NOTICE: When government or other drawings, specifications or other data are used for any purpose other than in connection with a definitely related government procurement operation, the U. S. Government thereby incurs no responsibility, nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use or sell any patented invention that may in any way be related thereto.

Technical Report No. 32-233

Feasibility of Interstellar Travel

Dwain F. Spencer
Leonard D. Jaffe

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

March 15, 1962

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CONTRACT NO. NAS 7-100

Technical Report No. 32-233

FEASIBILITY OF INTERSTELLAR TRAVEL

Dwain F. Spencer
Leonard D. Jaffe

Charles W. Cole, *Chief*
Engineering Mechanics Division

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA
March 15, 1962

CONTENTS

TABLES

FIGURES

ABSTRACT

The feasibility of interstellar flight is discussed. Mathematical equations for single-stage and multistage rocket propulsion are developed; velocity data and transit times are presented. The conclusions indicate that interstellar travel is theoretically feasible by utilizing known staged nuclear-energy systems.

I. INTRODUCTION

The earliest studies of relativistic rocket mechanics by Ackeret (Ref. 1 and 2), Tsien (Ref. 3), Bussard (Ref. 4), and others made two implicit assumptions that severely limit performance of the rockets considered. They assumed that nuclear-energy rockets are limited to a single stage and that the available energy corresponds to a fixed fraction of the final vehicle mass. The latter assumption apparently arose from the thought that spent nuclear fuel would either be retained on board or dumped, rather than exhausted at high velocity. These assumptions are neither necessary or desirable.

More recently, interstellar travel has been considered by Sanger (Ref. 5) and Stuhlinger (Ref. 6). They realized that the limitation regarding the amount of energy available being a function of the propellant mass rather than the final mass was unnecessary; however, they did not consider staging the vehicles as is done with chemical rockets. They concluded, therefore, that interstellar travel using nuclear reactions as an energy source is impossible because of fundamental limitations on the amount of energy available for rocket propulsion. In contradiction, the analysis presented in this report shows that nuclear fission or fusion rockets can be considered for interstellar travel.

II. BASIC EQUATIONS FOR SINGLE-STAGE ROCKET

The basic equations for single-stage rocket propulsion at relativistic velocities were derived by Ackeret and have been utilized by subsequent workers. Ackeret's work is inexact, however, in that he considers the rest mass exhausted to equal the rest mass of fuel consumed. More exactly, the rest mass of fuel consumed is

$$M_f = M_{ex} + \epsilon M_f \tag{1}$$

where M_{ex} = rest mass exhausted and ϵM_f = rest mass of fuel converted to kinetic energy. The initial rest mass of the vehicle is

$$M_0 = M_f + M_b \tag{2}$$

where M_b is the rest mass of the vehicle at burnout.

Let

$$\chi = \frac{M_b}{M_f} \tag{3}$$

Then

$$M_0 = M_f(1 + \chi) \tag{4}$$

The stage mass ratio is

$$\delta = \frac{M_0}{M_b} = \frac{1 + \chi}{\chi} \tag{5}$$

This is simply the result obtained with a chemical propulsion system.

To discuss the exterior energetics of the vehicle, a coordinate system fixed in space and a system relative to the vehicle may be used (Ref. 4, 5 and 6). Let u represent the velocity of the vehicle relative to the stationary system, v the velocity of the exhaust relative to the stationary system, and w the exhaust velocity relative to the vehicle. The exhaust velocity w is determined by the particular fuel employed and is taken as a constant. By employing conservation of momentum, mass, and energy, and the Lorentz addition of velocities, Ackeret showed that the final vehicle velocity is given by

$$\frac{u}{c} = \frac{\delta^{2w/c} - 1}{\delta^{2w/c} + 1} \tag{6}$$

A relationship between the exhaust velocity and the fraction of fuel converted to energy gives the desired form for the final velocity. In the coordinate system moving with the vehicle, the kinetic energy of the exhaust is

$$T_{ex} = \frac{dM_{ex}c^2}{\sqrt{1 - \frac{w^2}{c^2}}} - dM_{ex}c^2 \tag{7}$$

The kinetic energy results from the conversion of rest mass to energy within the engine. For every increment dM_{ex} exhausted, ϵdM_f is converted to energy and from Eq. (1) this is

$$\epsilon dM_f = \left(\frac{\epsilon}{1-\epsilon}\right) dM_{ex} \tag{8}$$

Then Eq. (7) has the form

$$\left(\frac{\epsilon}{1-\epsilon}\right) dM_{ex}c^2 = \left[\frac{c^2}{\sqrt{1 - \frac{w^2}{c^2}}} - c^2\right] dM_{ex} \tag{9}$$

Solution for w/c gives

$$\frac{w}{c} = \sqrt{\epsilon(2-\epsilon)} \tag{10}$$

or, in terms of the specific impulse

$$I = \frac{c}{g}\sqrt{\epsilon(2-\epsilon)} \tag{11}$$

Equations (10) and (11) were also given by Sanger and Huth (Ref. 7). The final form of Eq. (6) for a one-stage vehicle is

$$\frac{u}{c} = \frac{\left(\frac{1+\chi}{\chi}\right)^{2\sqrt{\epsilon(2-\epsilon)}} - 1}{\left(\frac{1+\chi}{\chi}\right)^{2\sqrt{\epsilon(2-\epsilon)}} + 1} \tag{12}$$

III. BASIC EQUATIONS FOR MULTISTAGE ROCKET

The kinematics of multistage relativistic rockets have been treated only by Subotowicz (Ref. 8); however, he did not examine energy requirements. As shown in Ref. 8, the burnout velocity u_n for the nth stage is given by

$$\frac{u_n}{c} = \frac{\prod_{i=1}^{n} \delta_i^{2 w_i/c} - 1}{\prod_{i=1}^{n} \delta_i^{2 w_i/c} + 1} \tag{13}$$

As in the classical case (Ref. 8), optimum staging occurs for equal step mass ratios or equal step burnout fractions if each step has the same exhaust velocity. Then Eq. (13) reduces to

$$\frac{u_n}{c} = \frac{\delta^{2n\, w/c} - 1}{\delta^{2n\, w/c} + 1} \tag{14}$$

where w/c is given by Eq. (10). Then

$$\lim_{n \to \infty} \frac{u_n}{c} = 1 \tag{15}$$

for a fixed step mass ratio. Thus, if enough stages are utilized, regardless of the exhaust velocity or mass ratio per stage, it is theoretically possible to attain a final velocity near that of light.

Another important aspect in the feasibility of interstellar travel is the final payload mass which can be delivered by a particular vehicle. Consider an n-stage vehicle with stage burnout rest mass $(\chi M_f)_i$ and stage structural or dead rest mass $(\beta M_f)_i$. Then the payload mass of the ith stage

$$(M_p)_i = (\chi M_f)_i - (\beta M_f)_i = (M_0)_{i+1} \tag{16}$$

the initial mass of the $(i + 1)$th stage.

From Eq. (4)

$$M_{0_1} = M_{f_1}(1 + \chi_1) \tag{17}$$

Now

$$M_{0_2} = (\chi_1 - \beta_1) M_{f_1} \tag{18}$$

But

$$M_{0_2} = M_{f_2} + \chi_2 M_{f_2} \tag{19}$$

and

$$M_{0_3} = (\chi_2 - \beta_2) M_{f_2} \tag{20}$$

Now

$$M_{f_2} = \frac{(\chi_1 - \beta_1)}{(1 + \chi_2)} M_{f_1} \tag{21}$$

Then

$$M_{0_3} = \frac{(\chi_1 - \beta_1)(\chi_2 - \beta_2)}{(1 + \chi_1)(1 + \chi_2)} M_{0_1} \tag{22}$$

Continuation of this procedure yields the desired result

$$M_p = \left[\frac{\prod_{i=1}^{n} (\chi_i - \beta_i)}{\prod_{i=1}^{n} (1 + \chi_i)} \right] M_{0_1} \tag{23}$$

Since the step fractions β_i and the stage fractions χ_i for optimum staging should be the same for all stages, Eq. (23) reduces to

$$\frac{M_p}{M_{0_1}} \equiv \Phi = \frac{(\chi - \beta)^n}{(1 + \chi)^n} \tag{24}$$

It may be of interest to determine the maximum vehicle burnout velocity for a given dead-weight fraction β and desired over-all payload fraction Φ. Algebraic solution for χ from Eq. (24) yields

$$\chi = (\Phi)^{1/n} \left[\frac{1 + \beta \left(\dfrac{1}{\Phi} \right)^{1/n}}{1 - (\Phi)^{1/n}} \right] \tag{25}$$

Substituting in Eq. (5) gives

$$\delta = \frac{1 + \beta}{\beta + \Phi^{1/n}} \tag{26}$$

and from Eq. (14)

$$\frac{u_n}{c} = \frac{\left(\dfrac{1 + \beta}{\beta + \Phi^{1/n}} \right)^{2nw/c} - 1}{\left(\dfrac{1 + \beta}{\beta + \Phi^{1/n}} \right)^{2nw/c} + 1} \tag{27}$$

Using Eq. (10), the final burnout velocity of the n-stage vehicle in terms of over-all payload fraction, dead-weight fraction, and fraction of mass converted to energy, is

$$\frac{u_n}{c} = \frac{\left(\dfrac{1+\beta}{\beta+\Phi^{1/n}}\right)^{2n\sqrt{\epsilon(2-\epsilon)}} - 1}{\left(\dfrac{1+\beta}{\beta+\Phi^{1/n}}\right)^{2n\sqrt{\epsilon(2-\epsilon)}} + 1} \tag{28}$$

Figure 1 is a plot showing the over-all mass ratio required versus energy fraction ϵ for various final vehicle velocity ratios u_n/c. The over-all mass ratio is given by

$$\Delta = \delta^n = \left(\frac{1+\beta}{\beta+\Phi^{1/n}}\right)^n \tag{29}$$

IV. EXAMPLES OF VELOCITIES AND TRANSIT TIMES

Following are some examples of velocities and transit times which may be attainable. The fraction of mass converted to energy by uranium fission is about 7×10^{-4}; by deuterium fusion, 4×10^{-3}. Table 1, obtained from Fig. 1, shows the over-all mass ratio Δ necessary to reach various velocities u_n/c for a fission rocket with $\epsilon = 7 \times 10^{-4}$. Table 2 shows the necessary mass ratio for a fusion rocket with an energy conversion fraction $\epsilon = 4 \times 10^{-3}$. If deceleration at the destination is required, the mass ratios must be squared; for a two-way trip with deceleration at each end, the mass ratios must be raised to the fourth power. These values are also shown in the tables.

Mass ratios of 10^3 to 10^6 seem quite feasible in principle. For unmanned probes, one-way trips without deceleration may well be adequate. Feasible velocity ratios corresponding to the mass ratios mentioned above are then 0.3 to 0.5 for uranium fission and 0.6 to 0.8 for deuterium fusion. The corresponding travel times depend on the acceleration used. If 1-g acceleration could be achieved, relativistic velocities would be reached within a few months and the spacecraft could then coast to its destination at the velocity indicated above. To reach Alpha Centauri at 4.3 light years, the transit times would be 9 to 14 years with a fission rocket and 6 to 7 years with a fusion rocket.

For two-way trips with deceleration at each end, as might be required for manned missions or sample returns, a multistage fission rocket could reach about $u_n/c = 0.13$. However, on this basis, the 8.6-light-year round trip to Alpha Centauri would require 66 years. With a deuterium fusion rocket, u_n/c of 0.3 seems attainable; the round trip to Alpha Centauri would then require 29 years.

Figures 2 to 4 show the attainable vehicle burnout velocity as a function of the number of stages for payload ratios of 10^{-1}, 10^{-3}, and 10^{-5} for a fusion rocket with $\epsilon = 4 \times 10^{-3}$. An interesting feature of these curves is the fact that a five-stage vehicle attains nearly the maximum possible velocity increment for a particular payload fraction.

Figure 5 displays the effect of the dead-weight fraction β for a five-stage fusion rocket at various payload ratios. The relatively small effect of the dead-weight fraction upon performance is a very significant feature in the design of this type of system. It indicates that a strong effort should be made to obtain 100% burnup even at the cost of additional structural weight.

V. CONCLUSION

It is concluded that if staged nuclear-energy rockets are used, relativistic velocities can be attained with reasonable mass ratios. Improvements in technology would be required, but no energy sources beyond the known fission and fusion reactions need be employed.

NOMENCLATURE

I	specific impulse
M_b	rest mass at burnout
M_{ex}	rest mass exhausted
M_f	rest mass of fuel consumed
M_p	payload mass
$(M_p)_i$	payload mass of the ith stage
M_0	initial rest mass
$(M_0)_{i+1}$	initial mass of the $(i + 1)$th stage
T_{ex}	kinetic energy of exhaust
u	vehicle velocity relative to stationary system
u_n	burnout velocity for nth stage
v	exhaust velocity relative to stationary system
w	exhaust velocity relative to vehicle
β	dead-weight fraction
$(\beta M_f)_i$	stage structural or dead rest mass for nth stage
β_i	step fractions
δ	stage mass ratio
Δ	over-all mass ratio
ϵ	energy conversion fraction
ϵM_f	rest mass of fuel converted to kinetic energy
χ	burnout fraction
$(\chi M_f)_i$	stage burnout rest mass for nth stage
χ_i	stage fractions
Φ	over-all payload fraction

Table 1. Mass ratios required for fission rockets, $\epsilon = 7 \times 10^{-4}$

Fraction of light velocity u_n/c	Required over-all mass ratio Λ		
	One-way trip, without deceleration	One-way trip, with deceleration	Two-way trip, with deceleration
0.1	1.4×10^1	2.0×10^2	3.8×10^4
0.2	2.2×10^2	4.8×10^4	2.3×10^9
0.3	4.1×10^3	1.7×10^7	...
0.4	9.0×10^4	8×10^9	...
0.5	2.1×10^6	...	...
0.6	1.0×10^8	...	...

Table 2. Mass ratios required for fusion rockets, $\epsilon = 4 \times 10^{-3}$

Fraction of light velocity v_n/c	Required over-all mass ratio Δ		
	One-way trip, without deceleration	One-way trip, with deceleration	Two-way trip, with deceleration
0.1	3.0×10^0	9.0×10^0	8.1×10^1
0.2	8.9×10^0	7.8×10^1	6.2×10^3
0.3	3.3×10^1	1.1×10^3	1.1×10^6
0.4	1.1×10^2	1.2×10^4	1.5×10^8
0.5	4.4×10^2	1.9×10^5	...
0.6	2.3×10^3	5.2×10^6	...
0.7	1.6×10^4	2.6×10^8	...
0.8	2.1×10^6	...	...
0.9	1.4×10^7	...	...

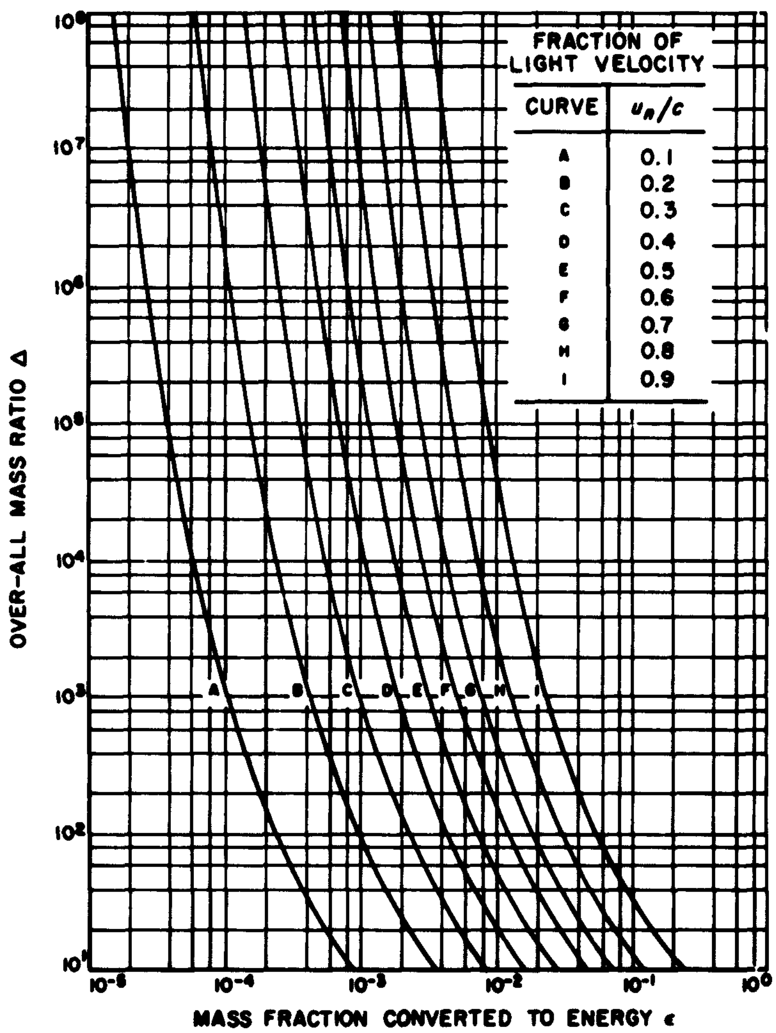

Fig. 1. Over-all mass ratio required versus energy fraction for various fractions of light velocity

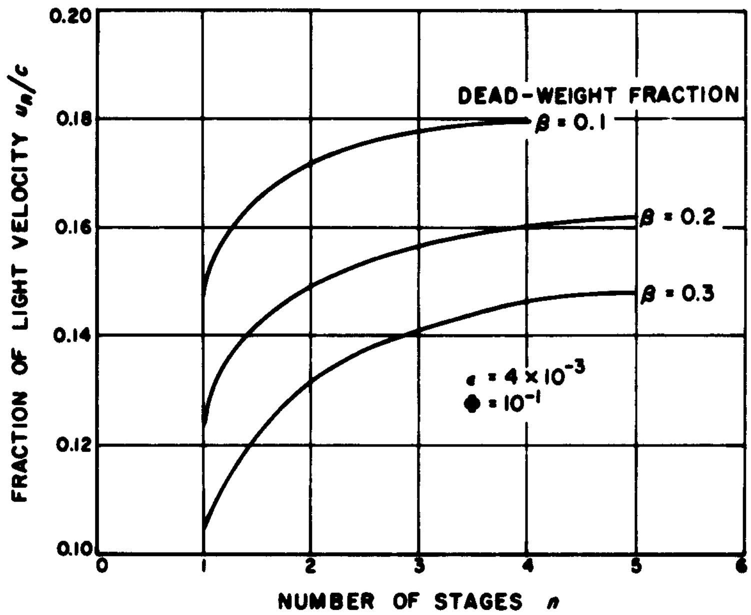

Fig. 2. Fractions of light velocity attainable for a deuterium fusion rocket versus number of stages for various dead-weight fractions (payload fraction = 10^{-1})

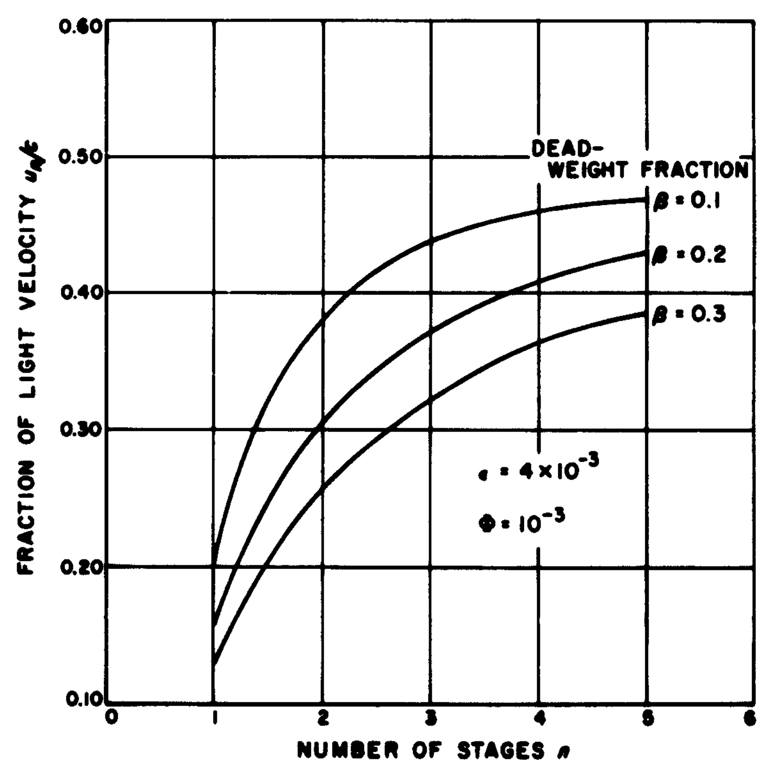

Fig. 3. Fractions of light velocity attainable for a deuterium fusion rocket versus number of stages for various dead-weight fractions (payload fraction = 10^{-3})

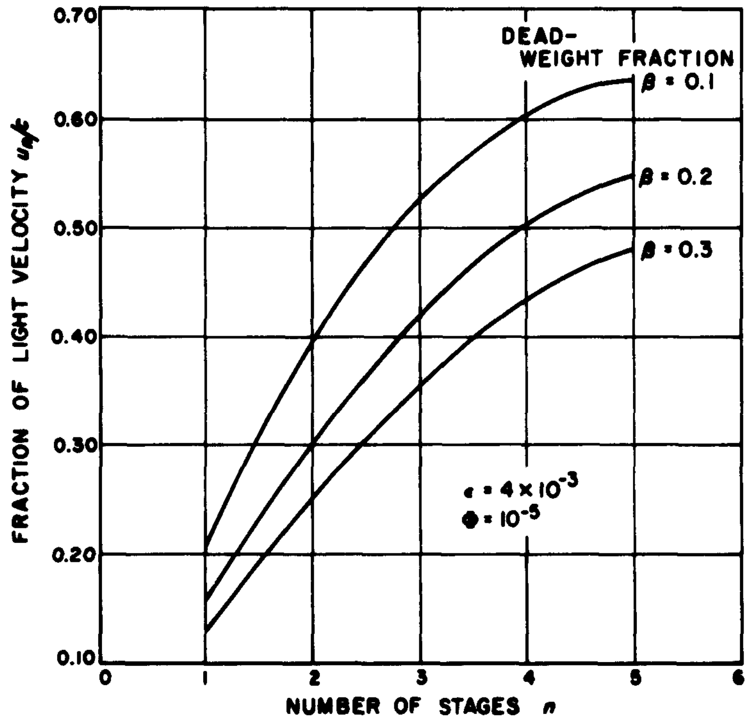

Fig. 4. Fractions of light velocity attainable for a deuterium fusion rocket versus number of stages for various dead-weight fractions (payload fraction = 10^{-5})

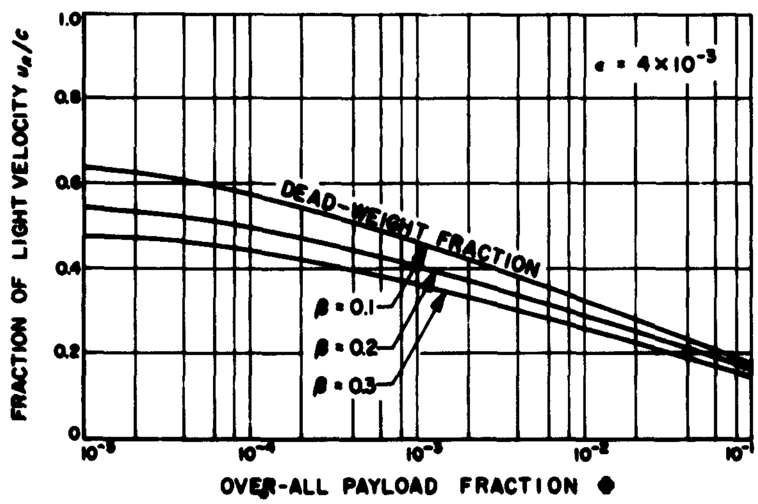

Fig. 5. Fractions of light velocity attainable for a five-stage deuterium fusion rocket versus over-all payload fractions for various dead-weight fractions

REFERENCES

1. Ackeret, J., "Theory of Rockets," *Helvetica Physica Acta,* Vol. 19, 1946, pp. 103-112.
2. .... "Theory of Rockets," *Journal of the British Interplanetary Society,* Vol. 6, 1947, pp. 116-123.
3. Tsien, H. S., "Rockets and Other Thermal Jets Using Nuclear Energy," *The Science and Engineering of Nuclear Power,* Addison Wesley, Cambridge, Vol. 11, 1949, pp. 177-195.
4. Bussard, R. W., "Galactic Matter and Interstellar Flight," *Astronautica Acta,* Vol. 6, 1960, pp. 179-194.
5. Sanger, E., "Atomic Rockets for Space Travel," *Astronautica Acta,* Vol. 6, No. 1, 1960, pp. 4-15.
6. Stuhlinger, E., "Photon Rocket Propulsion," *Astronautics,* Vol. 4, No. 10, October 1959, pp. 36, 69, 72, 74, 76, 78.
7. Huth, J., "Relativistic Theory of Rocket Flight with Advanced Propulsion Systems," *ARS Journal No. 30,* 1960, pp. 250-253.
8. Subotowicz, M., "Theorie der relativistischen n-Stufenrakete," *Proceedings of the 10th International Astronautical Congress,* London, 1959, Vol. 2, pp. 852-864.