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TABLES FOR RAPID DETERMINATION OF DAMPING AND NATURAL FREQUENCY OF A LINEAR SYSTEM

by

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ABSTRACT

A method is presented for rapid calculation of the damping and natural frequency of a galvanometer or any other linear, damped, one-degree-of-freedom system. Two equations are derived which relate the steady-state amplitude of response to the natural frequency and the damping. The solutions to these equations for a useful range of measured amplitudes are presented in tabular form. The technique involves measurements of steady-state amplitude of response at two different frequencies and is accurate to within two percent.

INTRODUCTION

Galvanometers and similar linear, damped, one-degree-of-freedom systems are frequently used as measuring and recording instruments. The damping and natural frequency of a galvanometer must be known to assess the accuracy of the data recorded by the galvanometer. A method, presented here, for determining the damping and natural frequency of a galvanometer has been found to be both accurate and extremely rapid in execution. With practice, we can estimate galvanometer damping and natural frequency at the rate of one galvanometer a minute. This particular method involves three direct measurements and the use of a table of constants computed by an IBM 7090 computer. This technique has an inherent error of less than two percent.

MATHEMATICAL ANALYSIS

We assumed that the galvanometer motion is described by the differential equation

$$\ddot{x} + 2(2\pi f)\,c\,\dot{x} + (2\pi f)^2 x = (2\pi f)^2 I \tag{1}$$

where x is galvanometer deflection,

f is galvanometer natural frequency in cps,

c is the ratio of the galvanometer damping to critical damping, and

I is current through galvanometer in units of d-c trace deflection.

If the current is a steady-state oscillation, $I = e^{2\pi i F t}$, then

$$x = Ae^{2\pi i F t}$$

is a solution to the differential equation [1] provided that

$$A = \left(1 - \frac{F^2}{f^2} + 2i\frac{F}{f}c\right)^{-1}$$

where A is the complex amplitude, and

F is the input test frequency in cps.

The magnitude of A is

$$|A| = \left[\left(1 - \frac{F^2}{f^2}\right)^2 + \left(2\frac{F}{f}c\right)^2\right]^{-1/2} \qquad [2]$$

If $|A|$ is measured at constant current for two different input frequencies F_1 and F_2, then two equations relate the parameters of natural frequency and damping to the amplitudes $|A_1|$ and $|A_2|$:

$$|A_1| = \left[\left(1 - \frac{F_1^2}{f^2}\right)^2 + \left(2\frac{F_1}{f}c\right)^2\right]^{-1/2} \qquad [3]$$

$$|A_2| = \left[\left(1 - \frac{F_2^2}{f^2}\right)^2 + \left(2\frac{F_2}{f}c\right)^2\right]^{-1/2} \qquad [4]$$

Equations [3] and [4] can be solved simultaneously for the natural frequency f, and the damping ratio c.

$$\frac{f}{F_1} = \left[\frac{\frac{F_2}{F_1} - 1}{\left(1 - \frac{1}{|A_1|^2}\right) - \frac{F_2}{F_1}\left(1 - \frac{1}{|A_2|^2}\right)}\right]^{1/4} \qquad [5]$$

$$c = \left\{\frac{1}{2} - \left[\frac{1}{4\frac{F_2}{F_1}\sqrt{\left(\frac{F_2}{F_1}\right)^2 - 1}}\right]\left[\frac{\left(\frac{F_2}{F_1}\right)^4\left(1 - \frac{1}{|A_1|^2}\right) - \left(1 - \frac{1}{|A_2|^2}\right)}{\sqrt{\left(\frac{F_2}{F_1}\right)^2\left(1 - \frac{1}{|A_1|^2}\right) - \left(1 - \frac{1}{|A_2|^2}\right)}}\right]\right\}^{1/2} \qquad [6]$$

Therefore, if the two test frequencies F_1 and F_2 and their corresponding amplitudes $|A_1|$ and $|A_2|$ are known, f and c may be determined from Equations [5] and [6].

Tables 1 and 2 show solutions of Equations [5] and [6] for the special case $F_2/F_1 = 2$.

APPLICATION

First, note that the amplitudes $|A_1|$ and $|A_2|$ in Equations [5] and [6] must be measured in terms of unit current. This unit current is that current whose amplitude will produce a static galvanometer deflection of one unit.

Establish unit current at a frequency F_0 low enough so that F_0/f is less than 0.1 (i.e., at a frequency less than one-tenth the expected natural frequency of the galvanometer). Then, the amplitude predicted by Equation [2] for damping between 0.5 and 0.7 is within 1 percent of unit amplitude. Note that if F_0/f is 0.1 and the damping ratio is 0.5, the amplitude is 1.0101. Accuracy improves with increased damping: if F_0/f is 0.1 and the damping ratio is 0.7, $|A|$ is 1.0002.

The response produced by the unit current at F_0 is noted or measured and succeeding measurements are in terms of this amplitude and at the same value of the current.

After a convenient measuring unit of response has been determined, a galvanometer may be tested in the following way.

The galvanometer is driven by an input test frequency F_1 and the amplitude produced, $|A_1|$, is measured in terms of the unit amplitude. $|A_1|$ should be between 0.95 and 1.10 times the unit amplitude to use the frequency table, Table 1, and between 0.90 and 1.10 times the unit amplitude to use the damping table, Table 2.

Next the galvanometer is driven by another input test frequency F_2 chosen as equal to $2F_1$ for calculating convenience. The amplitude produced, $|A_2|$, is measured in terms of the unit amplitude. $|A_2|$ should be between 0.40 and 0.90 times the unit amplitude to use the damping table and between 0.50 and 0.80 times the unit amplitude to use the natural frequency table.

These tables are useful only for recording instruments having damping between 0.50 and 0.70 if an accuracy of 2 percent is desired. This range of damping is generally provided for galvanometers, accelerometers, and similar instruments in an attempt to obtain maximum flat frequency response.

In general, the ranges included in both tables are large enough to accomodate the most common situations, and the values outside of the range are usually unacceptable in galvanometers.

After $|A_1|$ and $|A_2|$ have been measured in terms of the unit amplitude, the tabular value corresponding to $|A_2|$ and $|A_1|$ on the natural frequency table is f/F_1. Multiplying by F_1 gives f. The tabular value corresponding to $|A_2|$ and $|A_1|$ on the damping table is c.

The zeros at the lower left corner correspond to incompatible values of $|A_2|$ and $|A_1|$.

Brief instructions for use of the tables are in Appendix A.

CONCLUSIONS

If the damping and natural frequency tables are used acco' 'ing to instructions, a quick and simple method is available for use in calibrating galvanometers.

APPENDIX A

INSTRUCTIONS FOR USE OF TABLES FOR MEASURING DAMPING AND NATURAL FREQUENCY OF A GALVANOMETER

1. Drive the galvanometer at low frequency F_0 (frequency less than 1/10 of the nominal natural frequency of the galvanometer). Adjust the current so that the amplitude of galvanometer response is a convenient measuring unit.

2. Drive the galvanometer at a higher frequency, F_1, keeping the current the same, and measure amplitude A_1 of response using units chosen in Step 1.

3. Drive the galvanometer at frequency $F_2 = 2F_1$ (double the previous frequency), keeping the current the same, and measure amplitude of response A_2 using units chosen in Step 1.

4. Read the damping ratio from the damping table at the intersection of the column representing A_1 and the line representing A_2.

5. Read the frequency ratio from the frequency table at the intersection of the column representing A_1 and the line representing A_2. Multiply this frequency ratio by F_1 to obtain natural frequency of the galvanometer.

6. If no pair of frequencies F_1 and $F_2 = 2F_1$ can be found which produce amplitudes falling within the range of the tables, the galvanometer has damping 0.50 of critical or less.

TABLE 1

Natural Frequency of a Linear System in Relation to Test Frequency

A_2 \ A_1	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10
0.50	1.47	1.46	1.45	1.43	1.42	1.41	1.41	1.40	1.39	1.38	1.37	1.37	1.36	1.35	1.35	1.34
0.51	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38	1.38	1.37	1.36	1.36
0.52	1.52	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38	1.38	1.37
0.53	1.54	1.52	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.39
0.54	1.57	1.55	1.53	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40
0.55	1.59	1.57	1.55	1.54	1.52	1.51	1.50	1.49	1.47	1.46	1.45	1.45	1.44	1.43	1.42	1.41
0.56	1.62	1.60	1.58	1.56	1.54	1.53	1.52	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.44	1.43
0.57	1.64	1.62	1.60	1.58	1.57	1.55	1.54	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44
0.58	1.67	1.65	1.62	1.61	1.59	1.57	1.56	1.54	1.53	1.52	1.50	1.49	1.48	1.47	1.46	1.46
0.59	1.70	1.67	1.65	1.63	1.61	1.59	1.57	1.56	1.55	1.53	1.52	1.51	1.50	1.49	1.48	1.47
0.60	1.73	1.70	1.67	1.65	1.63	1.61	1.59	1.58	1.56	1.55	1.54	1.53	1.51	1.50	1.49	1.48
0.61	1.76	1.73	1.70	1.68	1.65	1.63	1.61	1.60	1.58	1.57	1.55	1.54	1.53	1.52	1.51	1.50
0.62	1.79	1.76	1.73	1.70	1.68	1.65	1.63	1.62	1.60	1.58	1.57	1.56	1.54	1.53	1.52	1.51
0.63	1.82	1.79	1.75	1.73	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.57	1.56	1.55	1.54	1.53
0.64	1.86	1.82	1.78	1.75	1.72	1.70	1.68	1.66	1.64	1.62	1.60	1.59	1.58	1.56	1.55	1.54
0.65	1.89	1.85	1.81	1.78	1.75	1.72	1.70	1.68	1.66	1.64	1.62	1.61	1.59	1.58	1.57	1.55
0.66	1.93	1.88	1.84	1.80	1.77	1.74	1.72	1.70	1.67	1.66	1.64	1.62	1.61	1.59	1.58	1.57
0.67	1.97	1.92	1.87	1.83	1.80	1.77	1.74	1.72	1.69	1.67	1.65	1.64	1.62	1.61	1.59	1.58
0.68	2.01	1.95	1.90	1.86	1.83	1.79	1.76	1.74	1.71	1.69	1.67	1.65	1.64	1.62	1.61	1.59
0.69	2.06	1.99	1.94	1.89	1.85	1.82	1.79	1.76	1.73	1.71	1.69	1.67	1.65	1.64	1.62	1.61
0.70	2.11	2.03	1.97	1.92	1.88	1.84	1.81	1.78	1.75	1.73	1.71	1.69	1.67	1.65	1.64	1.62
0.71	2.16	2.08	2.01	1.96	1.91	1.87	1.83	1.80	1.77	1.75	1.72	1.70	1.68	1.67	1.65	1.64
0.72	2.22	2.12	2.05	1.99	1.94	1.90	1.86	1.82	1.79	1.77	1.74	1.72	1.70	1.68	1.66	1.65
0.73	2.28	2.17	2.09	2.03	1.97	1.92	1.88	1.85	1.81	1.79	1.76	1.74	1.72	1.70	1.68	1.66
0.74	2.35	2.23	2.14	2.06	2.00	1.95	1.91	1.87	1.84	1.81	1.78	1.75	1.73	1.71	1.69	1.68
0.75	2.43	2.29	2.18	2.10	2.04	1.98	1.93	1.89	1.86	1.83	1.80	1.77	1.75	1.73	1.71	1.69
0.76	2.52	2.35	2.24	2.15	2.07	2.01	1.96	1.92	1.88	1.85	1.82	1.79	1.76	1.74	1.72	1.70
0.77	2.62	2.43	2.29	2.19	2.11	2.04	1.99	1.94	1.90	1.87	1.83	1.81	1.78	1.76	1.74	1.72
0.78	2.74	2.51	2.35	2.24	2.15	2.08	2.02	1.97	1.93	1.89	1.85	1.82	1.80	1.77	1.75	1.73
0.79	2.90	2.60	2.42	2.29	2.19	2.11	2.05	1.99	1.95	1.91	1.87	1.84	1.81	1.79	1.77	1.74
0.80	3.10	2.71	2.49	2.34	2.23	2.15	2.08	2.02	1.97	1.93	1.89	1.86	1.83	1.80	1.78	1.76

TABLE 2

Damping of a Linear System in Relation to Critical Damping

A_2 \ A_1	0.90	0.91	0.92	0.93	0.94	0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10
0.40	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.53	0.52	0.52	0.51
0.41	0.68	0.66	0.65	0.64	0.63	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.55	0.54	0.53	0.53	0.52	0.52
0.42	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.53	0.52	0.52
0.43	0.69	0.68	0.67	0.66	0.64	0.63	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.55	0.54	0.53	0.53	0.52
0.44	0.70	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.53	0.53	0.52
0.45	0.70	0.69	0.68	0.67	0.65	0.64	0.63	0.62	0.61	0.61	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54	0.54	0.53	0.52
0.46	0.71	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.58	0.57	0.56	0.55	0.55	0.54	0.53	0.53
0.47	0.71	0.70	0.69	0.68	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.53	0.53
0.48	0.72	0.71	0.69	0.68	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.54	0.53
0.49	0.73	0.71	0.70	0.69	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54	0.54	0.53
0.50	0.73	0.72	0.70	0.69	0.68	0.67	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.53
0.51	0.74	0.72	0.71	0.70	0.68	0.67	0.66	0.65	0.64	0.62	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54	0.53
0.52	0.75	0.73	0.71	0.70	0.69	0.67	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54	0.53
0.53	0.75	0.74	0.72	0.70	0.69	0.68	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54	0.54
0.54	0.76	0.74	0.73	0.71	0.70	0.68	0.67	0.66	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.58	0.57	0.56	0.55	0.54	0.54
0.55	0.77	0.75	0.73	0.71	0.70	0.69	0.67	0.66	0.65	0.64	0.62	0.61	0.60	0.59	0.59	0.58	0.57	0.56	0.55	0.54	0.54
0.56	0.78	0.76	0.74	0.72	0.70	0.69	0.68	0.66	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54
0.57	0.79	0.76	0.74	0.72	0.71	0.69	0.68	0.67	0.65	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54
0.58	0.79	0.77	0.75	0.73	0.71	0.70	0.68	0.67	0.66	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.55	0.54
0.59	[illegible]	[illegible]	0.75	0.74	0.72	0.70	0.69	0.67	0.66	0.65	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54
0.60	0.81	[illegible]	0.76	0.74	[illegible]	0.70	0.69	0.67	0.66	0.65	0.64	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54
0.61	0.82	[illegible]	0.77	0.75	[illegible]	0.71	0.69	0.68	0.66	0.65	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.56	0.55	0.54
0.62	0.83	[illegible]	0.77	0.75	[illegible]	0.71	0.70	0.68	0.67	0.65	0.64	0.63	0.62	0.60	0.59	0.58	0.57	0.57	0.56	0.55	0.54
0.63	0.84	0.81	[illegible]	0.76	0.74	0.72	0.70	0.69	0.67	0.65	0.64	0.63	0.62	0.61	0.60	0.58	0.58	0.57	0.56	0.55	0.54
0.64	[illegible]	[illegible]	0.79	0.76	0.74	0.72	0.70	0.69	0.67	0.66	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.65	0.87	[illegible]	0.80	0.77	[illegible]	0.73	0.71	0.69	0.67	0.66	0.64	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.66	[illegible]	[illegible]	0.81	0.78	[illegible]	0.73	0.71	0.69	0.68	0.66	0.65	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.67	0.92	0.86	0.82	0.77	[illegible]	0.74	0.71	0.70	0.68	0.66	0.65	0.63	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.68	0.95	0.88	0.83	0.79	[illegible]	0.74	0.72	0.70	0.68	0.66	0.65	0.64	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.69	[illegible]	[illegible]	[illegible]	0.80	[illegible]	0.75	0.72	0.70	0.68	0.67	0.65	0.64	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.70	1.05	[illegible]	0.86	0.81	[illegible]	0.75	0.73	0.71	0.69	0.67	0.65	0.64	0.62	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.71	[illegible]	0.94	0.86	[illegible]	[illegible]	0.76	0.73	0.71	0.69	0.67	0.65	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.72	0.	[illegible]	0.90	0.84	[illegible]	0.76	0.74	0.71	0.69	0.67	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.73	0.	[illegible]	0.94	0.86	[illegible]	0.77	0.74	0.72	0.69	0.67	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.74	0.	0.	0.99	0.88	0.82	0.78	0.75	0.72	0.70	0.68	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.75	0.	0.	1.08	0.90	0.83	0.79	0.75	0.72	0.70	0.68	0.66	0.64	0.63	0.61	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.76	0.	0.	1.62	0.94	0.85	0.79	0.76	0.73	0.70	0.68	0.66	0.65	0.63	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.77	0.	0.	0.	1.01	0.87	0.81	0.76	0.73	0.71	0.68	0.66	0.65	0.63	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.78	0.	0.	0.	1.21	0.90	0.82	0.77	0.74	0.71	0.69	0.66	0.65	0.63	0.62	0.60	0.59	0.58	0.57	0.56	0.55	0.54
0.79	0.	0.	0.	0.	0.95	0.83	0.78	0.74	0.71	0.69	0.67	0.65	0.63	0.62	0.60	0.59	0.58	0.57	0.55	0.54	0.53
0.80	0.	0.	0.	0.	1.04	0.86	0.79	0.75	0.71	0.69	0.67	0.65	0.63	0.62	0.60	0.59	0.58	0.57	0.55	0.54	0.53
0.81	0.	0.	0.	0.	0.	0.89	0.80	0.75	0.72	0.69	0.67	0.65	0.63	0.62	0.60	0.59	0.58	0.56	0.55	0.54	0.53
0.82	0.	0.	0.	0.	0.	0.94	0.82	0.76	0.72	0.69	0.67	0.65	0.63	0.62	0.60	0.59	0.58	0.56	0.55	0.54	0.53
0.83	0.	0.	0.	0.	0.	1.08	0.83	0.77	0.73	0.70	0.67	0.65	0.63	0.62	0.60	0.59	0.58	0.56	0.55	0.54	0.53
0.84	0.	0.	0.	0.	0.	0.	0.86	0.78	0.73	0.70	0.67	0.65	0.63	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53
0.85	0.	0.	0.	0.	0.	0.	0.91	0.79	0.74	0.70	0.67	0.65	0.63	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53
0.86	0.	0.	0.	0.	0.	0.	1.08	0.81	0.74	0.70	0.68	0.65	0.63	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53
0.87	0.	0.	0.	0.	0.	0.	0.	0.83	0.75	0.71	0.68	0.65	0.63	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53
0.88	0.	0.	0.	0.	0.	0.	0.	0.87	0.76	0.71	0.68	0.65	0.63	0.62	0.60	0.59	0.57	0.56	0.55	0.54	0.53
0.89	0.	0.	0.	0.	0.	0.	0.	1.00	0.77	0.71	0.68	0.65	0.63	0.62	0.60	0.58	0.57	0.56	0.55	0.54	0.53
0.90	0.	0.	0.	0.	0.	0.	0.	0.	0.79	0.72	0.68	0.66	0.63	0.61	0.60	0.58	0.57	0.56	0.55	0.53	0.52

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