs e (brickwork), p (roofing), q (gutters), v (grading) and w (landscaping) have a comfortable amount of total slack (nine days). The contractor can use these (and other slack jobs) as "fill in" jobs for workers who become available when their skills are not needed for currently critical jobs. This is a simple application of workload smoothing: juggling the jobs with slack in order to reduce peak demands for certain skilled workers or machines.

If the contractor were to affect changes in one or more of the critical jobs, the calculations would have to be performed again. This he can easily do; but in large projects with complex sequence relationships, hand calculations are considerably more difficult and liable to error.

Figure 7

Project Graph with Start and Finish Times



Computer programs have been developed, however, for calculating ES, LS, EF, LF, TS, and FS for each job in a project, given the set of immediate prerequisites and the job times for each job.[6]

Data Collection Problems with CPM

Information concerning job times and predecessor relationships is gathered, typically, by shop foreman, scheduling clerks, or others closely associated with a project. It is conceivable that several kinds of errors may occur in such job data:

(1) The estimated job times may be in error.

(2) The predecessor relationship may contain cycles; e.g. job A is a predecessor for B, B is a predecessor for C, and C is a predecessor for A.

(3) The list of prerequisites for a job may include more than the immediate prerequisites; e.g. job A is a predecessor of B, B is a predecessor of C, and A and B are predecessors of C.

(4) Some predecessor relationships may be overlooked.

(5) Some predecessor relationships may be listed that are spurious.

We first discuss errors of type (1). An accurate estimate of total project time depends, of course, upon accurate job time data. CPM eliminates the necessity (and expense) of careful time studies for <u>all</u> jobs, however. Instead the following procedure can be used. Given rough time estimates, construct a CPM graph of the project. Then those jobs that are on the critical path (together with jobs that have very small total slack, indicating that they are nearly critical) can be more closely checked, their times re-estimated, and another CPM graph constructed with the refined data. If the critical path has changed to include jobs still having rough time estimates, then the process is repeated. In many projects studied, it has been found that only a small fraction of jobs are critical; so it is likely that refined time studies will be needed for relatively few jobs in a project in order to arrive at a reasonably accurate estimate of the total project time. CPM thus can be used to reduce the problem of type (1) errors at a small total cost.

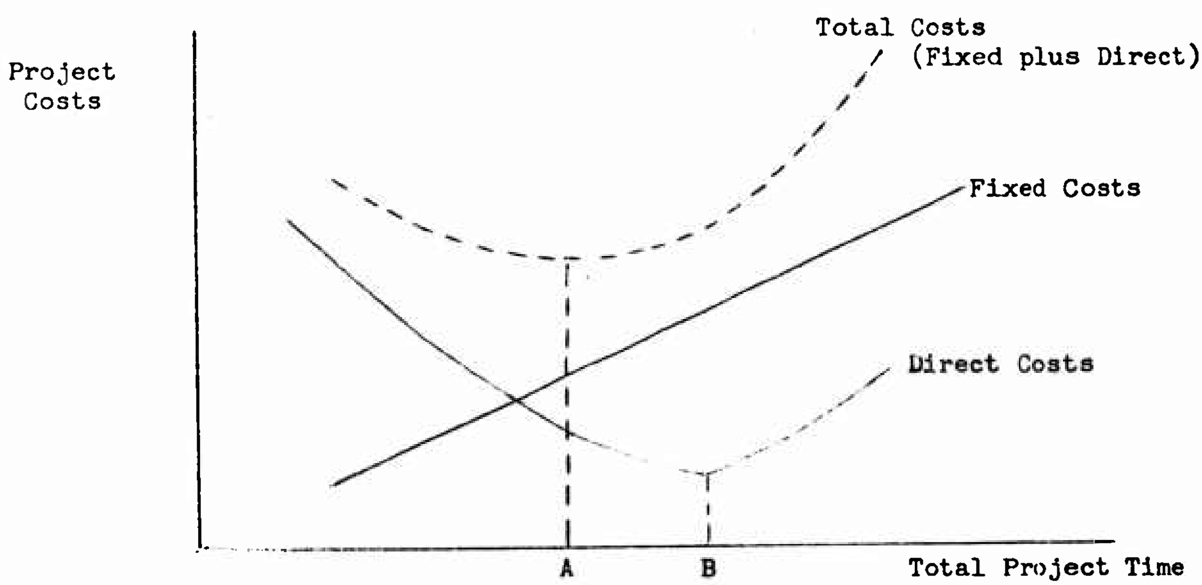
A computer algorithm[7/] has been developed to check for errors of types (2) and (3) above. The algorithm systematically examines the set of prerequisites for each job and cancels from the set all but immediate predecessor jobs. When an error of type 2 is present in the job data, the algorithm will signal a "cycle error" and print out the cycle in question.

Errors of type (4) and (5) cannot be discovered by computer routines. Instead, manual checking (perhaps by a committee) is necessary to see that prerequisites are accurately reported.

Cost Calculations with CPM

The cost of carrying out a project can be readily calculated from the job data if the cost of doing each job is included in the data. If jobs are done by crews, and the speed with which the job is done depends on the crew size, then it is possible to shorten or lengthen the project time by adding or removing men from crews. Other means for compressing job times might also be found; but any speed-up is likely to carry a price tag. Suppose that we assign to each job a "normal time" and a "crash time" and also calculate the associated costs necessary to carry the job in each time. If we want to shorten the project, we can assign some of the critical jobs to their crash time, and compute the corresponding direct cost. In this way it is possible to calculate the cost of completing the project in various total times, with the direct costs increasing as the overall time decreases.[8]

Added to direct costs are certain overhead expenses which are usually allocated on the basis of total project time. Fixed costs per project thus decrease as project time is shortened. A combination of fixed and direct costs as a function of total project time would probably appear as follows:



The minimum total cost (point A) would likely fall to the left of the minimum point on the direct cost curve (point B) indicating that the optimum project time is somewhat shorter than an analysis of direct costs only would indicate.

Other economic factors, of course, can be included in the analysis. For example:

A large chemical company starts to build a plant for producing a new chemical. After the construction schedule and completion date are established, an important potential customer indicates a willingness to pay a premium price for the new chemical if it can be made available earlier than scheduled. The chemical producer applies techniques of CPM to its construction schedule and calculates the additional costs associated with "crash" completion of jobs on the critical path. With a plot of costs vs. total project time, the producer is able to select a new completion date such that the increased costs are met by the additional revenue offered by the customer.

Conclusion

For the manager of large projects, CPM is a powerful and flexible tool, indeed, for decision making.

•It is useful at various stages of project management, from initial planning or analyzing of alternative programs, to scheduling and controlling the jobs (activities) that comprise a project.

•It can be applied to a great variety of project types (from our house-building example to the vastly more complicated design project for the Polaris) and at various levels of planning (from scheduling jobs in a single shop, or shops in a plant, to scheduling plants within a corporation).

•In a simple and direct way, it displays the interrelations in the complex of jobs that comprise a large project.

•It is easily explainable to the layman by means of the project graph. Data calculations for large projects, while tedious, are not difficult, and can readily be handled by a computer.

•It pinpoints attention to the small sub-set of jobs that are critical to project completion time, for more accurate planning and more precise control.

•It enables the manager to quickly study the effects of "crash" programs and to anticipate potential bottlenecks that might result from shortening certain critical jobs.

•It leads to reasonable estimates of total project costs for various completion dates, which enable the manager to select an optimum schedule.

Because of the above characteristics of CPM -- and especially its intuitive logic and graphic appeal -- it is a decision making tool which can find wide appreciation at all levels of management.[9/] Its use and an understanding of its operations are not limited to the skilled technician. The project graph helps the foreman to understand the sequencing of jobs and the necessity of pushing those that are critical. For the manager concerned with day-to-day operations in all departments, CPM enables him to measure progress (or lack of it) against plans and to quickly take appropriate action when needed. The underlying simplicity of CPM and its ability to focus attention on crucial problem areas of large projects make it an ideal tool for the top manager, on whose shoulders falls the ultimate responsibility for overall planning and coordination of such projects in the light of company-wide objectives.

FOOTNOTES

1/ For convenience in graphing, and as a check on certain kinds of data errors, the jobs may be arranged in "technological order," which means that no job appears on the list until all of its immediate predecessors (and hence all of its predecessors) have already been listed. Technological ordering is impossible if a cycle error exists in the job data (e.g., job A precedes B, B precedes C, and C precedes A).

2/ The "start" and "finish" circles may be considered pseudo jobs of zero time length.

3/ The above method of depicting a project graph differs in some respects from the representation used by James E. Kelley, Jr., and Morgan R. Walker who, perhaps more than anyone else, were responsible for the initial development of CPM. (For an interesting account of its early history, see their paper, "Critical-Path Planning and Scheduling," reported in Proceedings of the Eastern Joint Computer Conference, Boston, December 1-3, 1959.)

In the widely-used Kelley-Walker form, a project graph is just the opposite of that described above: jobs are shown as arrows, and the arrows are connected by means of circles (or dots) that indicate sequence relationships. Thus all immediate predecessors of a given job connect to a circle at the tail of the job arrow, and all immediate successor jobs eminate from the circle at the head of the job arrow. In essence, then, a circle marks an event: the completion of all jobs leading into the circle. Since these jobs are the immediate prerequisites for all jobs leading out of the circle, they must all be completed before <u>any</u> of the succeeding jobs can begin.

In order to accurately portray all predecessor relationships, "dummy jobs" must often be added to the project graph in the Kelley-Walker form. (For a detailed explanation of dummy jobs, see James E. Kelley, Jr., "Critical-Path Planning and Scheduling: Mathematical Basis," <u>Operations Research</u>, 301-302 (May-June, 1961). The method described above by the present authors avoids the necessity (and complexity) of dummy jobs, is easier to program for a computer, and seems more straightforward in explanation and application.

4/ The job list and project graph for the house-building example were developed by Peter R. Winters.

5/ A method for smoothing operations in a job shop, based on CPM and the use of slack, is reported by the authors in "Multi-Ship, Multi-Shop Production Smoothing Algorithm," <u>Naval Logistics Research Quarterly</u>, 9, March, 1962.

6/ An algorithm upon which one such computer program is based is discussed in a paper by the authors, "Mathematical Basis for the Critical Path Method," ONR No. 86, Graduate School of Industrial Administration, Carnegie Institute of Technology, 1962.

7/ See "Mathematical Basis for the Critical Path Method," Ibid.

8/ Algorithms for carrying out these cost calculations are discussed by Kelley in "Critical-Path Planning and Scheduling," Ibid.

9/ A. Charnes and W. W. Cooper have given a network in[...] of CPM, thus relating the technique to linear programming wi[...] extensive literature and richly developed managerial applicat[...] interpretations. See their paper, "A Network Interpretation[...] Directed Sub-Dual Algorithm for Critical Path Scheduling," f[...] in the Journal of Industrial Engineering.