Report Title: Feasibility Study of a Variable Firing Rate Function Generator for Analog Computers

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Approved: H. F. Hawthorne

Electronic Engr (Instr)

Chief, Res and Dev Div
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ABSTRACT

A feasibility study is presented of a function generator to simulate the firing pulses of an externally powered machine gun during the analog computer analysis of the recoil motion of the gun. The function generator has a rate that can be varied to simulate the change in firing rate that occurs during the firing acceleration interval of the operational cycle of the weapon. A heterodyne system with a voltage-controlled oscillator was one of the systems studied and found to be impractical. The second system was an electronic time interval generator. This system was found to be the most practical. A variable firing rate function generator can be developed to reproduce with reasonable accuracy the performance of an externally driven weapon during the acceleration period.
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SUBJECT
Improvement in Analog Simulation of Externally Driven Weapons

OBJECT
Feasibility study for a swept-frequency sub-audio oscillator.

CONCLUSION
A variable firing rate function generator can be developed to reproduce with reasonable accuracy the performance of an externally driven weapon during the acceleration period.
1. **SUBJECT**

Improvement in Analog Simulation of Externally Driven Weapons

2. **INTRODUCTION**

In order that the analog computer analysis of the recoil motion of an externally driven weapon be correct, it is necessary to simulate the change in firing rate which occurs during the weapon acceleration interval. It is recognized that this could be done in several ways. The firing impulse could be introduced manually by the computer operator or it could be introduced by some mechanical method. Neither of these methods seemed satisfactory from the standpoint of accuracy, flexibility, and repeatability.

The possibility of using a voltage-controlled oscillator for this purpose was considered. Initially, a heterodyne system was studied. One frequency was to be time-varied by a predetermined control signal shaped so that the variation between beat frequency cycles would simulate the change in firing rate. In essence, this is the condition of frequency modulation. Although this appears straightforward, several other aspects must be considered.

During the quiescent state a beat signal should not be evident, requiring that the oscillators be "locked" in phase. The condition of the controlled oscillator should always be the same when a control signal is applied to maintain the relationship between the start of motion and the time at which the first round is fired. Also, the generation of the control signal presents some additional problems. Finally the sub-audio oscillator output voltage would have to be shaped into a trigger pulse to drive the normal $F(t)$ impulse generator used in the computer. All this complexity makes the use of the heterodyne system for this express purpose impractical.

As a result of the critical review of the heterodyne system, a better understanding of the requirements to be imposed on this function generator was reached. This led to the conclusion that the use of an electronic time interval generator would be advantageous. Of the several types available, the phanastron was chosen. It can provide a very large ratio of time intervals; the generated time interval is a linear function of the applied control signal.

3. **RESULTS**

An analysis of the recycling action will not be given here since this can be found in most of the current texts for electronic waveform generators. During the generation of an interval, the phanastron behaves as an integrator; this portion of its operation will be analyzed here.

1. **MIT Radiation Series, Vol. 19, "Waveforms"**
The general expression for the output voltage is:

\[ v = V_0 - \frac{1}{R_1C_1} \int_{t_1}^{t_2} E_m f(t) \, dt \]

\( E_m f(t) \) = control voltage  
\( R_1C_1 \) = integrator time constant  
\( V_0 \) = initial value of output voltage  
\( v \) = instantaneous value of output voltage

As it is used here, the phanastron will recycle several times before the control voltage reaches a steady state value. Therefore, equation (1) should be rewritten to include a term representing the control voltage at the instant a new cycle is started.

\[ v = V_0 - \frac{1}{R_1C_1} \int_{t_1}^{t_2} e_i \, dt - \frac{1}{R_1C_1} \int_{t_1}^{t_2} (E_m - e_i)f(t) \, dt \]

where \( e_i = E_m f(t_1) \)

To prove the validity of the analysis, known parameters are substituted in equation (2). The cycle times are calculated and compared with measured values.

\[ f(t) = 1 - e^{-t/R_2C_2} \]

\[ v = V_0 - e/R_1C_1 \int_{t_1}^{t_2} dt - (E-e)/R_1C_1 \int_{t_1}^{t_2} (1 - e^{-t/R_2C_2}) \, dt \]

Solving (3) yields

\[ \frac{V_0R_1C_1}{(E-e)R_2C_2} - \frac{E(t_2-t_1)}{(E-e)(R_2C_2)} - e^{-t_2/R_2C_2} + e^{-t_1/R_2C_2} = 0 \]

The numerical solution is given in Appendix A. Comparison between the calculated and measured values gives an error not exceeding five per cent (Appendix B) for an individual cycle. Therefore, it is shown that the performance of the phanastron with a variable control voltage is readily predictable and that precision components are not required where moderate accuracy is acceptable. Appendix C shows pictorially the control and output signals.

The major disadvantage is the use of an analog control voltage to sweep the oscillator. This could be overcome by using an incremental control voltage that could be advanced during the recycling time of the phanastron. However, this aspect was not investigated.
4. CONCLUSIONS

In summary then, it is feasible to construct a variable firing rate function generator for the analog computer, and though the original heterodyne approach was discarded the second approach is practical.
Appendix A - Solution of Control Equation (3)
Appendix B - Comparison of Time Intervals
Appendix C - Typical Performance Curves
Appendix D - Drawing SA-C43345
Appendix E - Distribution
SOLUTION OF CONTROL EQUATION (3)

\[ v = V_0 - \frac{e}{R_1C_1} \int_{t_1}^{t_2} dt - \frac{(E-e)}{R_1C_1} \int_{t_1}^{t_2} (1 - e^{-\frac{t}{R_2C_2}}) dt \]

\[ v = V_0 - \frac{(E-R_1C_1)(t_2-t_1)}{R_1C_1} - \frac{(E-e)R_2C_2/R_1C_1}{(E-e)(R_2C_2)} \left( e^{-\frac{t_2}{R_2C_2}} - e^{-\frac{t_1}{R_2C_2}} \right) \]

A cycle ends when \( v = 0 \), therefore

\[ 0 = V_0 \frac{R_1C_1}{(E-e)(R_2C_2)} - \frac{E(t_2-t_1)}{(E-e)(R_2C_2)} - \frac{e^{-\frac{t_2}{R_2C_2}}}{R_2C_2} + \frac{e^{-\frac{t_1}{R_2C_2}}}{R_2C_2} \]

Solving (4) for \( t_2 \),

let \( t_1 = 0 \)

\[ R_2C_2 = 15 \text{ seconds} \]
\[ R_1C_1 = \frac{1}{3} R_2C_2 \]

\[ t_2 \]
\[ V_0 = 200 \text{ volts} \]
\[ E = 170 \text{ volts} \]
\[ e = E(1 - e^{-\frac{t_1}{R_2C_2}}) + 30 \]
\[ x = \frac{t_2}{R_2C_2} \]

\[ 1.477 - 1.212x - \frac{x}{.87} = 0 \]
\[ x = .87 \]
\[ t_2 = 13.9 \text{ seconds} \]

This procedure is continued for consecutive values of \( t \).
## Comparison of Time Intervals

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<th>Measured</th>
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<tr>
<td>t2</td>
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<tr>
<td>t3</td>
<td>20.8</td>
<td>&quot;</td>
<td>20.2</td>
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<td>t4</td>
<td>27.0</td>
<td>&quot;</td>
<td>26.4</td>
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<td>t5</td>
<td>33.1</td>
<td>&quot;</td>
<td>32.2</td>
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<td>t6</td>
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<td>38.8</td>
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<tr>
<td>t7</td>
<td>45.2</td>
<td>&quot;</td>
<td>44.8</td>
</tr>
<tr>
<td>t8</td>
<td>51.2</td>
<td>&quot;</td>
<td>49.6</td>
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TYPICAL PERFORMANCE CURVES

Upper trace: Generator output - 50v/div
Lower trace: Control voltage - 100v/div
Time scale: 5 sec/div
Appendix D

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Drawing SA-C43345
APPENDIX E

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