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FEASIBILITY STUDY AND EXPERIMENTAL MODEL OF AN OPTIMAL CONTROL SERVOMECHANISM.

Report No. 1  Contract SC 07256  1st Quarterly Report
DA 36-039

First Quarterly Progress Report, 15 June to 15 September 1961.

U.S. Army Signal Research and Development Laboratory
Fort Monmouth, New Jersey

Aeronca Manufacturing Corporation
Aerospace Division
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Baltimore, Maryland

62-1-5
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Feasibility Study and Experimental Model of an Optimal Control Servomechanism.

Report No. 1
Contract No. SC 87256
DA 36-039

First Quarterly Progress Report, 15 June to 15 September 61.

Object of Research: To develop an optimal servo-motor.

By Wm. S. Hare
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1.0 PURPOSE

The purpose of this project shall be the construction of a feasibility model demonstrating the principles of optimal discontinuous control for servo motors. The particular servomechanism to be constructed will be optimally designed with respect to inputs of the type, step plus ramp disturbance, and will be self adaptive to torque variations in the load. There appear to be two possible design approaches to the synthesis of the optimal switching function. One of these would be to construct an analog of the switching function using conventional analog computer techniques, such as logarithmic amplifiers, multipliers, dividers and other nonlinear elements to produce, in real time, subject to the state variations of the error and error rate in the phase-plane, an optimal discontinuous forcing function.

A second synthesis technique would be to construct the switching function faster than real time through the use of passive analog elements. This prediction technique would run the linear plant to the origin, in the phase-plane, many times faster than real time. Noting the error and switching the forcing function will match that predicted condition. This second technique has been adopted by the project managers of this contract for the following reasons:

1. It appears to be economically much more feasible than the fabrication of many expensive nonlinear computing elements.
2. It is expected that the accuracy of the passive elements involved will be higher than those of the equivalent analog synthesis techniques.
3. The technique appears to be new and therefore represents
a step forward in the state of the control system's art with respect to relay actuated control systems.

4. It is not felt that this technique is in any real sense a deviation from the statement of work of the contract in the sense that the optimal switching curve is indeed fabricated, faster than real time, and sampling techniques provide us with the answer as to whether or not the linear plant is following the optimal switching function.

While on the face of it there appears to be a loss of generality in constructing a faster than real time analog for the second order servomechanism, it is also true that in any generalized n-th order control system's problem, the use of logarithmic amplifiers in analog synthesis is mandatory. Once the system has been reduced to the n-1 order, in phase-space, the final solution could be mechanized in the same manner that it is being produced in this particular project. A completely general approach would be to use the logarithmic amplifier synthesis technique for the entire control system's problem. However, due to the unavailability of sufficiently accurate log amplifiers, it is felt that a feasibility determination can be made on the basis of this technique. Further studies will improve the techniques required to mechanize the completely general n-th order problem.
2.0 ABSTRACT

Using results obtained by Dr. R.W. Bass, who introduced the concept of relay feedback into the field of control theory, it has been possible to mathematically describe a phase plane portrait of the switching characteristic of an extremal torque, time optimal, positional-servomechanism which is self adaptive to torque load variations.

This problem has been of historical interest to workers in the field, and, at the present time, of accentuated practical interest in the field of guidance and control. Because of this new emphasis on extremal and optimal control the current feasibility effort was instituted. The simplifying assumptions and mathematical idealizations have been dropped and an engineering attack has been mounted on the practical, the realizable, the economic, and the uncertain factors.

The approach has been twofold; theoretical work which removes the original simplifications, introduces uncertain parameters and considers the inherent difficulties of extremal systems as immediate problem objectives; practical implementation, as the second major effort, has been devoted to consideration of the factors influencing the choice of design, generation of system block diagrams (both functional and operational) and the construction of feasibility hardware to justify our choice of system elements.

Both of these efforts have produced problem areas and compromise structures. In many cases the choice of solution is either arbitrary or must be based on inadequate engineering data or theoretical research which has not yet been accomplished. In view of these prob-
lem areas it is felt that several meetings with Mr. Carl Allen, the Contracting Officer's Designated Representative, would be advisable to resolve these issues.
3.0 CONFERENCES AND PUBLICATIONS

3.1 Conferences

Dr. R.W. Bass, principal investigator and W.S. Hare, project manager, attended the Joint Automatic Control Conference, 1961, with Carl Allen of the Signal Corps and had several important informal discussions with leaders in the field of optimal and adaptive control on subjects relating closely to this project.

Dr. Bass and Mr. Hare met on 16 October 1961 with Samuel Adler, Carl Allen and Peter Devreectes of the Signal Corps to discuss the status of the project and plan the future work.

3.2 Publications

No publications have been approached to date, however there appears to be a class of unusual problems represented in the multipli-connected phase-space difficulty which would justify publication in advanced journals.
4.0 FACTUAL DATA

4.1 Theoretical Work

The theoretical work has been devoted to mathematical effort to obtain a rigorous switching characteristic in closed form which at the same time includes consideration of the fact that any desired final position of a rotating servomechanism may be taken modulo $2\pi$. This means that we have a multipli connected phase space and this is a new consideration in this field.

In addition, computer simulation studies have been instituted, primarily as a means of establishing practical tolerances on the system hardware to attain an acceptable limit cycle, stability margins and the effect of stochastic variations on the input signal and other system parameters.

In our effort to derive a switching characteristic applicable to practical transducers in a medium power servomechanism, the following calculations have been made.

Consider the following configuration for motor and load, in which the following idealizations have been assumed:

a. Motor transfer function is linear and second order.
b. Gear train is free of backlash.
c. Load parameters are constant.
d. External torque variations never exceed the stall torque of the motor.

See Figure 1.
Figure 1

Pictorial of Motor and Load Configuration
If the servo torque-speed curves have the following form:

![Figure 2.](image)

end \( E_1 > E_2 > E_3 \ldots \)

and since

\[
T = \frac{\partial T}{\partial E} A + \frac{\partial T}{\partial \dot{\theta}} \dot{\theta}^i
\]

then let

\[
\frac{\partial T}{\partial E} = K_e; \quad \frac{\partial T}{\partial \dot{\theta}} = -K_n
\]

Hence the developed torque

\[
(1) \quad T_m = K_e E_{in} - K_n \dot{\theta}^i
\]

The load torque at the motor shaft

\[
(2) \quad T_L = \left( \frac{J_L}{n^2} + J_m \right) \ddot{\theta}^i + \left( \frac{f_L}{n^2} + f_m \right) \dot{\theta}^i - \tau_d
\]

Equating (1) and (2) we have the system differential equations

\[
(3) \quad \left( \frac{J_L}{n^2} + J_m \right) \dddot{\theta}^i + \left( \frac{f_L}{n^2} + f_m + K_n \right) \ddot{\theta}^i = K_e E_{in} + \tau_d
\]
Let
\[
\frac{J_L}{n^2} + J_m = j, \quad \frac{\tau_d}{n} = T_d
\]
\[
\frac{f_L}{n^2} + f_m + K_n = f
\]
\[
\theta' = n\theta
\]
\[
\frac{K_i E_{in}}{n} = F \, \text{sgn} \sigma \quad \text{(the extremal constraint)}
\]
Hence (3) becomes
\[
(4) \quad J \ddot{\theta} + f \dot{\theta} = F \, \text{sgn} \sigma + T_d
\]
If the solution of this equation is to follow a given input, \( \theta^*_1(t) \) so chosen that
\[
(4a) \quad \theta^*_1(t) = a + bt, \quad |b| < \frac{F + T_d}{f} \quad \text{(runaway velocity)}
\]
within any finite interval, then it is convenient to define a new variable, \( e(t) \), the actuating error
\[
(5) \quad e(t) = (\theta^*_1(t) - \theta(t))
\]
Clearly \( |e(t)| \to 0 \) as \( \theta(t) \to \theta^*_1(t) \).
After substitution of (5) and (4a), (4) becomes
\[
(6) \quad J \ddot{e} + f \dot{e} = (fb - T_d) - F \, \text{sgn} \sigma
\]
We then desire that choice of \( \sigma \) which will bring \( e(t) \) and its derivative to the origin from any initial position \( (e_0, \dot{e}_0) \).
Let us solve (6)

Set

\[ e(0) = e_0 \]
\[ \dot{e}(0) = \dot{e}_0 \]

Now

\[ e(t) = A + B e^{-Jt} + \frac{1}{J} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] t \]

\[ \dot{e}(t) = - \frac{f}{J} B e^{-Jt} + \frac{1}{J} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] \]

then

\[ e_0 = A + B \]
\[ \dot{e}_0 = - \frac{f}{J} B + \frac{1}{J} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] \]

hence

\[ B = \frac{J}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] - \frac{J}{f} \dot{e}_0 \]
\[ A = e_0 + \frac{J}{f} \dot{e}_0 - \frac{J}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] \]

Finally

(7) \[ e(t) = e_0 + \frac{J}{f} \dot{e}_0 - \frac{J}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] t \]
\[ + \left\{ \frac{J}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] - \frac{J}{f} \dot{e}_0 \right\} e^{-Jt} + \frac{1}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] t \]

and

(8) \[ \dot{e}(t) = \frac{1}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma \right] - \frac{1}{f} \left[ (fb - T_d) - F \operatorname{sgn} \sigma - f\dot{e}_0 \right] e^{-Jt} \]

If we eliminate the parameter, \( t \), the resultant equation defines two families of phase plane trajectories, where the phase plane is defined to be \( e(t), \dot{e}(t) \) plane. These two families differ in the assumed algebraic sign \( \sigma \). See the following sketches wherein it is assumed that \( b \), the input rate, and \( T_d \), the disturbing torque, are held constant throughout the trajectory.
Figure 3.

Phase-plane Semi-trajectories

- 9a -
Note that only that part of the trajectory which is heavily shaded in the sketch, will bring the system to the origin with no torque reversal. This then is the desired final trajectory. Let us solve equations (7) and (8) for this trajectory by eliminating $t$, and setting the endpoint equal to the origin, $e(t) = \dot{e}(t) = 0$. Hence, (8) becomes

$$e^{-\frac{f}{T}t} = \frac{1}{\frac{1}{T} \left[ (fb - T_d) - F \text{sgn} \sigma \right]} \left[ (fb - T_d) - F \text{sgn} \sigma \right] - \dot{e}_o$$

and

$$t = \frac{J}{T} \log \left( \frac{\frac{1}{T} \left[ (fb - T_d) - F \text{sgn} \sigma \right] - \dot{e}_o}{\frac{1}{T} \left[ (fb - T_d) - F \text{sgn} \sigma \right]} \right)$$

Clearing (7) and (8) we have

$$0 = e_o + \frac{J}{T} \dot{e}_o + \frac{J}{T^2} \left[ (fb - T_d) - F \text{sgn} \sigma \right] \log \left( \frac{(fb - T_d) - F \text{sgn} \sigma - \dot{e}_o}{(fb - T_d) - F \text{sgn} \sigma} \right)$$

Equation (10) is a double valued implicit function of $(e_o, \dot{e}_o, \sigma)$.

See illustration on page 10a.

Note that in the I and III quadrant solutions, it is implied by equation (9a) that we reach the origin in negative time, i.e., an unrealizable solution.

In other words, we require that equation (10) holds in reality only if

$$\text{sgn} \sigma = \text{sgn} \dot{e}_o$$

Hence

$$0 = e_o + \frac{J}{T} \dot{e}_o - \frac{J}{T^2} \left[ (T_d - fb) + F \text{sgn} \dot{e}_o \right] \log \left( 1 + \frac{\frac{f}{T} \dot{e}_o}{(T_d - fb) + F \text{sgn} \dot{e}_o} \right)$$
Figure 4.

Second Order Switching Boundary
becomes the optimal trajectory, and indeed it will be shown, is the switching curve.

If in the derivation of (12) we let the end point be given
\[ e(t) = e_1 \]
\[ \dot{e}(t) = 0 \]
then
\[ e_1 = e_0 + \frac{J \dot{e}_0}{J_0} - \frac{J}{f} \left[ (T_d - \tau_b) + F \text{ sgn } \dot{e}_0 \right] \log \left\{ 1 + \frac{f \dot{e}_0}{(T_d - \tau_b) + F \text{ sgn } \dot{e}_0} \right\} \]
which implies that the system comes to rest with an overshoot of \( e_1 \).

Note that the sign of \( e_1 \) is also the sign of the applied torque which will cause the system to evolve to a point of interception with the ideal trajectory (12).

Hence, if we set \( e_1 = 0 \) and the applied torque equal to, \( F \text{ sgn } \mathbf\sigma \), then we generate the complete closed loop ideal switching characteristic for arbitrary initial conditions.

\[ \mathbf\sigma = e_0 + \frac{J \dot{e}_0}{J_0} - \frac{J}{f} \left[ (T_d - \tau_b) + F \text{ sgn } \dot{e}_0 \right] \log \left\{ 1 + \frac{f \dot{e}_0}{(T_d - \tau_b) + F \text{ sgn } \dot{e}_0} \right\} \]

For step response only, with no torque adaptation we have

\[ \mathbf\sigma = e_0 + \frac{J \dot{e}_0}{J_0} - \frac{JF}{f} \left( \text{ sgn } \dot{e}_0 \right) \log \left( 1 + \frac{fe_0}{F} \right) \]

See illustration on following page, 11a.

Equations (13) and (14) are completely rigorous with respect to the assumptions listed at the beginning of this section; however, we note the sensors of a rotating servomechanism are really measuring the variables of a congruence class, namely

\[ \mathbf\theta_1 = \theta + 2\pi n \quad (n = \text{any integer}) \]

is a solution of the equations.

Representing \( \theta = \theta \mod 2\pi \) is equivalent to mapping the phase plane on a cylinder. See pages 11b, 11c, 11d.

- 11 -
Figure 5.

Phase-plane Trajectories
Figure 6.

Cartesian Map of Ambiguous Phase-plane.
Figure 7.

Truncated Phase-plane Plot
Figure 8.

Cylindrical Map of Phase-plane (ambiguity removed).
(The shaded regions represent \( o > 0 \))
The construction of a computer analog of this switching criterion is non-trivial. Several structures present themselves, however. Perhaps the most generally useful method is the analog synthesis.

This difficulty with the multipli-connected phase surface is avoided in part by the selection of components.

For example we note that the motor and load are in a closed system in which it is assumed

\[ |T_d - fb| \leq F. \]

Clearly then \( |\dot{e}| \leq \frac{2F}{\dot{e}} \) is the worst case condition. If the \( e \) axis is expanded through choice of gear ratio then the semi-infinite strip of Figure 9 (page 12a) can be represented in a finite box with as few as two discontinuities in traversing the range of \( e \) or \( \dot{e} \). See Figure 10 on page 12b.

Another source of difficulty arises from the fact that no synthesis will be perfectly realized. Analogous to this are the well known oversimplifications of the signum operator, the inherent nonlinearities of the plant, and the inaccuracies of the sensors.

These realities lead one to believe that the system will have several torque transitions in its evolution to the origin. Usually this multiple switching is a limit cycle or a chattering regime or both.

We have, in our analog synthesis program, observed these phenomena in an attempt to put practical limitations on the acceptable tolerances of the servo components. Because our multiplier cabinets have been inoperative since the outset of this program, we have not as yet closed the loop on the analog synthesis of equation (13). We have, however, approximated equation (14) with straight line segments.
Figure 9.

Cartesian Plot of Phase-plane Showing High Gear Ratio.
Figure 10.

Cylindrical Plot of Preceding Page Showing High Gear Ratio.
4.1.1 List of Symbols

\( T = \) torque
\( \tau_d = \) disturbance torque at motor shaft
\( T_d = \) disturbance torque at load
\( T_m = \) developed torque
\( T_L = \) load torque
\( J_L = \) inertia of load
\( J_m = \) inertia of armature
\( J = \) equivalent inertia at load
\( f_L = \) load friction
\( f_m = \) motor friction
\( f = \) equivalent friction at load
\( n = \) gear ratio
\( K_e = \) motor torque constant
\( K_n = \) motor damping constant
\( \theta' = \) shaft displacement
\( \theta = \) load displacement
\( E_{in} = \) terminal voltage
\( \sigma = \) switching function (or control function)
\( F = \frac{K_e E_{in}}{n} \)
\( \Theta_i(t) = \) input command
\( e(t) = \) error
\( a = \) initial displacement of input command
\( b = \) rate of change of \( \Theta_i(t) \)
\( e_o = \) initial error
\( e_o = \) initial error rate
\( A, B = \) undetermined coefficients of differential equations
\( e \) = base of natural logarithms  
\( t \) = time  
\( \text{sgn } \sigma \) = signum operator = 0 if \( \sigma = 0 \); = ±1 if \( \sigma \geq 0 \)
5.0 CONCLUSIONS AND PROGRAM FOR NEXT INTERVAL

5.1 Practical Implementation

The realization of equation (13) is most easily visualized as a real time analog synthesis. Fast log amplifiers or Schmitt triggers are required. Our first attack was to build up the switching function by this technique. It is now felt that a faster than real time network analog of the linear plant with a sampling circuit and comparator to close the loop will be both economically and technically better. This device will be described in Section 5.3.

Exhaustive catalog searches have led us to the Diehl Mfg. Corp. products as the most useful source of medium power servo components. We have in fact ordered:

a. Diehl No. FPF85-18-1 Servo-motor
b. Diehl No. FPF85-29-1 Servo-motor w/Tach
c. Diehl No. FDE-15-1 D.C. Tachometer

External torque is to be provided by a small hysteresis synchronous motor. For reasons of economy, the inertial load is to be placed at the motor shaft with a precision anti-backlash geartrain (Sterling T628, 299.7:1) driving the control transformer.

In order to maintain a flexible dual mode boundary it was deemed advisable to construct a full power saturating amplifier. This item is in design at present and will be completed by the motor delivery date, approximately 1 December 1961.

5.2 Problem Areas

a. Sensor measures Sin ϕ, not ϕ, this is demonstrably stable, but leads to non-optimality for steps larger than π/2 radians.
b. Sensor output is 60 cycle subcarrier, i.e., there exists an inherent 16 millisec time delay between samples.

c. Component drift and long term accuracy prohibits use of practical multipliers and log amplifiers in computation of real time switching functions.

d. Motor controller should have peak power capability of 600 watts to effect smooth torque transition across dual mode boundary. Solid state switching imposes a severe restriction at this power level.

e. Noise in tachometer, in input signals and computer cannot be compensated for by any known technique other than construction of a wide dual mode boundary. These effects in a nonlinear discontinuous controller have not been mathematically investigated and this is beyond the scope of the present project.

5.3 Proposed Solutions

a. The use of induction potentiometers would cure this difficulty, however, noise in the difference amplifier would introduce as many problems. This solution increases the cost of the system without improving performance noticeably. It has been observed that the choice of gear train can all but eliminate this ambiguity. In any event the problem arises only for large values of ε and causes only one ambiguous switching even then.

b. At runaway velocity, a 16 millisecond sampling delay could cause a 1.2° error in the output position of the device. Clearly this would not be a steady state error but would be a contribution to the expected transient. Smoothing and linear extrapolation can be accomplished cheaply and should go a long way toward a cure.
c. Analog techniques are avoided in the following scheme: Construct a network analog which operates faster than real time. This network which will involve active devices, is linear and simulates the physical plant. It will be shown rigorously in the next report that if the real time state variables are applied to the simulator as initial conditions then an a priori selection of the sgn $\sigma$ will enable determination of the actual sgn $\sigma$ as soon as the simulated error rate makes its zero crossing. This determination can be done with diode logic.

It appears to be possible to achieve good accuracy at sampling rates in excess of 400 pps. A detailed description will be provided of the entire simulator-computer.

d. Only rarely will the full reverse power of 600 watts be required. A compromise measure has been implemented. A saturating 250 watt transistorized controller is under construction and the dual mode region will be adapted to this.

e. The effects of noise will not be studied until our computer error analysis is instituted.

5.4 Present Status

a. Mathematical and computational work is 60% complete. The remainder will be devoted to computer synthesis.

b. Design of electronic components is 20% complete.

c. Mechanical design, block synthesis and component selection is 90% complete.

The remaining 40% of the mathematical and computational work will be in the area of error analysis. A computer routine
has been obtained which will enable us to construct the topology of the dual mode region in order to prevent extraneous switching of the servomechanism in the presence of random additive noise and errors in the calculation of the true switching curve. Computational work needs to be done in the calculation of the linear feedback function to be used within the dual mode region. This work is essentially state-of-the-art control systems engineering and can be done relatively quickly.

Those equipments which have already been designed are the error sampling device, the linear portions of the switching computer and part of the high power control amplifier. Design work remains to be done on the external torque drive source, the nonlinear portions of the motor control computer and the high power stage of the linear amplifier.

The remaining mechanical design that is to be completed is the establishment of error criterion and tolerances for the mechanical components associated with the gear train and external torque drive motor.

Test and evaluation will be performed, beginning as soon as the device has been fabricated. It is expected that two man-months of work will be involved.

5.5 Summary

In summary, we feel that great progress has been made in establishing a synthesis technique which will be useful for engineers. The mathematical work has uncovered new but apparently soluble problems connected with the multipli-connected phase-space.
The engineering work has disclosed a new but relatively simple mechanization of the switching function using sampled data techniques. Several practical problems have appeared in the project and have been isolated and either minimized or solved. The final device when delivered will represent a new step forward in the art of relay actuated control systems as well as pointing out several new mathematical difficulties and therefore should be a significant development in control system technology.
6.0 IDENTIFICATION OF PERSONNEL

Dr. R.W. Bass, Chief Scientist

Mr. Wm S. Hare, Manager, Guidance and Control Design Section

Mr. David Maxwell, Senior Engineer

Mr. J.E. Heller, Technician

Resumes of the above personnel are enclosed.
Robert W. Bass

**EDUCATION:**
- Ph.D., Mathematics, Johns Hopkins University
- M.A., Mathematics, Oxford University (Rhodes Scholar, Maryland and Wadham)
- B.A., Physics, Johns Hopkins University

**EXPERIENCE:**
- 1960 - Dat: Aeronca Manufacturing Corporation, Aerospace Division
- 1959 - 1960 RIAS (Research Institute for Advanced Study, a Division of Martin Co.). Member, Mathematics Center.

**MAJOR PROJECTS:**
- AEC - Project Matterhorn
- USAF - Non-linear Mechanics Research Contract
- USN - Non-linear Mechanics Research Contract

**PUBLICATIONS:**

"Remark on Cosmological Models" (with L. Witten), Reviews of Modern Physics, vol. 29 (1957), pp. 452-453.


"The Optimal Space Vehicle Attitude-Control System for Arbitrary Angular Variations, including Nonlinear Coupling" (with P. A. Castruccio and D. L. Slotnick), Aerona Technical Report 60-23.


BOOK:

SOCIENTIES:
Fellow of British Interplanetary Society
American Mathematical Society
American Astronautical Society
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Society Industrial and Applied Mathematics

AWARDS:
Phi Beta Kappa
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EDUCATION:  
MS  Electrical Engineering, Johns Hopkins University, Baltimore.
B.E.S  Electrical Engineering, Johns Hopkins University, Baltimore.

EXPERIENCE:  
1961  Aeronca Manufacturing Corporation, Aerospace Division.
1957 - 1961  Johns Hopkins University. Research Staff and Instructor, Electrical Engineering Department.

MAJOR PROJECTS:

U.S.N.  SEADAR System Study
U.S.A.F  E.C.C.M. Research Contract
A.E.C.  Random Number Generation (Biophysics Research Contract with McCollum Pratt Institute)

PUBLICATIONS:

"Random Search Patterns". Bendix Radio Tech Report
"Transient Analysis of a Pulse Power Supply" B.Radio Tech. Report
SOCIETIES:

Society of Sigma Xi
Eta Kappa Nu
Inst. of Radio Engineers
Amer. Inst. of Elec. Engineers

AWARDS:

Engineering Scholarships, 1951 - 1955
Johns Hopkins University, Gilman Fellowship, 1957 - 1959.
DAVID J. MAXWELL

EDUCATION: B.E.E Cornell University

Presently working on dissertation toward final requirement for Doctor of Engineering at Johns Hopkins University

EXPERIENCE:

1961 - Date Aeronca Manufacturing Corporation
   Aerospace Division

1955 - 1961 Bendix Aviation
   Bendix Radio Division
   (1/2 time employment)

MAJOR PROJECTS:

USAF - Electronic steerable array radar
USAF - Antijamming Systems study
COMM - Transistorized IF Amp

PUBLICATIONS:

Bendix Radio Technical Reports:

1. A serrodyne traveling wave tube amplifier
2. A wide-band transistorized IF amplifier
3. Effects of limiting before beam-forming on multiple target reception on ESAR
4. A scheme for CW anti-jamming
5. A discrete linear FM pulse compression scheme.

SOCIETIES:

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Commercial Radio Institute - Electronics
McCoy College (evening) 2½ years E.E.
Drexel Institute of Technology 3 years - Physics
Presently at McCoy working on B S in physics.

EXPERIENCE:

Nov. 1961 - date Aeronca Mfg. Corporation
Aerospace Division


June 1959 - April 1961 - Martin/Drexel Co-op student


MAJOR PROJECTS:

US Army - Lacrosse Missile Power System
USN - P6M GSE
USAF - Titan Trainers

PUBLICATIONS:

None

SOCIETIES: ARS
AIP
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