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SHIP VIBRATION

by

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PREFACE

In treating the subject of ship vibration it is necessary to recognize that it falls in the border region between exact science and empiricism. While it is true that intensive research has been conducted in this field in several parts of the world and much progress has been made, at the time of this writing (1960) it cannot be said that an adequate method of vibration analysis of a ship exists.

It is true that one can consider the hull as an ideal mass-elastic system and can write down a certain set of partial differential equations which govern the behavior of such a system, but, in attempting to predict the level of service vibration of a ship in the design stage, one must be well aware of the limitations of such a treatment.

It is also possible to present empirical data on the level of service vibration of ships of various types together with the principal design features of the ships involved. This approach is also inadequate since the level of vibration will vary with a number of parameters simultaneously.

In the preparation of this book an attempt has been made to follow a path midway between the theoretical and empirical approaches. This leads to what may be called a "rational theory of ship vibration." Use is made of the properties of ideal free-free beams to obtain an insight into the effects of various design changes on the vibratory response characteristics of hulls. However, it is also attempted to guard the reader and the user against extending the calculations into realms in which they have no validity. One aim is to show that the vibratory characteristics are closely related to the structural strength characteristics.

While intended principally for the naval architect, the book has been prepared also with the research worker and the student in mind. It has not been attempted, however, to include a treatment of the fundamentals of mechanical vibration. It is presupposed that the reader has or can acquire a background such as furnished by the courses in mechanical vibration now given in practically all colleges of engineering. Specific references are included at the end of each chapter and a general bibliography is given at the end of the book.

The contrast between the problem of avoiding serious steady-state vibration and withstanding the effects of severe transient vibrations due to heavy seas is pointed out. The problem of setting up design specifications with regard to vibration is also discussed. However, no attempt is made to disguise the fact that the present state of the art of predicting hull vibratory response characteristics is primitive. Where controversial issues arise only opinions can be furnished.

In the mathematical treatment of the subject and in the illustrations given in the appendixes, the aim has been to emphasize the physical principles involved without burdening the reader with too many details. It is assumed that the designer who makes use of the methods discussed in the book will assign the task of carrying out the actual vibration calculations to a member of his staff who can consult the references when further details are needed. Thus this book is not of the manual or handbook variety although concrete procedures for the designer are suggested. With the rapid pace of development of computational aids today it is clear that procedures that are written out in great detail may become obsolete almost overnight, but the principles involved are durable.

Except as otherwise noted, the statements made apply to either surface ships or submersibles. Although most of the information given was obtained from research sponsored by either the United States Navy or jointly by the Navy and the Society of Naval Architects and Marine Engineers, the book is not intended specifically for naval designers, and problems that are strictly naval are not discussed. Thus submarines are mentioned only because they may become future commercial carriers and questions that relate to the detection of undersea craft are omitted.

The relatively new field of hydroelasticity has been included since this is recognized as a field of growing importance. In fact, in the broadest definition of hydroelasticity, the subject of hull vibration itself would have to be included.

The book is based chiefly on the work of the U.S. Experimental Model Basin and the David Taylor Model Basin and an exhaustive commentary on the work of other agencies is not attempted here.

It is not overlooked that more elaborate analyses of the dynamical system comprising the hull and the surrounding water than the beam-theory analysis presented in this book are conceivable. However, it is felt that even after such analyses have been developed, the designer will still be restricted to the methods discussed here in the preliminary design stage. The data required for more elaborate analyses will, in general, be available only at a very advanced stage of the design.

While this book is concerned chiefly with the problem of hull vibration, there has been included among the chapters on design considerations, one dealing with the vibration of the propulsion system itself. Here, however, the treatment is relatively brief and intended to serve chiefly as a guide to other sources of information on this subject in the technical literature. In dealing with the hull itself, no attempt has been made to review all the available literature, but to concentrate on the techniques that appear most fruitful.

In choosing a notation it was found impossible to adhere strictly either to standards in naval architecture or in engineering since the subject involves both fields. In recent years the American Standards Association has extended its sphere from acoustics into the field of mechanical shock and vibration. Many of the symbols used conform to the ASA standards, but the common symbols for the principal dimensions of ships used in naval architecture are also retained. The common use of nondimensional notation in naval architecture has not been followed here, as this has not found such wide acceptance in the field of mechanical vibration. Nevertheless, it is pointed out in the chapter on hydroelasticity that the aeroelastician has also found such notation preferable.

Finally it seems in order to point out that vibration theory plays a central role in ship dynamics just as it does in mechanics in general. An acquaintance with the vibratory characteristics of the hull not only assists the designer in avoiding serious vibration difficulties when his product goes into service but also gives him a deeper insight into many factors involved in good structural design.

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NOTATION

As stated in the preface, the notation used in this book has been derived from the fields of both naval architecture and mechanical engineering. It has not been feasible at this stage to avoid using the same symbol with different meanings in different places. Hence, in listing symbols here with specific meanings, the page on which the symbol first appears with this meaning is given. Furthermore, in most cases the meanings of symbols are also given in the text as they are used. As far as possible, symbols in naval architecture conform with Reference N-1; see page N-13. In formulas proposed by various authors, the original notation has been converted in many cases to conform with that generally used in the book. In some cases, however, it was considered preferable to retain the notation of the original author.

Just as it was found impossible to produce a wholly consistent notation, it was also found necessary to use different systems of units in different places. The two principal systems used are the foot-ton-second system and the inch-pound-second system. A major exception is the frequent use of mils (thousandths of an inch) as a unit for displacement amplitude. Where specific formulas for numerical computation are given, the units applicable to it are given in the text.

Since this book is not intended as an instruction manual for the use of those preparing requests for vibration calculations to be made by the Applied Mathematics Laboratory of the David Taylor Model Basin, the reader should make sure what units are currently in use with the codings in operation at the time before initiating such requests. Reference N-2, page N-13, will be found helpful in this respect.

Symbol	Meaning	Page
A	Mechanical admittance based on displacement	4-11
A	A lift coefficient of a hydrofoil defined by the relation $F_l = AS^2\theta$	8- 7
A'	Area of that portion of the cross section of a hull contributed by the plating when plane is parallel to the direction of the shear load (called the "web" area)	A- 4
A	Level of amplitude of vibration under calm sea conditions used as a basis for comparison with rough sea conditions	H- 5
A_d	Mechanical admittance at driving point d (based on displacement)	4-12
a	Acceleration	G- 2
a_i	Coefficient of the ith normal mode function in the series representing an arbitrary displacement pattern of hull vibration	3-22
B	Beam of hull	3- 6
b	Half-breadth of a ship section at the waterline	A- 2

Symbol	Meaning	Page
b	Semichord length of a hydrofoil or an aircraft wing	15- 8
C	Schlick's empirical constant appearing in formula for the fundamental vertical frequency of a hull	8- 3
C	Lumped viscous damping constant of a hull (equal to $c\Delta x$)	4- 3
C	Viscous damping constant of a vibratory system of a single degree of freedom	4-10
C	Electrical capacitance	4-11
C	Linearized (viscous) damping constant applicable to the translational degree of freedom of a control surface system of two degrees of freedom at zero velocity	14- 4
C	Lewis' two-dimensional added mass coefficient giving the ratio of the added mass of a ship form (in vertical vibration) to that of a circular form of the same beam	A- 2
C_M	Moment coefficient of a spade rudder	14- 2
C_{di}	The effective viscous damping constant of a hull in its ith normal mode of vibration and with respect to driving point d	4-10
c	Viscous damping constant equivalent to nonviscous damping on the basis of energy dissipation, damping force per unit velocity	8- 2
c	Linearized (viscous) damping constant applicable to the rotational degree of freedom of a control surface system of two degrees of freedom at zero velocity	14- 4
c	Coefficient for the inertia effect of water appearing in Prohaska's formula for the fundamental vertical frequency of a hull	C- 4
c	Viscous damping constant, damping force per unit velocity in sense opposing the velocity	3- 1
c	Velocity of wave propagation	3- 5
c	Distributed viscous damping constant of a hull, damping force per unit length per unit velocity	4- 2
c'	Angular viscous damping constant	G- 2
c_c	Critical viscous damping constant	8- 3
c/c_c	Ratio of damping to critical damping	G- 2
cpm	Blade frequency in cycles per minute	4-13
D	Depth of hull	3- 6
d	Draft	A- 2
d	A driving point in a mass-elastic system	4- 9
d	Rectilinear displacement	G- 2

Symbol	Meaning	Page
d	Propeller diameter	7- 5
E	Young's modulus of elasticity	3- 2
E	Electrical voltage	D- 1
EI	Bending rigidity of a beam	3- 2
EV_H	Equivalent horizontal virtual inertia factor $\left(1 + \dfrac{\text{effective added weight of water for horizontal vibration}}{\text{displacement of ship}}\right)$	C- 6
e	Eccentricity of a rotating mass of a mechanical vibration generator	9- 4
F	A concentrated force acting on a hull at an arbitrary point and treated as constant over a short interval of time in the digital treatment of transient response	5-22
F	Force	G- 2
$F(t)$	A concentrated driving force (acting on a hull) which is an arbitrary function of time	5- 4
F_l	Lift force acting on a hydrofoil	8- 7
F_{vp}	Rotating time vector representing the vertical component of the blade frequency force due to the port propeller	9- 6
F_{vs}	Rotating time vector representing the vertical component of the blade frequency force due to the starboard propeller	9- 6
F_0	Area enclosed by the shell plating of the midship section of a hull (not the area of the material)	C- 7
f	Frequency	7-11
G	Shear modulus of elasticity	3- 4
GJ	Torsional rigidity of a shaft	3- 4
GJ_e	Effective torsional rigidity of a hull with respect to its longitudinal axis	3- 8
g	Acceleration of gravity	3- 6
H	Draft	7-11
H	Impulse applied to a hull at driving point d	5- 5
h	Distance from the axis of a control surface to the center of gravity of the rotating element (based on an allowance for added mass effect of water) considered positive if the c.g. is downstream	14- 4
h	Distance from the top of a polemast to the elastic axis of a hull	11- 4
I	Moment of inertia of the cross section of a beam with respect to its neutral axis (based on the area of the material)	3- 2

Symbol	Meaning	Page
I	Mass moment of inertia of the rotatable assembly of a control surface system of two degrees of freedom including the added mass moment of inertia effect of the water	14- 4
I	Electrical current	D- 1
I	Mass moment of inertia of the entire ship with respect to the longitudinal axis through its center of gravity without any allowance for the inertia effect of the surrounding water	D- 3
I	Mass moment of inertia	G- 2
I_a	Moment of inertia of an area	G- 2
I_H	Moment of inertia of the area of the midship section of a hull for bending in the horizontal plane	C- 6
I_s	Mass moment of inertia of a resiliently mounted assembly or "sprung mass" with respect to an axis through its center of gravity	6-10
I_V	Moment of inertia of the area of the midship section of a hull for bending in a vertical plane	C- 6
I_x, I_y, I_z	Mass moments of inertia of a resiliently mounted rigid assembly with respect to the X-, Y-, and Z-axes, respectively, with origin at the center of gravity of the assembly	6- 8
I_{xy}, I_{xz}, I_{yz}	Mass products of inertia of a resiliently mounted rigid assembly with respect to axes X-Y, X-Z, and Y-Z, respectively, with origin at the center of gravity of the assembly	6- 8
$I_{\mu x}$	Mass polar moment of inertia of a beam or shaft per unit length with respect to its longitudinal axis	3- 4
$I_{\mu x}$	Mass moment of inertia of a hull per unit length with respect to the x-axis including the allowance for the inertia effect of the surrounding water	3- 9
$I_{\mu z}$	Rotary inertia of hull per unit length (difference between the mass moment of inertia of the hull including the effect of added mass of water and the value that would apply if all the mass were concentrated at the longitudinal axis)	3- 3
J	Polar moment of inertia of the section area of a beam or shaft (based on the area of the material)	3- 4
J	Propeller advance ratio $= \dfrac{\text{inflow velocity}}{nd}$	7- 6
J	Longitudinal coefficient applied by F.M. Lewis to values of added mass of water in ship vibration to correct for departure from two-dimensional flow	A- 2
J_{e0}	Effective polar moment of inertia of the midship section area of a hull (based on the area of the material)	3- 6

Symbol	Meaning	Page
j	$\sqrt{-1}$ (imaginary unit)	3-16
K	Spring constant of a vibratory system of a single degree of freedom	4-10
K	Translational spring constant of a control surface system of two degrees of freedom	14- 4
K	Shear rigidity factor for beam or hull such that the slope of the deflection due to shear is equal to the total shearing force at the section divided by KAG, where A is the cross section area (of the material) and G is the shear modulus of elasticity	3- 8
K_p	Pressure coefficient $= \dfrac{p}{\rho n^2 d^2}$	7- 4
K_T	Thrust coefficient $= \dfrac{T}{\rho n^2 d^4}$	7- 5
K_{di}	The effective spring constant of a hull in its ith normal mode of vibration and with respect to driving point d	4- 9
K_{uv}	Spring constant of an entire set of resilient mountings relating a displacement of the mounted assembly in the Y-direction with the restoring force in the X-direction and conversely. A displacement v in the positive Y-direction evokes a force $-K_{uv}\,v$ in the X-direction; if v and K_{uv} are both positive the force is directed toward $-x$. Similarly, a displacement u toward $+x$ evokes a force $-K_{uv}\,u$ in the Y-direction	6- 7
K_{uw}, $K_{u\alpha}$, etc	Spring constants of an entire set of resilient mountings defined by obvious extension of definitions of K_{uv} and $K_{u\beta}$. For K_{uu}, $K_{\alpha\alpha}$ etc., the same axis is used twice	6- 7
$K_{u\beta}$	Spring constant of an entire set of resilient mountings giving either the restoring force in the X-direction due to a unit rotation of the mounted assembly about the Y-axis, or the restoring torque about the Y-axis due to unit displacement of the assembly in the X-direction. The sign convention corresponds to that for K_{uv}	6- 7
K^{I}	Spring constant of entire set of resilient mountings installed between the cradle and the hull in a compound isolation mounting system	6-12
K^{II}	Spring constant of an entire set of resilient mountings installed between the assembly and the cradle in a compound isolation mounting system determined by holding the cradle fixed	6-12
K^{III}	Spring constant of an entire set of resilient mountings installed between the assembly and the cradle in a compound isolation mounting system determined by holding the assembly fixed	6-12
KAG	Shear rigidity of a beam or hull	3- 8
$K.E.$	Kinetic energy	4- 8

Symbol	Meaning	Page
KW	Power	G- 2
k	Spring constant, restoring force per unit displacement	3- 1
k	Horn's empirical coefficient appearing in formula for fundamental torsional frequency of a hull	3- 6
k	Torsional spring constant of a rudder-steering system	14- 2
k	Torsional spring constant of a control surface system of two degrees of freedom	14- 4
k'	Angular spring constant	G- 2
k_a	Axial spring constant of a resilient mounting	6- 6
$k_{e_{cg}}$	Effective spring constant of a local ship structure referred to its center of gravity	6- 3
k_i	Generalized elastic constant of a mass-elastic system applicable to its ith normal mode of vibration	4- 8
k_r	Radial spring constant of a resilient mounting	6- 6
L	Length of a hull (usually assumed to be the distance between the forward and after perpendiculars)	3- 3
L	Electrical inductance	4-11
L	Distance from the axis of a control surface to the center of lift, considered positive if the center of lift is upstream	14- 5
L	A characteristic length or dimension of a ship	15- 8
M	Bending moment	3- 2
M	Mass of a vibratory system of a single degree of freedom	4-10
M	That part of the mass of a control surface system of two degrees of freedom which can vibrate only in translation	14- 4
M	Total mass of a uniform bar	C- 5
M'	Imaginary component of the rotating time vector representing a vibratory bending moment	4- 4
M_i	Generalized mass of a mass-elastic system applicable to its ith normal mode of vibration	4- 8
M_{di}	Effective mass of a hull in its ith normal mode of vibration and referred to the driving point d	4- 9
$M_{e_{cg}}$	Effective mass of a local ship structure referred to its center of gravity	6- 3
M_α	Hydrodynamic moment acting on a spade rudder	14- 2
m	Mass of a rigid body	3- 1

Symbol	Meaning	Page
m	Mass of the rotatable element of a control surface system of two degrees of freedom including an allowance for added mass effect of the water	14- 4
m_s	Mass of a resiliently mounted rigid assembly or "sprung mass" flexibly supported in a hull	G-10
N	Frequency of fundamental vertical flexural mode of a surface ship	3- 3
N	Maximum rpm of a rotating member	10-10
N'	Number of significant vertical flexural modes of a hull	4- 7
N_e	Fundamental torsional frequency of a hull	3- 6
N_H	Frequency of the 2-node horizontal flexural mode of a hull	C- 6
N_n	Predicted fundamental vertical natural frequency of a new ship	C- 2
N_V	Frequency of the 2-node vertical flexural mode of a hull	C- 6
N_0	Known fundamental vertical natural frequency of an old ship	C- 2
n	Frequency of a simple harmonic vibration	3-11
n	Revolutions per second (rps)	7- 5
n	Rpm of a rotating member	10-10
n_1	Frequency of the fundamental mode of vibration of a system	3- 5
P	Single amplitude of a simple harmonic driving force $P \sin \omega t$	4- 9
P_0	Single amplitude of the vertical component of the propeller exciting force (at blade frequency)	D- 2
P_0	Single amplitude of a simple harmonic driving force	3- 1
$P.E.$	Potential energy of a vibrating beam	4- 8
$P(t)$	Concentrated driving force acting on an element of a hull of length Δx	4- 3
$P(x, t)$	Driving force per unit length acting on a beam—in a direction normal to the X-axis	3- 2
$P_h(t)$	Horizontal component of the force produced by a rotating eccentric mass	9- 4
$P_v(t)$	Vertical component of the force produced by a rotating eccentric mass	9- 4
p	Pressure	7- 5
Q	Resonance magnification factor	8- 4
$Q_{di}(t)$	Generalized driving force on a hull applicable to the ith normal mode and referred to the driving point d	5- 4

Symbol	Meaning	Page
$Q_i(t)$	Generalized driving force on a hull applicable to the ith normal mode but without reference to any specific driving point	5- 3
q	Mass distribution coefficient appearing in Prohaska's formula for the fundamental vertical frequency of a hull	C- 3
q	A generalized displacement in matrix notation	6- 9
q	Number of cycles used in estimating the logarithmic decrement from a record of freely decaying vibration	8- 5
$q_i(t)$	Generalized displacement of a vibrating beam in its ith normal mode	4- 8
R	Electrical resistance	4-11
R	Lever arm of weight unbalance of a rotating member	10-10
R	Factor appearing in Prohaska's formula for the fundamental vertical frequency of a hull	C- 3
r	J. Lockwood Taylor's shear correction factor	C- 3
r_1	Correction factor for variable inertia used in applying Prohaska's formula for the fundamental vertical frequency of a hull	C- 3
r_2	Correction factor for shearing force used in applying Prohaska's formula for the fundamental vertical frequency of a hull	C- 3
r_3	Correction factor for transverse compression and dilitation used in applying Prohaska's formula for the fundamental vertical frequency of a hull	C- 3
S	Velocity of undisturbed water relative to a hydrofoil	8- 7
s	Distance from a fixed point measured along the shell plating of a hull in a plane normal to the longitudinal axis of the hull	C- 7
T	Torque with respect to the longitudinal axis of a cylindrical shaft	3- 4
T	Moment about the longitudinal axis of a hull due to all shearing stresses in the cross section	3- 8
T	Torque	G- 2
T	Propeller thrust	7- 5
T_0	Single amplitude of blade-frequency driving torque with respect to the longitudinal axis of a hull	4-13
T_0	Single amplitude of blade-frequency exciting couple with respect to the longitudinal axis of a hull	D- 3
t	Time	3- 1
t	Tip clearance between propeller and hull (in the plane of the propeller)	7- 5
U	Maximum allowable residual unbalance of a rotating member	10-10

Symbol	Meaning	Page
u	A displacement in the X-direction	6- 7
u, v, w	Displacement in the X-, Y-, and Z-directions, respectively	6- 7
V	The net shearing force in the direction of flexural vibration transmitted by one section of a hull to the adjoining section	3- 7
V	The real component of the rotating time vector representing a vibratory shearing force	4- 3
V	Velocity of an aircraft wing relative to the undisturbed air	15- 8
V	Velocity of a ship	15- 8
V	Volume	G- 2
V'	The imaginary component of the rotating time vector representing a vibratory shearing force	4- 4
V_v	Vertical virtual inertia factor = $\left(1 + \dfrac{\text{added weight of water for vertical vibration}}{\text{displacement of the ship}}\right)$	C- 6
v	A displacement in the Y-direction	6- 7
v	Rectilinear velocity	G- 2
W	Energy dissipated per cycle in a simple harmonic vibration in the presence of damping	8- 3
W	Weight of a rotating member	10-10
WR	Weight unbalance of a rotating member	10-10
w	Displacement in the Z-direction	6- 7
X, Y, Z	Rectangular coordinate axes fixed in space	2- 2
X_1, Y_1, Z_1	Rectangular coordinate axes with origin at the center of gravity of the cradle in a compound isolation mounting system	6-12
X_2, Y_2, Z_2	Rectangular coordinate axes with origin at the center of gravity of the assembly in a compound isolation mounting system	6-12
x	Distance in the longitudinal direction forward of the plane of the propeller	
x	Displacement in the X-direction	7- 6
x_r	The X-coordinate of a point r on a beam subject to vibration	4- 8
Y	A rectangular coordinate axis fixed in space	2- 2
Y	The displacement of points of a hull in the Y-direction when vibrating in one of its normal modes of vibration	2- 6

Symbol	Meaning	Page
Y	The single amplitude of a vibratory system of a single degree of freedom	4-11
Y	The steady-state single amplitude of vibration at the stern of a ship due to a simple harmonic driving force of single amplitude P_0	4-12
Y	Displacement in the Y-direction of the axis of rotation of a control surface member	14- 4
Y_{di}	The single amplitude of a hull in its ith normal mode of vibration at the driving point d	4- 9
$Y_a(x)$	Amplitude pattern assumed as a starting mode shape in the calculation of a hull flexural mode by the Stodola method	3-19
$Y_a'(x)$	Amplitude pattern used in the calculation of a hull flexural mode by the Stodola method and obtained from $Y_a(x)$ by a parallel shift of the X-axis	3-20
$Y_a''(x)$	Amplitude pattern used in the calculation of a hull flexural mode by the Stodola method and obtained from $Y_a(x)$ by a combination of a parallel shift and a rotation of the X-axis	3-20
$Y(x)$	An arbitrarily assumed normal mode pattern of vibration of a hull	3-22
$Y_i(x)$	Pattern of displacement in the Y-direction of a hull vibrating in its ith normal mode	3-22
y	Displacement in the Y-direction	3- 2
y	Real component of the rotating time vector representing a vibratory displacement in the Y-direction	4- 3
y'	Displacement in the Y-direction of the center of mass of an element of the hull of length Δx	3- 8
y'	Imaginary component of the rotating time vector representing a vibratory displacement in the Y-direction	4- 4
y''	Displacement in the Y-direction of the center of shear of the cross section of a hull	3- 8
$\dot{y}$	Velocity in the Y-direction	3-16
$\ddot{y}$	Acceleration in the Y-direction	3-12
y_{cg}	Single amplitude of vibration in the Y-direction of the center of gravity of a local ship structure	6- 3
y_{rs}	The amplitude of vibration at a point s of a beam due to a simple harmonic driving force applied at point r	4- 8
y_n^s	Displacement in the Y-direction at the nth station of the hull at the s interval of time in the digital calculation of transient response of a hull	5- 8
$\bar{y}(x)$	Mode shape obtained in the calculation of hull modes by the Stodola method on a graph in which its magnitude differs by the factor $1/\omega^2$ from that of the curve assumed in starting the calculation	3-20

Symbol	Meaning	Page
Z	Mechanical impedance based on displacement	4-11
Z	Electrical impedance	D- 1
Z	Electrical impedance of a circuit having resistance, inductance, and capacitance all in series	4-11
Z_v	Mechanical impedance based on velocity	4-11
Z_{di}	Mechanical displacement impedance of a hull in its ith normal mode of vibration at driving point d	4-11
z	Number of blades per propeller	7- 4
$\bar{z}$	Z-coordinate of the center of mass of an element of a hull of length Δx (including allowance for added mass of water)	3- 7
$\bar{\bar{z}}$	Z-coordinate of the center of shear of the cross section of a hull vibrating flexurally in the Y-direction	3- 7
α	Empirical constant appearing in impedance-type formulas for stern vibration of a ship	7-11
α	Angle of attack of a spade rudder	14- 2
α_A	Empirical constant in impedance-type formula for athwartship vibration of hulls	7-11
α_T	Empirical constant in impedance-type formula for torsional vibration of hulls	7-11
α_v	Empirical constant in impedance-type formula for vertical vibration of hulls	7-11
$\alpha, \beta, \gamma,$	Angular displacements with respect to the X-, Y-, and Z-axes, respectively	6- 7
β	Angular displacement with respect to the Y-axis	6- 7
β	Section area coefficient	A- 2
β	Empirical coefficient appearing in the formula of Todd and Marwood for the fundamental vertical frequency of a hull	C- 5
β'	Component of the slope of the elastic line of a hull due to shearing only	5- 2
γ	Real component of the rotating time-vector representing a vibratory angular displacement of the cross section of a hull with respect to a Z-axis	4- 3
γ	Angular displacement with respect to the Z-axis	3- 8
γ'	Imaginary component of the rotating time-vector representing a vibratory angular displacement of the cross section of a hull with respect to a Z-axis	4- 4
Δ	Displacement of a ship	3- 3

Symbol	Meaning	Page
ΔS	A small distance along the shell plating in the plane of the section	C- 6
Δx	Length of a small element of a hull measured in the direction of its longitudinal axis	3- 7
δ	Logarithmic decrement for a free vibration	8- 3
δ	Shell plating thickness	C- 6
θ	Angle of attack of a hydrofoil	8- 7
θ	Angular displacement of a control surface from its equilibrium position	14- 4
θ	Angular displacement	G- 2
λ	Complex exponential term in the expression for vibratory motion whose real part indicates the rate of decay or buildup and whose imaginary part indicates the circular frequency ($\lambda = \mu + j\,\omega$)	14- 5
λ	Scale factor by which a dimension of a ship is multiplied to obtain the corresponding dimension for the ship model	G- 1
λ	Wavelength	3- 5
λ_i	Frequency of free vibration in the ith normal mode in the presence of damping	5- 5
μ	Mass per unit length	3- 8
μ	Real part of complex exponential term in the expression for vibratory motion indicating the rate of decay or buildup of the vibration, that is, the degree of positive or negative damping	14- 6
ρ	Radius of curvature of the elastic line of a deformed beam	3- 2
ρ	Mass density of water	7- 5
τ	Time at any instant between 0 and t	5- 5
ϕ	Rotation of the cross section of a beam or hull with respect to its longitudinal axis	2- 4
ϕ	Phase angle by which the driving force leads the displacement in a simple harmonic vibration	4- 9
ϕ	Steady-state single angular amplitude of a hull at the stern and with respect to its longitudinal axis due to a simple harmonic driving torque of single amplitude T_0	4-13
ϕ	Empirical coefficient appearing in Burrill's formula for the fundamental vertical frequency of a hull	C- 3
ϕ	Single amplitude in rotation about the longitudinal axis of a hull	D- 3
$\phi_i\,(x)$	The ith function of a series of orthogonal functions of x	4- 8
ω	Circular frequency of a simple harmonic time-varying quantity	3- 1

Symbol	Meaning	Page
{ }	A column matrix	6- 9
[ ]	A matrix	6- 9
⨂	The midship section of a hull	A- 1
≈	Equals approximately	4-33
·	Designates differentiation with respect to time, when over a symbol	3- 1
··	Designates double differentiation with respect to time when over a symbol	3- 1
≤	Equal to or less than	5- 7

REFERENCES

N-1. Saunders, H.E., "Hydrodynamics in Ship Design," published by SNAME (1957).

N-2. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

CHAPTER 1

HISTORICAL BACKGROUND

Although ship vibration phenomena were undoubtedly encountered much earlier, the subject appears to have first attracted scientific study toward the end of the 19th century. As might be expected, the impetus came from the occurrence of resonance which indicated the need for a method of predicting the natural frequency of vibration of the hull.

In 1894 O. Schlick[1-1] proposed a formula for the fundamental vertical hull frequency and this provided the naval architect of that time with a guide for designing his propulsion system so that the operating propeller shaft rpm would not coincide with this hull frequency.

Other evidence of interest in the subject in this early period is the French textbook "Theorie du Navire,"[1-2] published in 1894 which included a chapter on ship vibration among its four volumes. In that work, examples were cited in which the rated speed of ships had to be reduced to avoid hull vibration. It does not detract from the pioneering contribution of the authors of that classical work that they were led astray in their speculations regarding hull vibration by the observation that the ratios of natural frequencies of hulls to the fundamental frequency corresponded more nearly to those of the string than to those of the solid bar with free ends.

Another early investigator in this field was A.N. Krylov who recorded hull vibration on a naval cruiser in 1900.[1-3] His work on both the theoretical and practical aspects of the subject led to a complete book on the subject of ship vibration published in 1936.[1-4] This work is devoted chiefly to the fundamentals of mechanical vibration and the application of classical beam theory to the hull vibration problem.

Increasing interest in the subject is evident in the technical literature from about 1900 to World War II. A picture of the status of the development of the theory of hull vibration around 1932 is given by the paper of E. Schadlofsky[1-5] where it is suggested that the fundamental vertical frequency of the hull can be estimated by a beam-type analysis involving graphical integration. This process, based on the method of Stodola,[1-6] is discussed in detail in Chapter 3. As indicated in the bibliography on page Bi-1, numerous other authors have explored the application of beam theory to the analysis of hull vibration.

About the time Schadlofsky's paper was published, considerable impetus was given to the experimental phase of ship vibration research by the manufacture of machines capable of vibrating entire hulls. This development took place in Germany where such machines had been previously designed by the firm of Losenhausen in Dusseldorf for the dynamic testing of riveted and welded bridges. These machines contained adjustable eccentric masses so arranged that unidirectional sinusoidal forces and couples could be produced, as discussed in Chapter 15.

[1-1] References are listed at end of each chapter. For complete bibliography, see page Bi-1.

The largest machine of this type was delivered to the U.S. Experimental Model Basin in Washington, D.C. in 1931. Starting from that date, the U.S. Experimental Model Basin and its successor, the David Taylor Model Basin, continued to maintain and develop machines of this general type and to conduct experiments to verify theoretical predictions of hull vibration characteristics.

Since World War II ship vibration research has been carried on at an expanding rate by all the principal maritime nations of the world. This is evident from the bibliography. In the United States, the Society of Naval Architects and Marine Engineers has done much to stimulate interest in ship vibration and has cooperated closely with the Bureau of Ships of the Navy Department in this field. Two of its research panels, in particular, have been directly concerned with the ship vibration problem.

In recent years the development of analog and digital computers has contributed greatly to the development of hull vibration analysis.[1-7] Simultaneously, experimental techniques have been devised to determine the vibratory response characteristics of the hull[1-7] as well as the forces tending to excite vibration in the hull.[1-8] So broad is the horizon that has been made visible by modern developments in computing techniques that methods of vibration analysis entirely independent of the beam theory of the hull are now under investigation. These methods are along the line suggested by Professor H.A. Schade in his discussion of Reference 1-7. In these "three-dimensional" analyses, the restriction that all points at the same cross section of the hull partake of the lateral motion of the "hull girder" is removed. No results of these investigations, however, are available at this time (1960).

REFERENCES

1-1. Schlick, O., Series of Articles on Ship Vibration in TINA (1884, 1893, 1894, 1901, and 1911).

1-2. Pollard, J. and Dudebout, A., "Theorie du Navire,"–Treizieme Partie, "Vibrations des Coques des Navires a Helice," Vol. IV, Chapt. LXIX, "Causes et Periodes des Vibrations," Paris (1894).

1-3. Babaev, N.N., "Contemporary Status of Development of the Problem of Ship Vibration," TMB Translation 291 (Jul 1959) from Russian paper in Sudostroenie, No. 3 (161) (Mar 1956).

1-4. Krylov, A.N., "Vibration of Ships," in Russian (1936).

1-5. Schadlofsky, E., "The Calculation and Measurement of Elastic Natural Frequencies of Ship Hulls," STG, Vol. 33 (1932). Also EMB Translation 7 (Jun 1934). Supplement (Nov 1935).

1-6. Stodola, A., "Steam and Gas Turbines," (Dampf-und Gasturbinen). Translated by Dr. Lewis C. Lowenstein, P. Smith, New York (1945).

1-7. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

1-8. Pien, P.C. and Ficken, N.L., "The Measurement of Propeller Induced Vibratory Forces on Scale Ship Models," Paper presented at ATTC Conference (Sep 1959).

CHAPTER 2

QUALITATIVE DISCUSSION OF SHIP DYNAMICS

A. INTRODUCTION

Unlike the case of a body free in space, the dynamical system considered in this book must include both the body itself and its surrounding medium. The density of the medium in this case is comparable with the density of the material of which the vehicle is constructed; this is contrary to the situation in aeronautics. It is therefore to be expected that the water will have a marked effect on the dynamical behavior of the ship, and there is abundant evidence that this is the case.

The forces exerted by the water on the hull arise either from pressure, which acts in a direction normal to the hull surface at any point, or from friction or shear, which acts in a direction tangential to the surface. As far as rigid body motions are concerned, when these two sets of forces are integrated over the wetted surface of the hull, the entire system of forces can be reduced to effective forces acting at the center of gravity in each of the three principal directions (vertical, longitudinal, and athwartship) and effective moments about the three axes through this point. In general, these forces and moments depend not only on the rectilinear and angular displacements of the hull with respect to these axes but also on the rectilinear and angular velocities and accelerations; or, in the case of rough seas, on the motion of the water surface relative to the ship.

Concurrently with these rigid body motions the hull may execute elastic vibrations of numerous types. Although these latter vibrations are the main subject of this book, they cannot be considered as entirely independent of the rigid body motions. In fact, in rough seas the rigid body motions frequently lead to vibrations accompanying large hydrodynamic impacts, and, even in calm seas, the forward motion of the ship may generate hydrodynamic flow excitations of different types. It is shown in Reference 2-1 that, although the effect of buoyancy may be detectable for the frequency of the fundamental mode of vibration of long, slender hulls, it is in general justifiable to neglect the effect.

B. RIGID BODY MOTIONS

When considered as a rigid body, a ship has six degrees of freedom, and hence there are six displacement-like quantities to be taken into account in completely specifying its motion. The steady forward velocity, the only motion desired in the normal operation of the ship, is not ordinarily considered in discussing its rigid body motions. They are the motions superimposed on this steady forward velocity by the sea action, and always involve time-varying velocities and accelerations. With reference to the axes shown in Figure 2-1, the rigid body displacements in translation in the X-, Y-, and Z-directions are called, respectively, surge, sway (or sidling), and heave, whereas the angular displacements about the same axes are called roll, pitch, and yaw. Of these six displacements, the three most important are roll, pitch, and heave.

Although right-hand systems of coordinate axes are used throughout this book, it has been found convenient, where elastic vibrations are dealt with, to orient the Y-axis in the direction of the vibration. Thus when horizontal (athwartship) hull vibration is under discussion the axes are oriented as in Figure 2-1, but when vertical hull vibration is discussed the Y-axis is taken vertical and the Z-axis horizontal (with positive direction out of the paper.)

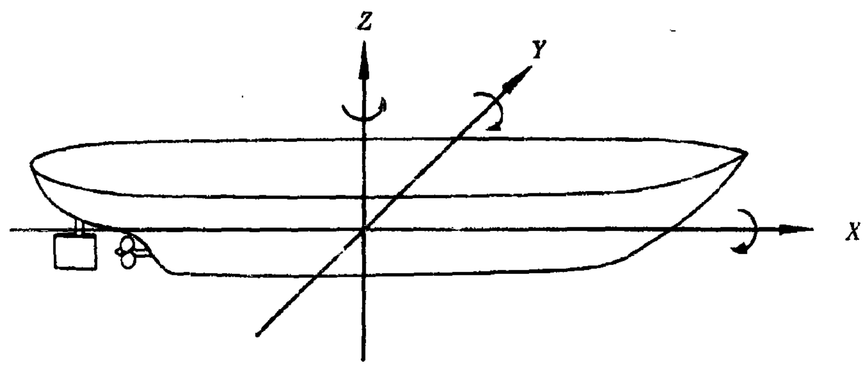


Figure 2-1 – Axes through the Center of Gravity of a Ship with Respect to Which Forces and Moments Exerted by the Water May Be Defined

In the dynamics of rigid bodies, motions are defined in terms of the translation of the center of mass and rotation about the center of mass. While this procedure is also applied to the ship, it must be realized that, since the hull is elastic, these relatively slow motions are also accompanied by elastic deformations. These elastic deformations are not the ones considered in the discussion of hull vibration. Furthermore, the hull by itself is not usually considered as an isolated body because the component of the water forces due to acceleration is usually accounted for by adding mass to the hull mass to take care of this inertia effect. Since the rolling, heaving, and pitching motions, although slow, are still oscillatory and thus have the essential characteristics of vibrations, it is important to distinguish them from the elastic vibrations which, as has been stated, are the main subject of this book.

In the absence of an external alternating force, a body in free space could not execute motions in which the displacement of the center of mass was oscillatory. The ship is subject to the constant force of gravity and to gravity moments which vary with its angular displacements about axes other than those through its center of gravity. The buoyancy moments accompanying rolling, heaving, and pitching are due to buoyancy forces that vary with these motions. Although elastic deformations in general accompany these motions, they are too small to play an essential role in determining these motions, and the term "rigid body motions" is retained to distinguish them from the motions of the hull in which the elastic deformations do play an essential role.

A ship stopped in a calm sea, if initially disturbed, will execute damped rigid body motions and eventually again come to rest. In disturbed seas, the surrounding water provides not only the restoring forces and moments necessary for oscillatory rigid body motions and the forces associated with the added mass, but also the forces and moments required to maintain these motions in the presence of damping.

C. ELASTIC VIBRATIONS OF THE ENTIRE HULL

When a ship is subjected to an impulsive load, such as occurs when a descending anchor is suddenly arrested, it will execute elastic vibrations in addition to whatever rigid body motions are excited. Of these vibrations some are observed only locally and some are observed throughout the hull. The latter, in general, are of the type that may exist in a beam free in space and so are called "beamlike." Although the surrounding water plays an important role in these vibrations, it does not destroy their beamlike characteristic and it is helpful to consider the vibrations of the ideal solid beam free in space. This is frequently spoken of as the free-free beam (both ends free).

As emphasized in standard works on mechanical vibration,[2-2, 2-3, 2-4] the two terms "modes" and "nodes" are used repeatedly in the discussion of continuous systems and must not be confused with each other in spite of the similarity in spelling. Thus the mode is the pattern or configuration which the body assumes periodically while in the vibratory condition, whereas the node is a point in the body which has no displacement when the vibration is confined to one particular mode. "Normal mode" of vibration is another very common term. The normal modes are the patterns in which the body can vibrate freely after the removal of external forces.

A beam free in space may undergo four principal types of elastic deformation designated as bending, twisting, shearing, and extensional deformations. These may all occur simultaneously. In a solid beam, these same types of deformation may exist with respect to any of the three principal directions even though the relative magnitudes of bending, shearing, and torsion may be very different with respect to the different axes. In the case of the ship, the elastic deformations that play a significant role in its vibration are limited to bending and shearing in both the vertical and horizontal planes through its longitudinal axis, and to torsion about the longitudinal axis. The identification of extensional (longitudinal) beamlike vibrations of hulls has so far been inconclusive, and this type of vibration is ordinarily considered insignificant in ships although it may be quite significant in the propulsion systems themselves, as shown in Chapter 12.

In a symmetrical beam the bending and shearing effects combine to produce what are usually called the flexural modes, as illustrated in Figure 2-2.

The curves plotted in Figure 2-2 indicate the displacements in the Y-direction of points falling on the X-axis when the bar is at rest. Similar modes exist for displacements in the Z-direction.

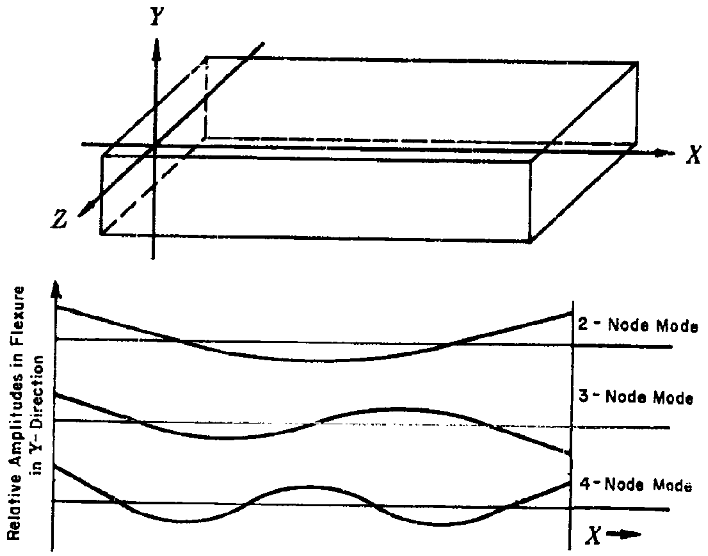


Figure 2-2 – Flexural Modes of a Free-Free Uniform Bar

Figure 2-3 illustrates the torsional modes in which a uniform beam may vibrate, and the curves plotted show angular displacement versus distance from the end.

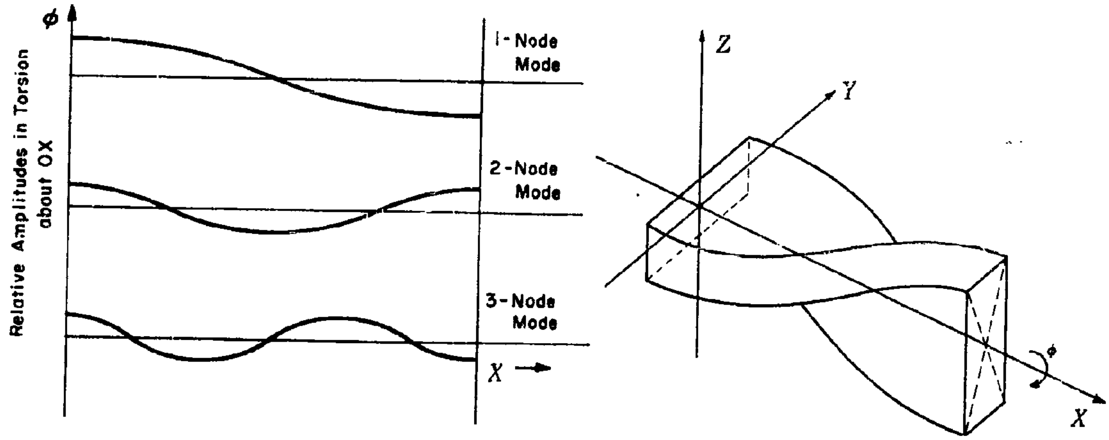
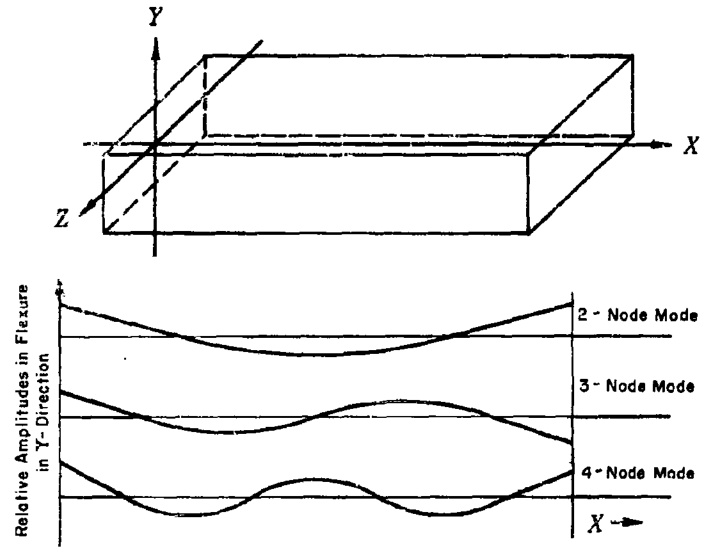


Figure 2-3 – Torsional Modes of a Free-Free Uniform Bar

In both the flexural and torsional types of vibration, a natural frequency is associated with each pattern of vibration and the natural frequencies increase as the number of nodes (points at which the curves cross the X-axis) increases.



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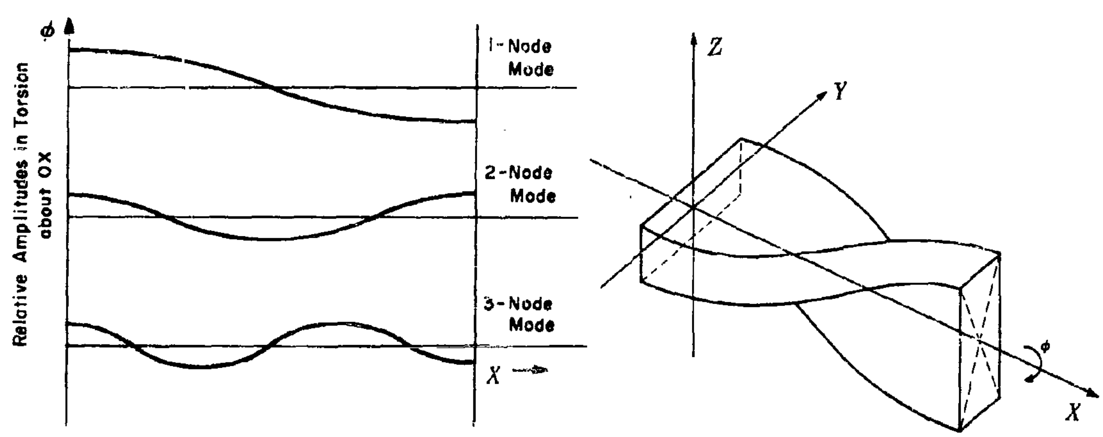


Figure 2-3 – Torsional Modes of a Free-Free Uniform Bar

In both the flexural and torsional types of vibration, a natural frequency is associated with each pattern of vibration and the natural frequencies increase as the number of nodes (points at which the curves cross the X-axis) increases.

If a free-free beam is unsymmetrical with respect to either the vertical or horizontal plane through its longitudinal axis, it will be found that its natural modes of vibration involve torsion, bending, and shearing simultaneously. Thus from Figure 2-4 it is clear that if the left end of the bar accelerates in the Y-direction, the bar will tend to twist because of the inertia of the vertical member whose center of mass lies above the X-axis.

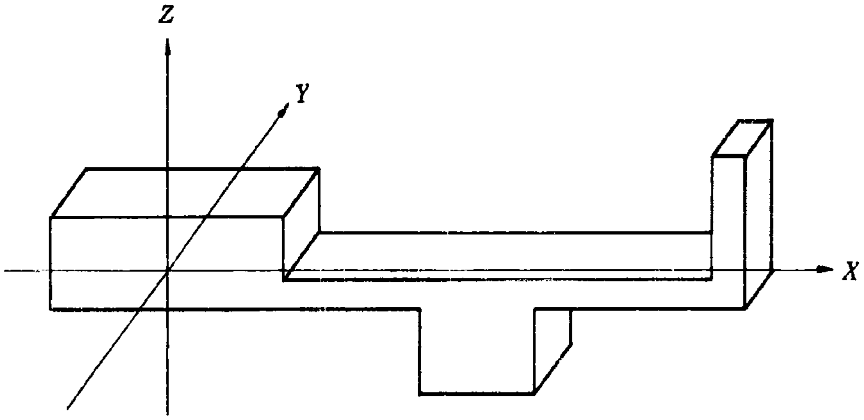


Figure 2-4 – Free-Free Bar Whose Normal Modes of Vibration Involve Combined Torsion, Bending, and Shearing

The normal modes of vibration of the ideal free-free beam are independent of one another, and, under an impact, the beam may vibrate in several of these modes simultaneously. However, such a system has the property that, if it is initially deformed into a pattern corresponding to any of its normal modes of vibration, it will thereafter vibrate only in that mode and at the frequency associated with that mode.

Patterns of two typical torsion-bending modes of a hull are illustrated in Figure 2-5.

A hull, of course, is a much more complicated structure than a solid beam. It behaves like the free-free beam only in its lower modes of vibration. Hence these modes are said to be beamlike,and they may be excited by either transient or steady-state disturbances. The transient disturbances are due to wave or slamming impacts which induce trains of damped vibrations in one or more of these modes simultaneously. Steady-state vibrations are caused by rotating unbalanced engine or machine elements, unbalanced propellers, or unbalanced shafting. Vibration may also be set up by nonuniformity of pitch among the blades of a propeller and, above all, by the variation in load on the individual blades as they rotate in the nonuniform velocity field in the propeller race. The propellers also cause pressure fluctuations on the surface of the hull and appendages in their immediate vicinity. Propeller blade excitation is the chief cause of steady-state ship vibration at this time (1960).

A common characteristic of the forced propeller-excited vibration of ships is that it is concentrated in the stern. Since the beamlike modes of vibration involve large amplitudes at both ends of the hull, this phenomenon obviously does not result from vibration in a single normal mode. It is apparently due to the resultant of the nonresonant responses in several normal modes, as explained in Chapter 4.

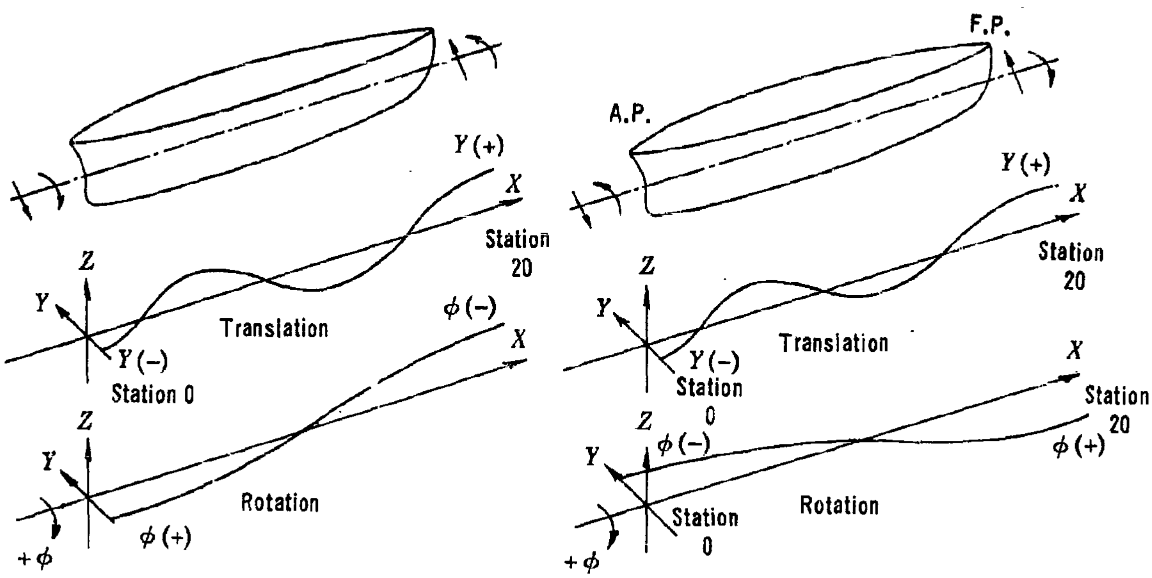


Figure 2-5 – Two Torsion-Horizontal Bending Modes of a Hull Having Reversed Phase Relations between Rotation and Translation

D. ADDED MASS

Before proceeding with the qualitative discussion of ship dynamics, it is necessary to give further attention to the inertia effect of the surrounding water as it relates to the elastic vibrations of the hull. While it is true that the water can actually exert only normal pressures and frictional forces on the hull, these forces may be broken down into components which have different time rates of change. A component of the water forces that is proportional to the acceleration of the hull at the point of interest and opposite to the acceleration in direction yields here an effect of increased inertia. The relatively high density of water makes this inertia effect of serious concern in the vibration of ships and underwater ship components.

Unfortunately, a variety of terms have been used in the technical literature for designating this water inertia effect. There are two distinct concepts responsible for some of the confusion in terminology: (1) the apparent increase in mass of a body vibrating in water; and (2) the apparent total mass of the body (including the effect of the water). Such terms as "added mass," "virtual mass," "apparent mass," and "apparent added mass" will all be found in various publications on the subject. Although reference must inevitably be made to papers in which such terms as "virtual mass" and "apparent mass" appear, only the term "added mass" will be used hereafter in this book for the inertia effect of the water. This conforms to Reference 2-5.

Not only is there much confusion about terminology regarding water inertia but there is also a growing feeling that the concept of added mass of bodies vibrating in water has outlived its usefulness. This arises from the fact that as the frequency increases, the assumption of incompressibility of the water on which the added mass concept is based becomes untenable and the vibrating body becomes, in effect, a source of underwater sound.

In the theoretical treatment of the added mass effect, the flow pattern for vibratory rigid body motion is considered the same as for steady or unidirectional motion. Moreover, when a rigid circular cylinder with its axis lying in the plane of a free surface of water is vibrating vertically, it is assumed that the flow pattern is the same below the surface as if the cylinder were deeply submerged. This justifies the treatment as one for a circular cylinder in infinite fluid and the subsequent discarding of half of the added mass that would apply to the deeply submerged case; see Figure 2-6.

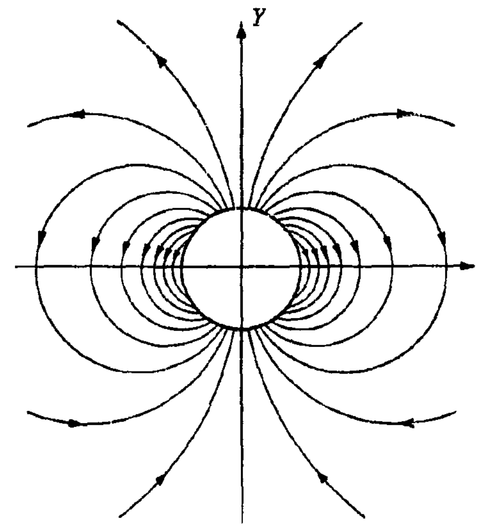


Figure 2-6 – Pattern of Flow about a Circular Cylinder Moving with Constant Velocity in the Positive *Y*-Direction in an Unbounded Fluid Medium

The theoretical derivation of added mass for vibrating hulls (by considering the flow about ideal bodies in incompressible fluids) is based on the assumptions used in classical hydrodynamics (potential flow). (See References 2-6 and 2-7.) The values derived on the basis of two-dimensional incompressible flow are corrected for three-dimensional effects and then applied as ordinary added masses in the hull vibration calculations as will be discussed in the next chapter. It must be pointed out here, however, that, because of the complex form of hulls, when motion of an underwater form takes place in a given direction, inertia effects are developed not only in that direction but also in other directions. This phenomenon is spoken of as an inertia coupling between the various degrees of freedom. As shown in Reference 2-8, in the most general case for a submerged rigid body having six degrees of freedom, there are 21 such inertia terms. Fortunately, in practice most of these either vanish because of symmetry or can be neglected.

It may seem surprising that the added mass effects for vibratory motion do not vary widely with the ship's forward velocity, but the potential flow theory indicates no variation;

this has been borne out by experimental observations. For further discussion of this point, see pages 59 and 60 of Reference 2-7.

E. LOCAL EFFECTS

In view of the complexity of a ship's structure, the number of "local" hull structures is enormous. However, these structures may be divided into categories of different relative importance. The most important distinction to be made as far as hull vibration is concerned is between those structures that have an appreciable effect on the vibration characteristics of the ship as a whole and those that do not. The possibility of affecting the ship as a whole obviously depends primarily on the mass of the local structure, but it also depends on its location and its stiffness.

When a mass and spring are attached to a free-free beam, the "sprung mass" participates in the normal mode vibrations of the combined system. It can introduce an extra mode so that, as far as the beam itself is concerned, there may then exist two flexural modes with the same number of nodes. Its effect on the previously existing modes depends on both its mass and the proximity of the beam frequencies to the natural frequency of the mass-spring combination, that is, to the natural frequency of the mass when the end of the spring is held fixed.

When local flexibilities of ship structures produce a sprung mass effect the normal modes of the hull tend to depart from beamlike form, and modes of vibration of the ship may be found in which the local vibration is excessive, whereas at the ends of the ship the vibration is well within tolerable limits.

When local structures are of relatively small mass in comparison to the mass of the ship, their effect on the vibratory characteristics of the ship as a whole will be negligible. However, because of resonance, they may themselves vibrate excessively. If their natural frequencies coincide with the frequency of some source of excitation prevailing at the operating speed of the ship, these structures may respond to an imperceptible hull vibration so as to produce an intolerable local condition. Obviously, the cure for such a condition is to change the natural frequency of the local structure.

F. SHALLOW WATER EFFECTS

The vibration characteristics of ships are materially modified in passing from deep to shallow water. In the first place, there is a marked increase in the added mass effect for vertical vibration, and in the second place, the propeller exciting forces may be greatly changed.

The change in added mass effect is due to the alteration of the noncirculatory flow pattern. A rough rule for the limit of depth at which this effect is no longer evident is six times the draft. When the depth is less than this a lowering of the frequencies of the vertical modes of the hull is observed.

The variation in propeller excitation in restricted waters arises from a modification of the steady flow due to the restriction of the channel around the ship. As will be pointed out in Chapter 7, any effect that disturbs the uniformity of flow into the propeller races will set up lateral forces which are transmitted to the hull through the propeller shaft bearings.

In addition to these two effects there will usually be a reduction in operating speed on entering shallow water. Thus the vibratory level may vary because of any one of these separate effects.

Although it is not inconceivable under the circumstances that a particular ship might experience less vibration while operating in shallow water, the chances are that it will experience more. Specific examples of increased vibration in shallow water are cited in References 2-9 and 2-5. If a particular hull happens to be subject to resonant vibration when operating at its designed speed in deep water, then it is quite possible that resonance will be avoided at the speed assigned to shallow water operation.

REFERENCES

2-1. McGoldrick, R.T., "Buoyancy Effect on Natural Frequency of Vertical Modes of Hull Vibration," JSR (Jul 1957).

2-2. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

2-3. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

2-4. Rayleigh, Lord, "Theory of Sound," Second Edition, Dover Publications, New York (1945).

2-5. Saunders, H.E., "Hydrodynamics in Ship Design," published by SNAME (1957).

2-6. Lewis, F.M., "The Inertia of Water Surrounding a Vibrating Ship," Trans. SNAME, Vol. 37 (1929).

2-7. Wendel, K., "Hydrodynamic Masses and Hydrodynamic Moments of Inertia," STG, Vol. 44 (1950). Also TMB Translation 260 (Jul 1956).

2-8. Lamb, H., "Hydrodynamics," Dover Publications, New York (1945).

2-9. Noonan, E.F., et al., "Vibration Measurements on Ship S6-2 during Builders Trials," prepared for Hull Structure Committee of SNAME by Noonan, Knopfle, and Feldman, Professional Engineers (Nov 1959).

CHAPTER 3

BASIC BEAM THEORY OF SHIP VIBRATION

A. INTRODUCTION

The fundamental system considered in all texts,[3-1, 3-2] on mechanical vibration is the lumped mass-spring system of one degree of freedom shown schematically in Figure 3-1.

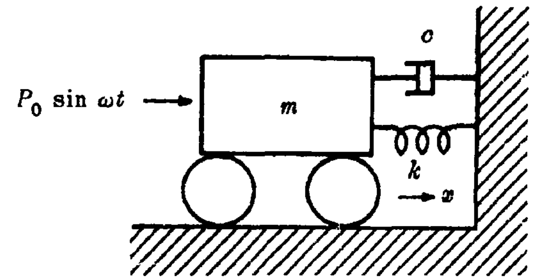


Figure 3-1 – Rectilinear Vibratory System of One Degree of Freedom

This system has mass m, spring constant k, viscous damping constant c, and in this case is acted upon by a simple harmonic driving force $P_0 \sin \omega t$ in the x-direction. The mass m is so restrained that it can move only in the x-direction.

The differential equation governing this case; namely,

$$m\ddot{x} + c\dot{x} + kx = P_0 \sin \omega t \qquad [3\text{-}1]$$

[where the dot denotes differentiation with respect to t (time)] is the most widely discussed equation in mechanical vibration theory. Its steady-state solution yields the familiar resonance curves of forced vibration. These indicate that very large amplitudes of vibration of the mass m will result when ω is close to the natural circular frequency of the system $\left(\omega \approx \sqrt{\frac{k}{m}}\right)$, and the damping constant c is "small."

Also quite important in vibration theory is the solution of Equation [3-1] when the driving force is absent ($P_0 = 0$). This yields an exponentially decaying free vibration at a frequency which approaches the undamped natural frequency as $c \to 0$.

Just as the lumped system of one degree of freedom provides the basis for the understanding of the vibratory characteristics of many familiar mechanical systems (for example, the pickup units of many vibration instruments), so the uniform free-free beam provides a basis for an understanding of the essential vibratory characteristics of ships.

The free-free uniform bar or beam is, of course, a continuous system (as contrasted with the lumped system of Figure 3-1), and, although it also has the properties of inertia and elasticity possessed by the system of Figure 3-1, the differential equation governing its vibratory motion is considerably more complicated.

The beam of Figure 3-2 is assumed to have a mass per unit length μ and a bending stiffness EI in the xy-plane. In the terminology of the Euler-Bernoulli beam theory this means that, if the beam is so slender that it can be bent into circular form in the XY-plane, with the two ends joined together, certain simple relations exist. Thus the bending moment M, due to the normal internal stresses acting at any cross section, will be related to the mean radius of curvature ρ by the equation

$$M = \frac{EI}{\rho} \qquad [3\text{-}2]$$

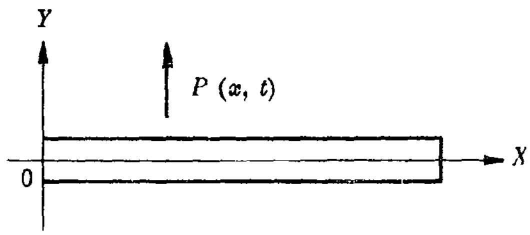


Figure 3-2 – Slender Beam Free in Space Subjected to a Lateral Forcing Function $P(x, t)$

When small deflections y of the beam of Figure 3-2 take place in the Y-direction, the approximation that the curvature (reciprocal of the radius of curvature) is equal to the second derivative of y with respect to x can be used. The familiar equation relating bending moment and deflection in simple beam theory is then

$$M = EI \frac{\partial^2 y}{\partial x^2} \qquad [3\text{-}3]$$

From this relation it can be shown that, in contrast with the ordinary second-order differential equation governing the system of Figure 3-1, the equation governing the system of Figure 3-2 is a partial differential equation which is of the fourth order with respect to x and the second order with respect to t (time). This equation is

$$EI \frac{\partial^4 y}{\partial x^4} + \mu \frac{\partial^2 y}{\partial t^2} = P(x, t) \qquad [3\text{-}4]$$

where $P(x, t)$ is the driving force per unit length in the Y-direction. This equation is widely discussed in the literature,[3-1, 3-2] and in its homogenous form $[P(x, t) = 0]$ leads to the well-known formulas for the natural frequencies of uniform slender beams with various end conditions.

It was natural that, since the ship when advancing through waves is loaded in bending and hence is essentially a beam, the early attempts to develop formulas for its natural frequencies should be based on the formula for the natural frequency of the free-free uniform beam.

The best known of such formulas is the Schlick formula for the fundamental vertical frequency of a surface ship:

$$N = C\sqrt{\frac{I}{\Delta L^3}} \qquad [3\text{-}5]$$

This formula is given in mixed units for convenience in practical application; thus

N is the fundamental vertical frequency in cpm,

C is Schlick's empirical "constant,"

I is the area moment of inertia of the midship section in ft^2-in.^{2} units,

Δ is the displacement in long tons, and

L is the length in ft.

For ranges of values of C and further discussion of this formula, see Appendix C. Other well-known formulas such as those of Burrill,[3-3] Todd and Marwood,[3-4] or Prohaska[3-5] are also discussed in that appendix.

Here the empirical formulas are contrived to account for the many ways in which the ship departs from the free-free uniform beam. Aside from its nonuniformity, one of the chief respects in which a ship departs from a slender beam in its vibratory characteristics is in the relatively much greater shearing flexibility of the ship. This is because the ship is not, in fact, as slender as the beams for which the Euler-Bernoulli assumptions are valid.

The modification of the Euler-Bernoulli uniform beam to allow for shearing flexibility yields what is now generally referred to as the "Timoshenko beam." This is still a uniform solid beam, but when it is deformed, the slope of its elastic line is considered to have one component due to bending and another due to shearing. In the actual equation discussed by Timoshenko,[3-2] there was included not only a term for shearing rigidity but also a term for rotary inertia, neither of which appear in the equation for the Euler-Bernoulli vibrating beam. The rotary inertia represents the increased inertia effect because the mass of the ship is not concentrated along its longitudinal axis.

The homogeneous form of Timoshenko's equation in the notation adopted for this book is:

$$EI\,\frac{\partial^4 y}{\partial x^4} - I_{\mu z}\,\frac{\partial^4 y}{\partial x^2 \partial t^2} + \mu\,\frac{\partial^2 y}{\partial t^2} + \frac{I_{\mu z}}{KAG}\,\mu\,\frac{\partial^4 y}{\partial t^4} - \frac{\mu EI}{KAG}\,\frac{\partial^4 y}{\partial x^2 \partial t^2} = 0 \qquad [3\text{-}6]$$

This equation has been discussed in many publications in addition to Reference 3-2. Even when the term for rotary inertia $(I_{\mu z})$ is omitted, it has not been found possible to derive from it a direct formula for the natural frequency of the free-free beam. However, curves can be plotted showing how the frequencies of various modes vary with the ratios $\frac{\mu L^4}{EI}$ and $\frac{EI}{2KAGL^2}$, as shown in Reference 3-6. The first of these ratios appears in the formula for the slender uniform beam in which shearing flexibility is neglected. This formula is

$$\omega_1 = 22.4\sqrt{\frac{EI}{\mu L^4}} \qquad [3\text{-}7]$$

The second ratio involves the relative magnitudes of the bending and shearing rigidities.

Many questions may be raised as to the interpretation of Equation [3-6] and of the wave solutions to which it gives rise. Since the ship is not a uniform beam, and analytical expressions cannot be given for the parameters EI, μ, $I_{\mu z}$, and KAG as functions of x, the reader is referred to the literature for further discussion of this equation. (See the bibliography at the end of the book.)

Before considering the equations that provide the basis for the rational theory of ship vibration proposed in this book, it is necessary to discuss briefly the torsional vibrations of the free-free uniform beam. As in the case of the flexural vibrations, in which the Euler-Bernoulli assumptions provided an integrable equation, a simplified theory of torsional vibration of beams or hulls is based on the torsional equations for the uniform (cylindrical) shaft.

Figure 3-3 shows a solid cylindrical shaft with axis coinciding with the X-axis. It is

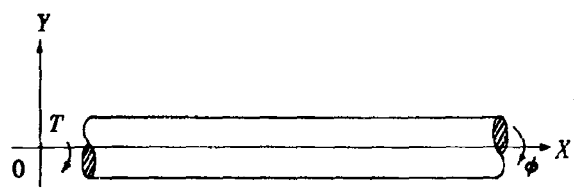


Figure 3-3 – Cylindrical Shaft with Axis Coinciding with OX and Twisting about OX

shown in texts on "strength of materials"[3-7] that the torsional rigidity of such a shaft is GJ, where G is the shear modulus of elasticity and J is the polar moment of inertia of the area of the cross section with respect to the X-axis. This means that, if one end is held fixed and a torque T is applied to the other end, the resulting twist at the point of application of the torque is given by the equation

$$\phi = \frac{TL}{GJ} \qquad [3\text{-}8]$$

where L is the length.

It can be shown that the torsional oscillations of such a shaft are governed by a partial differential equation of the second order with respect to both x and time. This equation is

$$GJ\,\frac{\partial^2\phi}{\partial x^2} = I_{\mu x}\,\frac{\partial^2\phi}{\partial t^2} \qquad [3\text{-}9]$$

where $I_{\mu x}$ is the mass polar moment of inertia of the shaft per unit length.

This equation merits special attention because it has the form of the one-dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \qquad [3\text{-}10]$$

In this well-known equation u may represent any displacement-like quantity, and c, the velocity of wave propagation. Thus, if in Figure 3-2, u is the displacement in the X-direction and c is the velocity of propagation of longitudinal waves along the beam, the longitudinal vibrations of the beam are governed by Equation [3-10]. In the torsional case of Figure 3-3 the velocity of propagation is

$$\sqrt{\frac{GJ}{I_{\mu x}}}$$

As shown in texts on acoustics,[3-8] when the governing differential equation for the vibration of the mechanical system can be expressed in the form of Equation [3-10], the formulas for the natural frequencies can be readily derived from the relations among frequency, wave velocity, and wavelength

$$c = n\lambda \qquad [3\text{-}11]$$

where n is the frequency and λ is the wavelength. The normal mode patterns in such cases are sinusoidal, and the boundary conditions determine what fraction of a wavelength is included in the distance between the boundaries of the system.

For a cylindrical shaft free in space, the frequency of the fundamental torsional mode which has one node at midlength is given by the formula:

$$n_1 = \frac{1}{2L} \sqrt{\frac{GJ}{I_{\mu x}}} \qquad [3\text{-}12]$$

The mode shape shown in Figure 3-4 is a half cosine. It will be noted that in this case the

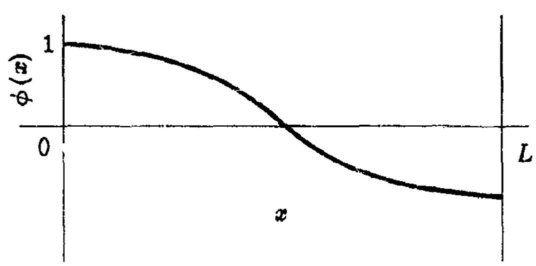


Figure 3-4 – Normal Mode Shape for Fundamental Torsional Mode of a Uniform Cylindrical Shaft with Free Ends

length of the shaft comprises a half wavelength (half of a full cosine) and that Equation [3-12] conforms to Equation [3-11] if

$$c = \sqrt{\frac{GJ}{I_{\mu x}}}$$

Just as the empirical formula of Schlick for the fundamental vertical flexural mode of a hull was based on the corresponding formula for the free-free uniform bar, so an empirical formula for the frequency of the fundamental mode of torsional vibration of the hull resembling the corresponding formula for the free-free cylindrical shaft was proposed by Horn.[3-9]

Horn's formula is:

$$N_e = 60k \sqrt{\frac{gGJ_{e0}}{\Delta (B^2 + D^2) L}} \qquad [3\text{-}13]$$

This formula is given here in mixed units for convenience in practical application; thus:

- N_e is the natural frequency in cpm,
- g is the acceleration of gravity in ft/sec^2,
- J_{e0} is the effective polar moment of inertia of midship section area,
- Δ is the displacement in long tons,
- B is the beam in ft,
- D is the depth in ft,
- L is the length in ft,
- G is the shear modulus of elasticity in tons/ft^2, and
- k is Horn's empirical coefficient.

For discussion of the evaluation of k and J_{e0}, see Appendix C.

Not only does the hull depart from the ideal free-free beam in the nonuniformity of mass and stiffness distributions, but it also lacks symmetry, and this property complicates its vibration characteristics to an extent that has so far been but little explored. Because of the close approximation to symmetry of most ships with respect to a vertical plane passing through the longitudinal centerline, the prototype beam for the equations to be derived here is assumed to have one plane of symmetry. Here the term symmetry means mirror symmetry. It is shown in Reference 3-10 that mirror symmetry is a sufficient condition for vibrational symmetry, but not a necessary one.

B. BASIC DIFFERENTIAL EQUATIONS FOR THE SHIP

The "rational" theory of ship vibration proposed in this book is based on the assumption that the hull may be considered as a free-free beam with three principal types of flexibility; namely, bending, shearing, and torsional (or twisting). In this theory the inertia effect of the water is treated as equivalent to mass added to the mass of the hull at suitable locations. The elastic axes for bending are assumed to be vertical and horizontal.

The longitudinal (X) axis is taken as passing through a point halfway between the main deck and the keel at the midship section; this is considered to be parallel to the keel whether or not the ship is actually at zero trim. The orientation of the Y- and Z-axes are dependent on the modes of vibration considered.

This analysis provides for a coupling of horizontal vibration with torsional vibration, but, because of the symmetry, no coupling of vertical vibration with either torsional or horizontal vibration is considered here.

The basic equations are derived by treating the forces and moments acting on the element shown in Figure 3-5 when it is taken as uniform with all parameters equal to the mean value over the length Δx.

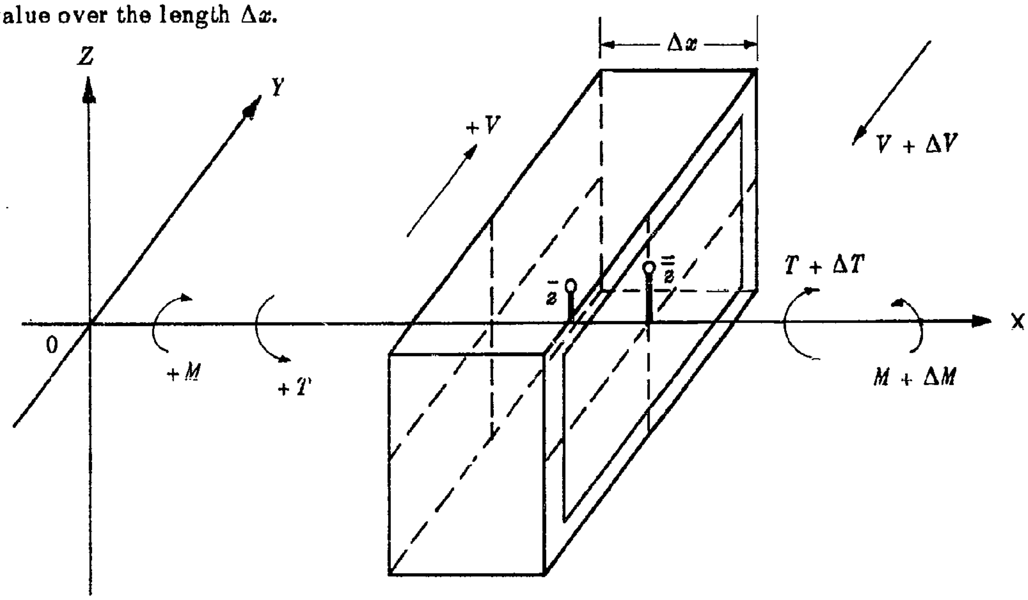


Figure 3-5 – Element of Ship of Length Δx Subject to Forces and Moments Imposed by Adjoining Elements while Executing Torsion – Horizontal Bending Vibration

The equations are derived by considering that the element of the ship of length Δx possesses a combination of the properties of the Timoshenko beam in flexure and the cylindrical shaft in torsion, but with account taken of the lack of both inertial and elastic symmetry. Thus, a term $\bar{z}$ is introduced to represent the vertical distance from the x-axis to the center of mass, and a term $\bar{\bar{z}}$ to represent the distance from the x-axis to the center of horizontal shear (which is here assumed to be the centroid of the area of the cross section of the hull).

The axes chosen form a right-hand set, and the sign convention is as indicated by the plus signs at the left end of the element in Figure 3-5. Thus the horizontal shear force V is positive when the part of the hull to the left of the element exerts a force on the element in the direction of positive Y.

Application of Newton's law to motion in the Y-direction gives

$$\mu \Delta x \frac{\partial^2 y'}{\partial t^2} = -\Delta V \qquad [3\text{-}14]$$

where y' is the displacement in the Y-direction of the center of mass of the element. Since $y' = y - \bar{z}\,\phi$, for small motions

$$\frac{\partial^2 y'}{\partial t^2} = \frac{\partial^2 y}{\partial t^2} - \bar{z} \frac{\partial^2 \phi}{\partial t^2} \qquad [3\text{-}15]$$

Hence

$$-\Delta V = \mu \Delta x \frac{\partial^2 y}{\partial t^2} - \mu \Delta x \bar{z} \frac{\partial^2 \phi}{\partial t^2} \qquad [3\text{-}16]$$

Newton's equation for moments about a Z-axis gives

$$\Delta M = V \Delta x + I_{\mu z} \Delta x \frac{\partial^2 \gamma}{\partial t^2} \qquad [3\text{-}17]$$

where M is the moment with respect to a Z-axis exerted on the element by the part of the hull to its left. From the Timoshenko beam properties

$$\Delta \gamma = \frac{M \Delta x}{EI} \qquad [3\text{-}18]$$

where γ is the rotation of the cross section with respect to a Z-axis.

$$\Delta y'' = \gamma \Delta x - \frac{V \Delta x}{KAG} \qquad [3\text{-}19]$$

where y'' is the displacement in the Y-direction of the center of shear and KAG is the shear rigidity. The last equation also implies that the displacement of the center of shear depends only on shearing and bending action and not on torsion, that is, that the hull twists about the center of shear.

The simple concept of torsional rigidity defines the quantity GJ_e by the torque required to produce a given rate of twist with respect to the longitudinal axis. The complete set of shear stresses in the cross section has a moment about the X-axis designated in Figure 3-5 as T. In the absence of a net vertical shear force, the torque with respect to the longitudinal axis is obtained by subtracting from T the moment due to the net horizontal shear force. This is $-V\,\bar{z}$. Hence, from the definition of torsional rigidity GJ_e, and the sign conventions indicated in Figure 3-5

$$GJ_e \frac{\Delta\phi}{\Delta x} = -(T + V\bar{z}) \qquad [3\text{-}20]$$

whence

$$\Delta\phi = -\frac{(T + V\bar{z})\Delta x}{GJ_e} \qquad [3\text{-}21]$$

Since $y'' = y - \bar{z}\phi$, and the element of length Δx is treated as uniform in the approximation to the continuously varying hull, $\Delta y'' = \Delta y - \bar{z}\Delta\phi$. So

$$\Delta y = \gamma\Delta x - \frac{V\Delta x}{KAG} - \frac{\bar{z}T\Delta x}{GJ_e} - \frac{V\bar{z}^2\Delta x}{GJ_e} \qquad [3\text{-}22]$$

Since $I_{\mu x}$ is specified with respect to the arbitrarily chosen X-axis, the equation for the time rate of change of moment of momentum about the X-axis gives

$$-\Delta T = -\mu\Delta x\bar{z}\frac{\partial^2 y'}{\partial t^2} + (I_{\mu x}\Delta x - \mu\Delta x\bar{z}^2)\frac{\partial^2\phi}{\partial t^2} \qquad [3\text{-}23]$$

where $I_{\mu x}$ is the mass moment of inertia per unit length with respect to the X-axis, including the allowance for the inertia effect of the water.

The first term in this equation is the rate of change with respect to the X-axis of the moment of momentum associated with the center of mass; the second term is the rate of change of moment of momentum about a parallel axis through the center of mass. Therefore

$$\Delta T = \mu\Delta x\bar{z}\frac{\partial^2 y}{\partial t^2} - I_{\mu x}\Delta x\frac{\partial^2\phi}{\partial t^2} \qquad [3\text{-}24]$$

If the foregoing equations are set in differential form with respect to x, the following set of partial differential equations is obtained:

$$\frac{\partial V}{\partial x} = -\mu\frac{\partial^2 y}{\partial t^2} + \mu\bar{z}\frac{\partial^2\phi}{\partial t^2} \qquad [3\text{-}25]$$

$$\frac{\partial M}{\partial x} = V + I_{\mu z}\frac{\partial^2\gamma}{\partial t^2} \qquad [3\text{-}26]$$

$$\frac{\partial\gamma}{\partial x} = \frac{M}{EI} \qquad [3\text{-}27]$$

$$\frac{\partial\phi}{\partial x} = -\frac{(T + V\bar{z})}{GJ_e} \qquad [3\text{-}28]$$

$$\frac{\partial y}{\partial x} = \gamma - V\left(\frac{1}{KAG} + \frac{\bar{z}^2}{GJ_e}\right) - \frac{\bar{z}\,T}{GJ_e} \qquad [3\text{-}29]$$

$$\frac{\partial T}{\partial x} = \mu\bar{z}\,\frac{\partial^2 y}{\partial t^2} - I_{\mu x}\,\frac{\partial^2 \phi}{\partial t^2} \qquad [3\text{-}30]$$

In these equations a term involving $\Delta\bar{z}$ is omitted since the element is treated as uniform over the length Δx. Also the X-axis is treated as a principal axis of inertia of the element.

The boundary conditions for free vibrations are

$$V = M = T = 0 \text{ for } \begin{Bmatrix} x = 0 \\ x = L \end{Bmatrix}$$

C. METHODS OF CALCULATING NATURAL FREQUENCIES AND NORMAL MODES OF VIBRATION OF SHIPS

The set of partial differential equations given in the foregoing section cannot, in general, be treated analytically for the nonuniform beam representing the ship since the coefficients which are functions of x are not given in mathematical form. They are available only in the form of graphs or tables. However, these equations furnish the basis for approximate methods of calculation of considerable practical importance. These methods are classified here as the digital, the analog, and the graphical methods.

In all three methods the problem is greatly simplified if only vertical hull vibration is of concern, for here, unless the ship is of unusual design, the symmetry eliminates the coupling effects. In this book, only the vertical case is used for illustration of the general methods of solution. For the vertical modes, the Y- and Z-axes are then rotated 90 degrees clockwise so that positive Y is upward and Z points outward from the plane of the paper. The axes still form a right-hand set and the flexural vibration is still in the direction of Y so that the same equations are valid. Here, however, $\bar{z}$ and $\bar{z}$ are made zero and the equations involving y are independent of ϕ. The reader is referred to Reference 3-11 for the treatment of the horizontal and torsion-bending cases.

1. DIGITAL METHOD

The digital process that has shown the most promise so far in computing natural frequencies and normal modes is referred to in David Taylor Model Basin reports as the Prohl-Myklestad method. This method utilizes the equations in their finite difference form and is closely related to the Holzer method widely used in torsional vibration analysis.

The process of treating the hull vibration problem by finite differences involves consideration of the validity of various approximations. Naturally the aim is to obtain the greatest possible accuracy with the minimum amount of computation. Some prefer to regard the process as one of first converting the ship into an equivalent "lumped" system in which the inertias are concentrated at certain points and the elastic properties are assigned to massless elastic members joining these lumped elements. Others insist that no such concept is necessary and that the representation of the continuous system by finite difference equations is entirely mathematical. From either point of view it seems obvious that the greater the number of lumps (or equations) used, the greater the attainable accuracy but sometimes, in the case of the computing machines, the accumulation of roundoff errors gives less accuracy with a large number of lumps.

Accuracy of computation, however, does not necessarily imply absolute accuracy. The latter is limited not only by the uncertainty in the evaluation of all parameters appearing in the equations, but also by the factors which cause a ship to depart from beamlike behavior. Here are included such properties as local flexibility, structural discontinuities, concentrated masses, hatch openings, and large superstructures.

The minimum number of sections for obtaining the accuracy warranted by the reliability of the input data appears to be 20. The point of diminishing returns appears to be reached at 40 sections. The basic process is the same regardless of the number of sections used.

The case of vertical flexural vibration used here as an illustration requires only four of the six equations given on pages 3-9 and 3-10. The parameters and variables dropping out are $\bar{z}$, $\bar{\bar{z}}$, $I_{\mu x}$, GJ_e, T, and ϕ.

A lumped system having approximately the vibratory characteristics of the ship is illustrated in Figure 3-6. This is based on a 20-section breakdown which requires 21 mass elements with one at each end.

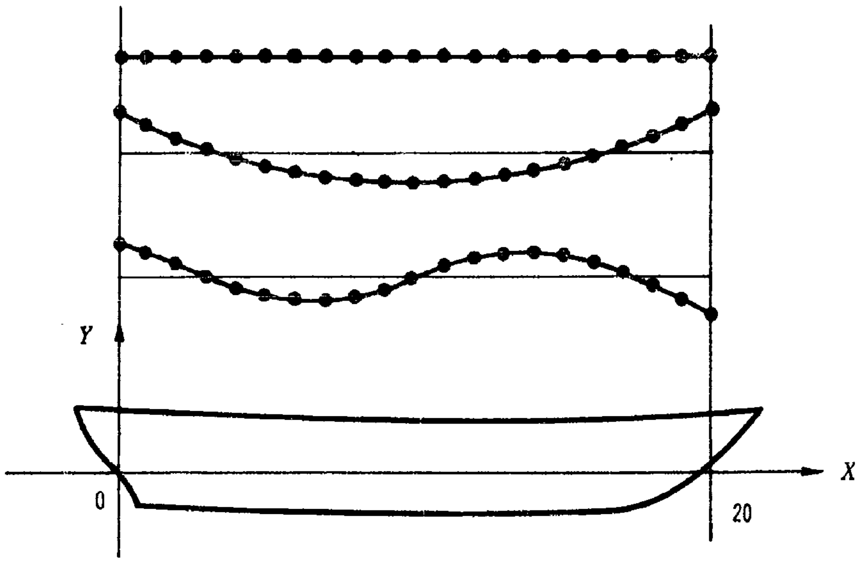


Figure 3-6 – Illustration of Breakdown of a Ship for Digital Calculation of Vertical Modes of Vibration

The mass elements are points of concentration of the translational inertia $\mu \Delta x$ and the rotational inertia $I_{\mu z} \Delta x$. The elastic properties of the hull are concentrated in the massless connecting members. In this case there are the bending rigidity EI and the shearing rigidity KAG. Hence, two principal modifications are made of the equations given on pages 3-9 and 3-10 for this particular calculation. First, they are converted from differential form to algebraic form with respect to time by using the fact that the desired solutions will be simple harmonic vibrations which can be expressed in the form

$$y(x, t) = Y_i(x) \sin \omega_i t \qquad [3\text{-}31]$$

which gives

$$\ddot{y}(x, t) = -\omega_i^2 \, Y_i(x) \sin \omega_i t = -\omega_i^2 \, y(x, t) \qquad [3\text{-}32]$$

where $Y_i(x)$ is the normal mode pattern for the ith normal mode. Second, they are expressed in finite difference form with respect to x.

The equations are then converted from differential form to finite difference form with respect to x. As given here, they are in their simplest form.

$$\Delta V = \mu \, \Delta x \, y \, \omega^2 \qquad [3\text{-}33]$$

$$\Delta M = V \Delta x - I_{\mu z} \, \Delta x \gamma \, \omega^2 \qquad [3\text{-}34]$$

$$\Delta \gamma = \frac{M \Delta x}{EI} \qquad [3\text{-}35]$$

$$\Delta y = \gamma \Delta x - \frac{V \Delta x}{KAG} \qquad [3\text{-}36]$$

As shown in Reference 3-11, a staggering system is usually used for increased accuracy of calculation. This requires introducing half stations and considering forces and displacements at staggered intervals with the boundary conditions $V = M = 0$ at each end. In this simplified form the subscript notation required to identify a value at any particular station along the length of the hull is unnecessary, as is also the use of half-station staggering of certain terms for increased accuracy of computation. Each of the four equations indicates the difference between two values of the variable on the left side of the equation in advancing a distance Δx to the right when the hull is executing a simple harmonic vibration of circular frequency ω.

The computation is started by assigning an arbitrary value to ω and unit value to y at the left-end station. It is known that $V_0 = 0$ and $M_0 = 0$ but the value of γ_0 must remain undetermined temporarily. The set of four difference equations will then permit advancing from Station 0 to Station 1 with V, M, γ, and y all known in terms of γ_0. This process can then be continued cyclicly until the right end of the hull is reached. Here it is known that if the frequency ω assumed was a natural frequency, it will be possible to find a value of γ_0 such that the boundary conditions, that V and M are both zero at this end, will be satisfied. Once γ_0 is determined numerical values are then fixed for V, M, γ, and y throughout the ship.

There are a number of ways in which this basic process can be carried out. Thus, use can be made of the fact that the desired solution can be formed from a linear combination of calculations made by starting at the left end, first with the values $y = 1$ and $\gamma = 0$ and then with the values $y = 0$, $\gamma = 1$. A range of values of ω is then explored for which it is possible to find a linear combination of these two calculations, which will make V and M both zero at the right end of the hull.

Since the lumped system used here to represent the hull has only 21 degrees of freedom, it can have only 21 natural frequencies and normal modes, whereas a continuous free-free beam has a theoretically infinite number of modes. In practice, such a calculation would not be carried beyond about the sixth or eighth mode, the number depending on the length to depth ratio of the hull. If the method were valid beyond this number of modes it would be necessary to use a larger number of elements in the breakdown in order to realize its potentialities.

Several points should be noted concerning this method of computation. The natural frequencies are found by a process of searching for values of ω at which the boundary conditions can be satisfied. At each frequency thus found the calculation automatically gives a normal mode pattern. There is no significance to the absolute amplitudes used in such a calculation, but only to their relative values. Thus, the calculation shows what values V, M, γ, and y would have throughout the hull if it were possible for it to vibrate with unit amplitude at the left end at the particular frequency in question. In the foot-ton-second system of units this would mean an amplitude of 1 ft at station zero, clearly an excessive value. However, since the equations used are linear the result can be used to obtain the values associated with amplitudes of practical magnitude, that is, fractions of an inch.

The Model Basin has for the present (1960) adopted the station designations as 0 at the stern and 20 at the bow for the vibration calculations. This is the opposite to that used in the naval architects' lines drawings but preserves the convention of having the bow to the right in the ship's profile.

In spite of the increased number of variables and parameters, the basic process of calculating torsion bending modes by digital calculation is the same as for the simpler case just illustrated. When a normal mode of this type is found by digital calculation there will be obtained sets of values of y, γ, ϕ, V, M, and T for each natural frequency on the basis of unit amplitude in y at station zero. When y and ϕ are plotted there may be found pairs of torsion-horizontal bending modes in which there are the same number of torsional and flexural nodes in each mode. However, there is reversal of phase between rotation and translation as illustrated in Figure 2-5.

2. ANALOG METHOD

The subject of electrical analogs of mechanical systems is very broad and is of great importance in vibration analysis in general, as is well known. The equations applicable to electrical networks in which alternating currents are flowing are, in general, of the same form as the equations applicable to vibrating mechanical systems. This at once suggests the use of electrical circuits as computing devices for studying the vibratory characteristics of mechanical systems.

An important distinction is to be made between those electronic computing devices that perform purely mathematical operations such as integration and differentiation, and those in which there is a truly physical analogy between the currents or voltages in various branches of a circuit and the amplitudes of vibration or forces existing in the corresponding mechanical system. The former are frequently called operational analogs and the latter are spoken of here as network analyzers. This chapter is concerned only with the latter.

Direct electrical analogs of vibrating mechanical systems are not unique and two such analogs have been used at the David Taylor Model Basin in making vibration analyses on the same network analyzer. These are known as the conventional or "classical" analog and the "mobility" analog. The analogous quantities in these two systems are listed in Table 3-1.

TABLE 3-1
Analogous Quantities in Two Types of Electrical Analog of a Vibrating Mechanical System

Mechanical Values	Conventional Analog Values	Mobility Analog Values
Mass	Inductance	Capacitance
Spring constant	Reciprocal of capacitance	Reciprocal of inductance
Viscous damping constant	Resistance	Reciprocal of resistance
Force	Voltage	Current
Displacement	Charge	Integral of voltage
Velocity	Current	Voltage
Frequency	Frequency	Frequency

In general, mechanical units and electrical units are of such magnitudes in practice that neither of these analogs can be used without a scaling transformation. The electrical circuit is driven at a much higher frequency than the mechanical frequencies met in practice. This reduces the size of inductances and capacitances required in either analog.

In dealing with the flexural vibration of hulls, the Model Basin has used a mobility analog in the form of a pair of coupled transmission lines; in one the current represents the shearing force in the hull, in the other the current represents the bending moment. This circuit is discussed in considerable detail in Reference 3-12. When a separate ground line is drawn parallel to each of these ungrounded lines, one section of this network, which then represents one of the sections into which the hull is broken down in the analysis, will have eight terminals, four at each end. Such a section is shown in Figure 3-7. This circuit comprises a "passive network," that is, one within which there are no sources of energy so that the network by itself is dissipative because of internal losses. When external voltages are removed, only transient oscillations remain which eventually die out just as the vibrations of the mechanical system die out after the exciting force is removed.

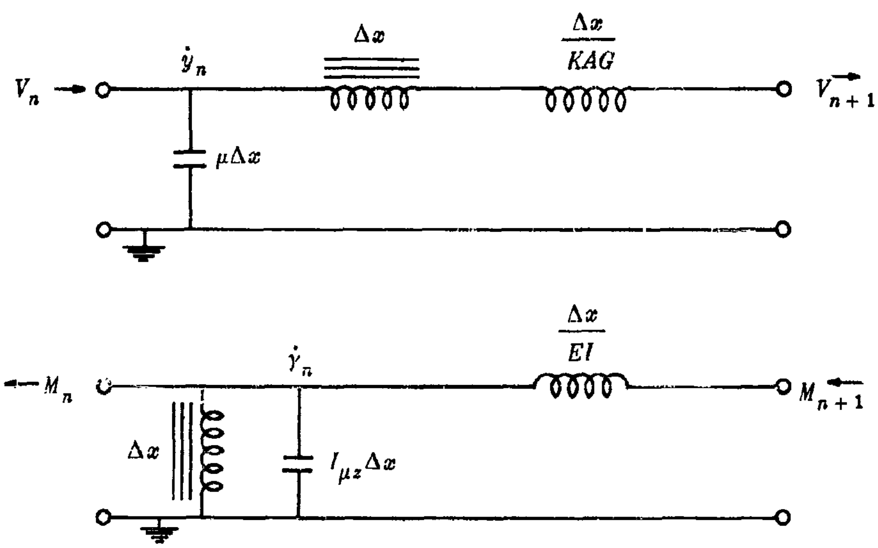


Figure 3-7 – One Section of an Electrical Circuit Analogous to a Section of a Vibrating Ship

In Figure 3-7 currents flowing to the right in the upper line represent vertical shearing forces in the hull, and currents flowing to the left in the lower ungrounded line represent bending moments in the hull. Voltages in the shear line are analogous to mechanical translational velocities and voltages in the moment line are analogous to mechanical angular velocities. It will be noted that inductances are proportional to the reciprocal of mechanical shearing or bending rigidities whereas capacitances are analogous to mass or mass moment of inertia. One winding of the transformer is in series with the shear line; the other winding is connected between the moment line and ground.

To find the natural frequencies of a hull experimentally, the usual procedure is to install a vibration generator that is operated over a wide range of speeds and to observe those speeds at which resonance peaks of amplitude are found. Similarly, with the analog the natural frequencies of the circuit are found not by observing free oscillations but by applying continuous excitation by means of an oscillator. In the case of the mobility analog the condition corresponding to resonance in the mechanical system is actually an electrical antiresonance. This is advantageous in that little power is taken from the oscillator. A high resistance is inserted between the ungrounded terminal of the oscillator and one end of the shear line in the circuit. If this resistance is high enough, the current fed into the shear line will remain practically constant regardless of the frequency since the change in impedance of the circuit will have little effect on the total impedance. At each antiresonance the voltage will rise at the ends of the shear line, and the normal mode pattern of the hull may then be found by plotting the voltage measured along the shear line at the 20 stations.

To show that this electrical system is analogous to the vibrating hull it is required merely to show that the application of Kirchhoff's laws for electrical circuits when applied to the section shown in Figure 3-7 will yield the same set of difference equations as Equations [3-33] through [3-36]. Thus, the difference in currents at the two ends of the shear line of the section must equal the current flowing from the shear line to ground. The latter current, however, is the product of the voltage $\dot{y}$ and the admittance $j\omega\mu\Delta x$, where $j = \sqrt{-1}$. In a-c circuit theory, differentiation with respect to time is effected by multiplying by the operator $j\omega$, and integration with respect to time by dividing by $j\omega$. Hence the current to ground is

$$j\omega\mu\Delta x\dot{y} = -\mu\Delta x\omega^2 y$$

Hence

$$V_n = -\mu\Delta x\omega^2 y + V_{n+1}$$

or

$$\Delta V = \mu\Delta x y\omega^2 \qquad [3\text{-}33]$$

In the moment line Kirchhoff's current equation involves four currents: M_n, M_{n+1}, $V_n\Delta x$ (through the transformer winding which is coupled to the shear line V), and $\dot{\gamma}$ $(j\omega I_{\mu z}\Delta x)$ through the capacitance. When the directions of these currents are taken into account there results the equation

$$\Delta M = V\Delta x - I_{\mu z}\Delta x\gamma\omega^2 \qquad [3\text{-}34]$$

The remaining two equations follow from Kirchhoff's potential relations. Thus the voltage drop in the inductance $\frac{\Delta x}{EI}$ is equal to the inductance times the rate of change of current, i.e., $\frac{\Delta x}{EI}\dot{M}$.

Hence

$$\dot{\gamma}_{n+1} - \dot{\gamma}_n = \frac{\Delta x}{EI}\dot{M}$$

or, on integrating with respect to time,

$$\Delta\gamma = \frac{M\Delta x}{EI} \qquad [3\text{-}35]$$

Finally, since the voltage drop in the winding of the transformer in the shear line is Δx times the voltage across the other winding ($\dot{\gamma}$) and since the voltage drop in the inductance $\frac{\Delta x}{KAG}$ is equal to $\frac{\Delta x}{KAG}\dot{V}$, the potential equation is

$$\dot{y}_n - \dot{y}_{n+1} = -\dot{\gamma}\Delta x + \frac{\Delta x\dot{V}}{KAG}$$

On integrating this gives

$$\Delta y = \gamma\Delta x - \frac{V\Delta x}{KAG} \qquad [3\text{-}36]$$

It is seen that the boundary conditions of the free-free hull are satisfied in the analog if the shear and moment lines are isolated at the ends since this makes $V = M = 0$. Of course, when an oscillator is connected to one end, it gives an input current which is analogous to applying an external simple harmonic driving force to one end of the hull. The voltages developed at the ends correspond to the rectilinear and angular velocities at the ends of the hull.

A photograph of the TMB Network Analyzer used for vibration analysis by means of the electrical analog is given in Figure 3-8. A complete description of this network analyzer is given in Reference 3-13.

3. GRAPHICAL INTEGRATION – STODOLA METHOD

Some success has been attained in computing the lower modes of hull vibration by graphical integration. This is of practical importance since analog or digital computing facilities are not universally available. In this chapter the application of graphical integration to the Stodola method is discussed, but it must be emphasized that numerical integration can also be applied to this method. In applying this method the added mass must have been previously computed and added to the mass of the hull as a distributed mass. Here the hull is considered as a beam free in space and the added mass is assumed to remain constant in time.

The case of vertical vibration is again used to illustrate the method, since in this case coupling with torsion is negligible. As a further simplification the effect of rotary inertia is neglected here. The set of differential equations to be integrated is then:

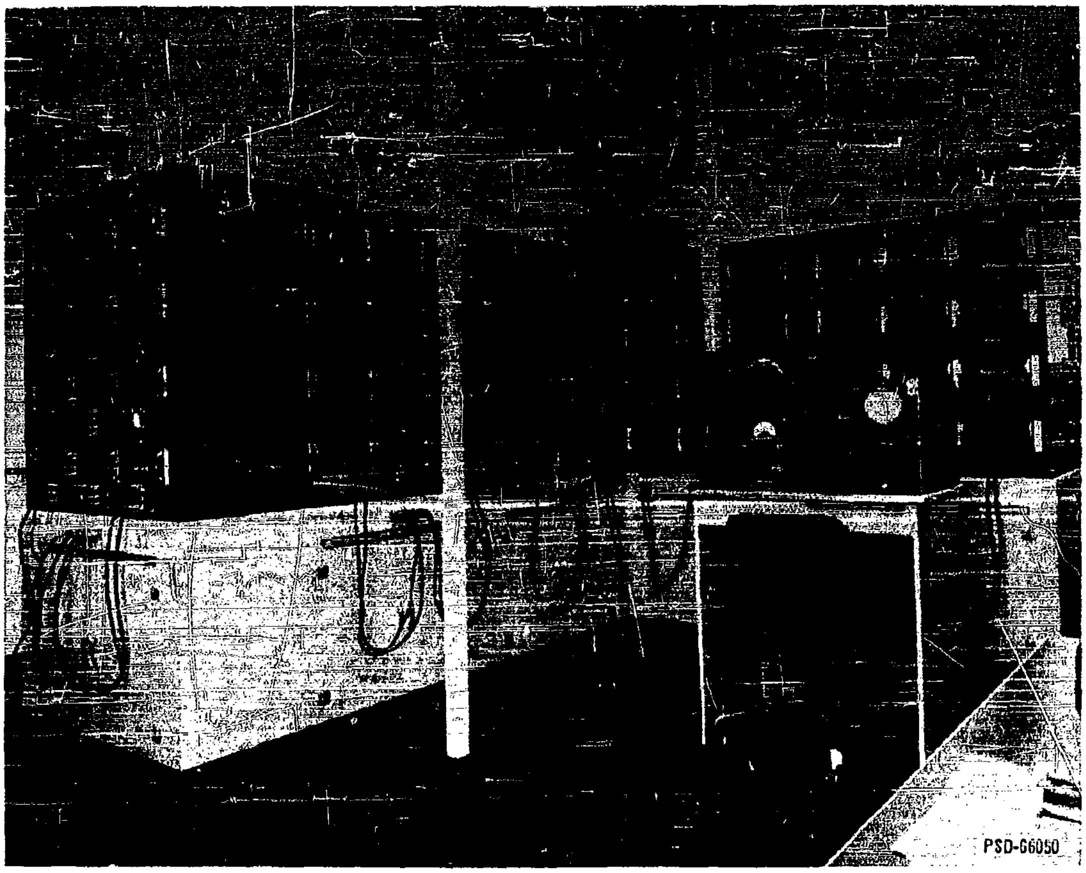


Figure 3-8 – Network Analyzer Used in Making Hull Vibration Calculations by Means of Electrical Analogs

$$\frac{\partial V}{\partial x} = \mu y \omega^2 \tag{3-37}$$

$$\frac{\partial M}{\partial x} = V \tag{3-38}$$

$$\frac{\partial \gamma}{\partial x} = \frac{M}{EI} \tag{3-39}$$

$$\frac{\partial y}{\partial x} = \gamma - \frac{V}{KAG} \tag{3-40}$$

With boundary conditions

$$V = M = 0 \quad \text{for} \begin{cases} x = 0 \\ x = L \end{cases}$$

In these equations μ, EI, and KAG all vary with x.

The curve of μ versus x is first plotted from the weight curve of the ship and the estimated added mass of water. This may be either a smooth or a stepped curve.

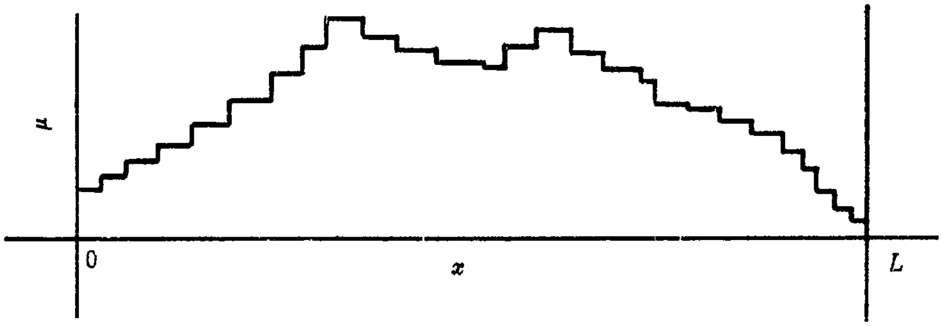


Figure 3-9 – Sample Plot of μ versus x Required for Use of Stodola Method

A normal mode shape is then assumed for the 2-node vertical mode. The nearer this is to the true mode shape the less labor is involved in the calculation, but, in the absence of a basis for a more realistic curve, the normal mode pattern Y_a for the free-free uniform bar is used as a starting curve. The value of Y_a may be taken as unity at $x = 0$. A set of values for plotting the uniform bar curve is given under Case 11A of Reference 3-14. The μ values are then multiplied by the corresponding values of Y_a to give the curve μY_a shown in Figure 3-11.

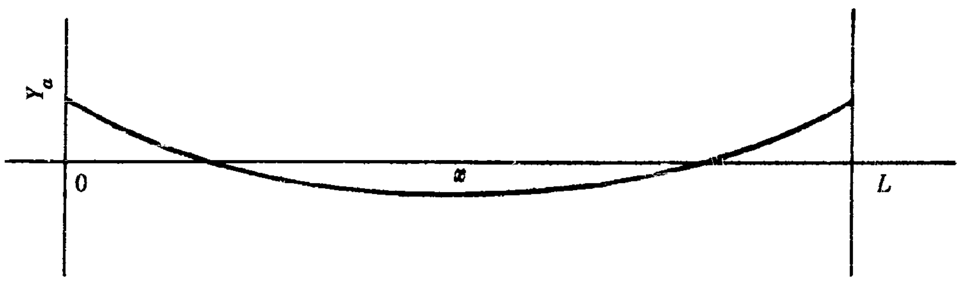


Figure 3-10 – Sample Plot of 2-Node Pattern of Free-Free Uniform Bar for Use in Stodola Method

If Y_a were the true normal mode pattern for the ship in question it would be found that

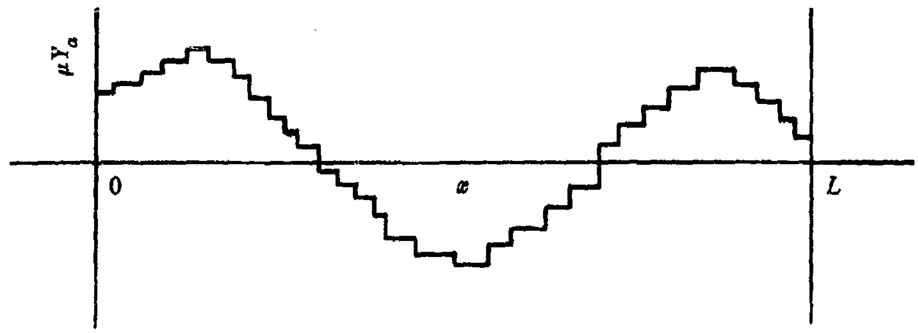


Figure 3-11 -- Plot of μY_a versus x for Use in Stodola Method

$$\int_0^L \mu Y_a \, dx = 0 \tag{3-41}$$

$$\int_0^L \int_0^x \mu Y_a \, dx \, dx = 0 \tag{3-42}$$

since the first integral is proportional to the shearing force $V(x)$ in the hull during vibration (actually $V(x) = \int \mu Y \omega^2 \, dx$), and the second is proportional to the bending moment $M(x)$. If the first integral is not zero it may be made zero by a parallel shift of the X-axis in the plot of $Y_a(x)$ to obtain a second approximation $Y_a'(x)$. If, after this modification, the second integral does not vanish, it may be made to vanish by a rotation of the X-axis about the point whose abscissa is that of the centroid of the area under the μ curve. The curve of amplitudes, obtained after shifting and rotating the base of the Y_a-curve and replotting on the basis of unit value at Station 0, is labeled $Y_a''(x)$. A mechanical integraph will draw the integral curve when the curve to be integrated is traced by the stylus.

Although the modifications that have been made so far in the starting mode shape of the free-free uniform bar will now make it satisfy the boundary condition on V and M, as yet no account has been taken of the elastic properties of the hull.

The four differential equations on page 3-18 yield the following set of integral equations:

$$\int \mu y \omega^2 \, dx = V \tag{3-43}$$

$$\int\int \mu y \omega^2 \, dx \, dx = M \tag{3-44}$$

$$\int \frac{dx}{EI} \int\int \mu y \omega^2 \, dx \, dx = y \tag{3-45}$$

$$\int\int \frac{dx \, dx}{EI} \int\int \mu y \omega^2 \, dx \, dx - \int \frac{dx}{KAG} \int \mu y \omega^2 \, dx = \bar{y} \tag{3-46}$$

After each integration the constant of integration must be found before proceeding to the next. In the case of the first two integrations the constants are furnished by the boundary conditions, and the value of y to be used in Equation [3-43] has already been adjusted so that the first two constants of integration will be zero. Thus V and M are zero for $x = 0$ and $x = L$.

The last equation of the above set may be written

$$\left[\iint \frac{dx\,dx}{EI} \iint \mu y\,dx\,dx - \int \frac{dx}{KAG} \int \mu y\,dx\right] = \frac{\bar{y}}{\omega^2} \qquad [3\text{-}47]$$

where it may be seen that, if all the constants of integration were zero, and, if the starting mode shape were the true mode shape, the curve finally obtained by carrying out these integrations would differ from the starting curve only by the scale factor $\frac{1}{\omega^2}$. Hence the natural frequency could be found directly from this scale factor.

As long as KAG and EI remain finite, all integrations may be carried out starting with zero values at $x = 0$. When the final integrations on the left side of Equation [3-47] are carried out, the two curves plotted with zero value at $x = 0$ may be combined to give a curve for $\frac{\bar{y}}{\omega^2}$.

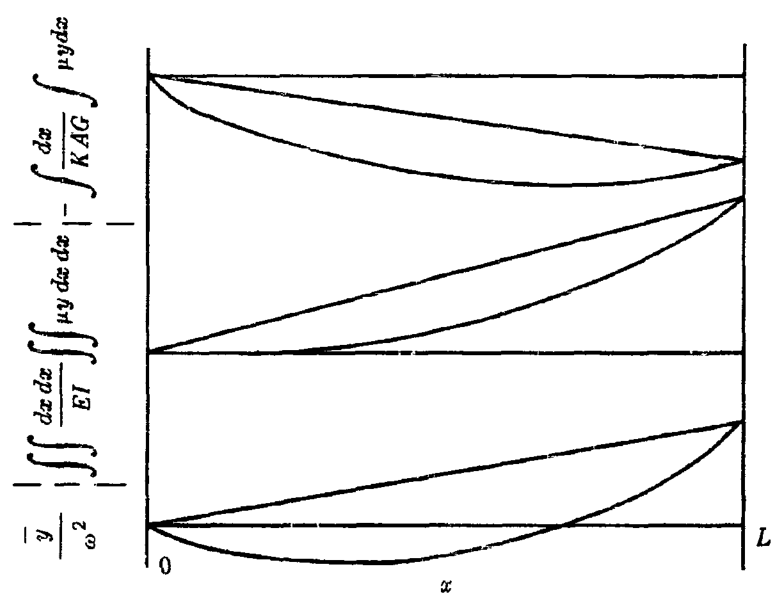


Figure 3-12 – Combining of Components of Bending and Shearing Deflection

First the ends of each curve are joined by straight lines. Then the ordinates measured vertically from the curve to the inclined line are combined. The frequency is found by comparing the magnitude of this final curve with the similar curve obtained by joining the ends of the curve $Y_a''(x)$ used as a basis for the μy curve. This curve was previously obtained from the curve $Y_a(x)$ by a parallel shift and rotation of the X-axis. To find the normal mode pattern, however, it is still necessary to find the true base of the $\frac{\bar{y}}{\omega^2}$ curve. As in deriving the curve $Y_a''(x)$, this is again accomplished by a parallel shift of the X-axis and a rotation about a point whose abscissa is that of the centroid of the area under the μ curve. The true base must again be such as to make

$$\frac{1}{\omega^2}\int_0^L \mu y dx \quad \text{and} \quad \int_0^L \int_0^X \mu y \, dx \, dx$$

both equal to zero.

If the curve $Y_a''(x)$ is the true normal mode curve, the process will yield a $\frac{\bar{y}}{\omega^2}$ curve differing from Y_a'' only in scale. It can be proven[3-1] that the Stodola process is convergent to the lowest mode; that is, that, if the whole procedure is repeated using the finally obtained Y curve as a second starting curve, the second calculation will come closer to giving the same shape than previously. It can be shown also that no matter what shape is initially assumed, unless it coincides exactly with the shape of a higher mode, the process will eventually converge to the 2-node mode shape.

If the Stodola method as previously outlined is applied to the calculation of the second mode, the process will in general converge to the first mode. To make it converge to the second mode it is necessary to make use of the orthogonality relations applicable to normal modes.[3-1] As a result of this property, if Y_i and Y_j are two of the normal mode shapes, then

$$\int \mu \, Y_i \, Y_j \, dx = 0$$

if $i \neq j$.

An arbitrarily assumed mode pattern can, in general, be resolved into a series of normal functions. This is,

$$Y(x) = \sum_{i=1}^{\infty} a_i \, Y_i(x) \qquad [3\text{-}48]$$

For the Stodola process to converge to the second mode the first mode component in the assumed function $Y(x)$ must be eliminated.

It has been shown that for a true normal mode the Stodola process gives a result that differs from the initial pattern only in scale; that is, it gives for $Y_i(x)$ the value $\frac{Y_i(x)}{\omega^2}$. Once the first mode shape is known, the curve assumed for the second mode can be corrected for any first-mode component present by subtracting the function

$$\frac{\int_0^L \mu Y Y_1 \, dx}{\int_0^L \mu Y_1^2 \, dx} \times Y_1(x)$$

It is seen that this quantity is equal to $a_1 Y_1(x)$ of the summation in Equation [3-48] since

$$\int_0^L \mu Y Y_1 \, dx = \int_0^L \mu (a_1 Y_1 + a_2 Y_2 + \text{etc}) \, Y_1 \, dx \qquad [3\text{-}49]$$

By the orthogonality relation all terms of the integration vanish except

$$\int_0^L \mu \, a_1 Y_1^2 \, dx$$

For further discussion of the Stodola method as applied to this problem, see Reference 3-6.

4. GENERAL

This chapter presents only the essential elements of the methods used so far at the David Taylor Model Basin to calculate the natural frequencies and normal modes of vibration of a hull. A recent TMB report[3-11] goes much further into the mathematical details of such calculations than is feasible in a book dealing with the general subject of ship vibration. This reference also goes into the extension of the beam theory to the cases in which part of the mass contributing to the displacement of the ship is considered flexibly attached to the hull girder. This question is also discussed in Chapters 5 and 13 of this book.

REFERENCES

3-1. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

3-2. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

3-3. Burrill, L.C., "Ship Vibration: Simple Methods of Estimating Critical Frequencies," NECI Trans, Vol. 51 (1934–35).

3-4. Todd, F.H. and Marwood, W.J., "Ship Vibration," NECI Trans, Vol. 64 (1948).

3-5. Prohaska, C.W., "The Vertical Vibration of Ships," Shipbuilder, Vol. 54 (Oct, Nov 1947).

3-6. Ormondroyd, J., et al., "Dynamics of a Ship's Structure," Engin Res Inst, Univ of Michigan, Ann Arbor (Mar 1949) (out of print).

3-7. Boyd, J.E., "Strength of Materials," McGraw-Hill Book Co., New York (1924).

3- 8. Rayleigh, Lord, "Theory of Sound," Second Edition, Dover Publications, New York (1945).

3- 9. Horn, F., "Horizontal-und Torsions–Schiffschwingungen auf Frachtschiffen," WRH, Vol. 6, No. 18 (1925).

3-10. Vane, F.F., "A Guide for the Selection and Application of Resilient Mountings to Shipboard Equipment," TMB Report 880, Revised Edition (Feb 1958).

3-11. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

3-12. Kapiloff, E., "Calculation of Normal Modes and Natural Frequencies of Ship Hulls by Means of the Electrical Analog," TMB Report 742 (Jul 1954).

3-13. Kenny, J.H. and Leibowitz, R.C., "Design Details and Operating Procedure for the TMB Network Analyzer," TMB Report 1272 (Apr 1959).

3-14. McGoldrick, R.T., "A Vibration Manual for Engineers," TMB Report R-189, Second Edition (Dec 1957).

CHAPTER 4

BEAM THEORY OF STEADY-STATE SHIP VIBRATION

A. INTRODUCTION

As in the previous chapter the treatment of hulls given here is based on beam theory. However, at this point, account must be taken of the limitations of this theory. This leads to what is called here the "rational" theory of ship vibration. The methods presented in this chapter are thus essentially heuristic and "quasi-mathematical."

When there is applied to the ideal Euler-Bernoulli beam (otherwise free in space) an external forcing function, the differential equation applicable to the system is

$$EI\,\frac{\partial^4 y}{\partial x^4} + \mu\,\frac{\partial^2 y}{\partial t^2} = P(x, t) \qquad [4\text{-}1]$$

where $P(x, t)$ is the driving force in the Y-direction per unit length of the beam. It is to be noted that in general P varies both with distance from the left end of the beam and with time. When the forcing function is specified mathematically particular solutions of Equation [4-1] can be given, as shown in References 4-1 and 4-2.

For the nonuniform Euler-Bernoulli beam the differential equation has the more general form

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\,\frac{\partial^2 y}{\partial x^2}\right) + \mu(x)\,\frac{\partial^2 y}{\partial t^2} = P(x, t) \qquad [4\text{-}2]$$

where EI and μ now vary with x. Even if the ship were of such construction that $EI(x)$ and $\mu(x)$ could be expressed mathematically it can be appreciated that Equation [4-2] would have severe limitations in indicating the manner in which the hull would vibrate under a given exciting force. In the first place, the inertia effect of the surrounding water is accounted for simply by the added mass component of μ; second, there is no dissipation or damping term in the equation; and third, there is no provision for deflection due to shearing. Last, but not least, there is nothing to indicate that the equation is not equally valid regardless of the frequency of the driving force. Thus, whether the driving force has a frequency of 1 cps or 10,000 cps, the patterns of vibratory response should be expected to be beamlike.

On the basis of experience the rational theory must recognize both the beamlike behavior of ships in their lower modes of vibration and the sharp departure from beamlike response characteristics in the higher modes. The beam theory itself cannot automatically do this. Although approaches to the vibration analysis of ships other than the beam theory approach have been suggested (see example in Reference 4-3), one of the chief aims in this book is to show that the beam theory can be combined with past experience to yield a prediction of hull vibratory response characteristics of practical use to both the ship designer and the ship vibration research worker. While a flexural mode of a hull may be excited by either a force or a moment, only the lateral force is considered at this stage.

In the first method of calculating forced vibration to be discussed, namely, the digital method, the ship is approximated by a lumped system exactly as in the digital method of calculating normal modes, but now a simple harmonic driving force is applied at one station and damping forces must be introduced. These damping forces limit the amplitude calculated at resonance, that is, when the driving frequency coincides with one of the natural frequencies of the system. The precise nature of the damping process in hull vibration is not well understood at the present time. The subject is discussed at some length in Chapter 8. Here it is only pointed out that in those calculations presented in this book which involve damping, the damping action is visualized as produced by equivalent viscous dampers distributed along the hull. These produce forces opposing the velocity at each point and proportional to that velocity. This involves the use of distributed damping constants c having the dimensions of force per unit length per unit velocity, and lumped damping constants C having the dimensions of force per unit velocity. In this book the damping constant is also usually assumed to be of the Rayleigh type, that is, viscous and proportional to mass (which makes $\frac{c}{\mu}$ a constant). However, there is also used in places a frequency-dependent damping defined by the relation $\frac{c}{\mu\omega}$ = constant.

The importance of damping in the attempt to calculate forced vibration of hulls cannot be overemphasized since it is the damping alone that limits the amplitude of a mass-elastic system when resonance is encountered. This subject is discussed in more detail in Chapter 8 where it is pointed out that experience has shown the utility of introducing equivalent viscous damping constants based on the rate of energy dissipation even when the actual damping process is believed to be much more complicated.

A characteristic of the forced vibration of mass-elastic systems in general is that, if the driving frequency is steadily increased, the forced response exhibits a succession of resonance peaks which indicate that the driving frequency coincides with one of the natural frequencies of the system. The minimum points between these peaks are designated as points of antiresonance. Although the response at the driving point passes through a minimum at an antiresonance, this is not necessarily true of the response at all points of the system. In fact, at certain points, there may be a peak of response at the antiresonance frequency.

Just as in the case of free vibrations the differential equations for forced vibration of ships cannot be integrated directly, and indirect methods must be devised. The methods discussed in this chapter are the finite difference or digital method, the analog method, and the normal mode method. For further details of the theory of beam vibration other than discussed in this chapter, the reader should see Reference 4-4.

B. DIGITAL METHOD

The digital method of calculating forced vibration is illustrated here for the case of vertical flexural vibration of the hull with no torsion involved. The lumped system to be used for a 20-section lumping is shown in Figure 4-1. In this case, in addition to the elastic member with the rigidities EI and KAG connecting the mass elements, there is also inserted an idealized dashpot between each element and a frame of reference fixed in space. The lumped damping constant is $C = c\Delta x$ and, in addition, there is an external force $P_0 e^{j\omega t}$ acting on one of the elements.

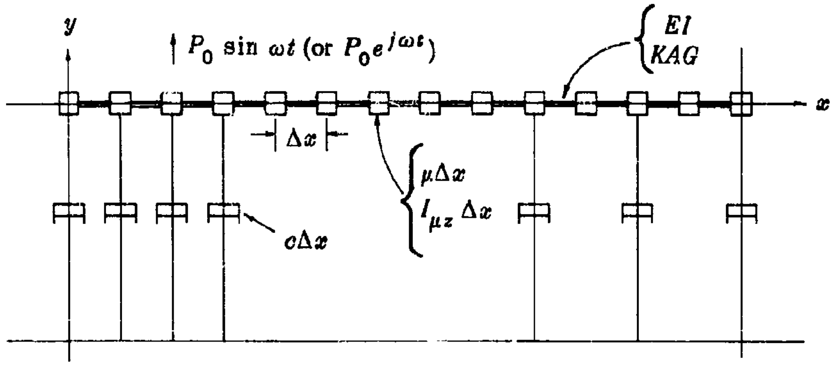


Figure 4-1 – Illustration of Lumped Approximation to a Ship for Digital Calculation of Forced Vibration

In this problem the digital calculation uses rotating time vectors for all time varying quantities. These vectors are resolved into real and imaginary components on an Argand diagram as discussed in Appendix 2 of Reference 4-3. The driving force vector is used as the reference vector for phase, and this is taken as falling instantaneously on the real axis. The calculation requires evaluating the real and imaginary components of V, M, γ, and y at all stations to give the steady-state forced response pattern of the hull.

In real form the difference equations are the same as for the normal mode calculation with the exception of the first (the shear force equation). The latter equations contain the additional damping and driving force terms. Thus:

$$\Delta V = -\mu \Delta x \frac{\partial^2 y}{\partial t^2} - C \frac{\partial y}{\partial t} + P(t) \qquad [4\text{-}3]$$

$$\Delta M = V\Delta x + I_{\mu z} \Delta x \frac{\partial^2 \gamma}{\partial t^2} \qquad [4\text{-}4]$$

$$\Delta \gamma = \frac{M\Delta x}{EI} \qquad [4\text{-}5]$$

$$\Delta y = \gamma\Delta x - \frac{V\Delta x}{KAG} \qquad [4\text{-}6]$$

In the rotating time-vector notation there must now be substituted for V the complex value $V + jV'$, and similarly for M, $M + jM'$, etc. If these substitutions are made in Equations [4-3] through [4-6], and the differences in the real and imaginary parts are separated out, there results the following set of eight difference equations:

$$\Delta V = \mu\Delta x y\omega^2 + C\omega y' + P_0 \qquad [4\text{-}7]$$

$$\Delta V' = \mu\Delta x y'\omega^2 - C\omega y \qquad [4\text{-}8]$$

$$\Delta M = V\Delta x - I_{\mu z}\,\Delta x\gamma\omega^2 \qquad [4\text{-}9]$$

$$\Delta M' = V'\Delta x - I_{\mu z}\,\gamma'\,\omega^2 \qquad [4\text{-}10]$$

$$\Delta\gamma = M\,\frac{\Delta x}{EI} \qquad [4\text{-}11]$$

$$\Delta\gamma' = M'\,\frac{\Delta x}{EI} \qquad [4\text{-}12]$$

$$\Delta y = \gamma - \frac{V\Delta x}{KAG} \qquad [4\text{-}13]$$

$$\Delta y' = \gamma' - \frac{V'\Delta x}{KAG} \qquad [4\text{-}14]$$

The boundary conditions are

$$V = V' = M = M' = 0 \text{ for } \begin{cases} x = 0 \\ x = L \end{cases}$$

The linearity of the equations makes it convenient to set the driving force amplitude P_0 equal to unity. Hence P_0 will equal one at the driving point and zero at all other points. Various schemes of coding this problem for a digital computer may be used as in the normal mode problem. Here there are four unknown quantities at Station 0 ($x = 0$), namely, y, y', γ and γ'. One scheme is to solve the complete set of equations successively with each of the following sets of initial values and then to find a linear combination of these solutions which makes V, V', M, and M' all zero at the right end of the system (Station 20).

The solution will give the real and imaginary components of V, M, γ, and y at all stations. The magnitudes (absolute values) will be the square root of the sum of the squares of the real and imaginary components, and the arctan $\left(\frac{\text{imaginary component}}{\text{real component}}\right)$ will give the phase angle by which the vector representing the variable in question leads the driving force vector. If the two vectors are in phase the imaginary component will be zero.

y	y'	γ	γ'
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

If the damping values used are not too high, the calculation will show a relatively large amplitude when the frequency of the driving force is set equal to or near to one of the natural frequencies of the hull, as computed in the normal mode calculation. Furthermore, if the amplitude at this frequency is plotted against x, the pattern obtained will be very close to that obtained in the normal mode calculation. Further details of the calculation of forced vibration by the digital method are given in Reference 4-5.

C. ANALOG METHOD

The analog method of calculating the forced vibratory response of a hull on the basis of beam theory differs from that outlined in Chapter 3 for computing the normal modes only in regard to the representation of damping. The method for finding normal modes by means of the analog was based on setting up forced oscillations in the circuit since steady-state conditions furnish a more practical method of electrical measurement than decaying oscillations. When the analog is to be used to predict forced vibration *per se*, however, not only the steady-state response patterns are important, but also the magnitudes of the voltages developed for given input currents which yield the forced amplitude of the mechanical system.

Practical difficulties are encountered here in the use of a network analyzer for there is always undesired dissipation in the inductances, capacitances, and transformers which are treated mathematically as dissipationless. Hence, to set up circuits that are strictly analogous to the mechanical systems defined by fixed parameters, it is necessary to take these dissipation effects into account. However, these precautions in the use of the network analyzer apply principally to predicting the resonant amplitudes of the hull under given exciting forces. For off-resonance frequencies the forced response patterns are determined chiefly by the elastic and inertia parameters. (See Reference 4-6.)

Figure 4-2 shows one section of a circuit analogous to a hull subjected to a simple harmonic driving force $P_0 e^{j\omega t}$. This differs from the section of the circuit previously considered for the normal mode calculation (Figure 3-7) in two essential respects. First, a resistance $\dfrac{1}{c\Delta x}$ is inserted between the shear line and the ground to allow for the mechanical damping. In the ideal circuit shown here no allowance is made for the dissipation inherent in

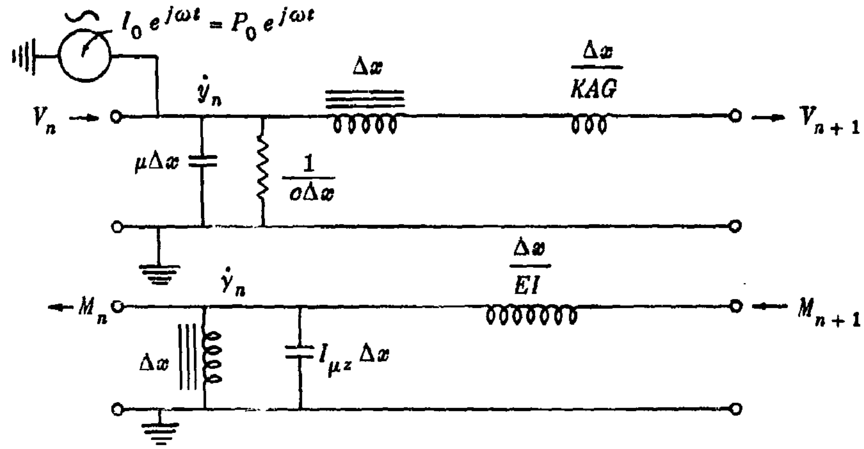


Figure 4-2 – One Section of an Electrical Circuit Analogous to a Ship Subjected to a Simple Harmonic Exciting Force

the inductances, capacitances, and transformers (as discussed in Reference 4-6). Second, there is injected into the shear line at this section a current I equal to the mechanical force $P_0 e^{j\omega t}$. The scaling factors applied in converting from mechanical to electrical values are not considered here. This current injection device is also an idealization since it has the property to maintain the current $I_0 e^{j\omega t}$ going into the shear line at this point regardless of the current flowing in the shear line due to any other sources of excitation present.

Of the four finite difference equations involved in the forced vibration calculations Equations [4-3] through [4-6], the only one that differs from the equations involved in the free vibration calculation is Equation [4-3] which involves the driving force $P(t)$ and the damping term $C \frac{\partial y}{\partial t}$. Hence, to show that the circuit of Figure 4-2 is analogous to the ship in forced vibration (just as the circuit of Figure 3-7 is analogous to the ship in free vibration), it is necessary only to show that the Kirchhoff equation for currents in the shear line is identical with Equation [4-3]. Equation [3-33] involves three currents, namely, the currents flowing into and out of the section (V_n and V_{n+1}, respectively) and the current flowing from the shear line to ground through the condenser ($-\mu \Delta x \omega^2 y$). In Figure 4-2 there are two additional currents, $I = P_0 e^{j\omega t} = P(t)$ and the current through the resistance $\frac{1}{c\Delta x} = \frac{1}{C}$. The latter is equal to the voltage $\dot{y}_n$ divided by the resistance, hence equal to $C\dot{y}$. Instead of Equation [3-33], the current equation for Figure 4-2 becomes

$$V_n = -\mu \Delta x \omega^2 y + C \frac{\partial y}{\partial t} - P(t) + V_{n+1} \qquad [4\text{-}15]$$

which is identical with Equation [4-1] since

$$\frac{\partial^2 y}{\partial t^2} = -\omega^2 y \quad \text{and} \quad \Delta V = V_{n+1} - V_n$$

It must be emphasized that when the network analyzer is used to compute the forced response of a continuous system, such as a hull, the circuit setup is actually the analog of a lumped approximation to the hull. Hence, if a 20-section breakdown of the hull is used, the circuit will in reality have only 20 degrees of freedom. Thus, while the off-resonance forced response may agree very well with digital calculations, it is subject to the same limitations as the digital calculations in giving a less and less realistic picture of the hull behavior as the driving frequency is raised above the frequency range of the first few normal modes of flexural vibration of the hull. The practical upper limit has been found to be the sixth vertical mode for hulls of usual length to depth ratios, as indicated in the following section.

D. NORMAL MODE METHOD

The normal mode method of calculating the forced vibratory response of hulls presented here is quasi-mathematical in that it combines only a limited number of terms of a series that is not converging rapidly in the mathematical sense. The justification is that it has been found by experience that the higher terms of the series are insignificant in the physical sense. As a matter of fact, it can readily be shown that in many cases the terms that are discarded are mathematically larger than the terms that are included.

First it is assumed (on the basis of experience only) that under a simple harmonic driving force the hull can respond in only a limited number of normal mode components. As a guide to the number of normal mode components to be used in forced vibration calculations, the formula originally proposed by Baier and Ormondroyd[4-7] for the number of significant vertical modes of a hull N' may be used as a guide. Assume

$$N' \approx \frac{5}{9}\,\frac{L}{D} \approx \frac{L}{B}$$

where the letters L, D, and B refer to ship length, depth, and beam, respectively. The ratio of the number of significant horizontal modes to the number of significant vertical modes may be taken as 2/3, and of torsional to vertical as 1/2.

While the method involves the selection of the number of terms to be used in the series on the basis of experience, it still retains certain important properties of the normal modes which are found in the theoretical analysis of ideal systems.

Three properties of the normal modes of ideal beams are of particular significance (see Reference 4-8, page 6), namely, the properties of orthogonality, influence function, and reciprocity.

Orthogonality of mathematical functions in general is discussed in Reference 4-9, and if a series of functions of a single variable possess this property within certain limits of this variable, then the integral of the product of any two is zero. Thus

$$\int_a^b \phi_i (x) \phi_j (x)\, dx = 0$$

if $i \neq j$. In the case of the ideal beam the orthogonality property involves the "weighting function" $\mu (x)$ and is

$$\int_0^L \mu (x)\, Y_i (x)\, Y_j (x)\, dx = 0$$

if $i \neq j$.

When a beam possesses this property its kinetic and potential energies can be expressed in terms of vibrations in its normal modes, and, in fact, as shown by Rayleigh,[4-10] the kinetic and potential energies of mass-elastic systems in general can be expressed in terms of the squares of generalized coordinates each involving the deformation or velocity in one of the normal modes. Thus

$$K.\,E. = \frac{1}{2} \sum M_i\, \dot{q}_i^2 \qquad [4\text{-}16]$$

and

$$P.\,E. = \frac{1}{2} \sum k_i\, q_i^2 \qquad [4\text{-}17]$$

where $K.\,E.$ is the kinetic energy,

$P.\,E.$ is the potential energy,

$\dot{q}_i$ is the generalized velocity in the ith normal mode,

q_i is the generalized displacement in the ith normal mode,

M_i is a generalized mass applicable to the ith normal mode, and

k_i is a generalized elastic constant applicable to the ith normal mode.

A more precise definition of these terms is not essential here.

The normal mode influence relation states that the normal mode pattern determines the influence of the point of application of a simple harmonic driving force on the magnitude of the amplitude excited in that mode. Thus, a given force will not excite a mode which has a nodal point at its point of application, and will excite the maximum amplitude in this mode when applied at the point at which the normal mode pattern is a maximum.

The reciprocity relation implies that, if a simple harmonic driving force applied at a point x_r of the ideal beam produces an amplitude of vibration y_{rs} at x_s, then

$$y_{rs} = y_{sr} \qquad [4\text{-}18]$$

In other words, the same amplitude will be measured if the points of application of the driving force and measurement of response are interchanged.

Without further discussion of these very important properties of ideal beams it is merely emphasized here that the great simplicity of dealing with mass elastic systems in terms of normal modes was demonstrated by Rayleigh in his "Theory of Sound."[4-10] It has been shown in recent years that the free-free nonuniform beam with shearing and bending flexibility, a close relative of the Timoshenko beam, retains these properties (see References 4-8 and 4-11).

In the case of the ship, which retains beamlike characteristics only within a limited frequency range, the use of these concepts, the use of a limited number of terms in normal mode response series summation, and the use of Rayleigh damping is largely intuitive. This, however, does not impair its utility. Regardless of how the normal modes of the hull are derived, if only flexural modes of vibration are involved, they may be used to compute the response to a simple harmonic driving force by means of the equation derived from the beam theory

$$y(x, t) = \sum_{i=1}^{i=N'} \frac{P Y_i(x_0)\, Y_i(x) \sin(\omega t - \phi)}{\omega_i^2 \sqrt{\left[1 - \left(\frac{\omega}{\omega_i}\right)^2\right]^2 + \left(\frac{c\omega}{\mu\omega_i}\right)^2} \int_0^L \mu Y_i^2(x)\, dx} \qquad [4\text{-}19]$$

where the force $P \sin \omega t$ is acting at x_0 and N' is the number of significant normal modes. This equation is discussed in detail in Reference 4-8.

The independent behavior of the normal modes of mass elastic systems permits the use of the convenient concept of effective systems of one degree of freedom, each representing a normal mode. There are various ways of defining such effective systems. They may be defined with respect to a particular driving point, or without this restriction. When the hull is represented by an equivalent lumped system, its effective mass at a driving point d in the ith normal mode is

$$M_{di} = \frac{\Sigma m Y_i^2}{Y_{di}^2} \qquad [4\text{-}20]$$

where Y_i is the amplitude at any station in the ith normal mode pattern, Y_{di} is the normal node amplitude at the driving point, and the summation includes the total number of lumps. The effective spring constant of the one-degree system is given by the equation

$$K_{di} = M_{di}\, \omega_i^2 \qquad [4\text{-}21]$$

and the effective damping constant is given for Rayleigh damping by the equation

$$C_{di} = (\text{constant}) \times M_{di} \tag{4-22}$$

The effective system can thus be visualized as the familiar one-degree-of-freedom system usually considered in vibration theory and shown in Figure 4-3.

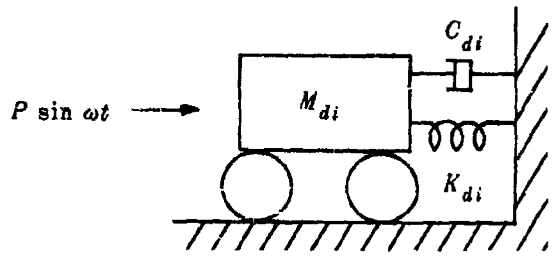


Figure 4-3 – Effective System of One Degree of Freedom Representing a Hull in One of Its Normal Modes

If each normal mode is thus treated, the component of driving point amplitude in each mode is found from the familiar equation for the one-degree system:

$$y = \frac{P \sin (\omega t - \phi)}{\sqrt{(K - M\omega^2)^2 + C^2 \omega^2}} \tag{4-23}$$

where the effective values are used in each case for K, M, and C. To obtain the net response the component of response at the driving point is multiplied by its normal mode function, and the patterns are combined with account taken of the algebraic signs. The result will be the same as given by Equation [4-19] if the same number of terms is used in both cases.

E. METHOD OF MECHANICAL IMPEDANCE

The term impedance, well known in electrical engineering, has become familiar in mechanical vibration analysis only recently. It is most commonly used in connection with the rotating vector to represent a steady-state vibratory component. Although a mechanical impedance may be defined on the basis of either displacement amplitude or velocity amplitude, the American Standards Association's Committee on mechanical shock and vibration prefers the latter. This stems from the fact that this committee was formerly affiliated with the ASA Committee on Acoustics. In the latter field the acoustic impedance concept has been in wide use for some time.

When the mechanical impedance is based on displacement it is the ratio of the amplitude of the driving force to the displacement amplitude at the driving point. Thus

$$Z = \frac{P}{Y}$$

In this equation all three symbols represent rotating time vectors and are complex numbers. For the system of Figure 4-3 it then is found, if the subscripts are omitted, that

$$Z = K - M\omega^2 + j\omega C \tag{4-24}$$

Since, in complex notation, the vibratory velocity amplitude is obtained by multiplying the displacement amplitude by $j\omega$, it follows that the mechanical impedance based on velocity can be obtained from the mechanical impedance based on the displacement by dividing by $j\omega$. Thus

$$Z_v = C + j\,\omega M - j\,\frac{K}{\omega} \tag{4-25}$$

The similarity between this and the well-known expression for the electric impedance of a circuit having resistance R, inductance L, and capacitance C in series, namely,

$$Z = R + j\,\omega L - \frac{j}{\omega C} \tag{4-26}$$

is noteworthy.

If the ship actually possessed only N' normal modes, in each of which its behavior conformed with the beam theory, the displacement impedance in each mode referred to the driving point could be found by computing

$$Z_{di} = K_{di} - M_{di}\,\omega^2 + j\omega C_{di} \tag{4-27}$$

To find the net impedance, the impedances in the individual modes must be combined reciprocally since the net amplitude is the vector sum of the amplitudes in the individual modes. Hence

$$Z_d = \frac{1}{\frac{1}{Z_{d1}} + \frac{1}{Z_{d2}} + \cdots\cdots \frac{1}{Z_{dn}}} \tag{4-28}$$

The awkwardness of dealing with the reciprocals of complex quantities can be circumvented by resorting to the concept of mechanical admittance which is the reciprocal of mechanical impedance. Thus

$$A = \frac{1}{Z} = \frac{Y}{P} \tag{4-29}$$

and

$$Y = PA \tag{4-30}$$

The admittances in the various modes combine by direct addition to give the net driving point admittance; that is,

$$A_d = \sum_{i=1}^{n} A_{di} \tag{4-31}$$

It is a common observation that, when the blade frequency of the propulsion device is well above the range of significant hull mode frequencies, the forced vibration of the hull is concentrated in the stern of the ship, and settles down to a fairly constant level regardless of the speed, unless a local resonance of some structure in the stern is encountered.

From Equation [4-24] it can be seen that at high frequencies, the inertial component is the major component of impedance when the damping is of the Rayleigh type. If the elastic and damping components are then neglected, the admittance becomes $\frac{1}{M_{di}\,\omega^2}$ and the net admittance has the form

$$A_d = \sum \frac{1}{M_{di}\,\omega^2} = \frac{1}{\omega^2} \sum \frac{1}{M_{di}} \tag{4-32}$$

This indicates that under forces increasing as the square of the frequency, the stern amplitude will remain constant at shaft speeds above the range of significant hull criticals.

On the basis of this reasoning, formulas have been proposed for estimating stern amplitudes for given driving forces when the prescribed conditions are met; see Reference 4-3. These formulas use empirical constants and the ship's displacement. It is hoped that data accumulated in the future will indicate to what extent a single constant can be used for ships of widely varying types. The formulas are as follows:

For vertical vibration

$$Y = \frac{P_0}{3.4 \times 10^{-6} \times D \times (\text{cpm})^2} \tag{4-33}$$

where Y is the single amplitude at stern in mils,

P_0 is the amplitude of the driving force in lb,

D is the displacement of ship in long tons, and

cpm is the blade frequency in cycles per minute.

For horizontal vibration

$$Y \approx \frac{P_0}{1.9 \times 10^{-6} \times D \times (\text{cpm})^2} \qquad [4\text{-}34]$$

For torsional vibration

$$\phi \approx \frac{T_0}{0.46 \times I \times (\text{cpm})^2} \qquad [4\text{-}35]$$

where ϕ is the single amplitude at the stern in radians,

T_0 is the blade frequency driving torque in lb-ft, (single amplitude),

I is the mass moment of inertia of entire ship about longitudinal axis through its c.g. (with no allowance for added mass) in ton-sec^2-ft, and

cpm is the blade frequency in cycles per minute.

It is seen that in all three cases, if the driving force increases as the square of the frequency, the formula gives a constant amplitude. These formulas are readily converted to formulas for vibratory velocities in lieu of vibratory displacements.

REFERENCES

4-1. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

4-2. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

4-3. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

4-4. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

4-5. Adams, E.J., "The Steady-State Response of a Ship Hull to a Simple Harmonic Driving Force Computed by a Digital Process," TMB Report 715 (May 1950).

4-6. McCann, G.D. and MacNeal, R.H., "Beam-Vibration Analysis with the Electric-Analog Computer," Journal of Appl Mech, Vol. 17, No. 1 (Mar 1950).

4-7. Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single-Screw Vessels," Trans. SNAME, Vol. 60 (1952).

4- 8. McGoldrick, R.T., et al., "Recent Developments in the Theory of Ship Vibration," TMB Report 739, Revised Edition (Oct 1958).

4- 9. Wylie, C.R., "Advanced Engineering Mathematics," McGraw-Hill Book Co., New York (1951).

4-10. Rayleigh, Lord, "Theory of Sound," Second Edition, Dover Publications, New York (1945).

4-11. Ormondroyd, J., et al., "Dynamics of a Ship's Structure," Engin Res Inst, Univ of Michigan, Ann Arbor (Mar 1949) (out of print).

CHAPTER 5

BEAM THEORY OF TRANSIENT SHIP VIBRATION

A. INTRODUCTION

Although the steady-state vibration of ships under normal operating conditions is a very important consideration in naval architocture, it is clear that under rough sea conditions much more severe vibrations of a transient nature are encountered. If quasi-mathematical methods must be resorted to in dealing with steady-state ship vibration, this applies to a much greater degree in dealing with its transient vibrations. It is to be noted at once that the transient vibrations of ships are occasionally of such large amplitude as to deform the hull girder beyond the linear range within which the beam theory of hull vibration is considered to have a fair degree of validity.

Obviously, just as in the case of steady-state vibration, the transient vibrations of ships depend on both the exciting forces and the dynamical properties or response characteristics of the hull. The forces, however, in this case are quite complex and cannot be expressed in such simple mathematical terms as can the steady-state forces.

Much progress has been made in recent years in correlating the stress variations and motions encountered in ships in a seaway with the statistical data available on ocean waves; see Reference 5-1. It is merely pointed out here that useful correlations have been discovered between the statistical distributions of wave heights encountered at sea and the distributions of motions and stresses in hulls. All that is attempted in this chapter is to indicate that, within the linear range of deflections, the rational theory applied to steady-state forced vibration in Chapter 4 should also be applicable to transient vibrations. The linear range is even lower than the range of deflections within which hull girder stresses reach the yield point of shipbuilding steel, since buckling of members in compression will ordinarily occur before this point is reached.

The value of the consideration here of the transient response of hulls to low magnitude excitation is that it provides the naval architect with insight into the processes that take place before damage actually occurs under slamming conditions in a seaway. Furthermore, as far as possible damage to local structures and to equipment installed in the ship is concerned, it points the way to avoid some of the alterations that frequently have to be made after the builders' trials of a ship. There is no implication intended here that, up to this time (1960), experimental verification has been obtained of the adequacy of the treatment of the response of ship hulls to transient loading by the rational beam theory advocated in this book, or in fact by any other theory. This must await the vordict of investigations still underway. Although it is shown in this chapter how the same general methods applied in Chapter 4 to steady-state vibration are extended to the transient case, the presentation is only analytical and does not encompass a practical evaluation of these methods.

For simplicity and emphasis on basic principles, the discussion is restricted to flexural vibrations of the hull.

B. NORMAL MODE METHOD

The normal mode method of dealing with the transient response of ship hulls presented here is discussed in more detail in Reference 5-2. Basically, it rests on the assumption that in its flexural response, even when account is taken of its shearing flexibility and damping characteristics, the hull satisfies the conditions of a Rayleigh system. This means that the complex vibrations excited by transient loads can be analyzed by considering the separate response in each of its normal modes in the general scheme applied to mass-elastic systems by Rayleigh.[5-3]

The system comprising the hull and the surrounding water is idealized as an unrestrained beam loaded by an arbitrary forcing function and governed by the following set of partial differential equations:

$$\mu \frac{\partial^2 y}{\partial t^2} + c \frac{\partial y}{\partial t} + \frac{\partial V}{\partial x} = P(x, t) \qquad [5\text{-}1]$$

$$V = \frac{\partial M}{\partial x} \qquad [5\text{-}2]$$

$$\frac{\partial y}{\partial x} = \beta' + \gamma \qquad [5\text{-}3]$$

$$V = -KAG\,\beta' \qquad [5\text{-}4]$$

$$M = EI \frac{\partial \gamma}{\partial x} \qquad [5\text{-}5]$$

where c is the viscous damping force per unit length per unit velocity,

$P(x, t)$ is the external forcing function giving load per unit length varying arbitrarily with respect to both x and t, and

β' is the component of slope of elastic line due to shearing only.

The other quantities have been defined previously.

As indicated previously, the modes of free vibrations of such a system (when $c = 0$, and $P(x, t) = 0$) can be found only by graphical, finite difference, or analog methods if EI and KAG vary with x. Nevertheless, it is shown in Reference 5-4 that the normal modes of such a system retain the property of orthogonality; that is,

$$\int_0^L \mu Y_i(x)\, Y_j(x)\, dx = 0 \qquad [5\text{-}6]$$

where $i \neq j$.

It is shown in References 5-2 and 5-4 that, in accordance with the orthogonality relation, the dynamical behavior of this system can be treated in terms of series of responses in each of its normal modes.

It is also shown in Reference 5-2 that even with damping ($c \neq 0$), if it is of the Rayleigh type, that is, viscous and proportional to mass $\left(\frac{c}{\mu} = \text{constant}\right)$, the response to the arbitrary forcing function $P(x, t)$ can still be treated in a series of normal mode responses.

With no pretense at mathematical rigor it has been assumed, on the basis of practical experience, that under transient loading the hull will respond in only a limited number of beamlike modes. Here it is presupposed that these modes have already been determined by methods outlined in Chapter 3.

As shown in Chapter 4, in the dynamics of the ideal beam system representing the hull, each normal mode of vibration is treated as a vibratory system of a single degree of freedom having definite values of mass, spring constant, and viscous damping constant suitably derived.

Such systems have been derived and discussed on the basis of two slightly different concepts which it is well to clarify at this point.

In one scheme (illustrated on page 21 of Reference 5-2), a generalized coordinate $q_i(t)$, with the dimension of length, is used to represent the displacement of the system in its ith normal mode. The motion of the system in that mode is then given by multiplying $q_i(t)$ by the normal mode function where the latter is considered dimensionless (that is, merely a pattern of relative displacements at different distances from the left end of the system). Then the response of the system is given in terms of its normal modes by the series relation

$$y(x, t) = \sum_{i=1}^{N'} q_i(t)\, Y_i(x) \qquad [5\text{-}7]$$

where N' is the number of significant normal modes. The generalized force $Q_i(t)$, which in this case also has the dimension of force, is defined by the relation

$$Q_i(t) = \int_0^L P(x, t)\, Y_i(x)\, dx \qquad [5\text{-}8]$$

In this scheme there are associated with each normal mode generalized or effective masses, damping, and spring constants defined by the relations

$$M_i = \int_0^L \mu(x)\, Y_i^2(x)\, dx \qquad [5\text{-}9]$$

$$C_i = \int_0^L c(x)\, Y_i^2(x)\, dx \qquad [5\text{-}10]$$

and

$$K_i = \omega_i^2\, M_i \qquad [5\text{-}11]$$

where ω_i is the undamped natural frequency associated with the ith normal mode.

The other scheme, based on the same general theory, is more useful. Here the forcing function is concentrated at one point (called the driving point d). In this case the behavior of the system is derived by visualizing each normal mode of the system as presenting to the driving force at d an effective inertia, spring constant, and damping constant, all based not only on the normal mode pattern of each mode but also on the location of the driving point itself.

Thus, if the external force acting at d is $F(t)$, then

$$Q_{di}(t) = F(t)\, Y_i(x_d) \qquad [5\text{-}12]$$

$$M_{di} = \frac{\int_0^L \mu(x)\, Y_i^2(x)\, dx}{Y_i^2(x_d)} \qquad [5\text{-}13]$$

$$C_{di} = \frac{\int_0^L c(x)\, Y_i^2(x)\, dx}{Y_i^2(x_d)} \qquad [5\text{-}14]$$

and

$$K_{di} = \omega_i^2\, M_i \qquad [5\text{-}15]$$

It then results that the response in each normal mode is governed by the same equation as the vibratory system of one degree of freedom shown in Figure 3-1; that is,

$$M_{di}\, y_{di} + C_{di}\, y_{di} + K_{di}\, y_{di} = Q_{di}(t) \qquad [5\text{-}16]$$

It is shown in numerous text books (e.g., Reference 5-5) that when an external force starts to act on such a system, if at rest at $t = 0$, the response at time t is given by the equation

$$y_{di}(t) = \int_0^t \frac{Q_{di}(\tau)}{\lambda_i M_{di}} e^{-(C_{di}/2M_{di})(t-\tau)} \sin \lambda_i (t-\tau)\, d\tau \qquad [5\text{-}17]$$

where

$$\lambda_i = \sqrt{\omega_i^2 - \frac{1}{4}\left(\frac{C_{di}}{M_{di}}\right)^2} \qquad [5\text{-}18]$$

Hence the general response of the system is given by the equation

$$y(x, t) = \sum_{i=1}^{n} y_{di}(t)\, Y_i(x) \qquad [5\text{-}19]$$

If a single impulse H is applied at the driving point d, the same theory yields the equation for the response in the ith normal mode:

$$y_{di}(t) = \frac{H\, Y_i(x_d)}{\lambda_i M_{di}} e^{-C_{di}/2M_{di}} \sin \lambda_i t \qquad [5\text{-}20]$$

As in the steady-state problem, the summation is to be carried out only for the number of modes for which the beam theory is considered valid. Although the normal mode patterns are treated here as continuous functions, the same basic equations may be applied when the normal modes have been computed by the digital method and are available only in tabular form. In this case summations are substituted for integrations in evaluating the effective parameters. Thus

$$M_{di} = \frac{\Sigma\, m_x Y_{i_x}^{\ 2}}{Y_{di}^{\ 2}} \qquad [5\text{-}21]$$

where the subscript x indicates the X-coordinate of an individual lumped mass.

It must be remembered that this transient analysis is based on the treatment of the hull as a beam free in space. Such a beam has two rigid body modes of zero frequency, namely, the heaving mode and the pitching mode. If the components in these modes are included in the series summation, the calculation will not be realistic if carried out over an interval of time long enough for large rigid body rotations to develop. In the actual ship case, heaving and pitching are controlled by the buoyancy of the water and the effect of gravity, and the corresponding natural periods are finite. In practical transient problems, the principal vibrations have been executed by the hull before appreciable rigid body motions have had time to build up.

C. DIGITAL METHOD

The digital method of treating the hull transient response problem requires no prior computation of the normal modes. It uses finite differences in both time and space, and, in the time domain, requires the establishment of stability criteria which will ensure that the time steps are small enough.

The process, of course, involves the same basic concepts of the beamlike behavior of the hull as discussed previously. As elsewhere in this book, the hull is treated here as a beam free in space with a distributed mass added to allow for the inertia effect of the surrounding water. Distributed viscous damping is also included. The physical picture of the process is as follows.

The hull is approximated by the lumped system shown in Figure 5-1 as having 20 sections. As in the steady-state forced vibration problem, an ideal viscous damper is inserted between each lumped mass and a reference frame fixed in space.

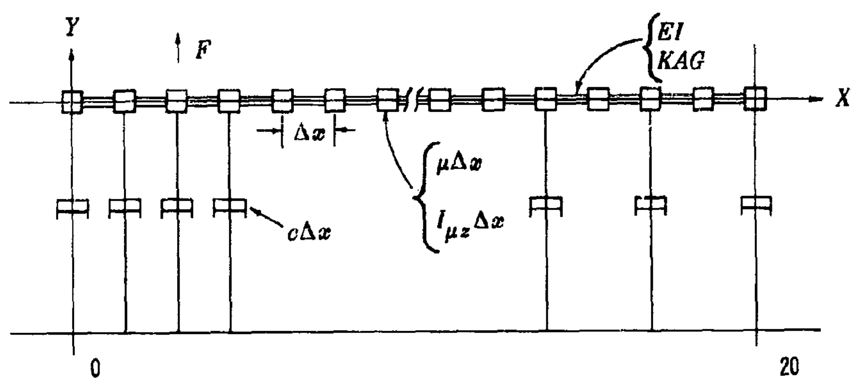


Figure 5-1 – Lumped System Used in Treating the Transient Response of the Hull by the Digital Method

At the instant when the transient load is applied, all stations of the hull have specified displacements and velocities relative to a set of axes fixed in space. In general, both these displacements and velocities will be zero. However, there will be accelerations, since external forces are now acting. The forces which actually may be varying continually with respect to time are considered as held at fixed values for a short interval of time, and then as changing instantaneously to another value, and so on. It is then possible, by the use of the beam equations and the boundary conditions, to compute the acceleration at all stations at $t = 0$. If then a step is taken in time during which these accelerations are assumed constant, the velocities at all points can be calculated, and, a short time later, the displacements due to these velocities can be calculated. Thus, as long as the external forces all along the hull are known at any instant of time, it is possible to repeat this cycle of operations indefinitely and thus to compute displacement at all points of the hull as they vary in time. This also involves computing the elastic deformations, and so the shearing stresses and bending moments are also found in the process.

The mathematical problem of establishing the criteria for stability of the finite difference calculation involved here is discussed in Reference 5-6. The stability criteria given there are:

$$\Delta t << \frac{\mu}{c}$$

and

$$(\Delta t)^2 \leq \frac{\mu I_{\mu z} (\Delta x)^2}{I_{\mu z}(KAG) + \mu(EI) + 0.25\ \mu\ (\Delta x)^2\ (KAG)}$$

The finite difference equations in this case involve both differences with respect to x and differences with respect to t. The former are the same as in the forced vibration problem except that simple harmonic motions and complex number notation are not required here. In Figure 5-2 a section of the hull of length Δx, the distance between stations indicated in Figure 5-1, is shown uncompressed (not lumped at one point) so that its elastic properties may

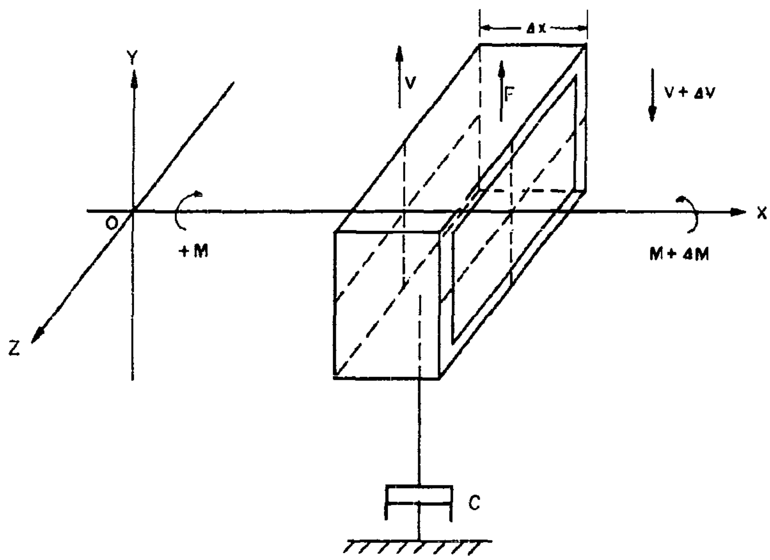


Figure 5-2 – Free-Body Diagram of an Element of a Ship of Length Δx Subject to Forces and Moments Accompanying Transient Vibrations be visualized. Here its mass could be conceived as concentrated at the midspan. It must be emphasized that in any scheme of representation of the hull as an equivalent lumped system, for the purpose of setting up difference equations for a digital treatment, there will be conflicting requirements in the schematic representation. It is not attempted here to indicate the notation which will yield the maximum accuracy for a given number of sections. Accordingly, the equations in the differences with respect to x are given in simplified form as follows:

$$\Delta V = -\mu \Delta x \ddot{y} - Cy + F \qquad [5\text{-}22]$$

$$\Delta M = V \Delta x + I_{\mu z} \Delta x \ddot{\gamma} \qquad [5\text{-}23]$$

$$\Delta \gamma = \frac{M \Delta x}{EI} \qquad [5\text{-}24]$$

$$\Delta y = \gamma \Delta x - \frac{V \Delta x}{KAG} \qquad [5\text{-}25]$$

It should be noted that in Equation [5-23] the term for rotary inertia is included, whereas in Equation [5-2] it is neglected. Experience so far has shown that its inclusion makes little difference in hull vibration calculations, and the demonstration of orthogonality of the normal modes given in Reference 5-4 was based on the set of equations in which the term was omitted. Also, although it is not exploited here, it might also be noted that the finite difference method is not restricted to Rayleigh damping but can be applied to both nonlinear and linear systems.

The equations in the differences with respect to time (also in simplified form) are

$$\Delta \dot{y}_n^s = \ddot{y}_n \, \Delta t \qquad [5\text{-}26]$$

$$\Delta y_n^s = \dot{y}_n \, \Delta t \qquad [5\text{-}27]$$

It is necessary in this case to use both subscripts and superscripts. The former indicate the space coordinate, the latter the time coordinate.

If the calculation starts from the rest condition, the boundary conditions are

$$\dot{\gamma}_n^0 = \gamma_n^0 = y_n^0 = \dot{y}_n^0 = 0$$

$$V_0^s = M_0^s = V_{20}^s = M_{20}^s = 0$$

At any instant of time there are sufficient equations to compute the accelerations at all points along the beam. It is then possible to compute the velocities and displacements at the next time step, and thus to continue the process indefinitely.

In this form of the problem, since the hull is considered free in space, no gravity or buoyancy forces are acting. Although the calculation will give the net result of the combination of rigid body motions and elastic deformations, the rigid body motions will not be realistic if it is carried out for a total time comparable with the pitching or heaving periods of the ship because there are no restoring forces to limit the rigid body displacements.

Obviously, many variations of this problem are possible. Thus the damping forces need not be restricted to the Rayleigh type and gravitational and buoyancy effects can be included. As given here the virtual mass is constant in time and thus does not vary with amplitude, frequency, or mode shape. While the method theoretically takes care of any elastic wave effects that may exist in such a system, no use is made of the traveling wave concept in setting up the problem.

Further discussion of this calculation is given in References 5-6 and 5-7 and a sample calculation is given in Appendix B.

D. ANALOG METHOD

Both the classical (conventional) and the mobility analogs discussed in Chapter 3 are applicable to the hull transient response problem as computing devices. It cannot be said, however, at this time that their potentialities in this field have been more than superficially explored.

While the networks representing the inertial and elastic parameters of the hull considered as a beam are the same as for the steady-state vibration problem considered in Chapter 4, the techniques of exciting the network and measuring its response are obviously radically different.

Instead of a simple oscillator capable of injecting a sinusoidal current (in the mobility analog) at any frequency desired within a given range, the transient problem requires a special transient injection circuit which can deliver a current of the desired waveform regardless of the impedance characteristics of the network. Thus it may be desired to inject a single half-sine pulse, a rectangular pulse, a triangular pulse, or a current pulse that rises "instantly" to a given value and then decays exponentially.

The measurement of transient response is also more complicated than in the steady-state case. In that case, an oscillogram is not really necessary if both an indicating a-c voltmeter and a phasemeter are available. In the transient case, a record must be made on an oscillograph or on magnetic tape of the transient signal at a number of points along the network.

In lieu of a single transient injection, it is often preferable to inject a series of input signals at intervals far enough apart to permit the transient to die out between intervals. If these intervals are not too far apart they will permit direct observation of the response at any point in the network on a cathode ray oscilloscope since the pattern will be retained both by the screen and the retina.

Further details of the use of the electrical analog for such calculations will be found in Reference 5-8. Obviously, at best, the analog predictions of hull response cannot be any more reliable than the theory from which the analogous circuit is derived. Furthermore, the analog may give a distorted picture of the true response as predicted by the mathematical theory used. The advantages of the analog, however, in permitting the operator to vary the inertia and stiffness parameters of the hull simply by turning dials make this method an attractive one.

A recent proposal by Dr. N.H. Jasper considered by the David Taylor Model Basin is the development of a transient computer for ship hulls which forecasts the stresses and vibrations encountered in seas of various wave heights. This is discussed in Reference 5-9. The idea is to use an electrical network to represent the hull as a beam, but, instead of treating it as an ideal system free in space to which loads that are known with respect to time are applied, the computer automatically applies the loads that are exerted by the sea, allowing for the fact that the buoyancy and added mass effects vary with the rigid body motions of the ship.

REFERENCES

5-1. Jasper, N.H., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," Trans. SNAME, Vol. 64 (1956). Also TMB Report 921 (Oct 1957).

5-2. McGoldrick, R.T., et al., "Recent Developments in the Theory of Ship Vibration," TMB Report 739, Revised Edition (Oct 1953).

5-3. Rayleigh, Lord, "Theory of Sound," Second Edition, Dover Publications, New York (1945).

5-4. Ormondroyd, J., et al., "Dynamics of a Ship's Structure," Engin Res Inst, Univ of Michigan, Ann Arbor (Mar 1949) (out of print).

5-5. von Karman, T. and Biot, M.A., "Mathematical Methods in Engineering," McGraw-Hill Book Co. (1940).

5-6. Polachek, H., "Calculation of Transient Excitation of Ship Hulls by Finite Difference Methods," TMB Report 1120 (Jul 1957).

5-7. McGoldrick, R.T., "Calculation of the Response of a Ship Hull to a Transient Load by a Digital Process," TMB Report 1119 (Mar 1957).

5-8. McCann, G.D. and MacNeal, R.H., "Bean-Vibration Analysis with the Electric-Analog Computer," Journal of Appl Mech, Vol. 17, No. 1 (Mar 1950).

5-9. Hoyt, E.D., "Simulation of the Sea and Analog Computation of the Forces on a Ship in Waves," Report to TMB on Contract Nobs 72376, T.O. 9, Project RR-1458-N by Reed Research, Inc (Nov 1959).

CHAPTER 6

EFFECT OF LOCAL FLEXIBILITY ON THE VIBRATORY CHARACTERISTICS OF A HULL

A. INTRODUCTION – THE "SPRUNG MASS" EFFECT

While the chief aim in this book is to exploit the dynamics of a ship's hull when considered as a nonuniform free-free Timoshenko beam, it has been pointed out that in its vibratory behavior, a ship departs from such a beam to an increasing extent as the driving frequency rises. In the response to a steady-state simple harmonic driving force, this departure from ideal beam behavior is evident in two respects of special significance to the naval architect: First, at high driving frequencies the amplitudes of vibration at points in the hull lying in the same transverse plane (normal to the longitudinal axis of the ship) cease to be the same. Second, when the blade frequency is higher than the frequency of (roughly) the sixth vertical flexural mode, the propeller-excited vibration is usually concentrated at the stern of the ship.

When the amplitudes of vertical vibration at points lying in the same transverse plane are not the same the situation is ascribed colloquially to "local flexibility." This local flexibility, which at low frequencies may completely escape attention, may become so pronounced at high frequencies as to fully control the vibratory response. For instance, if a mechanical vibration generator were installed on the main deck of a ship in either the bow or the stern, but in the middle of a panel of deck plating, it would be found that the lower flexural modes of the hull could be readily excited, but that beyond a certain running speed (driving frequency), the machine would be quite incapable of exciting the hull. This is true in spite of the fact that its driving force amplitude increases as the square of the speed. If, however, there are installed under this deck heavy shoring members that transmit the load to points on the deck below, where there are either vertical bulkheads or sections of shell plating, then the range of frequencies over which the hull can be excited by the vibration generator is greatly extended.

In dealing analytically with local flexibility, it has proven fruitful to consider the vibratory characteristics of an ideal beam having one or more masses elastically attached to it. The attached mass has been designated a "sprung mass" and thus the effect of local flexibility is often spoken of as the "sprung mass effect." In this book the term sprung mass is applied to local elastic structures themselves; to relatively rigid assemblies that are supported in the hull by means of resilient mountings; and to heavy items of equipment that are installed on foundations nominally rigid, but which in practice exhibit flexibility as a consequence of the large mass attached to them. A discussion of the properties of the ideal beam with an attached sprung mass is given in Reference 6-1.

B. LOCAL ELASTIC STRUCTURES

The chief structures falling in the category of local elastic structures are deck panels (including plating and stiffeners supported only at the edges), longitudinal and transverse bulkheads, panels of shell plating, stiffened shell plating panels supported only at the edges, deck houses or superstructures, masts, docking keels or skegs, shaft struts, and control surface members such as rudders or (in the case of submarines) diving planes.

The effect of these structures on the vibratory characteristics of the hull as a whole depends chiefly on two properties, namely, the local natural frequency and the local effective mass. Both properties require special consideration here. By the "local natural frequency" is meant the natural frequency that would be measured if the surrounding structure could not move. The identification of local natural frequencies in practice is not always clear, and, strictly speaking, any natural frequency observed may be considered as the frequency associated with a mode of vibration of the entire hull system. Obviously, large local structures cannot vibrate independently of the ship as a whole when the hull is unrestrained. When they are excited by impact, the frequency measured locally is then definitely a frequency of one of the modes of vibration of the hull considered as a beam, with the local structure acting as a sprung mass. If, however, the test is made in drydock, in which case the hull is restrained, the observed frequency may be a true local natural frequency.

The mass of the local structure, or rather, the ratio of the sprung mass to the mass of the ship, furnishes a criterion of the distinction between a local natural frequency and the local manifestation of a hull natural frequency. Unfortunately, the precise determination of such a criterion is not feasible. Moreover, it is not the actual mass of the local structure but its effective mass that furnishes the criterion.

The concept of effective mass was discussed in Chapter 4 in connection with the determination of the driving point mechanical impedance of the hull. It will be recalled that, from the impedance point of view, at any desired driving point the hull may be considered to present an effective mass for each of its normal modes of vibration. This mass has such a value that, if vibrating with a given amplitude at the frequency corresponding to the normal mode in question, it will have the same kinetic energy as will the entire hull if vibrating in this normal mode with the same amplitude at this driving point. The same concepts are applicable to the effective mass of the local structure. Although it is usually the fundamental mode of vibration of the local structure that is of concern, this is not always the case. Where more than one mode of vibration of the local structure is of concern, an effective mass must be evaluated for each mode.

In the mode involved the local structure presents to the hull the effect of a sprung mass attached at a selected point. The point at which the equivalent sprung mass is considered attached is somewhat arbitrary, and the value to be assigned to the effective mass depends on the point selected. In general, this point may be taken at the center of gravity of the local structure. The normal mode pattern and also the local natural frequency for the local structure must be known approximately. The effective mass is then such a mass that, if vibrating at this frequency and with unit amplitude, it will have the same kinetic energy as the local structure would have when vibrating in the mode in question with unit amplitude of the center of gravity; the ship itself is considered at rest (restrained) in both cases.

In general, the determination of the effective mass of the local structure would have to be carried out by an approximate method in which its volume was broken down into a large number of elements; to each element an amplitude was assigned in accordance with the known or assumed normal mode pattern. Then

$$m_{e_{cg}} = \frac{\Sigma m y^2}{{y_{cg}}^2} \qquad [6\text{-}1]$$

where $m_{e_{cg}}$ is the effective mass of the local structure at its center of gravity for the local mode of vibration in question,

m is the mass of one of the elements into which the local structure is broken down for the evaluation of effective mass,

y is the amplitude (in any arbitrary units) of the element m when the local structure is vibrating in this mode, and

y_{cg} is the amplitude in the same arbitrary units at the *c.g.* of the local structure when vibrating in this mode.

Once the effective mass has been evaluated, the effective spring constant can be found from the relation

$$\omega_0 = \sqrt{\frac{k_{e_{cg}}}{m_{e_{cg}}}} \qquad [6\text{-}2]$$

where ω_0 is the natural circular frequency of the local structure and $k_{e_{cg}}$ is the effective spring constant to be associated with $m_{e_{cg}}$.

Reference 6-2 discusses experiments conducted on a cargo ship in which the experimental data indicated a considerable departure in the vibration characteristics of the hull from those predicted by the beam theory. This reference indicates that there was reason to believe that local flexibility accounted for the departure from beamlike vibratory response characteristics. Exploratory calculations were later made by the electrical analog in which a circuit was set up to represent this hull as a beam but with part of the mass flexibility attached to the main hull girder. As indicated in Reference 6-3, these calculations showed qualitatively that the sprung mass effect could account for the observed vibratory response characteristics.

While sufficient information has not been obtained at this time to fix definitely the magnitude required of flexibly attached masses before their effect on the normal modes of hull vibration becomes perceptible, the following criteria are suggested here: (1) the effective mass of the local structure is of the order of 1/2 percent or more of the displacement of the ship, and (2) the local natural frequency is within the range of significant hull frequencies (roughly below 800 cpm). When both of these conditions are met, it is not expected that the usual normal mode pattern of beamlike form as discussed in Chapter 3 will apply to the ship in question.

It is then necessary to redefine the problem in terms of a beam with sprung masses. In doing this, account must also be taken of the fact that the local structure may react so as to affect not only flexural modes but also torsion-bending modes of the hull. In the latter case, the concept of the sprung mass representing the local structure must be generalized to take care of both the translational and rotational effects. This extension of the sprung mass concept is discussed in Reference 6-4.

C. RESILIENTLY MOUNTED ASSEMBLIES

The case of resiliently mounted assemblies requires special consideration here since in recent years there has been a trend toward the resilient mounting of massive elements such as diesel engines, auxiliary turbines, and turbogenerator sets. Since the isolation of such assemblies from the hull requires a relatively "soft" mounting system, both of the conditions previously stated for effecting the vibratory response characteristics of the hull are met. That is, the local item is significantly massive relative to the hull and its natural frequency on its resilient mountings falls below the upper limit of significant hull mode frequencies. A typical resilient mounting for shipboard equipment is shown in Figure 6-1. Usually at least four such

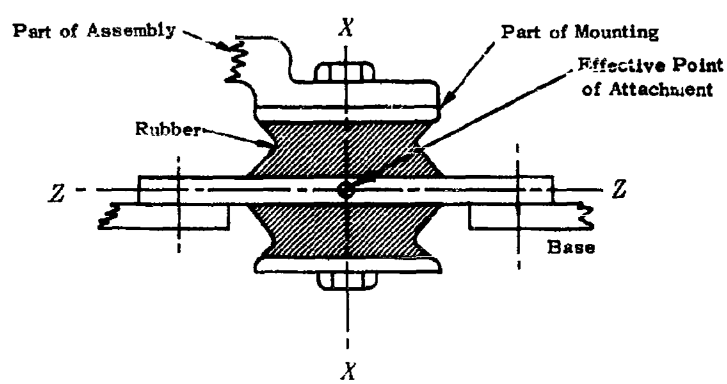


Figure 6-1 – A BST-Type Resilient Mounting for Shipboard Equipment

mountings will comprise the set supporting a single assembly. Figure 6-2 shows schematically a very common mounting arrangement. For further information on resilient mounting of shipboard equipment see Reference 6-5.

A resiliently mounted rigid assembly has six degrees of freedom and thus there may be six local sprung mass effects to consider. Fortunately, it is usually possible to design the local system to have planes of "vibrational symmetry." A plane is said to be a plane of vibrational symmetry when vibrations in this plane produce no tendency for the system to vibrate in translation in directions normal to the plane or in rotation about axes lying in the

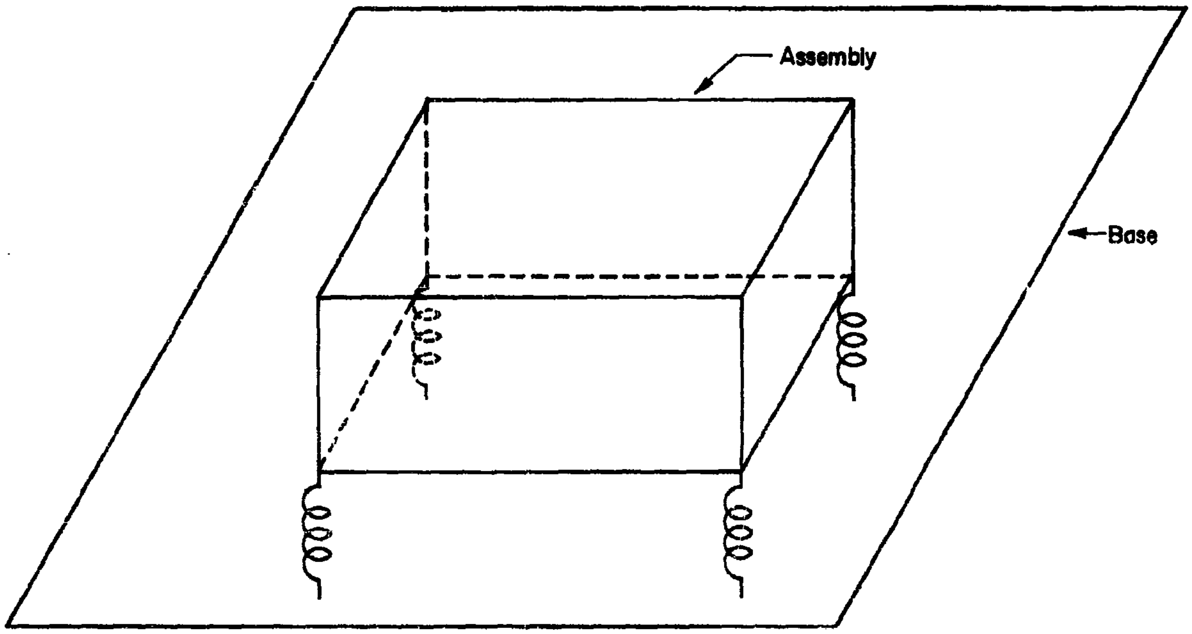


Figure 6-2 – Schematic Illustration of a Resiliently Mounted Assembly with a Typical Base-Mounting Arrangement

plane. As shown in Reference 6-5, the presence of this condition greatly simplifies the vibration analysis.

Even in the simplest type of ship vibration; namely, the vertical, which in general is independent of horizontal and torsional effects, it is clear that a resiliently mounted assembly, if sufficiently massive, can excite the hull in more than one way. If motion confined to a vertical plane through the longitudinal axis of the ship is considered, it is clear that such an assembly can affect the flexural hull modes when the ship is slamming. Moreover, if the assembly is located at a node of a certain flexural mode, which therefore it cannot excite by heaving, it may readily excite this mode by pitching or rocking (a combination of pitching

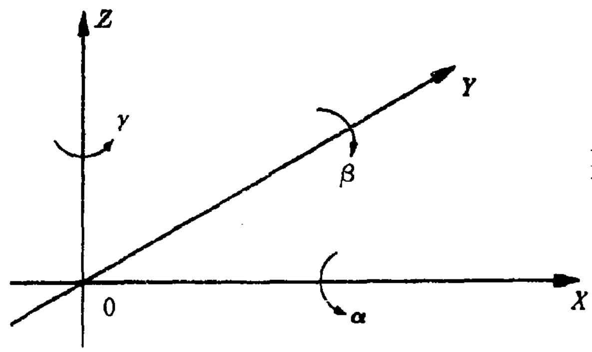


Figure 6-3 – Right-Hand Coordinate System Used in Calculation of Natural Frequencies and Normal Modes of Vibration of a Resiliently Mounted Rigid Assembly

about the c.g. with translation of the c.g.). As a matter of fact, in this case it is impossible to find a location in the hull at which neither pitching nor heaving of the assembly would induce flexural vibration of the hull.

In such a case, therefore, the effect of the local structure on the hull vibration characteristics cannot be represented by a sprung mass of a single degree of freedom. Both the translational and rotational effects must be taken into account. To do this the mass moment of inertia of the assembly with respect to an axis through its c.g. normal to the vertical plane through the centerline of the ship, as well as its mass, must be known.

To analyze the vibration of a rigid assembly supported by a single set of resilient mountings, a fixed set of axes is taken with origin at the c.g. of the mounted assembly in its rest position. A right-hand system is used and the X-axis is taken parallel to the longitudinal axis of the ship; see Figure 6-3. The individual mountings will have principal elastic axes that, in the most general cases, may not be parallel to the axes chosen for the calculation. Since a very common type of resilient mounting has an axis of polar symmetry (such as that shown in Figure 6-1), this is generally called the axis of the mounting; such a mounting is said to have an axial spring constant k_a and a radial spring constant k_r, each being the restoring force for unit displacement in the appropriate direction. If the mounting is displaced into its nonlinear range under the gravitational load of the mounted assembly, then the values of k_a and k_r to be used must be based on small displacements from the loaded position. Mountings not having an axis of polar symmetry can be treated as combinations of mountings having only axial stiffness.

If the mounting illustrated in Figure 6-1 is taken as an example, it is apparent that an arbitrary motion of the assembly relative to the base (involving both translation and rotation) will evoke not only restoring forces in the three principal directions, but also restoring moments about the three axes. In deriving the dynamical equations for the assembly, a great simplication results if two assumptions can be justified:

1. When the assembly moves relative to the base, the restoring forces developed in an individual mounting in the axial and radial directions can be evaluated from the displacements in these directions of a definite point within the mounting called the "effective point of attachment," which remains fixed relative to the assembly.

2. When the assembly moves relative to the base, the moments (with respect to axes with origin at its c.g. in the rest position) of the forces developed by the mountings are so large relative to any couples developed within the mountings themselves that the latter can be neglected.

It is shown in Appendix 5 of Reference 6-5 that from the k_a's and k_r's of the individual mountings, the coordinates of their effective points of attachment, and the orientation of their axes relative to the fixed XYZ-axes, there can be derived a set of elastic constants of the type K_{uv}, $K_{u\alpha}$, etc., which characterize the elastic properties of the entire set of mountings. There will actually be a total of 21 distinct values of the K's, 15 of the form K_{ij} where $i \neq j$, and 6 of the form K_{ii}.

The dynamical equations which yield the six natural frequencies and six normal modes of vibration of the system are Newton equations giving either (1) the relation between the rate of change of rectilinear momentum in a given direction and the forces in that direction; or (2) the relation between the rate of change of moment of momentum about a given axis and the moments about that axis. Instead of expressing the dynamical equations in differential form they are given here in algebraic form, as is commonly done in vibration theory. Thus, on the assumption of simple harmonic vibrations, terms of the type $-mu\omega^2$ are substituted for terms of the type $m\ddot{u}$.

The six dynamical equations for the resiliently mounted assembly, when converted to algebraic form, are:

$$K_{uu}u + K_{uv}v + K_{uw}w + K_{u\alpha}\alpha + K_{u\beta}\beta + K_{u\gamma}\gamma - mu\omega^2 = 0 \qquad [6\text{-}3]$$

$$K_{uv}u + K_{vv}v + K_{vw}w + K_{v\alpha}\alpha + K_{v\beta}\beta + K_{v\gamma}\gamma - mv\omega^2 = 0 \qquad [6\text{-}4]$$

$$K_{uw}u + K_{vw}v + K_{ww}w + K_{w\alpha}\alpha + K_{w\beta}\beta + K_{w\gamma}\gamma - mw\omega^2 = 0 \qquad [6\text{-}5]$$

$$K_{u\alpha}u + K_{v\alpha}v + K_{w\alpha}w + K_{\alpha\alpha}\alpha + K_{\alpha\beta}\beta + K_{\alpha\gamma}\gamma - I_x\alpha\omega^2 + I_{xy}\beta\omega^2 + I_{xz}\gamma\omega^2 = 0 \qquad [6\text{-}6]$$

$$K_{u\beta}u + K_{v\beta}v + K_{w\beta}w + K_{\alpha\beta}\alpha + K_{\beta\beta}\beta + K_{\beta\gamma}\gamma - I_y\beta\omega^2 + I_{xy}\alpha\omega^2 + I_{yz}\gamma\omega^2 = 0 \qquad [6\text{-}7]$$

$$K_{u\gamma}u + K_{v\gamma}v + K_{w\gamma}w + K_{\alpha\gamma}\alpha + K_{\beta\gamma}\beta + K_{\gamma\gamma}\gamma - I_z\gamma\omega^2 + I_{xz}\alpha\omega^2 + I_{yz}\beta\omega^2 = 0 \qquad [6\text{-}8]$$

where m is the mass of the entire assembly;

u, v, and w are displacements of the c.g. of the assembly in the X-, Y-, and Z-directions, respectively;

the K's are elastic constants for the entire set of mountings as defined in Reference 6-5;

I_x, I_y, and I_z are mass moments of inertia of the assembly about the X-, Y-, and Z-axes, respectively, which have their origin at the c.g. of the assembly and are not here restricted to principal axes of inertia;

I_{xy}, I_{xz}, and I_{yz} are mass products of inertia with respect to axes X-Y, X-Z, and Y-Z, respectively; and

ω is the circular frequency (2π times the frequency).

Such a set of equations, which reveals the vibratory characteristics of a mass-elastic system, may be reduced to a single symbolic equation in which matrices are used to represent entire sets of values in the initial set of equations. The use of matrix algebra in vibration analysis is discussed in considerable detail in Reference 6-6. It is sufficient to point out here that, in the present instance, there are six displacement coordinates involved, namely, u, v, w, α, β, and γ. These, if put in the form of a column matrix, can be represented by a single coordinate q. Thus

$$\begin{bmatrix} u \\ v \\ w \\ \alpha \\ \beta \\ \gamma \end{bmatrix} = \{q\} \qquad [6\text{-}9]$$

The array of quantities, which in the set of equations [6-3] through [6-8] represent inertias, in this case yields a matrix, called the inertia matrix, which can also be represented by a single matrix symbol. Thus

$$\begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_x & -I_{xy} & -I_{xz} \\ 0 & 0 & 0 & -I_{xy} & I_y & -I_{yz} \\ 0 & 0 & 0 & -I_{xz} & -I_{yz} & I_z \end{bmatrix} = [M] \qquad [6\text{-}10]$$

Finally, the array of quantities, which in these equations represent elastic constants, in this case yields a stiffness matrix which can be represented by a single matrix symbol. Thus

$$
\begin{bmatrix}
K_{uu} & K_{uv} & K_{uw} & K_{u\alpha} & K_{u\beta} & K_{u\gamma} \\
K_{uv} & K_{vv} & K_{vw} & K_{v\alpha} & K_{v\beta} & K_{v\gamma} \\
K_{uw} & K_{vw} & K_{ww} & K_{w\alpha} & K_{w\beta} & K_{w\gamma} \\
K_{u\alpha} & K_{v\alpha} & K_{w\alpha} & K_{\alpha\alpha} & K_{\alpha\beta} & K_{\alpha\gamma} \\
K_{u\beta} & K_{v\beta} & K_{w\beta} & K_{\alpha\beta} & K_{\beta\beta} & K_{\beta\gamma} \\
K_{u\gamma} & K_{v\gamma} & K_{w\gamma} & K_{\alpha\gamma} & K_{\beta\gamma} & K_{\gamma\gamma}
\end{bmatrix} = [K] \qquad [6\text{-}11]
$$

Then the entire set of six equations [6-3] through [6-8] can be represented by the single matrix equation

$$
-\omega^2 [M] \{q\} + [K] \{q\} = 0 \qquad [6\text{-}12]
$$

If Equation [6-12] is expanded by the rules for matrix multiplication (Reference 6-6), the set of equations [6-3] through [6-8] will be reproduced. It should be noted that Equation [6-12] is identical in form with the equation for the mass-spring combination having a single degree of freedom.

As shown in Reference 6-6, the matrix representation of the dynamical equations applicable to vibratory systems is not restricted to the free vibrations of undamped systems but is applicable to damped systems and forced vibration as well.

The "dynamical matrix," which is obtained by combining the two terms on the left side of Equation [6-12], is shown below.

$K_{uu} - m\omega^2$	K_{uv}	K_{uw}	$K_{u\alpha}$	$K_{u\beta}$	$K_{u\gamma}$
K_{vu}	$K_{vv} - m\omega^2$	K_{vw}	$K_{v\alpha}$	$K_{v\beta}$	$K_{v\gamma}$
K_{wu}	K_{wv}	$K_{ww} - m\omega^2$	$K_{w\alpha}$	$K_{w\beta}$	$K_{w\gamma}$
$K_{\alpha u}$	$K_{v\alpha}$	$K_{w\alpha}$	$K_{\alpha\alpha} - I_x\omega^2$	$K_{\alpha\beta} + I_{xy}\omega^2$	$K_{\alpha\gamma} + I_{xz}\omega^2$
$K_{\beta u}$	$K_{\beta v}$	$K_{\beta w}$	$K_{\beta\alpha} + I_{xy}\omega^2$	$K_{\beta\beta} - I_y\omega^2$	$K_{\beta\gamma} + I_{yz}\omega^2$
$K_{\gamma u}$	$K_{\gamma v}$	$K_{\gamma w}$	$K_{\gamma\alpha} + I_{xz}\omega^2$	$K_{\gamma\beta} + I_{yz}\omega^2$	$K_{\gamma\gamma} - I_z\omega^2$

This matrix has the important property of symmetry with respect to the main diagonal; also both the stiffness matrix and the inertia matrix are individually symmetrical. Since this matrix yields the determinant of the coefficients of the variables of the six simultaneous equations, it furnishes the so-called frequency equation since solutions of the simultaneous equations are only possible for values of ω for which the determinant vanishes. When the determinant is expanded and set equal to zero, the resulting equation is of the sixth degree in ω^2. The positive values of ω for which the determinant vanishes will be the natural frequencies of the assembly. Corresponding to each root there will be a set of relative values of u, v, w, α, β, and γ constituting a normal mode pattern. These patterns are found by solving the simultaneous equations obtained when each of the natural circular frequencies is substituted for ω in the set of equations [6-3] through [6-8].

When planes of vibrational symmetry exist many of the K's and the mass products of inertia may be zero. Then it will be found that all equations of the resulting set may be independent or will reduce to small groups of equations which are independent of the other groups.

As an illustration of the manner in which the effect of a relatively massive item of equipment, resiliently mounted in a hull, may be taken into account in the hull vibration analysis, a special case, chosen for its simplicity, will be considered here. The case selected is the analysis of the vertical vibration of a ship in which there is to be installed a heavy resiliently mounted assembly whose center of gravity will fall on the longitudinal centerline of the hull, and of such design that the vertical plane through the longitudinal axis of the hull will be a plane of vibrational symmetry of the resiliently mounted system.

The assembly now has three normal modes of vibration in this vertical plane when the hull is held fixed. In general, each of these modes will involve the displacements u, w, and β, and thus a combination of heaving, pitching, and surging motions is involved. Here, for consistency with Reference 6-5, the Y-axis is taken as horizontal and not vertical, as in the treatment of vertical hull vibration in Chapter 3.

In the finite difference equations used for finding the vertical normal modes of the hull, the only equations requiring modification due to the presence of the resiliently mounted assembly are those for the element of the hull of length Δx within which the assembly is mounted.

If the mass of the assembly is designated m_s and its mass moment of inertia about the athwartship axis through its c.g. (the Y-axis in this case) is designated I_s, the finite difference equations (in simplified form) become for this element:

$$\Delta V = -\mu \Delta x z \omega^2 - K_{ww}\,(z - z_s) + K_{uw}\,u_s - K_{w\beta}\,(\beta - \beta_s) \qquad [6\text{-}13]$$

$$\Delta M = -I_{\mu\gamma}\,\Delta x \beta \omega^2 + V\Delta x + K_{u\beta}\,u_s - K_{\beta\omega}\,(z - z_s) - K_{\beta\beta}\,(\beta - \beta_s) \qquad [6\text{-}14]$$

$$\Delta\beta = \frac{M\Delta x}{EI} \qquad [6\text{-}15]$$

$$\Delta z = -\beta\Delta x - \frac{V\Delta x}{KAG} \qquad [6\text{-}16]$$

Here the displacement amplitude of the assembly in the Z-direction is designated z_s rather than w_s in conformity with the notation for the displacement amplitude of the hull in that direction; whereas for the displacement amplitude at the assembly in the X-direction the designation u_s is retained. The force and moment equations for the assembly are, respectively:

$$- m_s z_s \omega^2 = K_{ww} (z - z_s) - K_{uw} u_s + K_{w\beta} (\beta - \beta_s) \qquad [6\text{-}17]$$

and

$$- I_s \beta_s \omega^2 = - K_{u\beta} u_s + K_{\beta w} (z - z_s) + K_{\beta\beta} (\beta - \beta_s) \qquad [6\text{-}18]$$

When these equations for the section containing the resiliently mounted assembly are combined with the usual equations for the remainder of the hull, and the digital process is carried out as described in Chapter 3, the solution then shows not only how the isolation mounting affects the natural frequencies of the ship but also how it affects the normal modes of the hull. It also shows how the resiliently mounted assembly vibrates in each of the normal modes of the entire system. From such a calculation it can be predicted whether the local vibration of the resiliently mounted assembly will be excessive when the level of vibration of the hull in its vicinity is within permissible limits. A more general treatment of the hull as a beam with sprung masses is given in Reference 6-4. It can readily be seen that the additional equations involved in the hull calculation to allow for the sprung mass effect of a resiliently mounted assembly (Equations [6-13] through [6-18]) are directly derivable from the dynamical matrix shown on page 6-9.

A further development in the isolation mounting of shipboard equipment has been the use of a compound mounting system. Here the equipment to be isolated is supported by one set of mountings attached to a cradle. The latter, in turn, is supported in the hull by another set of isolation mountings. Such a system is shown schematically in Figure 6-4. It has twelve degrees of freedom, six for each body, and accordingly has twelve natural frequencies and twelve normal modes of vibration.

A discussion of the calculation of the normal modes and natural frequencies of a compound isolation mounting system is given in Reference 6-7. A treatment of the same problem by means of the electrical analog, which in this case was derived from the Lagrangian equations, is given in Reference 6-8.

D. SUMMARY OF EFFECTS OF SPRUNG MASSES ON HULL VIBRATION

Both analytical studies and available experimental data indicate that the local flexibility or the sprung mass effect can cause a considerable modification of the beamlike vibratory response characteristics of a hull. The general effects of lowering hull natural frequencies that are below the local natural frequency and raising hull frequencies that are above

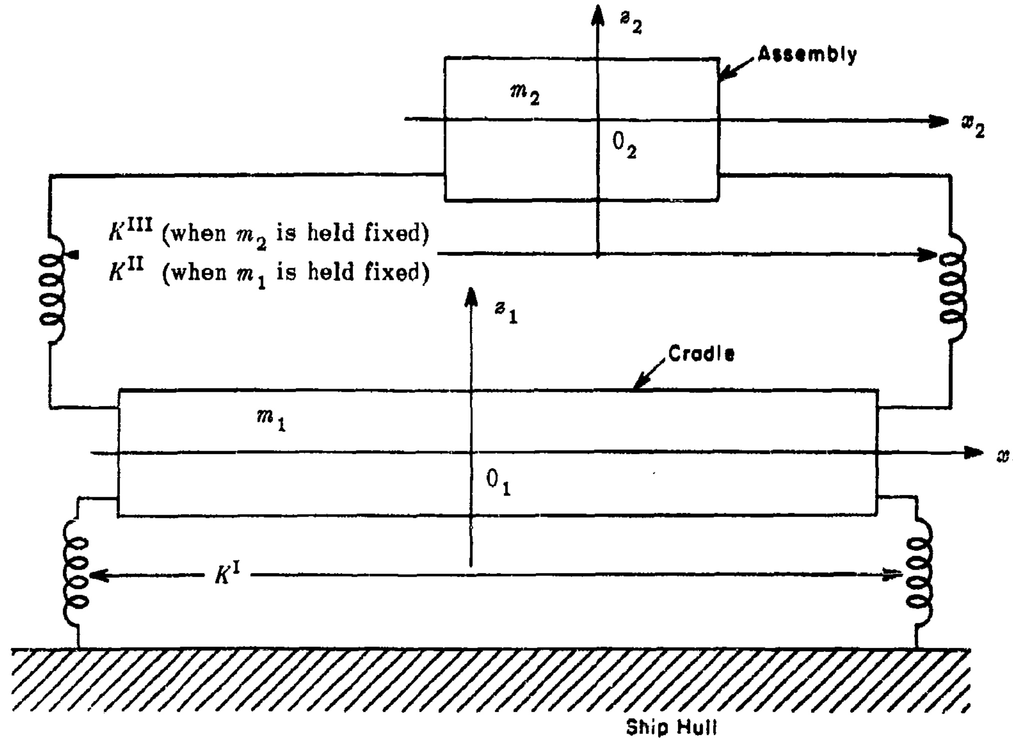


Figure 6-4 – Schematic Elevation of a Compound Isolation Mounting System

this frequency are of considerable practical importance. It is also significant that the effects on frequencies decrease the further they are from the local natural frequency itself. In general, if the sprung mass is large enough to have a significant effect on the hull vibratory response characteristics it will introduce an extra mode. Thus there may be found two modes of the overall system showing the same number of nodes in the displacement pattern of the hull girder proper. In such cases, the phase relation between the displacement of the sprung mass and that of the hull in its vicinity will be reversed in these two modes.

Sprung mass effects may produce marked changes in the response of a hull to an external simple harmonic driving force since the sprung mass may act in the role of a vibration neutralizer or dynamic vibration absorber. The properties of the latter are discussed in Chapter 9.

Hence, if the designer knows in advance that a large mass is to be resiliently mounted in the hull, he must take this into account in any vibration analysis that he attempts. The means for doing this have been indicated in this chapter and are discussed in further detail in Reference 6-4.

REFERENCES

6-1. Kennard, E.H., "Forced Vibrations of Beams and the Effect of Sprung Masses," TMB Report 955 (Jul 1955).

6-2. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

6-3. Leibowitz, R.C., "Natural Modes and Frequencies of Vertical Vibration of a Beam with an Attached Sprung Mass," TMB Report 1215 (Sep 1958).

6-4. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

6-5. Vane, F.F., "A Guide for the Selection and Application of Resilient Mountings to Shipboard Equipment," TMB Report 880, Revised Edition (Feb 1958).

6-6. Pipes, L.A., "Applied Mathematics for Engineers and Physicists," McGraw-Hill Book Co. (1946).

6-7. McGoldrick, R.T., "Calculation of Natural Frequencies and Normal Modes of Vibration for a Compound Isolation Mounting System," TMB Report 1420 (Jul 1960).

6-8. Ryder, F.L. and Zaid, M., "Direct Electrical Analogs of Double 6-Degree-of-Freedom Systems," Report on Contract Nonr-2952 (00) (X), Scientific Research Staff, Republic Aviation Corp., Farmingdale, New York (Feb 1960).

CHAPTER 7

PROPELLER-EXCITING FORCES

A. INTRODUCTION

This chapter is devoted chiefly to propeller-exciting forces, but it must be recognized that propulsion devices other than screw propellers may also generate vibratory forces. It is only the lack of information on the time-varying forces from paddle wheels and other propulsion devices of infrequent use in naval architecture that necessarily confines the discussion to screw propellers at this writing. A change in propulsion system design practice would necessitate detailed study of the forces produced by the new propulsion devices. A further restriction in this chapter is to hydrodynamic forces only. The forces arising from mass unbalance are discussed in Chapter 10.

The generic term "forces" is used in the title of this chapter to cover any type of hydrodynamic excitation of the hull arising from propeller action whether this be a force or a moment, and it may be recalled here that the significant flexural modes of the hull may be excited by either a force, a moment, or a combination of both.

The vibratory hydrodynamic forces arising from the operation of propellers may be separated into pitch-unbalance forces and blade-frequency forces. The former are due to irregularities in the manufacture of the propeller and appear even when the flow into the propeller is perfectly uniform.

The term "blade-frequency forces" is used here in a broad sense to cover the hydrodynamic forces which will exist regardless of the degree of perfection in the manufacture of the propeller. These forces are usually understood when the term "propeller-exciting forces" is used and their fundamental frequency is the blade frequency (rpm times the number of blades per propeller).

It is clearly of great value to the designer to be able to predict whether, under a proposed design, the propeller-exciting forces will be excessive. However, it is not the absolute magnitude of these forces that is of prime significance but their magnitude relative to the mechanical impedance of the hull at the "point" where these forces act. The vibratory response characteristics of the hull are discussed in Chapter 4.

The designer would like to be able not only to predict whether the level of hull vibration will be excessive for a proposed design, but to say what changes should be made in the design to reduce the level of service vibration to acceptable limits. This chapter, however, is concerned only with the forces themselves.

The propeller hydrodynamic forces exciting hull vibration operate both directly on the hull in the vicinity of the propeller as a fluctuating pressure and indirectly through the propeller shaft bearing as a result of lift, drag, and moment on the individual blades. In the latter case, all three components may vary in time even though the propeller maintains a constant angular velocity.

B. PITCH UNBALANCE FORCES

The first-order hull vibration due to pitch unbalance is usually strikingly in evidence when a propeller blade has been bent because of running into an obstruction. When this unbalance is due merely to imperfection in manufacture of the propeller, it is of much smaller amplitude than in the case of damage but is still of first-order frequency (frequency same as the shaft rpm). When a perfectly formed screw propeller operates at constant angular velocity in a uniform axial wake, its polar symmetry requires that it develop constant torque, constant thrust, zero lateral force variation at its bearing, and zero moment variation about any axis normal to its shaft axis.

The effect of nonuniformity of the blades may be seen qualitatively by considering a one-bladed propeller. While the latter will still produce constant thrust and constant torque if the velocity field is uniform, the bearing will now be subject to simple harmonic vertical and athwartship forces and moments about both the vertical and horizontal axes. These forces and moments will be of a frequency which is the same as the shaft rpm (one cycle per revolution). Moreover, if the velocity field in the propeller race is not uniform there will be a superimposed variation effect whose fundamental frequency is also of the first order, but which may have harmonic components depending on the irregularity of the velocity field (wake pattern). Thus, pitch unbalance gives rise to first-order hydrodynamic forces and moments at the propeller shaft bearing and to harmonics of the first-order frequency.

Since, under current practice in the manufacture of propellers, forces due to pitch unbalance are usually within acceptable limits, this phase of hydrodynamic propeller excitation has not attracted much attention so far. However, when first-order hull vibration is encountered on the trials of a new class of ship, the naval architect must always recognize that either mass unbalance or pitch unbalance may be the culprit.

C. BLADE-FREQUENCY FORCES

1. FIRST PRINCIPLES

Although the term "blade frequency" is now in general use for the forces under discussion in this section, it must be emphasized that harmonics of this frequency may be important in many hull vibration problems. Although it had been under investigation several years before, the subject of blade-frequency excitation of hulls attracted increased attention after World War II; see References 7-1 and 7-2. In spite of this, the information available on this subject at the present time must be considered relatively scant.

The forces that vary at blade frequency or harmonics of the blade frequency will exist in spite of extreme precision in the manufacture of a screw propeller and are directly chargeable to the hydrodynamics of the ship design. While they may be greatly magnified by nonuniformity of the wake, it is important to recognize that a fluctuating pressure field will exist forward of the propeller even in a uniform wake. In fact, the hull pressure forces (called the "surface forces" in Reference 7-1) and the bearing forces are generally treated quite independently. Thus, to predict the blade-frequency exciting forces, the surface forces and the bearing forces must be combined, taking account of any relative phase shifts.

Although the theoretical treatment of the pressure fields in the vicinity of the propeller (from which the hull forces must be deduced) is usually based on incompressible potential flow, which implies an infinite velocity of propagation, it must be recognized that such low intensity fields cannot actually propagate at a velocity greater than the velocity of sound. Since the latter is in the neighborhood of 5000 ft/sec, it is clear that if the pressure field extended over 100 ft from the propeller there could be appreciable phase shifts between the forces transmitted through the propeller shaft bearing and the surface forces. However, because these pressure fields usually are not significant beyond a distance of one-half diameter forward of the propeller (see Reference 7-3), and blade frequencies above 3000 cpm are rare for ships of 2000 tons or more, this phase shift is not a serious consideration at present.

The phase relation between the bearing forces and the surface forces, however, involves not only the consideration of the effect of finite velocity of propagation but the effect of algebraic sign or direction in space as well. Thus, if a one-bladed outboard-turning propeller is considered, when the blade is in the 3 o'clock position looking from astern, the bearing force will be directed upward. The pressure field forward of the blade, however, will be negative (suction) and will peak when the blade is in the 12 o'clock position. For instantaneous propagation, therefore, the bearing force and the vertical component of the surface force forward of the propeller would be 90 deg out of phase (with the bearing force leading) if represented on a rotating time-vector diagram such as described in Chapter 4. Thus, it is obvious that the separate determination of bearing and surface blade-frequency components is insufficient to predict the resultant hull driving force, and their phase relations must be taken into account.

On multiple screw ships the phase relations between the blade-frequency-force components are still further complicated by the fact that the forces from the different propellers will continually shift in phase unless the propulsion system has a synchronizing device. This effect and the pitching of the hull in a seaway are the principal causes of the fluctuation in the amplitude of propeller-excited vibration at the stern of a ship.

When propellers with different numbers of blades are used on the same ship, there will also be a "beating" at a frequency equal to the difference in the two blade frequencies. Thus, if four- and five-bladed propellers were operating at 200 rpm, there would be a beating at a frequency of $200 \times (5 - 4) = 200$ beats per min or 3.67 beats per sec, and the beat frequency would be of the same order of magnitude as the component frequencies. In such a case, the usual form of beats, which appears when the difference in the two frequencies is very small relative to their absolute values, would not be in evidence in the signal from a vibration pickup. Since the shaft speeds would actually be varying, the hull vibration record would appear quite irregular.

When the propeller blades are uniform, and the inflow to the propellers is uniform in time (but not in space), the variations in thrust or torque that occur as the propeller rotates will have a fundamental period equal to the time required for the propeller to rotate through an angle equal to 360 deg/s, where s is the number of blades per propeller. This is obvious since at intervals of this duration the propeller always has the same orientation, or better, the same "attitude" as seen from any point of the hull. The reciprocal of this interval is called the blade frequency, and clearly this is equal to the propeller rpm times s. Whereas the blade frequency is the fundamental frequency for propeller-exciting forces and moments, the number of blades and the after body arrangement may be such that very large harmonic force and moment components exist. Hence the term "propeller-exciting forces" includes both blade-frequency forces and forces whose frequencies are multiples of the blade frequency.

2. ANALYTICAL PREDICTIONS

At the present time there is available to the designer no purely analytical procedure by which he can start with a given hull and screw propeller design and calculate the blade-frequency forces and moments that would exist under service conditions. Reference 7-4 indicates the progress that had been made in the analytical prediction of marine propeller pressure fields up to 1959. Future progress in such analyses is to be expected, as indicated by Reference 7-5.

In the problem of surface forces, the first aim in the theoretical attack is to derive the free-space pressure field due to the operation of the propeller. To make use of theoretically derived free-space pressure fields, the designer must then be able not only to correct for the effect on the free-space pressure of the presence of the hull itself as well as its vibratory motion, but to integrate these pressures over the curved surface of the hull in the vicinity of the propeller. Finally, he must be able to compute the bearing forces and to combine these with the hull surface forces, taking account of algebraic signs or phase shifts. Thus, it is not surprising that a purely analytical prediction of the blade-frequency exciting forces has not as yet been achieved.

At the present stage of the art, such analytical predictions of blade-frequency exciting forces as can be made are essentially comparative; see Reference 7-6. To make a prediction the designer must have at hand force data on a previous ship of the same general type. He can then make estimates of the relative magnitude of the surface forces for the new ship by comparison of the axial and radial tip clearances from curves such as given in Figures 7-1 and 7-2 (from Reference 7-3). The parameters involved in this process (also from Reference 7-3) are:

$$\text{Pressure Coefficient: } K_p = \frac{p}{\rho n^2 d^2}$$

$$\text{Thrust Coefficient: } K_T = \frac{T}{\rho n^2 d^4}$$

where p is the free-space oscillating pressure at blade frequency (single amplitude) in psf,

n is the number of revolutions per sec,

ρ is the mass density of water in lb-sec^2/ft^4,

d is the propeller diameter in ft, and

T is the propeller thrust in lb.

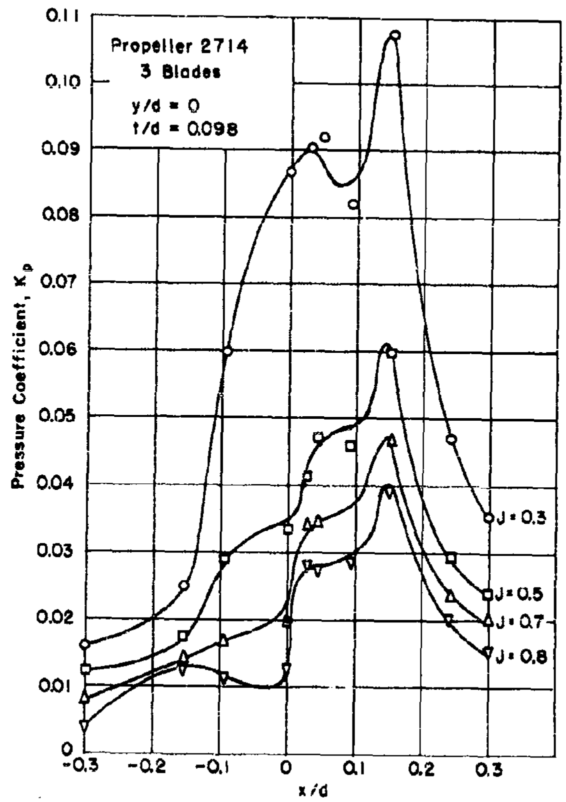


Figure 7-1 – Variation of Pressure Amplitude with Axial Distance from Propeller

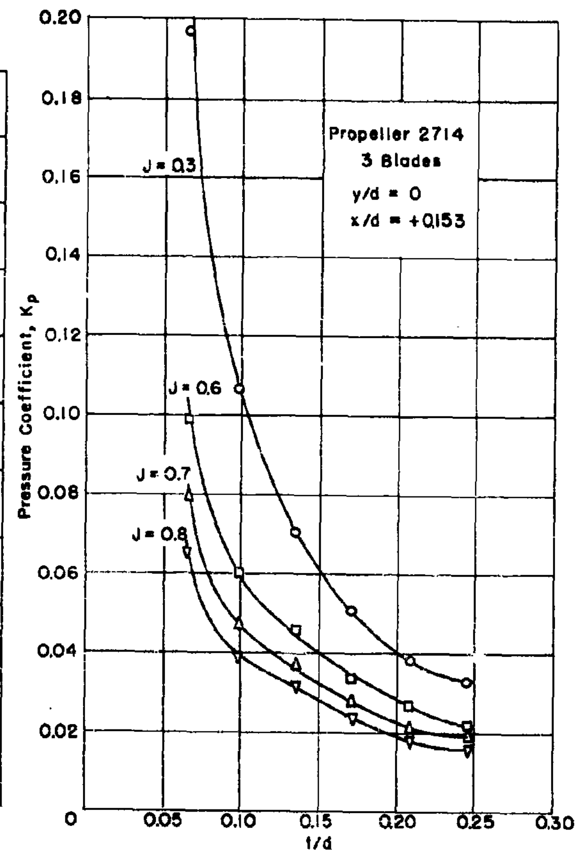


Figure 7-2 – Variation of Pressure Amplitude with Radial Tip Clearance t (in a Plane Ahead of the Propeller Plane)

The coordinate system applicable to Figure 7-1 and 7-2 is indicated in Figure 7-3.

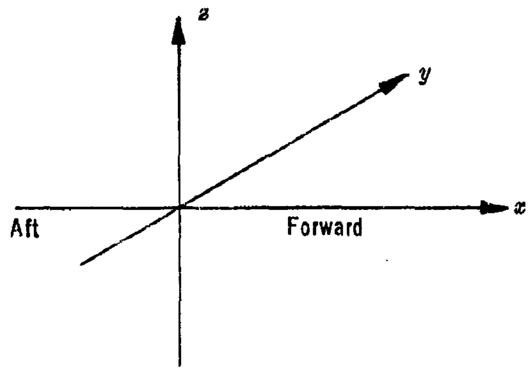


Figure 7-3 – Rectangular Coordinate System Applicable to the Pressure Amplitude Data Presented in Figures 7-1 and 7-2

Figures 7-1 and 7-2 were plotted for a three-bladed propeller. The effect on the pressure amplitude in going to increased numbers of blades may be seen from Figure 7-4 (also taken from Reference 7-3) in which J is the propeller advance ratio $\left(J = \frac{\text{inflow velocity}}{nd}\right)$.

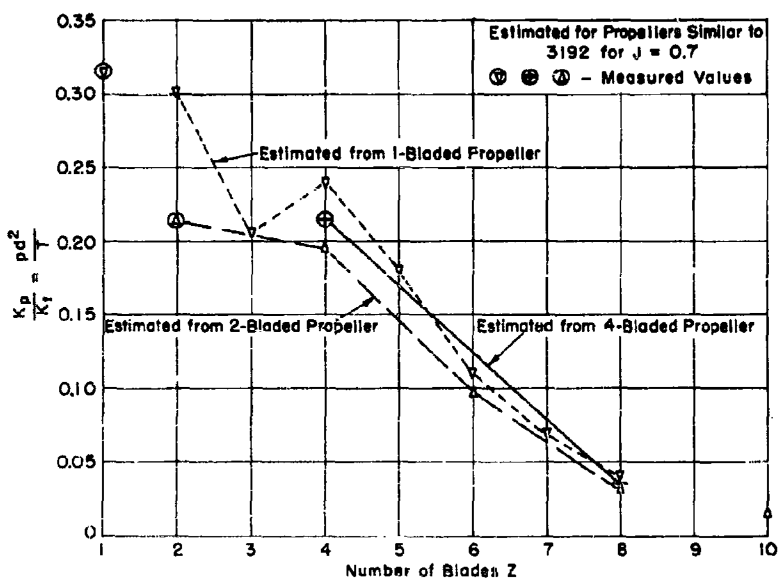


Figure 7-4 – Effect of Number of Propeller Blades on Free-Space Pressure Amplitude

$$\left(\frac{y}{d} = 0;\ \frac{x}{d} = 0.15\right)$$

The "analytical" prediction of the bearing forces at present requires the availability of a wake survey. There will be no thrust variation, torque variation, or bearing-force variation if the wake is uniform in the area of the propeller race and the propeller blades are uniform.

Thus, the process is similar to that of estimating the thrust and torque variation, the difference being that the lift and drag on the individual blades are resolved into components normal to the shaft axis. Details of the torque and thrust calculation are given in Reference 7-7. Although the bearing forces will be zero if the torque and thrust variation are zero, it does not follow that a wake pattern that gives high blade-frequency thrust variation will necessarily give large bearing-force variations. This is because the components of force on the individual blades may be additive as far as thrust is concerned while canceling as far as vertical or horizontal forces in the plane of the propeller are concerned.

It is clear that, regardless of the complexity of the wake pattern, as long as it is constant in time and the blades are uniform in pitch, such an analysis requires advancing the propeller through an angle of only 360 deg/z. The time taken for this angular displacement is the fundamental period of the bearing-force variation.

Thus, for a four-bladed propeller the estimate can be started with one blade in the 12 o'clock position. The lift and drag on each blade for this position are then estimated from the wake survey just as the thrust variation is estimated. Instead of combining the blade forces to give the net thrust, however, they are now combined to give the net vertical force (or horizontal force as the case may be). The propeller is then advanced a few degrees; this process is repeated until the propeller (here assumed four-bladed) which was originally in the 12 o'clock position has advanced to the 3 o'clock position. If the tabulated values of net vertical force are then plotted against time, a smooth curve through the plotted points gives one cycle of the bearing-force variation. The time interval will be the reciprocal of the blade frequency and in that period the four-bladed propeller will have rotated 90 deg.

The total hull driving force is then estimated by combining the bearing and surface forces. Although the estimate of their phase relation is uncertain, the maximum condition will obviously be obtained if they are assumed in phase.

3. MODEL PREDICTIONS

In view of the difficulties involved in the analytical prediction of blade-frequency exciting forces, it was inevitable that the possibilities of model predictions would be explored. The problem, however, also involves numerous difficulties which up to now have not been completely resolved.

In the United States the attempt to determine blade-frequency exciting forces from model tests was initiated by Professor F.M. Lewis under the auspices of the Society of Naval Architects and Marine Engineers. The measurements were made at the U.S. Experimental Model Basin and the results are discussed in References 7-8 and 7-9.

The basic idea of the method was to measure the overall or effective driving force acting on a self-propelled model by nullification of the model vibration by means of a mechanical vibration generator installed in the model and geared to the propeller drive shaft. Both the unbalance of the eccentrics of the vibration generator and their phase relative to propeller could be varied while the model was underway.

In experiments with this apparatus at the U.S. Experimental Model Basin, Professor Lewis was able not only to determine the effective blade-frequency vertical driving force but also to demonstrate that a large part of the effective force was due to the pressure pulsations acting on the hull itself in the vicinity of the propeller. This was shown by installing in one model a long propeller shaft which placed the propeller far enough astern for the pressure field of this propeller to act on another model without a propeller and thus not subject to bearing forces.

The model work on propeller-exciting forces was reactivated at the David Taylor Model Basin in the post-World War II period, and the Society of Naval Architects and Marine Engineers established two research panels in the field of hull vibration, one under its Hydrodynamics Committee and one under its Hull Structure Committee. A comprehensive paper giving the results achieved up to that time was presented to the Society by F.M. Lewis and A.J. Tachmindji in 1954.[7-1] Although this paper reported marked progress in the model determination of propeller-exciting forces, it emphasized the difficulties due to resonance effects in the wooden models themselves in spite of the efforts made to reinforce them. Such a model with the test gear installed in the stern is shown in Figure 7-5.

Figure 7-5 – Reinforced Wooden Model with Test Gear for the Determination of Propeller-Exciting Forces
(From Reference 7-1)

In Reference 7-1 it was pointed out that, although the force system was known to be considerably more complicated, for practical purposes the model determination of blade-frequency excitation was reduced to the prediction of a net vertical force, a net horizontal athwartships force, and a couple about the longitudinal axis of the hull. The horizontal and vertical forces are assumed to act at such locations as to cause no twisting moment on the hull, whereas the couple does apply such a moment. It is to be noted that, although there is no plane of symmetry for horizontal excitation, there can be a plane of symmetry for vertical excitation on multiple-screw ships if all propellers are rotating at the same speed and in phase. In the case of a single-screw ship, there will be no plane of symmetry for either vertical or horizontal excitation.

The latest development in the model determination of propeller-exciting forces up to this time is described in Reference 7-10. In the technique described in this reference, the difficulty with model resonance was circumvented by resorting to the use of a flexibly supported stern section designed to be nonresonant in the range of blade frequencies to be investigated. This is illustrated in Figure 7-6.

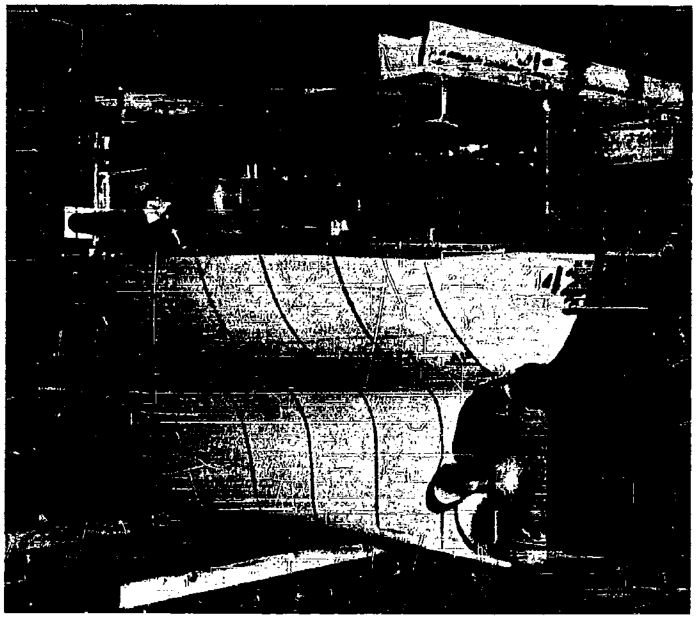


Figure 7-6 – Model for Propeller-Exciting-Force Determination Having Flexibly Suspended Stern Section

In the previous model technique a null method of force determination was used. Thus the propeller forces were neutralized by forces of known magnitude produced by a vibration generator. The later technique did not employ the null method but was based on producing, by means of a vibration generator, the same stern amplitude as produced by the propeller-exciting forces.

As in model testing for powering, the scaling laws are very important in determining propeller-exciting forces by means of model tests. In such model tests the model shaft speed is the same as in the ordinary self-propulsion test; that is, it is in accordance with the Froude scaling relation

$$\text{model rpm} = \text{ship rpm} \sqrt{\frac{\text{length of ship}}{\text{length of model}}}$$

If the boundary-layer distribution is assumed to be the same between ship and model, the experimentally determined model forces are then stepped up to full scale by multiplying by the factor:

$$\frac{\text{displacement of ship}}{\text{displacement of model}}$$

The results of a series of model tests with the flexible stern technique are given in Reference 7-11. As pointed out in this reference, in spite of the improvement in the model technique, unknown scale effects make the model determination of propeller-exciting forces still uncertain at this time.

D. EXPERIMENTAL FORCE DATA AVAILABLE

The experimental data on propeller-exciting forces at present are extremely meager in spite of the effort made to obtain such information. Such data as now exist indicate blade-frequency forces that are fairly large. To give a rough criterion of the order of magnitude of these forces, it may be pointed out that for single-screw cargo ships the vertical and horizontal forces have a single amplitude of the order of 10 percent of the steady thrust at the normal operating speed.

In the case of the Maritime C 4-Class dry cargo ship, both model and full-scale experiments were conducted, as pointed out in Reference 7-1. The full-scale thrust of these ships is 170,000 lb. Reference 7-1 gives the following values as determined from the model (all in single amplitude):

Net vertical force on hull	6 percent of mean thrust
Net athwartship force on hull	16 percent of mean thrust
Net couple on hull	70 percent of mean shaft torque
Thrust variation in shaft	5 to 8 percent of mean thrust
Shaft horsepower	22,000

These ships have the following principal characteristics:

L	525 ft
B	76 ft
D	44 ft 6 in.
H	31 ft 6 in.
Top speed	22 knots at 102 rpm
Propeller blades	4
Single screw	

A vibration-generator survey was conducted on SS GOPHER MARINER in the investigation of the vibratory response characteristics of the hull.[7-12] It was found during this survey that a vertical force of about 7 percent of the mean thrust would produce the amplitude in the stern observed at blade frequency under operating conditions. This would be about 12,000 lb.

In 1957 the David Taylor Model Basin made both an underway-vibration survey and a vibration-generator survey on a twin-screw naval destroyer, USS DECATUR (DD 936). These surveys are discussed in detail in Reference 7-13. In addition to operating the vibration generator at the critical frequencies of the hull, it was run up into the operating blade-frequency range (although not up to the maximum operating blade frequency). From the latter tests, impedance-type expressions were derived for the relation between driving force and stern amplitude at blade frequencies above the range of significant hull mode frequencies. As given in Reference 7-13, the constants to be used in the impedance-type formulas were:

$$\alpha_v = 3.5 \times 10^{-6} \frac{\text{lb-min}^2}{\text{mil-ton}}$$

$$\alpha_A = 2.0 \times 10^{-6} \frac{\text{lb-min}^2}{\text{mil-ton}}$$

$$\alpha_T = 0.46 \frac{\text{lb-min}^2}{\text{rad-ton-sec}^2}$$

For the vertical and athwartship vibration these constants are to be used in the approximate equation

$$Y_0 = \frac{P_0}{\alpha \Delta f^2}$$

where Y_0 is the single amplitude at the after perpendicular in mils,
P_0 is the single amplitude of the driving force in lb,
Δ is the ship's displacement in long tons, and
f is the frequency in cpm.

For torsional vibration the constant α_T is to be used in the approximate equation

$$\phi \approx \frac{T_0}{\alpha_T I f^2}$$

where T_0 is the single amplitude of the blade-frequency exciting couple in lb-ft, and I is as defined for Equation [D-6] of Appendix D.

The following values of propeller-exciting forces were deduced for DD 936 from the observed amplitudes in the stern in the underway-vibration survey in conjunction with the foregoing data.

Net vertical blade-frequency force, at 310 rpm = 82,000 lb
Net athwartships horizontal force, at 310 rpm = 33,000 lb

The DD 931-Class destroyer has the following principal characteristics:

L	407 ft
B	45 ft
D	25 ft
Full load displacement	3800 tons
Thrust per shaft	220,000 lb
Blades per propeller	4
Propeller diameter	13 ft 3 in.
Twin screws	
Twin rudders	

From the initial model experiments made at the U.S. Experimental Model Basin in the early 1930's, Professor F.M. Lewis[7-8] estimated the net vertical blade-frequency force for SS PRESIDENT HOOVER to be 24,000 lb single amplitude. This was 12½ percent of the total thrust. This ship has the following principal characteristics:

L	630 ft
B	81 ft
D	30 ft 3 in.
Displacement	29,000 tons
Blades per propeller	3
Twin Screws	

The ship was provided with propeller shaft bossings.

In Reference 7-14 the author suggested as a round number that the bearing forces account for 25 percent of the total blade-frequency exciting force acting on the hull, and that the remainder was due to the surface forces.

An example of reduction of blade-frequency vibration by the use of a fin to reduce the nonuniformity of flow to the propeller is discussed in Reference 7-15.

REFERENCES

7- 1. Lewis, F.M. and Tachmindji, A., "Propeller Forces Exciting Hull Vibration," Trans. SNAME, Vol. 62 (1954).

7- 2. Breslin, J.P., "The Pressure Field Near a Ship Propeller," JSR, Vol. 1, No. 4 (Mar 1958).

7- 3. Tachmindji, A.J. and McGoldrick, R.T., "Note on Propeller-Excited Hull Vibrations," JSR, Vol. 3, No. 1 (Jun 1959).

7- 4. Breslin, J.P. and Tsakonas, S., "Marine Propeller Pressure Field Including Effects of Loading and Thickness," Trans. SNAME, Vol. 67 (1959).

7- 5. Reed, F.E. and Bradshaw, R.T., "The Distribution of Exciting Forces Generated by Propellers," CONESCO Report F-101-2, Contract Nobs 77150 (Jun 1960).

7- 6. Breslin, J.P., "Ship Vibration. Part I – Propeller-Generated Excitations," Appl Mech Rev, Vol. 13, No. 7 (Jul 1960).

7- 7. Yeh, H.Y., "Thrust and Torque Fluctuations for APA 249, TMB Model 4414," TMB Report 1364 (Jan 1960).

7- 8. Lewis, F.M., "Propeller Vibration," Trans. SNAME, Vol. 43 (1935).

7- 9. Lewis, F.M., "Propeller Vibration," Trans. SNAME, Vol. 44 (1936).

7-10. Pien, P.C. and Ficken, N.L., "The Measurement of Propeller Induced Vibratory Forces on Scale Ship Models," Paper presented at ATTC Conference (Sep 1959).

7-11. Stuntz, G.R., et al., "Series 60 – The Effect of Variations in Afterbody Shape upon Resistance, Power, Wake Distribution, and Propeller-Excited Vibratory Forces," Paper prepared for Annual Meeting of SNAME (1960).

7-12. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

7-13. Paladino, A.R., "Vibration Survey on USS DECATUR (DD 936)," TMB Report 1271 (Mar 1960).

7-14. Babaev, N.N., "Contemporary Status of Development of the Problem of Ship Vibration," TMB Trans. 291 (Jul 1959) from Russian paper in Sudostroenie, No. 3 (161) (Mar 1956).

7-15. Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single-Screw Vessels," Trans. SNAME, Vol. 60 (1952).

CHAPTER 8

DAMPING OF HULL VIBRATION

A. INTRODUCTION

Damping plays a very important role in ship vibration, as in most other areas of mechanical vibration, and it is unfortunate that so little is known about the actual damping process at this time. It is not generally appreciated that the power required to maintain a quite perceptible vibration in a structure as large as a ship is relatively minute. It is also generally overlooked that an undamped mass-elastic system can be maintained in forced vibration at a nonresonant frequency with zero net power input to the system. Even under damped resonant conditions the power required to maintain noticeable hull vibration is amazingly small. Thus a ship of 5000-ton displacement can be maintained in vibration in its fundamental vertical mode with single amplitudes at bow and stern of the order of 10 mils with a mechanical power input of only 100 watts, as follows from the driving force and driving point amplitude data given in the first line of Table 5 of Reference 8-1.

While the designer will naturally aim to avoid resonance involving any of the significant hull modes with either the blade frequency, the shaft frequency, or the running rpm of any major piece of machinery, cases will inevitably arise in which this is not feasible. It may well be necessary in many cases to operate a ship at a shaft speed at which the blade frequency coincides with one of the natural frequencies of the hull. Furthermore, it must not be overlooked that, even when the operating blade frequencies are well clear of the range of significant hull frequencies, it will still be necessary to pass through critical speeds in coming up to the operating speed.

As shown in Chapter 4, the resonant amplitudes of the hull depend on the magnitude of the driving force, the location of its point of application relative to nodal points in the mode in question, and the damping itself. Clearly, the importance of being able to predict the damping values depends on the other factors determining the resonant amplitude. Thus, if there is assurance that the driving forces will be negligible, the need for the damping prediction is greatly diminished. If all the critical speeds with respect to hull vibration are known to fall well below the operating speed range of the ship, it may not be necessary to estimate the hull damping. In general, however, it is desirable to do so.

The difficulty in estimating damping values for the entire hull is due to the present scant knowledge of the actual damping mechanism. To cite obvious sources of damping is far easier than to assess their relative importance. Thus there is an immediate tendency to assume, since the hull cannot vibrate without imparting motion to the surrounding water, that this is an important source of hull damping. There is ample evidence, however, that in the range of frequencies of significant or beamlike hull modes, the damping action of the water is extremely small. Thus the decay rates for free vibration of ships are no greater than for steel structures in air.

Aside from the surrounding water there are two other obvious sources of hull damping; one is the internal friction in the hull structural material, the other is friction between slipping or sliding surfaces within the hull, associated with equipment or cargo.

The experiments on GOPHER MARINER [8-2] show that at the levels of vibration excited by propeller action, the most important source of damping appears to be the cargo friction, if by "cargo" there is included here everything movable inside the hull proper. The damping source next in importance seems to be the hysteresis in the hull itself if the structure is welded. With a riveted construction, the working of the joints can cause a dissipation of energy which is chargeable to the hull although not to the material. The least important source of damping appears to be the water. This statement applies to practical conditions under which the amplitudes are too small to set up appreciable surface waves, and to frequencies at which acoustics radiation is not an important factor. Here it should be noted that acoustic power levels are ordinarily extremely low as contrasted with the level of mechanical power required to vibrate a hull. Present evidence (see References 8-1 and 8-2) indicates that hull damping is actually dependent on both amplitude and frequency.

As in the case of the hull itself, damping also determines the magnitude of resonant vibration of local structures. Here both cargo and water damping effects are, in general, not involved, and the damping source must be attributed to the material of which the local structure is fabricated, the type of joints, or the type of support which determines the transmission of energy to other parts of the ship.

In harmony with the rational beam theory of hull vibration presented in Chapters 3 and 4, the treatment of damping in this book is also a rational or semiempirical treatment. Thus, in the case of flexural vibration, the damping coefficient c is used which represents a damping force per unit velocity, per unit length. This coefficient is further restricted to either of two types: the Rayleigh type which is viscous and proportional to mass, so that $\frac{c}{\mu}$ is constant (where μ is the mass per unit length of the hull including the allowance for added mass of the surrounding water); or the type increasing with frequency so that $\frac{c}{\mu\omega}$ is constant, where ω is the circular frequency.

In this scheme of treating hull damping, the c values to be used in actual calculations cannot be determined analytically from given hull design data but must be based on experimentally determined values obtained on other ships.

The reader will find in the technical literature numerous treatments of beam vibration with other types of damping; see, for example, References 8-3 and 8-4. In the belief that the principal source of damping of ship vibration is the cargo, these methods are not evaluated here. It is pointed out, however, that while the use of equivalent viscous damping constants based on energy dissipation is a makeshift expedient, the viscous damping constant that is almost universally used in the treatment of lumped vibratory systems in standard textbooks on mechanical vibration also involves an idealization of the actual damping process.

B. ANALYTICAL TREATMENT OF HULL DAMPING

The most widely used assumption in the analytical treatment of the damping of a mechanical system in vibration is that it is of the viscous type, as indicated in the previous section. As applied to the elementary system of one degree of freedom, this is the type of damping produced by a frictional force proportional to the velocity and having a direction opposite to that velocity. This gives for the free vibrations the familiar differential equation,

$$m\ddot{x} + c\dot{x} + kx = 0 \qquad [8\text{-}1]$$

where c is the viscous damping constant,

m is the mass, and

k is the spring constant.

In spite of the fact that mechanical damping is rarely of the true viscous type, an "equivalent viscous" constant is widely used because the solutions of the resulting linear differential equations are well known. The equivalent viscous constant is based on energy dissipation per cycle. If this is designated W, then

$$c = \frac{W}{\pi\omega Y^2} \qquad [8\text{-}2]$$

where Y is the single amplitude and ω is the circular frequency. The viscous damping concept is also commonly retained in establishing damping constants from the logarithmic decrements deduced from observations of decaying free vibrations. Thus, in the elementary system of one degree of freedom, the critical viscous damping constant is given by the equation

$$c_c = 2m\sqrt{\frac{k}{m}} \qquad [8\text{-}3]$$

and the logarithmic decrement is

$$\delta = \frac{2\pi \dfrac{c}{c_c}}{\sqrt{1 - \left(\dfrac{c}{c_c}\right)^2}} \qquad [8\text{-}4]$$

For small damping

$$\delta \approx 2\pi \frac{c}{c_c} \qquad [8\text{-}5]$$

A common criterion of the degree of damping is the resonance magnification factor. This is frequently designated by the symbol Q, widely used in electrical circuit theory as an index of dissipation for inductances. The lower the dissipation in the coil, the higher is its Q. For viscous damping

$$Q = \frac{1}{2 \dfrac{c}{c_c}} \qquad [8\text{-}6]$$

and for low damping

$$Q \approx \frac{\pi}{\delta} \qquad [8\text{-}7]$$

In the case of the hull, the complexity and uncertainty regarding the actual damping processes have forced an extension of these concepts. By assuming that the damping of the flexural vibration of the hull can be represented by a distributed viscous damping constant proportional to the mass per unit length (including the added mass of surrounding water), the ship can be treated as a "Rayleigh system," at least in dealing with vibration in its significant flexural modes. The Rayleigh-type of damping is then of the type c/μ = constant, where c is the equivalent damping force per unit velocity per unit length (axial), and μ is the mass per unit length including the added mass of water.

Under such assumptions, the free and forced vibrations of the hull may be treated in terms of normal mode responses. This effects great simplification. In this procedure the ith flexural mode of the hull is reduced to an effective system of one degree of freedom referred to a specific driving point d. According to the equations given on pages 4-9 to 4-11, the steady-state amplitude is

$$Y_{di} = \frac{P}{K_{di} - M_{di}\,\omega^2 + j\,C_{di}\,\omega} \qquad [8\text{-}8]$$

This gives not only the magnitude of the displacement amplitude in the ith mode but also the phase of its rotating vector in relation to that of the driving force of circular frequency ω.

Experience has indicated that the damping is actually dependent on both amplitude and frequency. For frequency dependence the indication is that the relation

$$\frac{c}{\mu\omega} = \text{constant}$$

is closer to reality than the relation

$$\frac{c}{\mu} = \text{constant}$$

In calculating steady-state forced vibration by the digital method discussed in Chapter 4, it is possible to use values of c satisfying either relation. All that is required in the former case is to adjust the value of c for each value of ω for which the calculation is made. However, this does not permit finding the response to an arbitrary excitation by the normal mode method or by the digital method discussed in Chapter 5. Thus, where a method of calculation requires the use of a true Rayleigh damping coefficient, its value must be based on a mean value of the frequencies involved; but where the calculation requires only the response of the hull at a single frequency, the damping coefficient can be selected according to that frequency.

In the case of local structures, the same general principles used in treating the damping of the hull may be applied, but the determination of normal modes and the subsequent reduction to effective systems for each normal mode referred to a specific driving point cannot, in general, be based on a beam-type analysis. Where the response in the fundamental mode is of prime concern and where a reasonable guess can be made as to the fundamental normal mode shape, the frequency may be estimated by the general Rayleigh method if the potential energy can be evaluated for a deformation in this pattern. The Rayleigh method of finding natural frequencies of systems is discussed in References 8-6 and 8-7 and requires the evaluation of both kinetic and potential energies. Thus, for a section of plating simply supported at the edges but with stiffeners in both the fore and aft and transverse directions, the bending energy may be evaluated in terms of the curvatures in the two principal directions and the rigidity factors for the stiffened plate. The effective mass values are then derived from the kinetic energy on the basis of the same concepts as applied to the hull girder.

C. EXPERIMENTAL METHODS OF DETERMINING DAMPING

Two standard methods of determining the damping of a vibratory system are (1) to excite the system by an impulse and to measure the rate of decay of the resulting free vibrations, and (2) to measure the resonant magnification in forced steady-state vibration. Both methods have been applied to the entire hull and also to local hull structures.

When the hull is excited by a vertical impulse at the bow, the predominant response is usually in the 2-node vertical flexural mode. This impulse is most conveniently applied by releasing an anchor and arresting it after a fall of a few feet. On one occasion it was found possible to measure the fundamental vertical frequency of a large naval vessel during calisthenics of the crew on the forward main deck. A horizontal impulse may be applied at the bow less conveniently by a bump applied by a tug boat. It is also possible to excite transient vibrations in the horizontal flexural modes by rudder maneuvering while underway.

The records from such tests usually show an initial complex vibration with high frequency components which shortly settles down to a train of damped sine waves of constant frequency. The logarithmic decrement is determined from the latter part of the record by measuring two peak displacements q cycles apart and using the relation

$$\delta = \frac{1}{q} \log_e \frac{A_n}{A_{n+q}} \qquad [8\text{-}9]$$

As applied to local structures, the impulse method of determining damping is similar, but in this case it is often difficult to produce a decaying vibration in a single mode. Blows with a heavy timber comprise the most common method of excitation.

The rational theory permits the evaluation of the damping constant to be used in the calculation of forced vibration of ships directly from the experimentally determined logarithmic decrement. Thus, suppose a calculation is to be made by the digital process outlined in Chapter 4. Here the Rayleigh damping constant is retained and this means that c/μ is treated as constant for all points along the hull and at all driving frequencies. Let M_{di} be the effective mass at any arbitrary point for the mode of vibration in which the logarithmic decrement was observed (here called the ith mode). Then the effective Rayleigh damping constant for this mode and at this point of the hull is given by the equation

$$\delta_i = \frac{\pi C_{di}}{M_{di}\,\omega_i} \qquad [8\text{-}10]$$

The damping constants c used in the digital calculation are then evaluated from the relation

$$\frac{c}{\mu} = \frac{C_{di}}{M_{di}} \qquad [8\text{-}11]$$

If hull damping were truly of the Rayleigh type, $\dfrac{C_{di}}{M_{di}}$ would be independent of the mode and it would follow from Equation [8-10] that the logarithmic decrement would vary inversely with the frequency of the mode. Since experience has shown it more feasible to assume that the logarithmic decrement remains constant, it is expedient to assume

$$\frac{C_{di}}{M_{di}\,\omega_i} = \text{constant} \qquad [8\text{-}12]$$

in any calculations dealing with response in a single mode.

Naturally, if experimental decrement values are available for a previous ship of generally similar design these are the best to use, but as pointed out in Reference 8-2, an average value of $\dfrac{C_{di}}{M_{di}\,\omega_i}$ obtained from experiments on a variety of ships is 0.03.

In deriving hull damping values from vibration generator tests, use can be made of the fact that at resonance the mechanical impedance (defined in Chapter 4) in the mode in question is solely the damping impedance $C_{di}\,\omega_i$. Hence, from the known exciting force of the vibration generator and the measured resonant amplitude at the driving point, C_{di} can be computed from the equation

$$C_{di} = \frac{P_0}{Y_{di}\,\omega_i} \qquad [8\text{-}13]$$

D. AVAILABLE DATA ON HULL DAMPING

Although experimental data on the damping of ship vibration are still relatively scarce, the designer is fortunate that such information is being accumulated at an accelerating pace. Obviously, in making estimates for a particular design the naval architect should seek data on ships of the same general type. Some typical data are presented in Tables 8-1, 8-2, and 8-3. Further data of this type can be found in Reference 8-5.

In Reference 8-8, the logarithmic decrement of an aircraft carrier in whipping following slamming in a rough sea was reported as 0.037.

E. DAMPING ACTION OF LIFTING SURFACES

In general, the damping of a hull, just as its inertia, will not change with the ship's forward speed. In the case of inertia, this is because the flow of water associated with the added mass effect discussed in Chapter 2 is noncirculatory. In hull damping, it is because the contribution of the water to the total damping effect is quite small in any case.

There is, however, a hydrodynamic damping effect which does depend on the ship's forward speed and which cannot be considered negligible. This effect, discussed in more detail in Chapter 14, involves the lifting surfaces such as rudders and submarine diving planes. In the present chapter the effect is discussed only in the simplest possible terms.

The function of the lifting surface, of course, is to produce a lift force derived from the flow whose moment will cause the rigid body rotation of the ship desired for a maneuver. This lift force is proportional to a "lift coefficient," the angle of attack, and the square of the relative velocity between the lifting surface and the water. In these simple terms the lift force is given by the relation

$$F_l = AS^2\theta \qquad [8\text{-}14]$$

where F_l is the lift force,

A is the lift coefficient,

S is the relative velocity, and

θ is the angle of attack.

As shown in Chapter 14, if the control surface acquires a component of velocity normal to the direction of the velocity S, there results a change in F_l due to the change in apparent angle of attack. A vibratory motion of the axis of the control surface, due to a vibration of the hull as a beam, will therefore cause a variation in F_l of the same frequency as that of the hull vibration. If the entire hull is reduced to an effective M, K, C system of one degree of freedom for the purpose of analysis, the governing equation for vibratory motion then becomes

$$M\ddot{Y} + C\dot{Y} + KY = F_l' \qquad [8\text{-}15]$$

where F_l' is the variation from the steady lift F_l due to any velocity of the lifting surface in the Y-direction.

Under the simplifying assumptions made here and discussed further in Chapter 14, it turns out that

$$F_l' = -AS\dot{Y} \qquad [8\text{-}16]$$

Hence Equation [8-15] becomes

$$M\ddot{Y} + (C + AS)\,\dot{Y} + KY = 0 \qquad [8\text{-}17]$$

This indicates that, when the vibration of the hull is accompanied only by a vibratory motion in translation of the control surface (θ remaining constant), the forward velocity of the ship causes a damping action in addition to the damping that would otherwise exist. The latter is represented by C in Equation [8-17].

It is shown in Reference 8-9 that the hydrofoil damping action of rudders at high ship speed can reach the same order of magnitude as the ordinary hull damping action. For rudders this, obviously, applies only to horizontal hull vibration. However, in the case of submarines the diving planes can produce a similar damping action in vertical hull vibration. When angular oscillations of the lifting surface are also present, the situation is radically altered, leading to the possibility of flutter as shown in Chapter 14.

TABLE 8-1

Principal Data on Ships for Which Damping Data Were Obtained by the David Taylor Model Basin

Ship	Type of Ship	Length between Perpendiculars, L	Beam, B	Mean Draft Full Load	Displacement Full Load	Test Displacement	Mean Test Draft	Depth, D	Moment of Inertia of Midship Section, ft^4		Shear Area, ft^2		L/D	B/D	L/B	Maximum Shaft	Number of Blades per Propeller	Maximum Blade Frequency
		ft	ft-in	ft-in	tons	tons	ft-in	ft-in	Vertical	Horizontal	Vertical	Horizontal				rpm		cpm
USS NIAGARA	Transport	400	58-0	15-5	6,740	5,500	12-11	37-0	2,620	5,832	2.46	8.20	10.8	1.57	6.9	188	4	752
USS CHAS. R. WARE	Destroyer	383	40-10	14-0	3,400	3,200	13-8	23-10	567	1,289	1.88	2.57	16.1	1.71	9.4	350	4	1400
SS E.J. KULAS	Ore Carrier	580	60-0	19-8½	*19,500	*19,500	19-8½	32-0	2,490	9,008	4.20	8.99	18.1	1.875	9.67	110	4	440
SS C.A. PAUL	Ore Carrier	520	54-0	21-4½	*16,200	*16,200	21-4½	31-0	1,825	3,631	4.32	4.63	16.8	1.74	9.62	123	4	492
SS PERE MARQUETTE 21	Car Ferry	348	56-0	14-7	* 5,900	* 5,900	14-7	21-6	2,482	6,900	3.90	4.63	16.2	2.6	6.21	110	4	440
SS OLD COLONY MARINER	Dry Cargo	528	76-0	31-6	21,000	16,400	24-0	44-6	7,621	16,000	4.41	16.13	11.9	1.71	6.95	105	4	420
USS NORTHAMPTON	Cruiser	664	70-3	24-0	17,100	16,200	24-0	52-9	10,600	20,600	8.03	27.1	12.59	1.33	9.45	340	4	1360
USS STATEN ISLAND	Icebreaker	250	63-6	25-9	6,660	4,500	22-9	39-1½	2,890	8,480	8.20	8.46	6.39	1.62	3.93	110	3	330

*Short tons.

TABLE 8-2

Damping Factors Derived from Vibration Generator Tests on Ships Listed in Table 8-1

(Vertical Modes Only)

Ship	Mode	ω rad/sec	c/μ 1/sec	$c/\mu\omega$	Driving Force tons	Driving Point Single Amplitude ft
NIAGARA	1st	11.5	0.49	0.043	0.51	0.0011
	2nd	20.9	0.41	0.019	1.68	0.0021
	3rd	30.5	0.83	0.027	3.57	0.0014
	4th	37.1	2.6	0.067	5.27	0.0058
	5th	46.8	2.20	0.047	8.44	0.0049
CHARLES R. WARE	1st	8.2	0.17	0.021	0.30	0.0050
	2nd	12.2	0.17	0.010	1.32	0.0070
	3rd	27.3	0.31	0.014	3.32	0.0031
	4th	37.6	1.3	0.035	6.29	0.0092
E.J. KULAS	5th	29.8	0.80	0.027	2.77	0.0006
C.A. PAUL	1st	4.71	0.029	0.006	0.16	0.0079
	2nd	11.1	0.114	0.010	0.76	0.0064
PERE MARQUETTE 21	1st	11.7	0.168	0.014	0.89	0.0052
NORTHAMPTON	2nd	13.9	0.298	0.021	1.21	0.0008
	3rd	21.4	0.512	0.024	2.86	0.0007
	4th	30.2	0.722	0.024	5.71	0.0004
	5th	37.6	1.33	0.035	8.84	0.0004
	6th	45.8	2.55	0.056	12.95	0.0001
	7th	52.4	7.80	0.149	17.19	0.0002
STATEN ISLAND	1st	29.3	0.976	0.033	2.81	0.0038
				(avg) 0.034		

TABLE 8-3

Experimental Values of Logarithmic Decrements for Fundamental Vertical Mode of Ships

Name of Ship	Type of Ship	Test Displacement tons	L ft-in	B ft-in	D ft-in	Test Draft ft-in	Logarithmic Decrement
NIAGARA	Transport	5,500	400	58	37	12-1	0.079
OCEAN VULCAN	Freighter	13,750	416	56-11	37-4	25-11	0.053
HAMILTON	Destroyer	1,380	310	31	20-9	10-8	0.023

REFERENCES

8-1. McGoldrick, R.T., "Comparison between Theoretically and Experimentally Determined Natural Frequencies and Modes of Vibration of Ships," TMB Report 906 (Aug 1954).

8-2. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

8-3. Kumai, T., "Damping Factors in the Higher Modes of Ship Vibrations," European Shipbldg, Vol. VII, No. 1 (1958).

8-4. Sezawa, K., "The Effect of the Distribution of Damping Resistance in a Ship on Her Vibration," J Soc Nav Arch Japan, Vol. 66 (Jun 1940).

8-5. Vane, F.F., "Shock and Vibration in Ships," Handbook of Shock and Vibration Control, Harris, C.M. and Crede, C.E., Editors, McGraw-Hill Book Co. (to be published in 1961).

8-6. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

8-7. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

8-8. Jasper, N.H. and Birmingham, J.T., "Strains and Motions of USS ESSEX (CVA 9) during Storms Near Cape Horn," TMB Report 1216 (Aug 1958).

8-9. McGoldrick, R.T., "Rudder-Excited Hull Vibration on USS FORREST SHERMAN (DD 931) – A Problem in Hydroelasticity," Trans. SNAME (1959). Also TMB Report 1431 (Jun 1960).

CHAPTER 9

ANTIVIBRATION DEVICES

A. INTRODUCTION

The technical literature reports many developments in the field of antirolling devices for ships, such as bilge keels, antirolling tanks, gyrostabilizers, and activated fin stabilizers. Aside from the fact that the rolling frequencies of ships are of the order of 1/10 or less of the lowest frequency normally associated with hull vibration, these devices perform the same basic function as antivibration devices. In one way or another they set up moments opposing the rolling motion of the ship. It may therefore seem surprising that relatively little attention has been given to the development of antivibration devices for hulls. This is true in spite of the fact that antivibration devices in the form of pendulum dampers have been used extensively in large internal combustion engines to suppress torsional vibration in the crankshaft system.

The presumption in seeking any antivibration device is that the source of disturbance cannot be eliminated. In the case of hull vibration (as shown in Chapter 7), while first-order exciting forces can be reduced by improved methods of balancing propellers, shafting, and machinery, or by closer tolerances for machining of propeller blades, the blade-frequency exciting forces cannot be reduced without changing the number of blades per propeller, or altering the stern configuration. Thus, a practical antivibration device for hulls could find application under the following situations: (1) when hull vibration develops unexpectedly on the initial trials of a new class of ship; (2) when the design study indicates that an excessive level of vibration will exist, but there are overriding advantages in the particular design adopted which warrant its retention; and (3) when an unusual type of propulsion system known to produce large vibratory exciting forces is required for special reasons.

In this chapter a brief discussion is given of certain antivibration devices that have actually been used to a limited extent for the purpose of reducing the level of service vibration of ships. These devices are the tuned vibration neutralizer, often spoken of in the literature as the "dynamic vibration absorber"; the adjustable rotating eccentric; shaft synchronizing devices; and flexible materials used in the vicinity of propellers. Both the tuned vibration neutralizer and the adjustable rotating eccentric set up forces equal and opposite to the external forces exciting the hull vibration. The synchronizing device, which is applicable to multiple-screw ships, in effect reduces the external exciting force; the flexible material attenuates the exciting force, that is, reduces the force transmitted to the hull.

B. THE TUNED VIBRATION NEUTRALIZER

The principle of the tuned vibration neutralizer is well known and is discussed rather thoroughly in most textbooks dealing with the fundamentals of mechanical vibration; see

Reference 9-1. The device is often referred to in the literature as the "dynamic vibration absorber."

The effect of a "sprung mass" on the vibratory characteristics of ship hulls is discussed in Chapter 6. The vibration neutralizer is in essence nothing but a sprung mass with provision for tuning its natural frequency and possibly having adjustable damping. It might seem that, if the designer is concerned about the prevention of a buildup of hull vibration in a particular normal mode, he has only to install in the ship a sprung mass tuned to the frequency of this mode. It follows from the discussion in Chapter 6, however, that the desired objective may not be attained that easily. If the sprung mass is large enough to counteract the external driving force, it will probably be large enough to substitute a pair of hull modes for the single mode otherwise existing. Hence, for practical application, it is incumbent on the designer to provide the vibration neutralizer with variable tuning so that its natural frequency can be adjusted to the frequency of the driving force over a considerable range of ship speeds.

Figure 9-1 shows a large vibration neutralizer that was actually installed on an Italian motorship and reported to have eliminated 94 percent of the previously existing vibration; see Reference 9-2. In this case the vibration was caused by machinery and not by propeller action. Of particular interest is the fact that the apparatus had a gross weight of 12 tons which was about 0.1 percent of the displacement of the ship. An interesting feature also was that the inertia element was a tank divided into many cells that could be flooded with sea water. Thus the tuning was adjusted by varying the mass, and, in service, an operator was required to manipulate the valves which flooded or emptied various cells.

The U.S. Experimental Model Basin conducted laboratory experiments with the vibration neutralizer around 1938 with a view toward exploring its potentialities for use on naval vessels; see Reference 9-3. While these experiments indicated that electronically controlled energizing devices could improve the performance of the vibration neutralizer under service conditions at varying operating speeds, they did not appear promising enough to warrant the development of a full-scale ship neutralizer at the time.

With regard to the feasibility of installing a full-scale shipboard vibration neutralizer, one point brought out in References 9-2 and 9-3 deserves emphasis here. It may not be necessary to install the apparatus in the immediate vicinity of the exciting source to obtain the desired neutralizing action. It is true that setting up an equal and opposite force at the driving point is the most direct method of neutralizing the driving force. However, it follows from the beam theory of hull vibration that a mass-spring combination will produce an antiresonance of the system at the frequency to which it is tuned when installed at either end. In the absence of damping it will maintain whatever amplitude is necessary to hold motionless that point of the system to which it is attached.

It will be of interest to the reader that, at a much later date than that of the experiments discussed in Reference 9-3, it was discovered, on the trials of a naval destroyer with

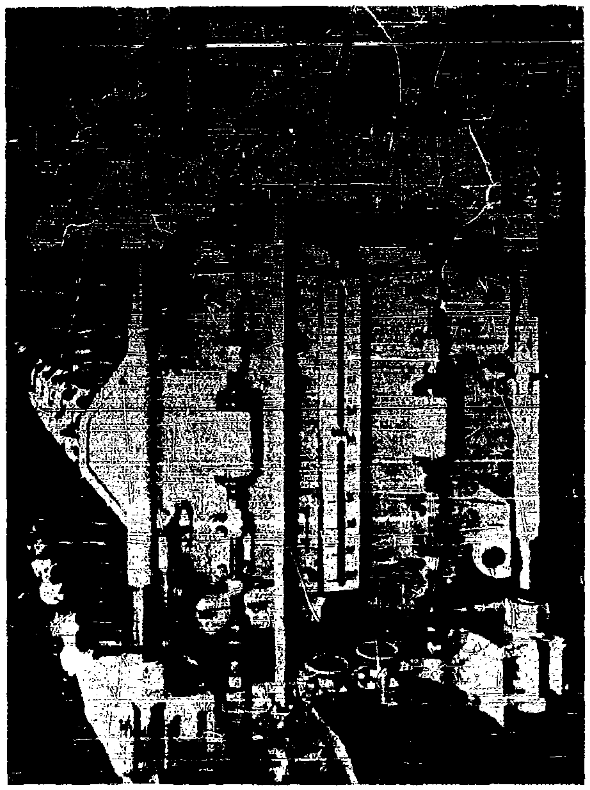


Figure 9-1 – Vibration Neutralizer Installed on Italian Motorship MARIA

twin rudders, that the natural frequency in torsional oscillation about the rudder-stock axes fell close to the frequency of the 3-node horizontal flexural mode of the hull; see Reference 9-4. The result was that the rudders acted as vibration neutralizers with respect to this mode of the hull and caused a peculiar forced response pattern when the hull was tested with a large mechanical vibration generator. This was only one phase of the unusual vibratory response characteristics observed on this particular class of ships. Further details are given in Chapter 14.

C. ADJUSTABLE ROTATING ECCENTRICS

The fact that adjustable rotating eccentrics have been used in the experimental determination of propeller-exciting forces on model scale (see Chapter 7) indicates the possibility of using such elements for the elimination of propeller-excited vibration on full-scale ships.

When an eccentric of mass m and eccentricity e rotates with a shaft, the reaction transmitted to the bearings in any one direction is

$$P = me\omega^2 \sin \omega t \qquad [9\text{-}1]$$

if the angle ωt is suitably specified. Thus, in Figure 9-2 the force in the vertical direction (positive upward) is given by Equation [9-1], and the force in the horizontal direction (positive to the right) is

$$P_h = me\omega^2 \cos \omega t \qquad [9\text{-}2]$$

In Figure 9-2 the bearing is fixed to its supporting structure or base. It might seem that if this base were part of a vibrating hull the expression for the bearing force would be much more complicated. It is readily shown, however, that, if the mass of the eccentric is

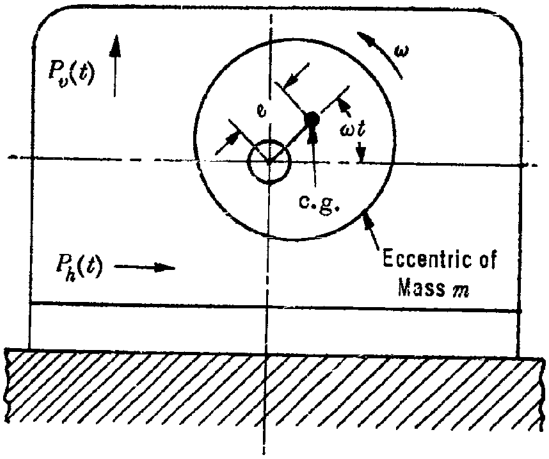


Figure 9-2 – Rotating Eccentric (Schematic) Exerting Sinusoidal Vertical and Horizontal Forces on Its Supporting Structure

added to the mass of the base or to the effective mass of the entire system on which the bearing force acts, then the effective forces acting on the system are still given by Equations [9-1] and [9-2], respectively.

If the rotating eccentric is to be used to cancel a force due to mass unbalance, it will perform the function of an ordinary balancing head and may in fact be attached to the unbalanced shaft itself. The important case for consideration here is that in which the rotating eccentric is to be used to cancel the blade-frequency exciting forces arising from propeller action. In this case, if the propeller shaft has an angular velocity ω, the propeller-exciting forces have a circular frequency $s\omega$ where s is the number of blades of the propeller. To cancel these forces, any rotating eccentric device installed in the hull must therefore be driven at an angular velocity s times that of the propeller shaft.

It is clear that, as contrasted with the vibration neutralizer which introduces a tuning problem, the adjustable rotating eccentric, if driven through a suitable gear train by the propeller shaft itself, will always synchronize in frequency with the blade-frequency exciting forces.

In spite of this advantage over the tuned vibration neutralizer, a number of problems need to be solved in designing a rotating eccentric device to cancel the blade-frequency propeller forces. A single eccentric such as shown in Figure 9-2 yields a rotating force with sinusoidal vertical and horizontal components. To obtain a pure sinusoidal force in one direction, a pair of eccentrics rotating in opposite directions must be used. Phase control is also necessary. Thus the device becomes essentially a vibration generator of the type used to vibrate hulls in ship vibration research, as discussed in Chapter 15.

A complete rotating eccentric device should be designed for both components of the blade-frequency force and for the couple with respect to the longitudinal axis of the ship as well. When it is considered how limited the space inside the hull may be in the vicinity of the propellers, it is apparent that a difficult design problem is involved with such a device. The problem is discussed further in Reference 9-5.

D. SYNCHRONIZING DEVICES

Synchronizing devices used as antivibration devices are applicable only to multiple-screw ships. Elementary considerations show that the scheme should be very effective in certain cases.

If a twin-screw design is considered for illustration, and, if there exists perfect symmetry of the geometry of the two propellers and of the flow with respect to the vertical plane through the longitudinal axis of the ship, then the vertical components of blade-frequency, propeller-exciting force will be equal for the two propellers. These forces will then be reinforcing when the two propellers, rotating at the same angular velocity and in opposite directions, are so phased that a blade of each propeller reaches the 12 o'clock position at the same instant. There is no implication here that the vertical force reaches its maximum value when a propeller blade passes through the 12 o'clock position. This is merely a convenient reference for phase between the port and starboard force vectors shown in Figure 9-3.

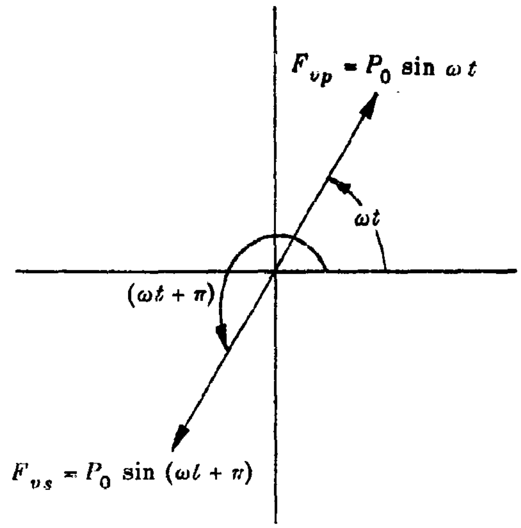


Figure 9-3 -- Time-Vector Diagram for Vertical Component of Blade-Frequency-Exciting Forces When Canceling Each Other

In the time-vector diagram (Figure 9-3), the blade position does not appear. This diagram merely indicates that time is taken as zero when the vertical component is zero for the port propeller. Whether a port propeller blade is in the 12 o'clock position or not when $\omega t = \pi/2$, if the assumed symmetry exists, both vectors in the diagram should coincide when port and starboard propeller blades pass through the 12 o'clock position simultaneously. If the vertical forces are canceling, so that the time vectors are as shown in Figure 9-3, then the reference blade of the starboard propeller should be advanced in the direction of its steady rotation by an angle of π/z radians from the 12 o'clock position when the reference blade of the port propeller is in its 12 o'clock position (where z is the number of blades per propeller).

In this simple illustration it follows that a synchronizing device that could maintain the propellers in this phase relation would ensure the cancellation of vertical blade-frequency exciting forces. The problem, however, is never this simple. There are horizontal force components to consider as well. Under the ideal condition assumed, since their vectors rotate at the same rate, but the horizontal force components are equal and opposite when both propellers have the reference vector in the 12 o'clock position, the phase condition for a cancellation of vertical blade frequency forces is the same as the condition for reinforcement of the horizontal components.

Thus, even in the ideal situation considered here, the synchronizing adjustment would have to be based on a compromise between canceling the vertical components and reinforcing the horizontal components. In practice, moreover, the conditions are much more complicated than this. Even if the synchronizing device can maintain a prescribed phase between propeller rotations, the flow conditions change with speed, heading, rolling, pitching, and trim of the ship. Nevertheless, if a synchronizing device can ameliorate one severe vibratory condition on a particular class of ship, it may well justify its installation.

Reference 9-6 discusses the synchronizing devices in detail and points out that the scheme is most feasible in connection with electric drives. Reference 9-7 describes a synchronizing gear applicable to diesel drives for which considerable success has been claimed. In this case it was found that the optimum synchronizing angle could best be obtained experimentally. It should also be noted that on multiple-screw ships it is invariably observed that the blade-frequency vibration is rarely steady but has a beating characteristic. When the ship is not pitching appreciably, this beating effect is due to the shifting phase accompanying the slight changes in speeds of the shafts. Thus the degree of benefit to be derived from a synchronizing device is indicated by vibration records that show this beating characteristic. The synchronizing device, if performing its function, holds the hull vibration level at the minimum value observed in the beating records.

E. FLEXIBLE MATERIALS IN THE VICINITY OF PROPELLERS

As pointed out in Chapter 7, a large fraction of the propeller-exciting forces acting on the hull is associated with the pressure field in the vicinity of the propellers. Just as the vibratory force due to an unbalanced piece of machinery may be attenuated by installing it on isolation mountings, so, theoretically at least, the effect of the blade-frequency pressure field at the stern of a ship can be attenuated by the use of flexible material at the stern.

Elementary considerations suggest that the benefit of such an expedient is highly frequency-dependent, and, by analogy with the simple problem of attenuating the force transmitted by a machine having a single degree of freedom, the natural frequency must be well below the frequency of the force that it is required to attenuate.

Blade frequencies are, in general, at the low-frequency extremity of the spectrum of mechanical vibration. To produce a stern structure with a frequency well below the lowest blade-frequency disturbance is a difficult design task. Nevertheless, according to Reference 9-8, this was accomplished on the survey ship NORD. In this case the pressure field directly over the propeller was attenuated sufficiently by the use of a rubber plate backed by an airtight box to attain a marked reduction in the level of hull vibration.

F. SUMMARY

This chapter mentions only a few of the most promising antivibration devices that have been suggested for ameliorating the effects of vibratory exciting forces acting on hulls. A friction damper such as the Lanchester damper discussed in Reference 9-1 could conceivably be developed for such a purpose and it has even been suggested that the propeller bearing forces be isolated from the hull by flexibly supported struts. No doubt many other schemes could be tried.

The question naturally arises as to whether or not antivibration devices should be considered in the early stages of the design of a ship. This is certainly a very important question, for, if the designer could be assured that vibration difficulties could be circumvented with such devices, he need not concern himself with the problems of estimating the magnitude of the propeller-exciting forces or of avoiding hull resonances. He could then concentrate on designing the afterbody and propellers strictly from the point of view of propulsive efficiency.

The available information on the results obtained so far with antivibration devices for ships and the difficulty of maintenance of such devices suggest that they should not be considered by the designer from the beginning. Rather, they should be regarded as a last resort to be used when a design study indicates that large vibratory exciting forces are unavoidable without alterations in the design which are otherwise inadmissible.

REFERENCES

9-1. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

9-2. Loser, U., "Vibration Neutralizer for Ships," Vol. 12, No. 1 (Jan-Feb 1934). EMB Translation 12 (Jan 1936).

9-3. McGoldrick, R.T., "Experiments with Vibration Neutralizers," EMB Report 449 (May 1938).

9-4. McGoldrick, R.T., "Rudder Excited Hull Vibration on USS FORREST SHERMAN (DD 931) – A Problem in Hydroelasticity," Trans. SNAME (1959). Also TMB Report 1431 (Jun 1960).

9-5. MacNaught, D.F., Discussion of paper "Hull Vibration Investigation on SS GOPHER MARINER," by R.T. McGoldrick and V.L. Russo, Trans. SNAME, Vol. 63 (1955).

9-6. Lewis, F.M., "Dynamic Effects," Chapter 2, Vol. 2 of Marine Engin, edited by H.L. Seward, published by SNAME (1944).

9-7. Description of Schelde Synchronizing Gear in Private Communication to SNAME (26 Aug 1957).

9-8. Waas, H. and Walter, H., "Technische Neuerungen auf dem Vermessungsschiff 'Nord' der Wasser-und Schiffahrtsverwaltung," Schiffstechnik, Vol. 3, No. 12/13 (Nov 1955).

CHAPTER 10

DESIGN CONSIDERATIONS RELATING TO STEADY-STATE HULL VIBRATION

A. INTRODUCTION

Up to the time of this writing, it cannot be said that the consideration of steady-state hull vibration has been a major item in what may be termed the paper stage of a ship design. Although, as shown by the existence of Reference 10-1, the subject could not escape the naval architect's attention, the simple fact is that so little concrete information on ship vibration has really been available to the naval architect that the question of hull vibration has been largely ignored. Only when serious vibration was encountered during the builder's trials of a new ship did its consideration assume importance. Then measures were taken to modify the hull or the propulsion machinery to reduce or eliminate the vibration.

While it is true that a large number of technical papers on the subject of ship vibration were available to the naval architect (as may be seen from the general bibliography included in this book), the practicing naval architect could not spare the time to digest the mass of scattered information contained in these papers. He had to rely on such summaries as given in Reference 10-1, and it is only because of the increasing tempo of research in this field since World War II that it is at all feasible to go beyond the limits of Reference 10-1 in this book.

There was, of course, for many years among naval architects a realization that unbalanced forces and moments set up by the propulsion machinery would cause hull vibration. In fact, studies leading to improved balancing of rotating and reciprocating machinery had been underway for over 50 years. It was indeed clearly recognized that, in avoiding serious first-order hull vibration (frequency same as the propeller shaft rpm), the remedy lay chiefly in reducing the forces rather than in trying to reduce the hull response to given forces.

Specifications for static and dynamic balancing of propellers and shafting as well as the specifications for finishing of propellers were gradually introduced over the years as the improvement in balancing and manufacturing techniques permitted.

The lack of information was most acute in the area of propeller hydrodynamic exciting forces, and, fortunately, some progress has been made in that area in recent years. It is obvious that, if the exciting forces can be reduced to negligible magnitude, steady-state vibration will not be a problem regardless of the natural frequencies of the hull. A possible exception to this is the production of "flow-excited" vibration which, under special circumstances, may produce a fairly steady vibration above a certain critical speed. This point is discussed further in Chapter 14. There it is also pointed out that control surfaces may induce flutter phenomena. The latter are not considered flow-excited vibrations in the ordinary sense. In general, the vibration due to sea action is of a transient nature and depends not only on the dynamical characteristics of the hull but also on the sea state and the seakeeping characteristics of the hull.

In the design of ships to be driven by conventional screw-propellers, at least one of the naval architect's chief concerns is the avoidance of serious vibration due to propeller-exciting forces of blade frequency or integer multiples of the blade frequency.

As will be shown in the next chapter, a serious vibration problem may develop when transient vibrations of the entire hull (resulting from rough seas) are magnified in local hull structures. This situation, however, is not chargeable to lack of attention to the items considered in the present chapter. Furthermore, the fact that transient vibrations may involve hull amplitudes anywhere from ten to a thousand times those considered under steady-state vibration does not give the steady-state hull-vibration problem a status of minor importance. A ship may be slowed down temporarily in rough seas, and its heading may be changed to minimize the effects of transient vibrations, but it must operate over long periods of time and at full speed under normal sea conditions in the presence of steady-state vibration.

It is not attempted in this chapter to lay out a rigid design schedule for avoiding steady-state hull vibration with conventional screw-propeller-driven ships. The subject of hull vibration is still too obscure to permit this. In fact, too many of the ideas introduced here are a matter of opinion to justify a manual-type presentation. It is attempted in this chapter, however, to furnish the naval architect with specific recommendations based on both experimental and analytical studies which, if followed, will reduce the "calculated risk" of hull vibration that must still be taken by the ship designer.

Although this chapter deals only with considerations relating to hull vibration, it must be pointed out that the selection of a propeller which is to be satisfactory from the hull-vibration standpoint must be based also on considerations relating to vibration in the propulsion system, as discussed in Chapter 12.

B. GENERAL PROCEDURE

Since the level of steady-state hull vibration that will be encountered in service depends on both the magnitude of the exciting forces and the response characteristics of the hull, the designer naturally first must consider whether there are any unusual specifications for the ship in question which would cause either abnormally large exciting forces or an unusual sensitivity of the hull to vibratory forces. It has been attempted to make it clear in previous chapters that in neither category can accuracy of calculation be expected at present. The only exception is the internal excitation due to mass unbalance. For this the designer can fall back on the specifications for balancing of machinery, shafting, and propellers (see Section F) to get an estimate of the first-order exciting forces and moments.

If the ship is to house some novel piece of machinery which develops large unbalanced forces, then vibration trouble is to be anticipated unless hull natural frequencies are kept well clear of the operating frequency of this machine.

It is also obvious that, if hull natural frequencies can be kept clear of the range of operating shaft rpm's, close attention to mass unbalance of propellers and shafting is less urgent than when they fall in this range.

If the propulsion system is not to include the conventional screw-propeller, a vibration problem for which little guidance is now available may be imminent. This applies to paddle wheels of either the side or stern type. The exception is the shrouded propeller, for the shrouding is intended to smooth out the flow in the propeller race.

If both the hull design and the design of the propulsion system are to be "conventional," the next consideration for the designer is the location of the range of operating blade frequencies with respect to the range of significant hull critical frequencies. When these two ranges do not overlap, the likelihood of propeller-excited hull vibration difficulties is greatly diminished. When they coincide, the reverse is true.

Presumably the designer will have on hand at this stage a design specification for a propulsion system with a particular range of operating shaft speeds and a particular number of blades per propeller. He should also have a general idea of the hull scantlings and an approximate weight distribution plot.

The operating range of blade frequencies is equal to the operating range of shaft rpm's multiplied by the number of blades per propeller s. The range of significant hull frequencies can only be estimated on the basis of the hull scantlings and weight distribution data available at this stage. A simple estimate of the frequencies can be made in accordance with the rational beam theory if: (1) all decks and expanses of shell plating are provided with longitudinal and transverse stiffeners, so spaced that local natural frequencies are not abnormally low; (2) no heavy pieces of equipment (heavier than ½ percent of the displacement of the ship) are installed on resilient mountings; (3) the cargo will not have unusual flexibility such as possessed by automobiles on inflated tires, large quantities of rubber or plastic material, or large quantities of springs; (4) there are no abnormally large expanses of deck unsupported by bulkheads or stanchions; and (5) the ship does not have abnormally large hatch openings or unusual structural discontinuities.

If the designer knows that the foregoing conditions can be met, he can then make a very simple estimate of the range of hull critical frequencies. First, he can estimate the fundamental or 2-node vertical frequency by one of the empirical formulas given in Appendix C. Better than this, he can use his own value of a constant to be substituted in the Schlick formula. This formula is

$$N = C \sqrt{\frac{I}{\Delta L^3}} \qquad [10\text{-}1]$$

where N is the frequency in cpm,

C is an empirical constant,

I is the area moment of inertia of the midship section in ft^2-in.^{2} units,

Δ is the displacement of the ship in long tons, and

L is the length in ft.

In lieu of assuming some value of C within the range of values given in Appendix C, the designer can devise his own value of C if he has available data on a previous ship of the same general type. Thus, if he knows N, I, D, and L for the other ship, he can solve Equation [10-1] for C. Then from the values of I, D, and L for the proposed design, he can use the same equation to find N for the new ship.

Unless the L/D exceeds 18, the designer can then assume that there will be not more than six vertical modes, four horizontal modes, or three torsional modes of significance. If the L/D exceeds 18, the formula suggested by Baier and Ormondroyd[10-3] can be used; namely

$$N' \approx \frac{5}{9}\frac{L}{D} \qquad [10\text{-}2]$$

where N' is the number of significant vertical modes,

L is the hull length, and

D is the hull depth.

The number of horizontal and torsional modes considered significant would then have to be increased proportionately.

At this stage the designer can make the assumption that the ratios of the frequencies of the vertical modes fall in the series 1, 2, 3, etc., so that he has only to double the 2-node frequency to estimate the 3-node frequency, and so on.

Next, the rough rule can be used that the 2-node horizontal frequency will be 1.5 times the 2-node vertical frequency and that the horizontal frequencies also follow the 1, 2, 3 rule. It is not intended to furnish here any empirical rules for the frequencies of submarines but such rules can be developed as information on such ships is accumulated.

Finally, an estimate can be made of the fundamental torsional frequency by Horn's formula given in Appendix C. While there is little information on which to base the ratios of the frequencies of the higher torsional modes to that of the fundamental, on the basis of the information obtained on GOPHER MARINER,[10-4] it is suggested here that the designer assume that the frequency of the third torsional mode will not be over 2.5 times the fundamental torsional frequency (for GOPHER MARINER the ratios were 1 : 1.6 : 2.2).

The next step depends on whether or not the range of hull critical frequencies is clear of the range of operating blade frequencies. For large ships it is a common occurrence that the highest significant hull critical frequency is lower than the lowest operating blade frequency. In this case resonance with the blade-frequency forces is not to be anticipated, and it is to be expected that the blade-frequency vibration will be concentrated in the stern. The vibration level, of course, will depend on both the driving force and the driving moment. The force and moment estimating is discussed in Chapter 7. The formulas for estimating the stern amplitude for known forces and moments are given in Appendix D.

When it is indicated that the frequency of any of the significant hull modes will fall in the operating range of blade frequencies, the designer should attempt to predict the level of resonant vibration that would exist if the two frequencies should coincide. Again the level will depend on the magnitude of the exciting force, but the response will depend also on the damping. The force can be estimated in accordance with the information given in Chapter 7, and the forced vibration calculation can be made by the methods given in Chapter 4.

If the design is such that the conditions specified on page 10-3 cannot be met, then no vibration predictions are feasible in the early design stage and the designer must wait until the design has advanced to the stage at which the parameters required for a more detailed vibration analysis are available. A sample calculation of a vertical hull mode by the digital process is given in Appendix A. For details of the calculation of hull modes when heavy units are to be installed on resilient mountings, see Reference 10-5.

In lieu of attempting to estimate the propeller-exciting forces, the designer should consider the possibility of negotiating for a model determination of these forces. The state of the art of doing this at the present time is indicated in Chapter 7 as well as in Reference 10-6.

C. REDUCING PROPELLER FORCES

The first-order propeller forces (those due to mass and pitch unbalance of the propeller) are considered in Section F of this chapter. The present discussion refers to the blade-frequency forces and to forces whose frequencies are multiples of the blade frequency.

As pointed out in Chapter 7, these forces depend on both the pressure fluctuations at the hull surfaces due to the individual propeller blades and the bearing forces. The latter forces in turn depend on the uniformity of the flow into the propeller races. Obviously, moving the propeller astern will usually reduce both components of the forces since the pressure field at the hull will be weakened and the wake variation (which is aggravated by the boundary layer when the propeller is close to the hull) will be reduced.

It is equally clear, however, that there are severe limitations to the process of moving the propellers astern. Some limitations are the danger of fouling the propeller in docking, the weakening of the support of the aftermost propeller shaft bearing, and the loss of thrust due to the high wake near the hull. It is possible, however, to obtain increased propeller tip clearance without moving the propeller by giving the propeller blades a rake in the aft direction.

The use of a propeller tunnel or shrouding is an obvious means of reducing propeller-exciting forces. This is discussed in some detail in Reference 10-7. If absence of hull vibration is an especially important requirement for the proposed ship, it may well warrant installation of a shrouded propeller for this reason alone.

In the past the most common expedient to ameliorate the effects of blade-frequency hull vibration (when encountered on the initial trials of a ship) was to substitute for the original propeller another of similar thrust and torque characteristics but with a different number of blades. Obviously, this expedient may also be introduced in the design stage. The trend has been from three to four-, five-, and six-bladed propellers.

Increasing the number of propeller blades will, in general, decrease both the forces due to the pressure field acting on the stern and the bearing forces. The effect of increasing the number of blades may be compared in a qualitative way to increasing the number of cylinders in an internal combustion engine. The thrust per blade and the contribution to the lateral force per blade are both reduced. Any given variation in wake will obviously cause less variation in the lift and drag on a single blade when the blade area or angle of attack is reduced. Thus, in general, the net effect on the resultant force due to all blades will be reduced. The exception occurs when the wake pattern is such that there is greater reinforcement of lateral force components with an increased number of blades. Hence, an analysis such as discussed in Chapter 7 is really necessary before it is assured that increasing the number of blades will reduce the exciting forces in the specific case in question. Thus, when the supporting arrangement is such that two or more blades pass through the wakes of obstructions simultaneously, large blade-frequency thrust variations may be expected. Of course, the reinforcement of the thrust variations does not necessarily mean that the lateral force variations will reinforce. The thrusts are unidirectional, whereas the lateral forces have different directions for different blades. Details of the calculation of bearing forces from the wake survey are given in Reference 10-8.

Hence, in seeking to reduce propeller-exciting forces without introducing unconventional changes, the designer must, in general, compromise between the expedients of increasing the propeller tip clearances and increasing the number of propeller blades.

D. AVOIDING HULL RESONANCE

The urgency of avoiding hull resonance depends, of course, on the magnitudes of the exciting forces and the damping. When either the forces are small or the damping is large, hull resonance is not intolerable and many a ship must operate under such a condition at particular speeds. While it may be noted here that first-order disturbing forces will steadily diminish as improved methods of balancing and machining are developed, the question at issue here is: How can resonance be avoided?

It is, in general, feasible for the designer to avoid first-order resonance if the ship is to operate at a fixed rpm. The reason is that, although propeller shaft speeds are relatively low, there is a considerable spread in the frequencies of the lower hull modes. The advantage of avoiding such resonance is obvious since, under such circumstances, a slight unbalance in propellers, shafting, or machinery will not cause excessive hull vibration.

Keeping the blade frequencies clear of the range of significant hull critical frequencies is usually relatively easy on large ships and relatively difficult on small ships. This is because on large ships the operating blade frequencies are usually well above the range of hull critical frequencies, whereas on small ships they are usually right in this range.

When the blade frequencies fall in the range of significant hull critical frequencies, the prospects of avoiding a hull resonance are not good. This is because first, as indicated in the previous section, there are normally about a dozen hull modes that may be significantly excited; and, second, the frequency of each mode will vary with hull displacement over a band which depends on the variations in loading encountered in the operation of the ship in question.

Figure 10-1, taken from Reference 10-4, illustrates the difficulty the designer may face in seeking to avoid hull resonance in the blade-frequency range. The ship in question (the

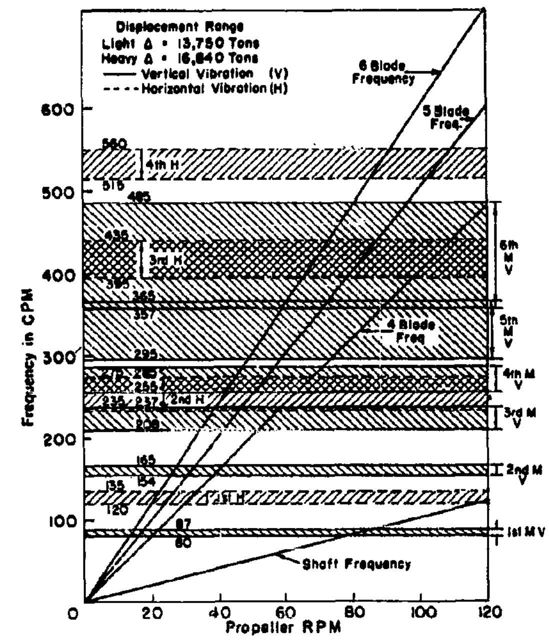


Figure 10-1 – Frequency of Hull Vibrations versus Propeller RPM Based on Experimental Data, GOPHER MARINER, with Torsional Effects Neglected

(From Figure 1 of Reference 10-4.)

MARINER class) was a cargo ship designed by the U.S. Maritime Administration, with the following principal dimensions:

L	525 ft
B	76 ft
D	44 ft 6 in.
Displacement	17,000 tons
Mean draft	24 ft
Maximum shaft rpm	105
Propeller blades	4
Single screw	

In this case, as in many other cases, the number of significant modes and the variation of natural frequencies with loading was such that resonance-free areas with propellers of the usual number of blades were almost nonexistent. The designer might, however, by resorting to a six-bladed propeller, have been able to boost the operating blade frequency above the range of significant hull mode frequencies. This was not done in the case of the MARINER Class, which operated satisfactorily with a four-bladed propeller, since the exciting forces were not excessive for that ship.

When any prospect of keeping the operating blade frequencies above the range of hull mode frequencies is indicated, the designer should attempt more refined calculations than are given by the preliminary estimates to more definitely pinpoint the critical speeds. If, after this, it is still indicated that the blade frequency will inevitably fall in the range of hull criticals, then every effort must be made to reduce the forces.

Of course, blade frequencies can also be increased by using higher shaft speeds. If the machinery design is not already frozen, this possibility should be considered. A faster turning propeller can develop the same thrust with a smaller diameter; this also gives increased tip clearance. A high-speed supercavitating propeller can bring the blade frequencies well above the range of significant hull critical frequencies, and this might be tried if not ruled out by other considerations.

E. AVOIDING LOCAL RESONANCE

As in the case of vibration of the entire hull, the amplitude of steady-state local vibration will also depend on both the magnitude of the exciting force and the damping. Although the classification "local structure" is somewhat arbitrary, here it means any structure that can be excited by a local disturbance so as to vibrate without appreciable vibration of the hull girder. Those are the structures considered in Chapter 6 and include masts, deck houses of short longitudinal extent, panels of deck plating, bulkheads, heavy items of machinery on nonrigid foundations, as well as items of equipment installed on resilient mountings.

Obviously, changing the natural frequency of a local structure to avoid a condition of resonance is a possibility, whereas changing a natural frequency of the hull itself after it is fabricated is extremely difficult.

One unfortunate aspect of the problem of local resonance is that the use of large numbers of propeller blades to reduce the propeller-exciting forces often brings the blade-frequency into the range of natural frequencies of many local structures whose frequencies would otherwise be well out of range.

There are, of course, some relatively large local structures for which major alterations would be required if it were considered necessary to raise their natural frequencies. In such cases, however, it will usually be found that the local natural frequencies fall in the range of significant hull mode frequencies, and, in fact, that these structures may even modify the vibratory response characteristics of the entire hull. Under such circumstances calculations should be made of the natural frequencies and normal modes of the hull with these local structures treated as equivalent sprung masses; see Chapter 6. The modes found in such calculations will indicate to what extent the amplitude of the local structure will exceed the hull amplitude in the modes in which it plays a significant role. These modes must now be considered modes of the combined system and the conditions which ensure that they will not be dangerously excited are the same considerations that apply to resonant vibration of the hull.

The fortunate aspect of the local resonance problem is that trouble due to this can be anticipated before the ship is ready for the builder's trials. The naval architect can use the relatively small and portable vibration generators such as described in Reference 10-9 to determine local natural frequencies. Even without these, such frequencies can be found by the "bop" or impact test in which a heavy timber is used to excite the structure and the transient vibrations following the impact are recorded with sensitive instruments. Some tests of this type can be made even before launching, whereas those in which the bottom plating may have an effect must be delayed until after launching.

F. BALANCING

The subject of balancing, in general, is a very important one for the naval architect concerned with avoiding ship vibration. It is treated in considerable detail in Reference 10-1 and the basic principles are discussed in standard textbooks on mechanical vibration; e.g., References 10-10 and 10-11. The naval architect (as contrasted with the marine engineer) is not so much concerned with the techniques of balancing propulsion machinery as with the specifications that should be set for balance. He must have assurance that, if he establishes a specification for maximum permissible unbalanced forces or moments on the basis of his estimate of the hull response characteristics, these specifications can be met by the manufacturer.

Since the advent of steam turbine power plants the chief concern of the naval architect with regard to balancing, at least for large ships, has been the specifications for balance of propellers and shafting.

It has been pointed out that a screw propeller is subject to both mass unbalance and pitch unbalance. The latter requires further consideration here. Pitch unbalance is caused by lack of uniformity or symmetry in the geometry of the propeller, and may exist even when mass unbalance is negligible.

If the pitch of one propeller blade is greater than that of the remaining blades, then, under uniform flow conditions in the propellor race, the lift, drag, and moments on small elements or strips of this blade are not the same as for the corresponding elements of any of the other blades. To exaggerate the pitch unbalance effect, one can consider the situation of a propeller having only one blade. The hydrodynamic force-system acting on the propellor could then be reduced to a constant axial force, a force lying in a plane normal to the shaft axis, a constant torque about the shaft axis, and a moment in a plane passing through the shaft axis. The vectors representing the lateral force and the moment would be constant in magnitude but would rotate with the propeller. It is seen that where the flow is uniform there is no thrust variation and no torque variation but that the propeller bearing will be subject to simple harmonic forces in both the horizontal and vertical directions as well as to simple harmonic moments in both the vertical and horizontal planes. The frequencies are first order; that is, the same as the rpm. When the velocity field is nonuniform, harmonics (integer multiples of first order) will also, in general, be present.

When the flow in the propeller race is nonuniform, all forces and moments previously considered for the one-bladed propeller will now be modulated by a "signal" due to the wake pattern whose fundamental period will be the time taken for the propeller to make one revolution. In this case there will be thrust and torque variations at the signal frequency, bearing forces in the vertical and horizontal directions, and moments in both planes of the first order. All harmonics of the wake pattern signal will also contribute so that the spectrum of bearing forces may have numerous frequency components. Thus, while pitch unbalance may be expected to produce first-order vibration, second and higher orders will also be present when the inflow to the propeller is nonuniform. For further discussion of excitation due to pitch unbalance, see Reference 10-8.

Military standards for mass balance of rotating members are given in Reference 10-2, Specification 3.2.3.2. The following is quoted from page 6 of this reference:

> "*Balance limits* – When balanced in accordance with 3.2.3.1 the residual unbalance, in *each* plane of correction, of any rotating part shall not exceed the value determined by:
>
> $$U = \frac{4W}{N} \text{ for speeds in excess of 1,000 rpm} \qquad [10\text{-}3]$$
>
> or
>
> $$U = \frac{5630}{N^2} \text{ for speeds between 150 rpm and 1,000 rpm} \qquad [10\text{-}4]$$
>
> or
>
> $$U = 0.25\,W \text{ for speeds below 150 rpm} \qquad [10\text{-}5]$$
>
> where U = maximum allowable residual unbalance in oz.-in.
> W = weight of rotating part in lb
> N = maximum operating rpm of unit."

The Westinghouse standard for naval equipment is expressed by the formula

$$WR < \frac{4W}{n} \qquad [10\text{-}6]$$

where n is the rpm and WR is the weight unbalance.

An intensive study of tolerance for shafting unbalance was made as a result of first-order hull vibration on a recent class of naval destroyers. The results of this study are discussed in Reference 10-12.

Reference 10-13, which deals with relatively small units rotating at high speed, also discusses the principles of dynamic balancing.

A frequent rule of thumb is that for the maximum running speed of the rotor

$$\frac{WR\omega^2}{g} < \frac{W}{100} \qquad [10\text{-}7]$$

or that, hence

$$\frac{R\omega^2}{g} < 0.01 \qquad [10\text{-}8]$$

In this formula a consistent set of units must be used. Thus, if R is in in. g must be in./sec^2, and ω must be the angular velocity in radi/sec.

In the case of reciprocating machinery, if cancellation cannot be obtained by arranging the crank angles suitably, it may be necessary to install auxiliary reciprocating masses driven by the machine to be balanced. This is discussed in Reference 10-1.

REFERENCES

10-1. Lewis, F.M., "Dynamic Effects," Chapter 2, Vol. 2 of Marine Engin, edited by H.L. Seward, published by SNAME (1944).

10-2. "Military Standard -- Mechanical Vibrations of Shipboard Equipment," MIL-STD-167 (Ships) (20 Dec 1954).

10-3. Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single-Screw Vessels," Trans. SNAME, Vol. 60 (1952).

10-4. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

10-5. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

10-6. Stuntz, G.R., et. al., "Series 60 – The Effect of Variations in Afterbody Shape upon Resistance, Power, Wake Distribution, and Propeller-Excited Vibratory Forces," Paper prepared for Annual Meeting of SNAME (1960).

10-7. Saunders, H.E., "Hydrodynamics in Ship Design," published by SNAME (1957).

10-8. Reed, F.E. and Bradshaw, R.T., "The Distribution of Exciting Forces Generated by Propellers," CONESCO Report F-101-2, Contract Nobs 77150 (Jun 1960).

10- 9. Robinson, Q.R., "Vibration Machines at the David W. Taylor Model Basin," TMB Report 821 (Jul 1952).

10-10. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York (1956).

10-11. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

10-12. Jasper, N.H., "Structural Vibration Problems of Ships – A Study of the DD 692 Class of Destroyers," TMB Report C-86 (Feb 1950) Unclassified.

10-13. Feldman, S., "Dynamic Balancing for Noise Reduction," Dept of the Navy, Bureau of Ships (Code 371), R & D Report 371-V-24 (Apr 1955).

CHAPTER 11

DESIGN CONSIDERATIONS RELATING TO TRANSIENT VIBRATIONS

A. INTRODUCTION

The basic beam theory of ship vibration discussed in Chapter 3 is a general dynamical theory and is applicable to the transient response characteristics of ships, as shown in Chapter 5. However, this theory cannot be applied indiscriminately to problems such as the slamming of ships in a seaway which may involve deflections so large that buckling and plastic strain of structural members occur.

No sharp line of demarcation can be drawn between the field of structural seaworthiness and the field of ship vibration. Certainly, in the structural strength of ships, the transient vibration problem is of far more importance than the steady-state vibration problem. In fact, the hull girder stresses due to steady-state vibration are almost negligible. Tolerances for steady-state vibration must be based, not on hull girder stress levels, but on permissible amplitudes determined from physiological factors, satisfactory operation of equipment installed in the ships, or by local stress and fatigue considerations.

All that is attempted in this chapter is to show that the rational beam theory of ship vibration advocated in this book can be used as a guide in considering, at least qualitatively, the transient response characteristics of a hull. Thus, for instance, after a severe slam in a seaway, a ship will execute a complex pattern of decaying vibrations which eventually settle down into vibrations in the normal modes considered in Chapter 3. Since these vibrations often persist for a large fraction of a minute, and are much larger in amplitude than those usually considered under steady-state vibration, they may cause intolerable vibrations of local structures whose natural frequencies happen to coincide with their frequencies.

It is also appropriate to point out in this chapter that, although the bending moments to which a hull is subjected in a seaway are actually of a transient nature, the components having the frequency of wave encounter vary at such a slow rate that they are treated as static in the standard strength calculation; see Reference 11-1. The static beam theory to be used in this case, however, should logically coincide with the rational beam theory considered to be valid in the low frequency range of hull vibration. Hence the designer should be able to derive certain conclusions as to the hull strength in a seaway from its vibratory response characteristics. Therefore, it is suggested here that the hull girder characteristics as determined from the vibration analysis are applicable regardless of whether the naval architect continues to rely on the time-honored standard strength calculation which uses a static load determined from an assumed wave profile, or undertakes to forecast the extreme loading conditions in a seaway.

The dynamical system treated in this book is actually an ideal system considered free in space with mass added to account for the inertia effect of the water and viscous dampers inserted to allow for the energy dissipation effects.

The transient loading in a seaway which imposes peak stresses in the hull involves rigid body motions that cause both large variations in the buoyant forces (which are entirely neglected in the vibration analysis) and large variations in the added mass effect of the surrounding water (which for convenience has so far been treated as constant in time in the vibration analysis).

Although this treatment of the hull as a system free in space for the purpose of the vibration analysis may seem highly artificial, the designer should realize that the hull by itself is actually a system free in space subject to a complex system of forces imposed by the surrounding water. If all these forces could be predicted in advance as functions of time, then the transient problem discussed in Chapter 5 would be valid, at least up to loadings that did not produce strains exceeding the elastic limit.

It is especially interesting to recall that the hull proper is never permitted to execute free vibrations, for, no matter how calm the sea may be, the pressures associated with the inertia of the surrounding water will always force it to vibrate at frequencies other than its natural frequencies in free space. As pointed out in Reference 11-2, it was a common practice around 1932 to compute the 2-node vertical frequency which a ship would have in air (free space) and then to apply to this frequency (called the "theoretical frequency") a number of correction factors. The largest of these was, of course, the factor for the inertia effect of the surrounding water. Roughly, this effect lowers the free space fundamental vertical frequency by 25 percent.

Although the free space natural frequencies of the hull are considerably higher than the natural frequencies in water, the corresponding mode shapes are not very different because the inertia effect of the water conforms roughly with the weight and rigidity distributions. Since the corresponding components of water pressure drive the hull at a frequency below its natural frequency, they are in phase with the vibratory displacement, whereas the vibratory buoyancy forces are 180 deg out of phase with the vibratory displacement.

B. THE HULL GIRDER

Although the designer has not been accustomed to think of the structural design of the hull girder as a vibration problem, he has been accustomed to regard the hull as a beam. If the hull is designed structurally as a beam, then it may be expected to exhibit the vibratory characteristics of a beam. In fact it is advocated here that the extent to which its vibratory response characteristics are beamlike may be used as a design criterion. This statement, of course, requires some clarification because it has been pointed out that if large items of equipment are deliberately installed on flexible mountings, they can distort the normal mode patterns considerably from beamlike form.

Clearly, the fundamental vertical frequency of a hull increases with its bending rigidity or EI. Although a thin-walled box girder will not be able to develop its nominal EI, an adequately stiffened thin-walled box girder can be made to approach this as a limit. As stiffeners are added to increase the bending rigidity, the fundamental flexural frequency of the free-free girder increases and thus the frequency is an indication of the flexural rigidity in this simple case.

The theory also shows, however, that the frequency depends on the mass distribution and hence it cannot be the sole criterion of bending rigidity. Furthermore, as suggested in the case of GOPHER MARINER,[11-3] nonbeamlike vibratory characteristics may be caused by the cargo which a ship is carrying. The exception in this case proves the rule; it does not invalidate it.

If the hull is designed so that its vibratory characteristics are beamlike, in the absence of heavy items of equipment resiliently mounted, they will then be predictable to a reasonable degree. If they are predictable, then the propulsion system can be designed to avoid hull resonance. Even when it is known that a ship must carry cargo that will make the prediction of its vibratory response characteristics difficult, there is still an advantage in producing a hull that will have predictable vibration characteristics with a "normal" cargo. It may be possible to operate at a slower speed whenever such abnormal cargo must be carried.

The transient loading of hulls in heavy seas can be treated only on a statistical basis as pointed out in Reference 11-4. It is not intended to imply here that designing to produce beamlike vibratory characteristics can reduce the slamming loads in a seaway. These depend on seakeeping characteristics which involve, among other factors, the shape of the hull and its speed. As a matter of fact, GOPHER MARINER experiments indicate that a cargo that introduces nonbeamlike vibratory characteristics may introduce a desirable damping action. The main contention here is that beamlike vibration characteristics for the hull proper are desirable.

Another factor pertinent to the present discussion is the presence of structural discontinuities. These are usually associated with local stress concentrations and are considered undesirable in design. In a beam simply supported at its ends and uniformly loaded statically, there would be an abrupt change of curvature in the deflection curve at the point of discontinuity. Similarly, if such a beam were set vibrating in its fundamental normal mode the pattern would also show a sharp change of curvature at this point.

Thus the rule (but a rule with exceptions) advocated here is that the vibratory response characteristics should be beamlike. The exceptions occur when large units are to be installed in the hull on resilient mountings or when unusual cargo having "springy" material is to be carried. In such cases the hull, even if designed for beamlike characteristics without such conditions, will not exhibit them in practice.

While it is not advocated here that the digital method of transient response calculation given in Chapter 5 is applicable to severe slamming load conditions, it might be noted that calculations made by the method discussed in Reference 11-5 predict that the vibratory response to an impulsive load at the bow will be predominantly in the 2-node flexural mode. This is generally confirmed by vibration measurements on ships in a seaway; see, for example, Reference 11-6.

C. LOCAL STRUCTURES

The transient motions of local structures when the ship is operating in heavy seas are of serious concern to the designer. Obviously, the vibratory motions of local structures which do not carry an internal source of excitation (such as an unbalanced piece of machinery) will depend on the motions of the hull girder in their vicinity. Even without considering the statistical problem of predicting the peak values of the transient motions which the hull will execute in a seaway, there can still be pointed out here the desirability of keeping the natural frequencies of local structures clear of the frequencies of the significant hull modes.

If it were a simple matter to raise the natural frequencies of local structures, the answer to this problem would be to reinforce them sufficiently so that all local natural frequencies were above the range of hull mode frequencies. Since this not feasible with the larger structures (such as masts), the designer must consider which hull modes are most effective in exciting the particular local structure in question. As an illustration, a pole mast will have relatively low cantilever natural frequencies in the fore-and-aft and athwartships directions. The fore-and-aft mode responds to hull girder rotation in the vertical plane passing through the longitudinal centerline of the hull. In the vertical flexural modes of the hull the rotation in the vertical plane is relatively large near the nodal points and a minimum at the antinodes. In the fundamental or 2-node mode the rotation is a maximum at the ends. Figure 11-1 illustrates a situation in which a pole mast is located at a point of relatively large rotation in the fundamental vertical flexural mode of the hull. On slamming in a seaway a train of decaying vibrations of large amplitude in this mode will be excited. If the pole mast were rigid, the fore-and-aft motions at the top of the mast due to this vibration of the hull would be $h\,\gamma$, where h is the distance from the top of the mast to the elastic axis of the hull and γ is the angular amplitude of the hull at the location of the mast. If, however, the fore-and-aft cantilever natural frequency of the mast happens to coincide with the frequency of this flexural mode of the hull, the fore-and-aft motion at the top of the mast will be greatly magnified. Thus, in this case, a special effort should be made to ensure that the fore-and-aft cantilever frequency of the mast is clear of the 2-node vertical frequency of the hull. Similarly, athwartship motion at the top of the mast is produced by torsional vibration of the hull at the base of the mast and thus the frequency of the 1-node torsional mode of the hull should also be avoided if possible.

It must be emphasized here again that when massive local structures have natural frequencies (estimated on the assumption of a restrained hull) which fall near a frequency of one of the modes which the hull would have if the local structure were rigid, the modes and

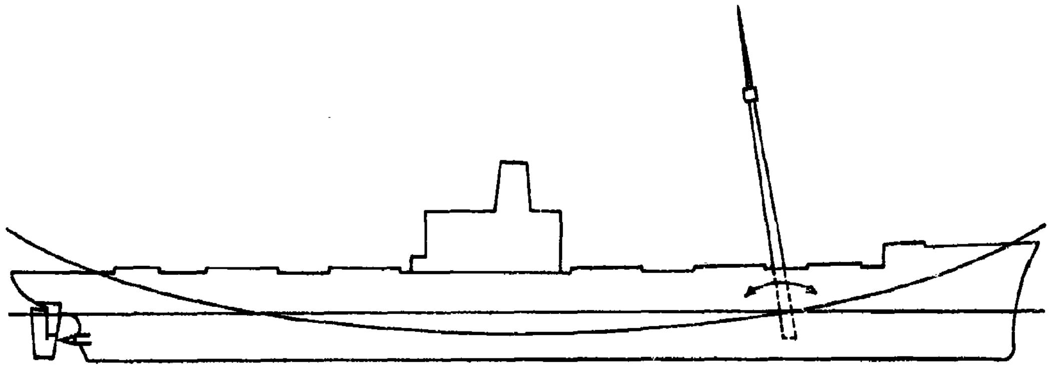


Figure 11-1 – Illustration of Case in Which a Pole Mast Is Located at a Nodal Point of the Fundamental Vertical Mode of a Hull

frequencies of the hull itself are modified by the "sprung mass" effect. Details of analytical methods of dealing with this situation are given in Reference 11-7. Unfortunately, it is only possible to carry out such analytical treatments when the hull design has reached a relatively advanced stage. Only then can the parameters required in the equations be evaluated.

D. RESILIENTLY MOUNTED ASSEMBLIES

When all six of the natural frequencies of a rigidly mounted assembly (computed on the assumption of an immovable hull) are well above the range of significant hull mode frequencies, it may then be assumed that under transient excitation of the hull the assembly will move with the hull. Under such circumstances the resilient mounting is installed for the purpose of isolating the assembly either from blade-frequency vibrations that are well above the hull natural frequency domain or from high-frequency vibrations accompanying shocks which produce local vibrations in the area of the mounting.

As in the case of local structures, a massive assembly such as an engine, if installed on isolation mountings, will affect the hull frequencies themselves, and then it must be considered as a sprung mass. For details of analysis in such cases, see Reference 11-7. In the case of machinery, the resilient mounting is used chiefly to isolate the hull from the unbalanced forces and moments due to the operation of the machine itself. Such isolation will be achieved only when all six natural frequencies of the assembly are below the operating frequencies (or speeds) of the machine.

A matter of chief concern in the selection of an isolation mounting system for an assembly is whether the mountings will bottom under rough sea conditions. Should this happen the equipment may suffer more severe damage than if it had been rigidly secured to the hull. As a safeguard against this, shock tests have been devised for military equipment, as discussed in Reference 11-8. The damaging effect of bottoming can also be reduced by the use of snubbers; see Reference 11-9.

REFERENCES

11-1. Rossell, H.E. and Chapman, L.B., Editors "Principles of Naval Architecture," published by SNAME, Vols. 1 and 2 (1939).

11-2. McGoldrick, R.T., "A Study of Ship Hull Vibration," EMB Report 395 (Feb 1935).

11-3. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

11-4. Jasper, N.H., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," Trans. SNAME, Vol. 64 (1956). Also TMB Report 921 (Oct 1957).

11-5. McGoldrick, R.T., "Calculation of the Response of a Ship Hull to a Transient Load by a Digital Process," TMB Report 1119 (Mar 1957).

11-6. Jasper, N.H. and Birmingham, J.T., "Strains and Motions of USS ESSEX (CVA 9) during Storms Near Cape Horn," TMB Report 1216 (Aug 1958).

11-7. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

11-8. Vane, F.F., "A Guide for the Selection and Application of Resilient Mountings to Shipboard Equipment," TMB Report 880, Revised Edition (Feb 1958).

11-9. Crede, C.E., "Vibration and Shock Isolation," John Wiley & Sons, Inc., New York (1951).

CHAPTER 12

DESIGN CONSIDERATIONS RELATING TO VIBRATION OF THE PROPULSION-SHAFTING SYSTEM

A. INTRODUCTION

Machinery vibration *per se* is a subject which is outside the domain of hull vibration. However, there are various types of vibration in the propulsion system of a ship which may be excited by the propeller itself regardless of the degree of mass balance attained in the manufacture of the propulsion machinery. Since the designer is not free to select a propeller on the basis of hull vibration characteristics alone but must also take such propeller-excited machinery vibration into account, some consideration of the latter has been included in this book. The vibration types in question are torsional vibration of the propulsion-shafting system, longitudinal vibration of the propulsion-shafting system, and lateral vibration of the propeller shaft.

Torsional vibration of propulsion systems is characterized by sinusoidal time variations in the angular velocity of the rotating members or, in other words, by the superposition of angular oscillations on the steady rotation of these members.

Longitudinal vibration is characterized by fore-and-aft oscillations of the propeller, the shafting, and the entire propulsion machinery, including both rotating and nonrotating members.

Lateral vibration of the propeller shaft is frequently spoken of as whirling. Under this type of vibration the shaft center describes a circular or elliptical path in a plane normal to the shaft axis. In some cases the ellipse may be very narrow and in the limit yield a rectilinear path.

In all three cases the basic phenomenon is now well understood and has been discussed in the technical literature in recent years. This does not mean, however, that the critical speeds can in all cases be predicted with high precision. As a matter of fact, in the evaluation of the parameters used in predicting such vibration, difficulties exist similar to those pertaining to the evaluation of the parameters used in the hull vibration calculations.

The analysis of all three types of vibration in the propulsion system follows the same general line as in the case of the hull. Thus, an estimate is usually made of the natural frequencies and normal modes of vibration of the system. Then an estimate is made of the exciting forces. It must be noted here that in the case of diesel drives, torsional excitation exists not only at the propeller but also in the prime mover. Next, an estimate is made of the damping in the system, and finally, a calculation is made of the forced vibratory response. This is then compared with permissible levels established on the basis of past experience, and, if predicted to be excessive, steps are taken to reduce it.

B. TORSIONAL VIBRATION

The subject of torsional vibration of propulsion systems has been treated extensively in the technical literature; see Reference 12-1. Historically, the phenomenon first became a serious problem in internal combustion engine systems and the literature on the subject in this connection is extensive; see Reference 12-2. The basic theory is the same as applied to the pure torsional modes of the hull itself. In the propulsion system, however, the analysis is complicated by the presence of reduction gears, high and low pressure turbine systems, or, in the case of diesel drives, by reciprocating members.

For the immediate problem of this book, the designer needs assurance that a selected propeller will not create a torsional vibration condition which would not exist if the propeller were replaced by a cylindrical mass of the same mass moment of inertia. The latter parameter, incidentally, must include an allowance for water inertia effect and this also poses a problem in evaluation of parameters; see Reference 12-3.

Fortunately, it is usually feasible to keep the operating range of blade frequencies clear of the range of torsional criticals. In the case of geared turbine drives, which at present comprise the majority of ocean-going ships, a common practice is for the designer to select a "nodal drive"; see Reference 12-1. In this case the first torsional critical speed is so low compared to the normal operating speed of the ship that excitation from the propeller is negligible. The second torsional mode is "tuned out." This is accomplished by designing the two turbine branches so that they will have the same torsional natural frequency (the bull gear being held fixed). This produces a mode such that the entire propulsion system from the propeller to the bull gear is a node and hence, torque variations at the propeller will not excite this mode. The third and higher torsional modes fall well above the maximum operating blade frequency.

A general scheme for the torsional vibration analysis of propulsion systems developed at the Bureau of Ships of the Navy Department is given in Reference 12-4. The principal aim of the designer in applying this method is to guarantee that significant torsional vibration will not exist in the propulsion system in normal operation of the ship.

C. LONGITUDINAL VIBRATION

Longitudinal vibration of propulsion systems has attracted general attention more recently than torsional vibration although the phenomenon undoubtedly occurred long before it was recognized. It was brought strikingly to attention when it appeared unexpectedly on large naval vessels during World War II.[12-5]

In this case, the analysis involves equations quite similar to those involved in torsional vibration analysis; the only difference is that forces and rectilinear displacements are involved instead of torques and angular displacements.

The chief difficulty in the analysis is in the evaluation of the longitudinal stiffness of the thrust bearing and machinery foundations, but, fortunately, in those propulsion systems in which the longitudinal natural frequencies are most likely to fall in the operating range of blade frequencies, the shafts are relatively long, and the frequencies of the lower longitudinal modes do not vary widely for large variations in the foundation stiffness.

Here, as elsewhere, the designer is more concerned with obtaining assurance that the longitudinal critical frequencies are clear of the operating range of blade frequencies than with being able to predict the frequencies with precision.

In general, longitudinal and torsional vibrations of propulsion systems may be considered as independent of one another, but this is not always the case, as pointed out in References 12-3 and 12-6. The propeller couples the longitudinal and torsional degrees of freedom of the system to some extent under all conditions, but the coupling effect is significant chiefly when the critical frequencies that would exist without this coupling effect are close to one another. In such cases the mode excited is actually a longitudinal-torsional mode and the excitation involves a generalized force which combines the effect of torque and thrust variations. However, for the designer, it is more important to ensure that the longitudinal and torsional frequencies are kept far apart than to be able to predict the amplitude when they are close together.

It is obvious that, where short drive shafts are involved, as when the machinery is installed far aft, the stiffnesses in the propulsion system are so high relative to the stiffness which the hull presents to the thrust bearing foundation that analyses of longitudinal vibration that consider the hull as fixed are not realistic. Here, as in numerous other cases, the longitudinal vibrations of the propulsion system merge into vibrations of the entire hull-machinery system for which, as yet, reliable methods of prediction have not been developed. The fortunate circumstance is that (at least in the case of surface ships) in such instances the highest operating blade frequency is usually below the first longitudinal critical frequency.

D. LATERAL VIBRATION OF PROPELLER SHAFTS

Considerable impetus was given recently to the study of lateral vibration of propeller shafts when it was suspected that the phenomenon of shaft whirl was the principal cause of the failures of tailshafts that occurred at an alarming rate on single-screw merchant ships shortly after World War II. Although it was later found that the chief stress variations in the tailshafts were due to thrust eccentricity,[12-7] the investigation of the problem led to improved methods of predicting tailshaft whirling speeds and stresses; see References 12-8 and 12-9.

As emphasized in Reference 12-9, the combination of a heavy propeller and a long overhang from the tailshaft bearing may result in a low whirling speed of the tailshaft. The Bureau of Ships procedure for ensuring against a whirling speed in the running range is outlined in Reference 12-10. A general discussion of design stage calculations for marine shafting is given in Reference 12-11. Inertia effects of the surrounding water on propellers are discussed in Reference 12-3.

REFERENCES

12- 1. Lewis, F.M., "Dynamic Effects," Chapter 2, Vol. 2 of Marine Engin, edited by H.L. Seward, published by SNAME (1944).

12- 2. Wilson, W.K., "Practical Solution of Torsional Vibration Problems," 2 Vols., John Wiley & Sons, Inc, New York (1956).

12- 3. Lewis, F.M. and Auslaender, J., "Vertical Inertia of Propellers," JSR (Mar 1960).

12- 4. Gariboldi, R.J., "Procedure for Torsional Vibration Analysis of Multimass Systems," Dept of the Navy, Bureau of Ships (Code 371), R & D Report 371-V-19 (Dec 1953).

12- 5. Kane, J.R. and McGoldrick, R.T., "Longitudinal Vibrations of Marine Propulsion-Shafting Systems," Trans. SNAME, Vol. 57 (1949). Also TMB Report 1088 (Nov 1956).

12- 6. Panagopulos, E.P., Discussion of Paper "Longitudinal Vibrations of Marine Propulsion-Shafting Systems," by J.R. Kane and R.T. McGoldrick, Trans. SNAME, Vol. 57 (1949).

12- 7. Jasper, N.H. and Rupp, L.A., "An Experimental and Theoretical Investigation of Propeller Shaft Failures," Trans. SNAME (1952).

12- 8. Neifert, H.R. and Robinson, J.H., "Further Results from the Society's Investigation of Tailshaft Failures," Trans. SNAME (1955).

12- 9. Jasper, N.H., "A Design Approach to the Problem of Critical Whirling Speeds of Shaft-Disk Systems," TMB Report 890 (Dec 1954).

12-10. Michel, R., "Interim Design Data Sheet DDS 4301, Propulsion Shafting," Dept of the Navy, Bureau of Ships, S 1/1-2 (415), DDS 4301 (1 May 1957).

12-11. Panagopulos, E.P., "Design-Stage Calculations of Torsional, Axial, and Lateral Vibrations of Marine Shafting," Trans. SNAME, Vol. 58 (1950).

CHAPTER 13

DESIGN CONSIDERATIONS RELATING TO RESILIENT MOUNTINGS

It was inevitable that in the initial attempts at the isolation mounting of equipment in ships it was found in many cases that the remedy was worse than the disease. For this reason many such installations were provided with locking arrangements so that, if the mounting were found to make conditions worse, it could be in effect eliminated. It also became a common practice to provide snubbers to ease the impact when the mounted assembly bottomed; see Reference 13-1.

Although a decision to install major items of shipboard equipment on resilient mountings is a very important one from the design standpoint, this chapter is concerned only with the questions which arise after such a decision has been made. The subject of isolation mounting of shipboard equipment, in general, is discussed in detail in References 13-1 and 13-2. The questions at issue here are: How does the use of resilient mountings in a ship affect the behavior of the ship from the vibration standpoint? What account of this should be taken in the hull design?

If the weight of an assembly to be supported by resilient mountings is of the order of ½ percent or more of the displacement of the ship, and any one of its six natural frequencies, computed on the assumption of fixed support of the ends of the mounts, falls within the range of significant hull frequencies (as estimated with the assembly rigidly attached to the hull), then it is advisable to consider the effect of the resilient mounting on the normal modes of vibration of the hull.

For simplicity it is assumed here that the mounted system can be designed to have two planes of vibrational symmetry and that these planes are the XZ- and YZ-planes shown in Figure 13-1 where 0 is at the c.g. of the mounted assembly when in its rest position, and the XZ-plane coincides with the vertical plane through the longitudinal axis of the ship. The concept of vibrational symmetry is discussed in Chapter 6.

In all cases considered in this book the vertical hull modes are treated as independent of the horizontal and torsional modes. However, it must be pointed out here that, if a heavy assembly is flexibly mounted at a considerable distance either to port or starboard of the ship's longitudinal axis and above or below it, there results a coupling of all three

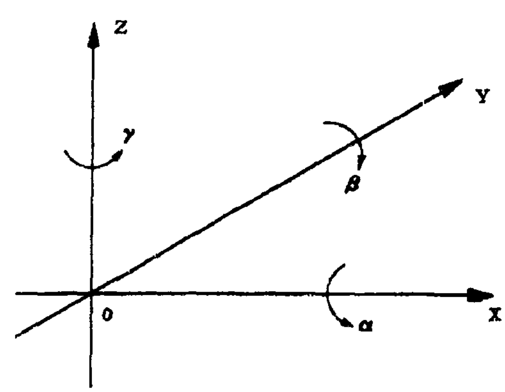


Figure 13-1 – Right-Hand Coordinate System Used in Calculation of Natural Frequencies and Normal Modes of Vibration of a Resiliently Mounted Rigid Assembly (vertical flexural, horizontal flexural, and torsional) types of vibration. The equations for the more general cases of hulls with sprung masses are given in Reference 13-3.

The designer first must carry out the calculation of the natural frequencies and normal modes of the mounted assembly on the assumption that the hull is immovable. This process is discussed in Chapter 6. Also, the estimates of natural frequencies and normal modes of the hull must be made on the assumption that the mounted assembly is blocked so that it cannot move relative to the hull.

If it is recalled that the effect of the sprung mass on the hull is significant only for modes whose frequency is near the natural frequency of the sprung mass, and that the effect is to lower the frequency of a mode below and to raise the frequency of a mode above, it is apparent that the designer can derive some guidance from the two foregoing calculations even without making the more elaborate calculations for the hull with the masses treated as sprung. Thus, if the operating blade frequencies fall well above the range of significant hull mode frequencies predicted with rigid mountings, no difficulty with steady-state, propeller-excited vibration is indicated when the natural frequencies of the assembly fall well within the range of hull mode frequencies. Of course, if an assembly has a natural frequency near that of the highest significant hull mode, and the lowest operating blade frequency is only slightly above this, a possibility of resonance exists and the need for further calculations is indicated. However, the designer is still faced with the possibility of excessive transient vibrations. If the mountings have locking devices it may still be possible to avoid bottoming of the mountings under rough sea conditions. When a sprung mass effect is due to the flexibility of a massive local structure itself, this possibility, of course, does not exist.

One might argue that, if the resilient mountings of an engine were to be locked out under rough sea conditions, they should not have been installed in the first place. Such reasoning, however, is not sound. If the mountings are intended primarily to isolate the hull from the relatively high frequency vibrations of the engine, then this will be accomplished under normal operating speed conditions in relatively calm seas. When the sea gets rough, the engines would be slowed down and thus their unbalanced forces and moments would be reduced. Furthermore, if severe transient vibrations should be set up in the hull under these sea conditions, the vibration induced by the engine would probably be relatively unimportant as long as these conditions prevailed. Thus, if an isolation mounting system is designed mainly for eliminating steady-state vibration, it may be justified to lock it out under slamming conditions in which it would be required to act as a shock mounting which should be designed for more severe conditions.

Although the subject of mass-unbalanced rudders is discussed in some detail in connection with hydroelasticity in the following chapter, it is in order to point out here that such appendages can have a marked sprung mass effect on the horizontal vibration of a hull. At the present time it is indicated that with a hydraulic steering gear the effective torsional stiffness of the rudder system may be relatively low. Thus, it cannot always be assumed that rudder torsional frequencies will be above the range of significant horizontal hull mode frequencies. The vibration analysis of such systems is discussed further in References 13-4 and 13-5.

In summary, whenever the isolation mounting of large masses (of the order of ½ percent or more of the mass of the ship) is under consideration, the designer should look into its effect on the vibration characteristics of the ship as a whole.

It must be kept in mind that the discussion of isolation mountings in this book does not extend into the range of acoustic frequencies in which wave effects, such as discussed in Reference 13-6, take place.

REFERENCES

13-1. Crede, C.E., "Vibration and Shock Isolation," John Wiley & Sons, Inc., New York (1951).

13-2. Vane, F.F., "A Guide for the Selection and Application of Resilient Mountings to Shipboard Equipment," TMB Report 880, Revised Edition (Feb 1958).

13-3. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

13-4. McGoldrick, R.T., "Rudder-Excited Hull Vibration on USS FORREST SHERMAN (DD 931) – A Problem in Hydroelasticity," Trans. SNAME (1959). Also TMB Report 1431 (Jun 1960).

13-5. Leibowitz, R.C., "Modes of Vibration of Rudder of USS ALBACORE (AGSS 569)," TMB Report C-952 (Feb 1959) CONFIDENTIAL.

13-6. Harrison, M., et al., "Wave Effects in Isolation Mounts," TMB Report 766 (Jan 1952).

CHAPTER 14

HYDROELASTICITY

A. INTRODUCTION

At this time hydroelasticity must be regarded as a new science for the naval architect. Although the basic phenomena dealt with in hydroelasticity are not new, for the most part their occurrence has been rare in the operation of ships, and until recently they have been dealt with individually as isolated phenomena.

What is really new is the recognition that these separate phenomena comprise a group falling under the definition of the term "hydroelasticity." Although various definitions of this term will be found in the literature (see References 14-1, 14-2, and 14-3), it seems sufficient to state here that hydroelasticity is concerned with those problems in which water vehicles are subjected to time-varying forces imposed by the water, but governed also by the elastic properties of the hull or its appendages. In general, hydroelasticity in naval architecture parallels the well-established field of aeroelasticity in aeronautical engineering; see Reference 14-4.

Note that the definition of hydroelasticity just given is really broad enough to include propeller-excited hull vibration itself, which is the chief topic discussed in this book. However, the term is currently restricted to problems with hulls or appendages which involve the water flow past the hull, with the exception of propeller-excited hull vibration, since the latter is of such common occurrence as to warrant individual consideration.

Lateral vibrations of circular cylinders moving in fluids are frequently observed, and this phenomenon has been found to be associated with the shedding of vortices. In particular, such vortices are commonly called Kármán vortices and the series of these observed in the wake of a towed cylinder is called a Kármán vortex street. The frequency of shedding such vortices depends on both the towing velocity and the diameter of the towed cylinder, and hence, in the design of appendages such as propeller struts, the use of a streamlined section is important not only in reducing drag but also in preventing vibration. The Kármán vortex shedding phenomenon is discussed in further detail in Reference 14-1.

Another hydroelastic phenomenon that can be troublesome to the naval architect is the so-called "Helmholtz resonator" phenomenon. It is illustrated very simply by the production of a musical tone by blowing over an opening in an air-filled chamber. In naval architecture the term is something of a misnomer since the chamber is an enclosure whose walls also may vibrate and play an important role in determining the frequency. Nevertheless, the hydroelastic phenomenon is similar to the one in air in the sense that it is the flow over an opening in a compartment that produces it.

Two other phenomena of prime importance in aeroelasticity have their counterparts in hydroelasticity although they occur less often in the latter case. These phenomena are "flutter" and "divergence" and apply particularly to hydrofoils , that is, to lifting surfaces or control surface members.

The basic phenomena of flutter and divergence are dealt with in great detail in textbooks on aeroelasticity; see, for example, References 14-4, 14-5, or 14-6. They arise from the variation in lift force and moment with velocity and angle of attack, and involve the question of the static or dynamic stability of the lifting surface member under its normal range of operating speeds.

As an illustration of divergence, consider the torsional stability of the spade rudder shown schematically in Figure 14-1. In this figure the rudder stock is intentionally placed

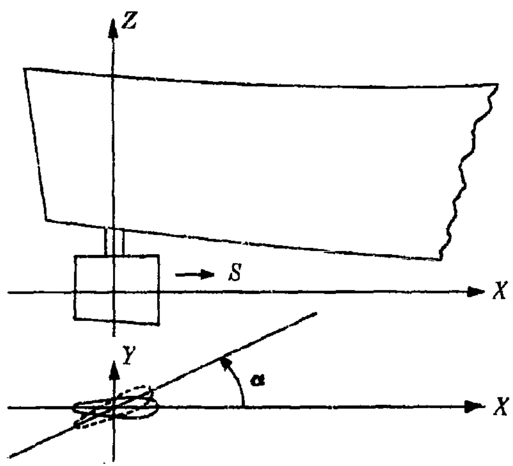


Figure 14-1 – Schematic Plan and Elevation of Spade Rudder Subject to Possible Torsional Divergence

near the mid-chord position to exaggerate the hydrodynamic instability effect. This axis would normally be near the forward quarter-chord position. For simplicity, assume that the water flow in this case is parallel to the ship's longitudinal axis and that the water velocity relative to the rudder is equal to $-S$, where S is the ship's forward velocity. Further, assume that the flow imposes on the rudder a moment M_α with respect to the rudder stock axis given by the relation

$$M_\alpha = C_M S^2 \alpha \qquad [14\text{-}1]$$

where α is the angle of attack and C_M is a moment coefficient for this particular rudder.

If the steering gear were locked, any change in the external steady moment applied to the rudder would evoke an angular displacement $\Delta\alpha$, and an elastic restoring moment equal to $-k\Delta\alpha$, where k is the torsional spring constant of the rudder-steering system. Thus, if equilibrium exists at an initial angle of attack α , any additional angular displacement $\Delta\alpha$ evokes a restoring moment $-k\Delta\alpha$ and an upsetting moment $C_M S^2 \Delta\alpha$, so that the net restoring moment is

$$-(k - C_M S^2)\,\Delta\alpha$$

Thus, in this case, the hydrodynamic moment produces a negative torsion spring effect which varies with the speed of the ship. If the ship should attain such a speed that

$$k - C_M S^2 = 0 \qquad [14\text{-}2]$$

the torsional rigidity of the system would vanish, and, at any speed higher than this, the hydrodynamic moment would twist the rudder to failure. In this simplified example the value of S obtained by solving Equation [14-2] would be called the speed of torsional divergence. Divergence is thus a condition of static instability or instability under a nonoscillatory flow condition.

Flutter is a condition of dynamic instability and is discussed in more detail later in this chapter. As applied to a control surface member, the phenomenon may be explained in simple terms also by reference to Figure 14-1. Suppose that, instead of steady conditions prevailing, the stern of the ship is vibrating horizontally (in the athwartship direction). Then, if the rudder lacks mass balance with respect to the rudder stock axis, angular oscillations of the rudder will also be generated. The latter will evoke variations in the hydrodynamic lift force acting on the rudder. The effect of these variations in lift force on the system will depend, of course, on their frequency, magnitude, and phase. If conditions are such that the lift force variation is in phase with the vibratory velocity and in the same direction, it will exert a negative damping effect on the system. Since the lift force variation increases as the ship's speed increases, it may happen that it is suitably phased and of such magnitude at a certain speed as to cancel out all positive damping effects in the system. A speed at which this condition was reached would be called a critical flutter speed. At a higher speed, if the phase did not shift, the system would have a net negative damping. Under such a circumstance an oscillation once started would build up exponentially until damage or failure occurred.

Still another phenomenon falling under the category of hydroelasticity is the "singing" propeller, also discussed in more detail later in this chapter. This is by no means a new phenomenon to the naval architect (see Reference 14-7), but it is still not thoroughly understood. It involves the development of vibration of a propeller blade when the shaft attains a certain speed, the frequencies usually being in the audible range. It is clearly a hydroelastic phenomenon since it depends on both the flow past the blade as it rotates and the mass-elastic properties of the propeller.

B. RUDDER-HULL VIBRATION

In spite of the fact that W. Ker Wilson (Reference 14-8) had cited rudder flutter as a recognized phenomenon in 1954, few cases of rudder vibration had been reported in the

United States prior to its occurrence on a new class of naval destroyer. The United States Navy encountered a case of hull vibration in 1956 which was traced to the behavior of a pair of twin rudders. Although the ship involved was a naval craft, the investigation of the problem revealed nothing that would restrict the phenomenon specifically to naval types, and hence it is cited here as a type of hull vibration problem other than propeller-excited with which any naval architect might be faced in the future. It is a typical problem in hydroelasticity. Its investigation is discussed in detail in Reference 14-9 and only a synopsis is given here.

On the initial trials of the ship in question, a 3-node horizontal vibration of the hull developed in the upper speed range and persisted with a practically constant frequency throughout this speed range. The steadiness of the frequency, of course, eliminated propeller excitation as the cause. The vibration was finally traced to the twin rudders which were initially set with the trailing edges "in" a few degrees to minimize power consumption. On reversal of this setting, that is, by placing the trailing edges a few degrees "out," this vibration diminished to a permissible level.

To make a long story short, the subsequent investigation showed that the torsional natural frequency of the rudders fell close to the frequency of the particular hull mode excited, and, more specifically, that the rudder-hull system had a normal mode in which the hull vibrated in 3-node flexure while the rudders executed torsional oscillations about the rudder stock axes. The situation satisfied the basic conditions that would make control surface flutter possible if the lift coefficient of the rudders and the speed were high enough.

The basic mechanism of control surface flutter can be explained in simple terms by considering the "classical" flutter system of two degrees of freedom illustrated schematically in Figure 14-2.

The control surface member or hydrofoil which may oscillate torsionally about a vertical axis is supported by a structure which can oscillate only in translation in a direction normal to the steady velocity. The latter structure of mass M is suspended by flexural springs of combined stiffness K from a rigid member which moves at uniform velocity S above the water surface. The rigid member also supports a surface plate (not shown) just above the hydrofoil which moves with uniform velocity and eliminates surface disturbances. The distance from the axis to the center of gravity of the rotating element (based on an allowance for added mass effect of water) is designated h which is considered positive if the c.g. is downstream.

The translational system has damping equivalent to a viscous damping constant C; the rotational system has mass moment of inertia I, which includes an added inertia for entrained water; an elastic stiffness k; an equivalent viscous damping constant c; and a mass m, including an allowance for added mass effect of the water. The displacement of the axis of the control surface in translation is designated Y and the rotation of the hydrofoil about the Z-axis (not shown) is designated θ.

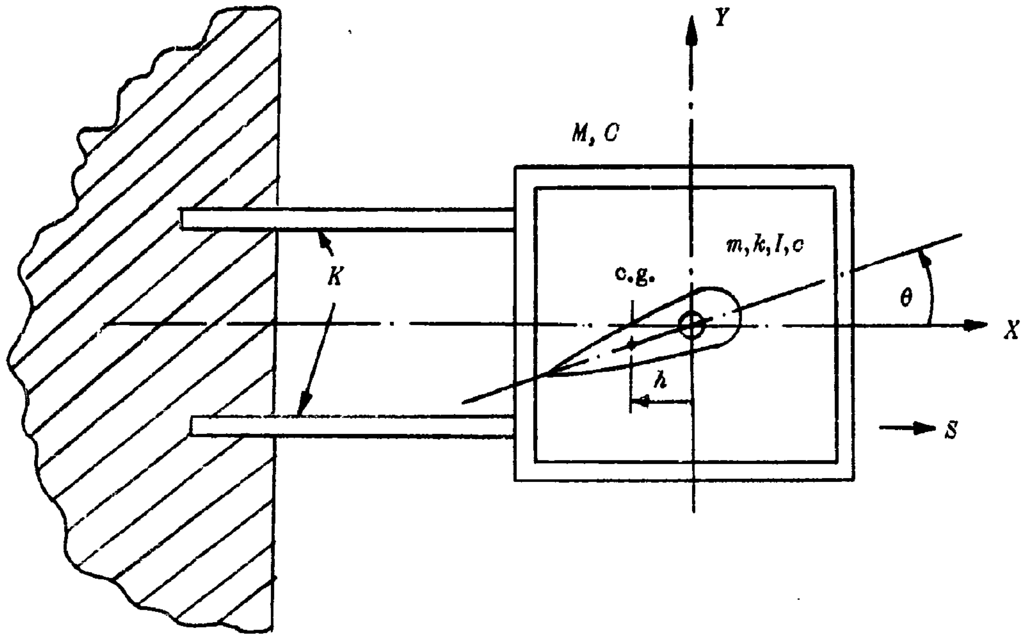


Figure 14-2 – Schematic Plan View of a Control Surface Flutter System of Two Degrees of Freedom

A simplified analysis of the oscillatory motion of such a system may be based on the following assumptions:

(a) The lift force during oscillatory motion may be taken from the lift coefficient for steady flow which is a linear function of angle of attack.

(b) The hydrodynamic moment with respect to the axis of rotation of the foil is given by multiplying the lift force by the distance L from the axis to the center of lift which is a constant L. L is considered positive if the center of lift is upstream.

This yields the following pair of simultaneous ordinary differential equations in the variables Y and θ:

$$I\ddot{\theta} + c\dot{\theta} + (k - ALS^2)\,\theta - mh\ddot{Y} + ALS\dot{Y} = 0 \qquad [14\text{-}3]$$

$$-mh\ddot{\theta} - AS^2\,\theta + (M + m)\,\ddot{Y} + (C + AS)\,\dot{Y} + KY = 0 \qquad [14\text{-}4]$$

Such a pair of differential equations can be solved by an analog computer, or they may be converted to algebraic equations by the assumptions $Y = Y_0 e^{\lambda t}$; $\theta = \theta_0 e^{\lambda t}$. If the resulting equations are then divided by $e^{\lambda t}$, the result will be a pair of simultaneous homogeneous algebraic equations in Y_0 and θ_0 with λ appearing in the coefficients. The determinant of the coefficients is called in this case the "flutter determinant." When it is set equal to zero the resulting equation is a quartic in λ.

The stability of the oscillations then is determined by the four roots of this equation. In general they will be complex. The real part of any complex root then indicates the exponential rate of decay or buildup of a possible mode of vibration. These roots, however, depend on the value of speed S and, in general, it will be found that their characteristics change markedly as S increases. If the roots of λ are computed for a series of values of S, the damping of the system may be plotted as a function of S. Thus, if $\lambda = \mu + j\omega$, plots of $\mu/2\pi\omega$ will show how the rate of decay or buildup in a possible mode of vibration varies with speed for a given set of values of the parameters of the system according to Equations [14-3] and [14-4].

Since the system has two degrees of freedom, it will, in general, have two normal modes of vibration, and, if it is given an arbitrary displacement, its subsequent motion will be a combination of oscillations in each mode. Any one root of λ gives a frequency and rate of decay or buildup in one of these modes. As flutter is approached, the motions in the separate modes may merge into a "flutter mode" in which the phase relation between θ and Y is such as to give zero net damping to the system. At the value of S corresponding to the critical flutter speed, the calculations give a pure imaginary root ($\lambda = j\omega$). This indicates that an oscillation at this frequency (ω) once started will persist indefinitely without damping. The analog computer has the advantage that it automatically combines the responses in the separate modes according to the initial conditions imposed by a given disturbance.

Of course the actual rudder-hull system is much more complex than the ideal system just discussed. The hull has not just one normal mode of vibration in which the rudder stocks move normal to the longitudinal axis of the ship, but several. The rudders also may oscillate in more than one mode since the rudder system may bend, twist, and shear. The flow to the rudders is not actually uniform since they lie in the outflow jet of the propellers. The lift and moment coefficients of the rudders may not be linear functions of the angle of attack even without the occurrence of cavitation. If cavitation occurs, nonlinearities may be expected to be much more pronounced.

In spite of all these complications it appears possible to use this simplified flutter system as a guide in forecasting the possibility of flutter of rudder-hull systems or of control surface-hull systems in general.

The investigation of the rudder-excited hull vibration on the DD 931-Class destroyer by the David Taylor Model Basin (as supplemented by experiments in the towing basin with an apparatus built especially for this purpose) suggested that in the field of naval architecture, as contrasted with the aircraft field, a "subcritical flutter" condition is more likely than unstable flutter.

The subcritical condition is revealed by the same basic analysis and is essentially part of the same general phenomenon. It is illustrated by a curve such as d in Figure 14-3 when the minimum fails to dip below the axis. Then the damping of the overall system is reduced to an undesirable level by the condition which produces critical flutter, but

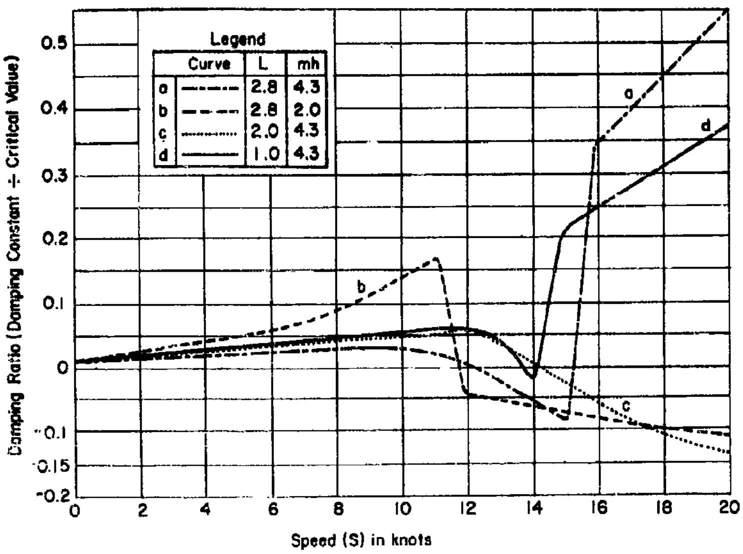


Figure 14-3 – Variation with Speed S in the Rates of Decay or Buildup of Possible Modes of Vibration of the Ideal System Shown in Figure 14-2 According to Equations [14-3] and [14-4]

Negative values of the damping ratio indicate oscillatory instability (critical flutter).

oscillatory instability is not actually produced. The objectionable feature of the situation arises from the fact that there are always other sources of disturbance on a ship underway. Hence, whenever the system is disturbed as by a wave impact, the vibrations in this particular mode of the rudder-hull system decay at a very slow rate. The other modes retain their normal damping characteristic.

Further discussion of the phenomenon of control surface flutter will be found in References 14-4, 14-5, 14-6, and 14-10. The calculation of hull modes when the rudders are treated as flexibly supported is discussed in Reference 14-11.

C. THE SINGING PROPELLER

The problem of the singing propeller is not usually considered a hull vibration problem, but it is certainly a hydroelastic problem and one with which a general naval architect may have to contend; see Reference 14-1.

The phenomenon has been investigated abroad both from the hydrofoil flutter point of view and from the vortex shedding point of view. The former is discussed in Reference 14-7, the latter in Reference 14-12. Much support for the latter point of view is furnished by the fact that critical speeds for singing can frequently be boosted beyond the top operating speed of the propeller shaft by sharpening the trailing edges of the propellers.

In the case of commercial ships, the chief objection to the singing propeller is the annoyance of the high frequency vibration to human ears rather than the absolute level of vibration amplitude produced. It is well-known that the levels of acoustic power that are intolerable from the physiological standpoint are extremely low from the mechanical vibration standpoint. This means that singing propellers may become intolerable because of noise at amplitudes far below those at which they are in danger of structural failure.

D. COMMENTARY

The naval architect will become increasingly aware of the science of hydroelasticity in the future. This will require an acquaintance not only with the subject of mechanical vibration but also with the hydrodynamics of flow about streamlined and unstreamlined forms. Only the barest outline of the subject of hydroelasticity has been sketched in this chapter. The subject is but little understood at this time. When one surveys the extensive literature available in aeroelasticity, the amount of literature on hydroelasticity published so far appears meager indeed.

Studies in hydroelasticity up to the present time indicate that there are large discrepancies between experimental results and analytical predictions based on the classical theory used in aeroelasticity. This is emphasized in the discussions of Reference 14-9. The physical magnitudes involved in the two fields are, of course, quite different, and thus it might be expected that effects that can be neglected in one field could play a dominant role in the other.

In spite of the present lack of information in the field of hydroelasticity, certain areas in which hydroelastic problems may be expected in the future can be pointed out here.

Control surface flutter has been discussed in this chapter; for surface ships the only control surface members are the rudders, except when activated fin stabilizers are used. However, should the submarine become a commercial ship, its diving planes are clearly subject to the same flutter mechanism as the rudders. Its periscopes, if it retains them, will of course be subject to vortex shedding excitation, as will, in fact, any protruding member not properly streamlined. Antipitching fins will introduce hydroelastic effects.

The designer of hydrofoil boats will be concerned not only with the static stability of the hydrofoils but also with their dynamic stability. Thus, they will warrant a hydroelastic analysis as detailed as the aeroelastic analyses currently made in aircraft design.

For a discussion of panel flutter and further details of hydroelasticity, the reader is referred to 14-13, 14-14, and 14-15. The classical treatment of flutter as used in aeroelasticity is given in Reference 14-16. Hydrofoil flutter is discussed in Reference 14-17.

REFERENCES

14- 1. Saunders, H.E., "Hydrodynamics in Ship Design," published by SNAME (1957).

14- 2. Heller, S.R., Jr. and Abramson, H.N., "Hydroelasticity: A New Naval Science," Journal ASNE (May 1959).

14- 3. Flax, A.H., "Aero-Hydro-Elasticity," Structural Mechanics, Proceedings of First Symposium on Naval Structural Mechanics, Pergamon Press, New York (1960).

14- 4. Bisplinghoff, R.L., et al., "Aeroelasticity," Addison-Wesley Publishing Co. (1955).

14- 5. Fung, Y.C., "An Introduction to the Theory of Aeroelasticity," John Wiley & Sons, New York (1955).

14- 6. Scanlan, R.H. and Rosenbaum, R., "Introduction to the Study of Aircraft Vibration and Flutter," MacMillan Co., New York (1951).

14- 7. Taylor, J. Lockwood, "Propeller Blade Vibration," TINA (1943).

14- 8. Wilson, W. Ker, "A Review of Ship Vibration Problems," Mar Engr and Nav Arch (Jul, Aug, Oct, Nov 1955). Also published in ASNE Journal (May 1954).

14- 9. McGoldrick, R.T., "Rudder-Excited Hull Vibration on USS FORREST SHERMAN (DD 931) – A Problem in Hydroelasticity," Trans. SNAME (1959). Also TMB Report 1431 (Jun 1960).

14-10. McGoldrick, R.T. and Jewell, D.A., "A Control-Surface Flutter Study in the Field of Naval Architecture," TMB Report 1222 (Sep 1959).

14-11. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

14-12. Krivtsov, Y.V. and Pernik, A.J., "The Singing of Propellers," TMB Translation 281 (Oct 1958).

14-13. Abramson, H.N. and Chu, W.H., "Problems of Hydroelasticity," JSR, Vol. 3, No. 1 (Jun 1959).

14-14. Henry, C.J., et al., "Aeroelastic Stability of Lifting Surfaces in High-Density Fluids," JSR (Mar 1959).

14-15. Jewell, D.A., "A Note on Hydroelasticity," JSR, Vol. 3, No. 4 (Mar 1960).

14-16. Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," NACA Report 496 (1934).

14-17. Abramson, H.N. and Chu, W.H., "A Discussion of the Flutter of Submerged Hydrofoils," JSR, Vol. 3, No. 2 (Oct 1959).

CHAPTER 15

SHIP VIBRATION RESEARCH

A. INTRODUCTION

Ship vibration research was initiated toward the end of the nineteenth century. Credit for pioneering is usually given to O. Schlick who proposed the now famous Schlick formula for the frequency of the 2-node flexural mode of surface ships in 1894. Among other early investigators was A. N. Krylov who reported on the recording of ship vibration in 1900. Although the analogy to string vibration cited therein was unsound, it is of historical interest that the subject was also discussed in a French treatise on naval architecture published in 1894; see Reference 15-1.

As in so many other fields of research, progress has been slow but at an accelerating pace, and the development of theory has proceeded hand in hand with the development of experimental techniques. This is a field in which full-scale experimentation is costly. The mere tying up of a ship for the length of time required for vibration experiments is often prohibitive, to say nothing of the cost of installing and operating the necessary equipment. However, it must be emphasized that the correction of vibration unexpectedly encountered on the trials of a new ship may be much more costly.

It is not attempted in this chapter to survey the ship vibration research work which is now carried on by all the principal maritime nations abroad. This discussion is confined chiefly to the ship vibration research in the United States which, for the most part, has been carried on by the United States Navy either independently or in collaboration with the Society of Naval Architects and Marine Engineers.

On the experimental side, this research has involved the following principal phases:

(a) The development of instrumentation for recording vibration of ships and ship models.

(b) The development of machines capable of vibrating ships and ship models.

(c) The development of apparatus for determining propeller-exciting forces on model scale.

(d) The conduct of full-scale experiments with vibration generators to determine the vibratory characteristics of ships in service.

(e) Running of vibration surveys on ships in service and the systematic storage of data so obtained.

(f) The conduct of model experiments for determining propeller-exciting forces.

(g) Model experiments for the determination of the added mass effect of the surrounding water.

On the theoretical side, ship vibration research in the United States has involved:

(a) The development of analytical methods of predicting the normal modes and natural frequencies of vibration of ships.

(b) The development of analytical methods of predicting the steady-state forced vibration of a ship under given exciting forces.

(c) The development of analytical methods of calculating transient vibrations within the elastic range.

(d) The development of analytical methods of predicting the vibratory exciting forces which will act on a ship of given design.

(e) The development of computing techniques, either analog or digital, for carrying out calculations by the analytical methods developed.

The research program, of course, has also included the comparison of analytical predictions with experimental results and the evaluation of the analytical methods on the basis of these comparisons. Most of the theoretical research accomplishments have been covered in previous chapters. Hence, the present chapter is devoted chiefly to the experimental phase of ship vibration research and to the attempts to correlate theory with experiment.

B. VIBRATION GENERATORS

The development of vibration generators for ship vibration research in the United States was based chiefly on the prior development of such machines for research on bridges by the firm of Losenhausen in Dusseldorf, Germany, prior to World War II. Their successful use in Germany led the U.S. Experimental Model Basin to purchase from Losenhausen in 1931 the largest machine of this type when built. These generators consist of eccentric masses which may be so unbalanced and so phased as to produce sinusoidal forces in one direction only, sufficient to maintain a vibration of the entire hull of a magnitude permitting experimental determination of the normal-mode pattern. The machine had a force amplitude of 49,000 lb and a deadweight of 25 tons. Its two eccentrics each weighed 6000 lb and could be offset up to about 12 in. It was operated by two d-c motors, each of 15-kw capacity.

There was also purchased from Losenhausen at the same time a very small vibration generator, weighing only about 140 lb, and having eccentric weights at each end of two parallel shafts. This machine was not only suitable for vibration experiments on model scale but was also capable of exciting local structures on full-scale ships and was used repeatedly for many years. As originally furnished, it had a maximum force rating of 440 lb and a maximum speed of 3600 rpm.

The David Taylor Model Basin later undertook the development of vibration generators for full-scale ship experiments of greater versatility than possible with the huge machine originally purchased from Losenhausen. The Taylor Model Basin medium vibration generator

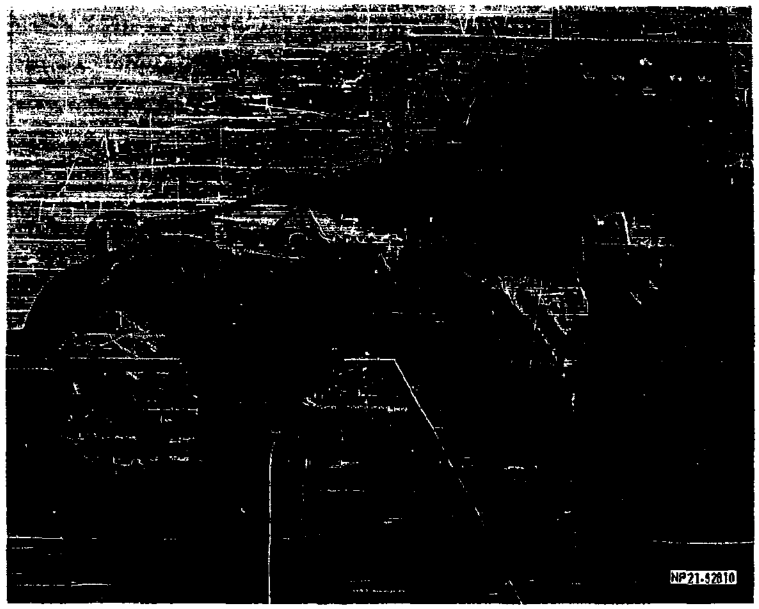


Figure 15-1 – Taylor Model Basin Medium Vibration Generator

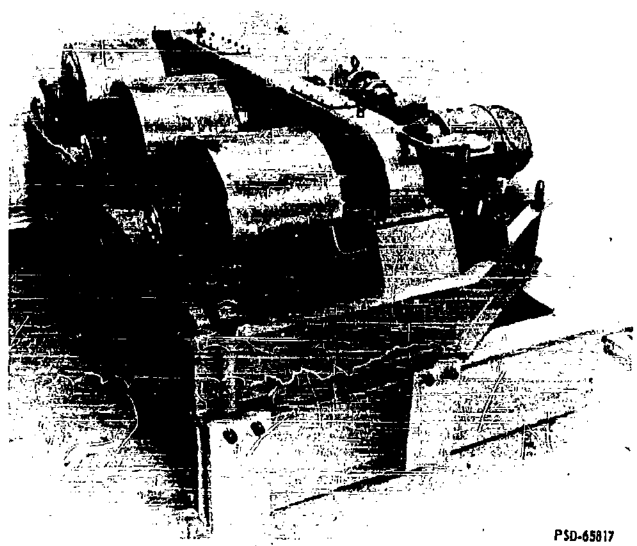


Figure 15-2 – Taylor Model Basin Three-Mass 40,000-Pound Vibration Generator

shown in Figure 15-1 and the Model Basin three-mass 40,000-lb vibration generator shown in Figure 15-2 were results of this effort.

The latter machine (shown in Figure 15-2) makes use of a scheme patented by Dr. R. K. Bernhard who had been associated with the design of the large Losenhausen machine in

Germany. It has three parallel shafts; the two outer shafts rotate in one direction while the inner rotates in the opposite direction. This permits the production of a sinusoidal exciting force in any desired direction in a plane normal to the shaft axes.

The operation of large vibration generators on full-scale ships has been greatly facilitated by the progress that has been made since their inception in the development of speed-control devices for direct-current motors. The difficulty in operation is due to the power versus speed characteristic that must be met by a machine for this service. If there were no damping, no power output would be required to maintain a constant amplitude of vibration in a mass-elastic system such as a ship. Actually, even with damping, very little power is absorbed except in the neighborhood of resonance. Since, on approaching a resonance speed from below, the power required will increase, the speed is stable on the low speed side of resonance. Thus, any change in line voltage which would tend to increase the speed will also tend to load the motors which will limit the speed. Above the resonance speed, however, the situation is reversed. As the speed increases, the motors are unloaded by the drop in amplitude of the system and hence tend to run away. An amplidyne-type of speed control is therefore beneficial.

For studying the vibratory characteristics of hulls, it is a prime requirement to maintain a selected speed of the vibration generator. If this speed can be maintained the hull may then be explored with portable instruments. This permits location of the nodal points and the plotting of normal mode patterns. It also facilitates the step-by-step plotting of curves of displacement amplitude, velocity amplitude, acceleration amplitude, or power versus speed.

Vibration generators for shipboard use must be able to produce not only sinusoidal forces in either the vertical or horizontal directions but also couples about the longitudinal axis for investigating torsional vibration of hulls. Some progress has been made in recent years in investigating the coupling of hull torsion with horizontal hull flexure, but much remains to be done. It is very difficult to design a single machine to produce a pure couple large enough to excite the torsional modes of the hull. Proposals have been made to use separate units synchronously driven, one installed on each side of the hull near the stern. With large machines, however, the synchronizing becomes a problem comparable with that of synchronizing propeller shafts. The design of the Taylor Model Basin three-mass, 40,000-lb vibration generator is a compromise as far as pure couples are concerned. The maximum couple attainable is 120,000 lb-ft.

Further details of vibration machines used by the Taylor Model Basin will be found in Reference 15-2.

C. SHIP VIBRATION INSTRUMENTS

Certain salient points in the design of vibration instruments for shipboard use are discussed here. For further details, the reader should see Reference 15-3. The basic principles used in the design of vibration instruments in general are discussed in standard texts on mechanical vibration; e.g., References 15-4 and 15-5. It must be pointed out in the beginning that instruments that perform satisfactorily in making a shipboard investigation with a vibration generator may be quite inoperable at conditions under which underway ship vibration surveys must be made. Vibration generator tests normally can be undertaken only under very calm sea conditions, and with the ship either dead in the water or advancing at a very slow speed.

The chief difficulty in the design and operation of vibration instruments for underway vibration surveys arises from the fact that the rigid body motions of the ship are extremely large in proportion to the displacement amplitudes in vibration which are to be measured and the accompanying accelerations are high. There is not only this problem to contend with but also the fact that the vibratory amplitudes themselves cover a very large range, roughly from a single amplitude of 0.001 in. to 1.0 in. This, of course, does not refer to attempts to measure vibration under slamming conditions in a seaway where amplitudes of a foot or more are occasionally encountered.

As may be seen from Reference 15-3, all the well-known types of vibration instruments have been tried on board ship. In spite of all the effort expended in this direction, standardization of instrumentation for underway vibration surveys has not been attained up to the present time. However, both the Bureau of Ships of the Navy Department and the Society of Naval Architects and Marine Engineers are continuing their efforts in this area.

The principal types of instruments now in use are:

(a) Amplitude indicating: dial gage-type vibrometers, optical vibrometers, Cordero vibrometers, velocity- or accelerometer-type transducers with integrating amplifiers, cathode ray or recording oscillographs, and rectifiers with d-c indicating meters.

(b) Frequency indicating: sets of Frahms reeds, single tunable reed, stroboscopes, oscilloscope, and direct-indicating electronic frequency meter.

(c) Amplitude and frequency recording: mechanical pallographs, crystal accelerometers with integrating amplifiers and oscillograph, velocity pickups with integrating amplifiers and oscillograph, transducers with frequency-modulation tape recording using playback, optical recording for film (mirragraph), and optical accelerometers.

(d) Phase measurement: electronic indicating and electronic recording.

A typical direct-recording instrument for shipboard use is illustrated in Figure 15-3 and a typical remote-recording system in Figure 15-4. In the former case, the instrument has two seismic elements; one can move only in the vertical direction, the other only in the horizontal direction. Each element is suspended in such a way as to have a natural frequency

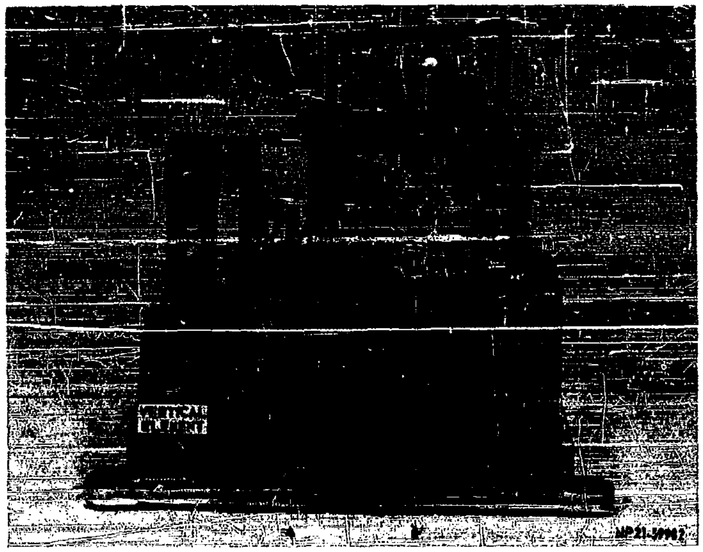


Figure 15-3 – TMB Two-Component Pallograph

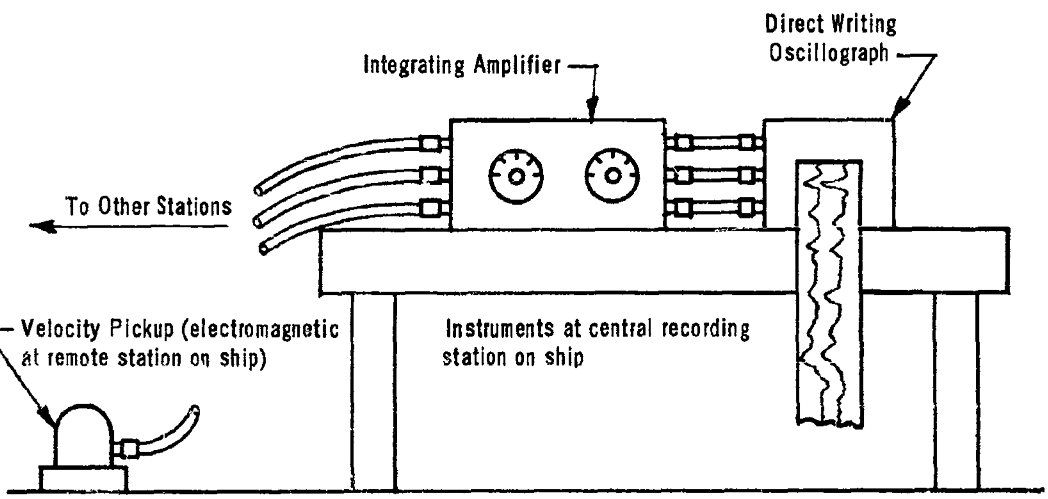


Figure 15-4 – Typical Remote Recording System for Shipboard Vibration Measurements

considerably lower than the vibration frequencies normally to be measured. Under such circumstances the element remains practically fixed in space whereas the rest of the instrument vibrates with the deck of the ship on which it is placed. The relative motion between the element and the base of the instrument is then magnified by a linkage and recorded on paper together with a timing signal. It is shown in Reference 15-3, however, that, by introducing controllable damping and with adequate calibration techniques, such instruments can be used at ranges of frequencies within which their own natural frequencies fall.

In the remote recording system shown in Figure 15-4, the pickup unit also contains a seismic element. In this instance the element is a magnet whose field cuts across a coil fixed to the base of the pickup. The voltage generated in the coil is proportional to the relative velocity between the base and the element. An integrating amplifier converts the signal to one proportional to vibratory displacement which is recorded on the oscillograph. If desired, the velocity signal can be recorded directly without integration.

In general, instrumentation satisfactory for underway vibration surveys can be used for vibration-generator surveys but the reverse does not hold. The limitation, if any, will be in its sensitivities and frequency ranges. However, although the earlier use of vibration generators on board ship was chiefly for determining the natural frequencies of the hull and its normal mode patterns, recently it has been found practical to use such machines for determining forced vibratory response characteristics over the range of blade frequencies encountered in the normal operation of the ship.

Under very calm water conditions and with very sensitive vibration-recording equipment, vibration-generator surveys may be made with amazingly low amplitudes, as in the case of PONTCHARTRAIN[15-6] in which normal modes were plotted with amplitudes of the order of 0.01 mil.

The investigation of torsion-bending modes and pure torsional modes of hulls requires special attention to instrumentation. Here it is required to separate the rotational and translational components of the hull vibration. This can be done with translational instruments only by simultaneous recording of signals from pickups at opposite deck edges and observation of phase. A recording angular accelerometer with integrating amplifier will give the angular amplitude directly if its sensitivity to rectilinear vibration is sufficiently low.

D. EXPERIMENTAL TECHNIQUES

1. MODEL

Considerable model experimentation has been carried out in the United States in the effort to clarify the added mass effect of the surrounding water on hull vibration characteristics; see Reference 15-7. At this writing, however, the effort on model scale in the field of ship vibration research is concentrated on the problem of determining model forces. As stated in Chapter 7, this work is an extension of the pioneering work of F.M. Lewis.

In all model work one encounters the problem of similitude or scaling. This subject is discussed further in Appendix G. It should be noted here that it is not feasible (because of conflicting scaling rules; see Appendix G) to build a complete self-propelled dynamic model of a ship from which the propeller-excited vibration of the prototype can be predicted from direct measurements of the vibration of the model. Quite aside from the difficulties of fabricating structural models of ships, the research scientist is confronted with scaling rules for ship vibration that do not conform with those for ship propulsion.

In F.M. Lewis' original scheme, a continuous wooden model was used for the propeller-exciting force determination and the line of reasoning was that, since the determination was made by nullification of the forces by forces produced by a balancing machine inside of the model, the dynamic characteristics of the wooden model itself were irrelevant.

In later experiments it was found that resonance of the wooden model was deleterious and a scheme of flexibly suspending the stern portion from the rest of the model was adopted. As shown in detail in Reference 15-8, the present setup yields the following values:

a. Blade-frequency vertical force,

b. Blade-frequency horizontal force, and

c. Blade-frequency torsional moment about the longitudinal axis.

In the field of hydroelasticity (see Chapter 14), the question of the feasibility of model experimentation is quite important. It must be decided whether it is feasible to predict a control surface flutter condition by model techniques. As shown in Reference 15-9, flutter model testing has been found practical in aeronautical engineering. In naval architecture, however, its use will have to be much more restricted. The criterion of similitude in flutter experiments is the nondimensional Strouhal number or "reduced frequency." In aeroelasticity, this is commonly expressed in the notation $\frac{b\omega}{V}$, where b is the half-chord dimension of a wing, ω is the circular frequency of the vibration, and V is the velocity of the wing relative to the undisturbed air. This relation indicates that for a scaled model of the same material as the prototype, the flutter speed should be the same as for the prototype.

In the hydroelastic field, if a dynamical model were manufactured of the same material as the ship (since the natural frequencies vary inversely as the scale), the numerator in the expression for the Strouhal number would be the same for model and prototype. Hence, the Strouhal number itself would be the same if the model speed were the same as the speed of the prototype. Obviously, for surface craft the flow patterns will not be similar if model and prototype run at the same speed since the law for similitude of bow waves is the Froude law; that is,

$$\frac{V^2}{Lg} = \text{constant}$$

where V is the velocity,

L is a characteristic length, and

g is the acceleration of gravity.

Although it has been pointed out that in the case of a deeply submerged submarine model the flow pattern is independent of the speed unless flow separation develops, there remains the problem of fabricating the dynamical model and of running it at such a high speed. In the aircraft field, flutter model testing is well established and here the high speed required is feasible. It is also possible in wind tunnels to vary the density to reduce discrepancies between model and full-scale Reynolds numbers. Furthermore, as pointed out in Appendix G, in the field of ship slamming it is feasible to experiment with dynamic models based on Froude's law since here the propulsion system itself is not involved. In such models the elastic characteristics are not represented in the shell but in a special girder installed along the longitudinal axis of the model.

In dealing with vibration problems of local structures it is easier to satisfy the conditions of similitude than when the entire hull is involved, since hydrodynamic forces are not involved. Thus, similitude for vibratory characteristics is theoretically satisfied if the model is made of the same material as the prototype and duplicates it to scale. The practical difficulties here lie in the fabrication of the model and in the simulation of boundary restraints and damping characteristics. These difficulties, however, are so great that comparatively little has been accomplished to date in solving vibration problems of local hull structures by dynamic model experiments.

2. FULL SCALE

Full-scale experiments in the field of ship vibration are at present confined chiefly to tests with mechanical vibration generators. These machines are installed at either end of the ship and operated over a sufficient range of speed and driving force to excite the normal modes of vibration of the hull, one at a time. During the operation of the vibration generator, the vibration of the hull is recorded either by a multichannel oscillograph system or tape recording system, or by exploring the hull with portable vibration instruments. The aim is usually to record enough data to permit plotting the normal mode patterns for all modes of the hull that can be excited. This includes the vertical, horizontal, torsional, or torsion-bending modes. It is also possible in such tests to determine phase relations among the displacements at stations along the hull. These show whether a standing or traveling wave condition exists.

A useful adjunct to the vibration generator test is the anchor drop test. While such a test usually excites only the 2-node vertical flexural mode of the hull, the recording of the subsequent vibration permits a determination of the logarithmic decrement; that is, the damping. Furthermore, since the 2-node vertical flexural is the mode of the hull lowest in frequency, it may be found in some cases that the vibration generator cannot produce sufficient force to excite this mode even when the machine is set at its maximum eccentricity. The anchor drop may then be the only means of determining the fundamental vertical frequency.

Although used less often for this purpose in the past, the vibration generator, if installed directly over the propellers, furnishes a means of determining the full-scale propeller-exciting forces. Conceivably, it could be used as a balancing device to cancel out the propeller forces when the ship is under way in the same manner as in the model determination of propeller-exciting forces by the null method. This technique has not been used full scale to any extent up to the present time, and synchronizing of the vibration generator with the ship's shafts is not easily attained. However, if with the ship dead in the water, the vibration generator is operated at speeds (rpm) coinciding with operating blade frequencies (cpm), the data so obtained may be used to estimate the underway propeller forces, provided the underway hull vibration is also recorded. This, of course, involves the assumption that the damping and added mass characteristics of the hull are the same underway as in still water. In the case of vertical vibration, only the hydrofoil action of the propellers appears to cause a variation in damping. In the case of horizontal vibration, the hydrofoil damping action of the rudders is to be taken into account. In the case of submarines, the diving planes provide a source of damping of vertical vibration. It should be noted, however, that where the operating blade frequency falls above the range of significant hull mode frequencies, the mechanical impedance at the stern is chiefly an inertial impedance.

Aside from the full-scale vibration-generator experiments just described, both the U.S. Navy and the Society of Naval Architects and Marine Engineers have initiated a systematic series of underway vibration surveys on new classes of ships. Such surveys play an important role in the overall ship vibration research program. Even if nothing unusual develops during such a survey from the ship vibration standpoint, the data obtained are still extremely valuable in establishing acceptable levels of vibration for various classes of ships. If a severe vibratory condition is discovered, the investigation of this is likely to lead to a definite advance in ship vibration theory. It is never desirable from the naval architect's point of view to dismiss a case of serious ship vibration which has been corrected by the trial and error process without really tracking down the phenomenon. Unfortunately, however, economic considerations sometimes impose such an outcome.

With the advent of the application of statistical methods to the study of the performance of ships in a seaway (see Reference 15-10), more attention has been given in recent years to the collection of data on the transient vibrations of ships. As pointed out in Chapter 2, these vibrations are much more important from the point of view of structural integrity of the hull than the steady-state vibrations which are excited by the propeller.

While, in principle, the instruments already mentioned in this chapter can be used for recording transient vibrations, it is in order to point out here that the severe sea conditions under which large transient hull vibrations are generated rule out an instrument of the type shown in Figure 15-3 for such a purpose. The trend at this time is in the direction of remote-recording systems using magnetic tape recording. This permits subsequent playback of the record into vibration analyzers in the laboratory.

E. CORRELATING THEORY AND EXPERIMENT

The complexity of the ship vibration problem is such that exact theories are nonexistent. While the theory of vibration of ideal mass-elastic systems can serve as a guide, it is only by continually comparing theoretical predictions with experimental results that a rational theory can be established.

The theory advocated in this book is a beam theory with important modifications. The idealized beam is considered to be free in space, but it carries added mass nonuniformly distributed to account for the inertia effect of the surrounding water. Thus it is nonuniform in both its mass and rigidity distributions along its length. It can bend, twist, and shear, but does not execute significant longitudinal vibration and its torsion is only about its longitudinal axis. If longitudinal vibrations of surface vessels, and especially submarines, are "significant" from some points of view, this arises from considerations not dealt with in this book and involves amplitudes of a lower order of magnitude. The damping of the idealized system is provided by external and distributed viscous damping between itself and axes fixed in space. It may have one or several flexibly attached masses and these may move relative to it in both rotation and translation. Finally, it has the unique property (for a beam) that it can vibrate in only a limited number of modes.

In checking this theory, the simplest criterion is the comparison between predicted and measured natural frequencies. Such a comparison requires merely tabulating the frequencies and identifying them by the node shapes, then listing the predicted frequencies and the percentage of errors in the predictions.

The second best criterion is the comparison of the experimental and computed normal mode patterns. Even if the frequencies check, the theory will have less and less validity, the greater the departure of the experimentally determined normal mode patterns from those predicted. In this connection, beamlike modal patterns are not to be expected if there is considerable local flexibility, especially when the local structure is massive. Hence, when the observed modal patterns are not beamlike, calculations must be tried with various local structures treated as flexibly attached, if correlation is to be improved.

The third area to be explored is that of the forced vibratory response of the hull. Here much depends on whether the vibration is resonant or nonresonant. The forced resonant response depends chiefly on the damping and the magnitude of the driving force. Since the latter is known when a vibration generator is used, the forced resonant response in this case yields an experimental value of damping.

In the range of frequencies above that of the significant hull modes, the observed forced response in the stern provides a check on the proposed formula for mechanical impedance at the stern of a ship discussed in Chapter 1.

If the experimentally determined horizontal modes are observed to be torsion-free, or very nearly so, then the same criteria for comparing theory and experiment in vertical vibration are applicable to horizontal vibration. When it comes to torsion-bending modes, it is necessary to convert the mechanical system to a generalized system which combines the effects of translation and rotation before the comparison can be made. The general scheme used for doing this is outlined in Chapter 4.

REFERENCES

15-1. Pollard, J. and Dudebout, A., "Theorie du Navire,"–Treizieme Partie, "Vibrations des Coques des Navires a Helice," Vol. IV, Chapt. LXIX, "Causes et Periodes des Vibrations," Paris (1894).

15-2. Robinson, Q.R., "Vibration Machines at the David W. Taylor Model Basin," TMB Report 821 (Jul 1952).

15-3. Allnutt, R.B. and Mintz, F., " Instruments at the David Taylor Model Basin for Measuring Vibration and Shock on Ship Structures and Machinery," TMB Report 563 (Jul 1948).

15-4. Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co, New York (1956).

15-5. Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co., New York (1955).

15-6. Allnutt, R.B., "Investigation of Hull Vibrations of USCGC PONTCHARTRAIN (WPG 70)," TMB Report R-294 (Aug 1946).

15-7. Kaplan, P., "A Study of the Virtual Mass Associated with the Vertical Vibration of Ships in Water," SIT, Davidson Lab Report 734 (Dec 1959).

15-8. Stuntz, G.R., et al., "Series 60 – The Effect of Variations in Afterbody Shape upon Resistance, Power, Wake Distribution, and Propeller-Excited Vibratory Forces," Paper prepared for Annual Meeting of SNAME (1960).

15-9. Bisplinghoff, R.L., et al., "Aeroelasticity," Addison-Wesley Publishing Co. (1955).

15-10. Jasper, N.H., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," Trans. SNAME, Vol. 64 (1956). Also TMB Report 921 (Oct 1957).

APPENDIX A

ILLUSTRATION OF A CALCULATION OF A VERTICAL NORMAL MODE BY THE DIGITAL PROCESS

It is essential to point out here that the illustrations used in this or other appendixes of this book are not to be construed as samples for use in preparing data for calculations to be made by the Applied Mathematics Laboratory of the David Taylor Model Basin. The problems that have been coded for solution by that Laboratory are continually being studied for improvement of the computing technique as new and improved computing facilities are installed. Although the preparation of data for such a calculation as given in this Appendix is discussed in considerable detail in Reference A-1, the reader is urged to check directly with the Applied Mathematics Laboratory for the latest instructions if a request for hull vibration calculations is to be made of the Taylor Model Basin.

One of the simpler hull vibration cases (the calculation of the 2-node vertical mode of a commercial cargo ship) is used for illustration in this Appendix to emphasize the basic elements of the method. For the application of the digital method to the calculation of torsion-bending modes, including consideration of the effect of sprung inertias; see Reference A-1. This illustration is based on the Maritime C-4 design cargo ship GOPHER MARINER whose vibration characteristics were the subject of Reference A-2. The design of this class of ship is discussed in detail in Reference A-3.

The method is based on the finite difference equations given in their simplest form in Chapter 3. The orientation of rectangular coordinate axes for the calculation is shown in Figure A-1. It was assumed in this case that both $\bar{z}$ and $\bar{\bar{z}}$ were zero because of the symmetry of the ship with respect to the XY-plane.

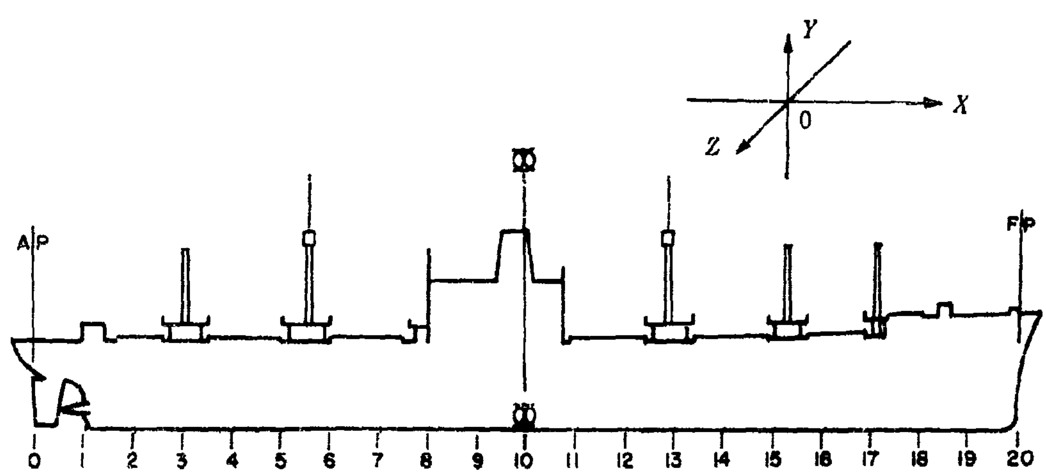


Figure A-1 – Diagram Showing the Division of the Ship into 20 Equal Sections for the Vibration Calculations

The hull was divided into 20 sections between the after and forward perpendiculars. The ship was visualized as shown in Figure A-1; that is, with the stern to the left. It should be noted that the station numbering here is from the after perpendicular forward for the reason stated in Chapter 3. As this numbering system is purely arbitrary, the reader should check on the numbering system in use at the time if a future request for calculations by the Taylor Model Basin is contemplated.

The weight corresponding to the added mass of the surrounding water was added to the weight of the vessel for the test loading condition. The weight added to each of the 20 sections was estimated from the formula:

$$\text{Added weight per unit length} = \frac{1}{2} J C \pi \rho b^2 \qquad \text{[A-1]}$$

where J is the longitudinal coefficient applied to correct for the departure of the vibratory motion of the water from two-dimensional flow in planes parallel to YZ and the C coefficients are factors, based on two-dimensional flow, which give the ratio of the added mass for a cylindrical form having the shape of a ship section to the added mass for a circular cylindrical section of the same beam.

The C coefficients were obtained as follows:

(a) The section-area coefficient β for the section was estimated by inspection (β is the ratio of the area of the underwater section to 2 bd, where d is the draft and b is the mean half-breadth of the section at the waterline, in ft).

(b) B/d for the section was then computed (where B is the whole beam at the section).

(c) The value of C corresponding to these values of β and B/d was found from Figure A-2.

The coefficient J was obtained from Figure A-3 after computing L/B (length over beam) for the ship.

Combined Mass, m

The values of the combined mass m used in the digital computation were derived as follows: A continuous curve of combined weight per unit length was plotted as in Figure A-4 by adding to the ordinates of the weight curve of the ship the values of added weight of water. The lumped masses used in the digital calculation were then derived by concentrating the weight indicated by the combined curve at the 20 stations after converting weight to mass by dividing by g (the acceleration of gravity).

Area Moment of Inertia, I

The area moments of inertia were evaluated for a sufficient number of sections of the hull to permit the plotting of a continuous curve. All the longitudinal structural members up to the uppermost continuous deck or weather deck were considered in the evaluation. If a ship has decks with expansion joints, which hence are not designed to carry hull bending

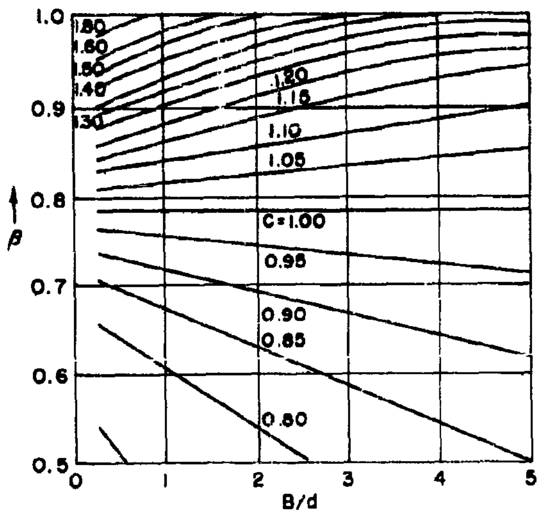


Figure A-2 – Curves for Estimating the Coefficient C Used in the Added Mass Evaluation

(From Reference A-4)

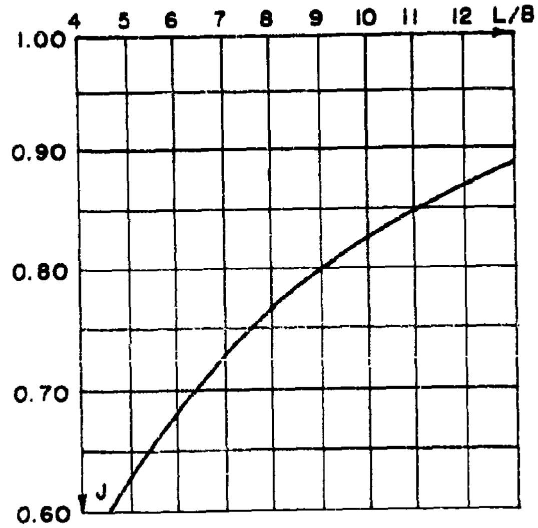


Figure A-3 – Curve for Estimating the Coefficient J Used in the Added Mass Evaluation

(From Reference A-4)

stresses, they should not be included. Superstructures extending less than 30 percent of the length of the hull may be completely omitted in computing the area moment of inertia of sections, as was done in this case. With longer superstructures their contribution should be neglected at their ends but added gradually so as to make them fully effective in their midlength, provided they are continuous and well tied into the main deck structure.

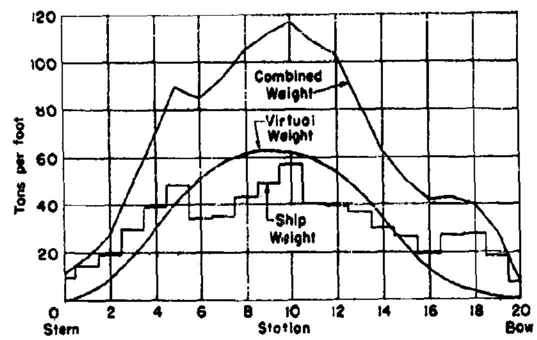


Figure A-4 – Weight Curves for GOPHER MARINER (Heavy Displacement) Used for Calculation of Vertical Modes

Longitudinal bulkheads or longitudinal stiffeners terminating at or near the section in question should not be included. Where hatch openings fall at the section, the hatch coaming section area should be added.

It is important to note that both the μ and I curves approach zero at the ends of the ship. While the extreme stations (0 and 20) are taken at the perpendiculars, so that there is a projection of the hull beyond these stations at each end, the μ and I curves may be plotted to give excessive values of μ/I near the ends of the ship. Such a condition can cause considerable error in the digital calculation of the natural frequencies because it effectively makes the ends of the ship very "flabby." If the ship actually had such a construction, the lowest mode would be determined by this local condition at the ends. On the other hand, no amount of overestimating of I at the ends can have much effect on the natural frequency since it simply means that the overhang at each end is rigidly attached to the ship at the perpendicular. Therefore, the rule is to check at the ends to make sure that the value of μ/I does not rise abruptly. This precaution was followed in this illustration.

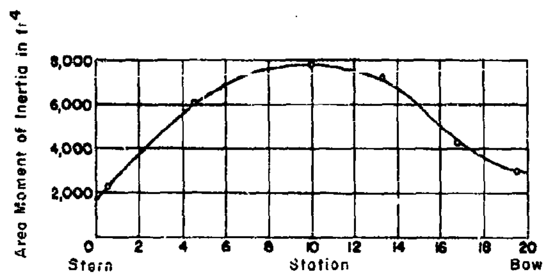


Figure A-5 – Area Moment of Inertia Curve Used for Calculation of Vertical Modes of GOPHER MARINER

Shear Rigidity Factor, K

For GOPHER MARINER the shear rigidity factor was evaluated by letting $KA = A'$, where A is the total area used in the evaluation of I, and A' is the area of the vertical plating only (such as the side shell plating and continuous longitudinal bulkhead plating). The "web" area A' was plotted for as many sections as were available (in this case six) and a smooth curve was drawn as shown in Figure A-6.

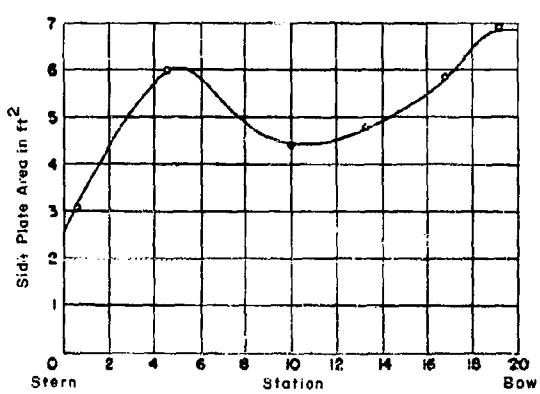


Figure A-6 – Side Plating Area Curve Used in Evaluating Vertical Shearing Rigidity of GOPHER MARINER

The values actually furnished for the digital calculation of the vertical modes of GOPHER MARINER for a heavy displacement loading are given in Table A-1. In this case, the term for rotary inertia was omitted on the assumption that its effect would be negligible. The reader is cautioned against using Table A-1 as a sample for furnishing data to the

TABLE A-1

Initial Data Furnished for the Digital Calculation of Vertical Modes of GOPHER MARINER for Heavy Loading Condition

n	m $\frac{\text{ton-sec}^2}{\text{ft}}$	$\frac{\Delta x}{EI} \times 10^8$ $\frac{1}{\text{ton-ft}}$	$\frac{\Delta x}{KAG} \times 10^6$ ft/ton
0	7.48	0.3160	11.35
1	13.97	0.5205	8.56
2	22.31	0.3615	7.17
3	38.76	0.2880	6.75
4	56.51	0.2455	5.72
5	73.04	0.2185	5.67
6	69.14	0.2005	6.08
7	75.89	0.1880	6.70
8	85.47	0.1800	7.23
9	90.79	0.1760	7.62
10	95.11	0.1750	7.73
11	80.35	0.1750	7.63
12	75.76	0.1790	7.37
13	67.02	0.1885	7.05
14	50.61	0.2040	6.73
15	41.14	0.2320	6.33
16	25.77	0.2740	5.98
17	26.61	0.3270	5.52
18	23.80	0.3805	5.06
19	14.94	0.5265	4.96
20	5.53	0.3240	-

$\Delta x = 26.25$ ft
$L = 525$ ft
$B = 76.0$ ft
$D = 44$ ft 6 in.

Displacement: 16,840 tons

Applied Mathematics Laboratory of the David Taylor Model Basin for future hull vibration calculations. The slightest changes made in recoding the problem for a new computing machine may make it impossible for the machine to carry out the computation with the initial data as presented, and current Applied Mathematics Laboratory instructions must be followed.

The results obtained from the digital calculation of the 2-node vertical mode of GOPHER MARINER for the heavy displacement are given in Table A-2.

It will be noted that, although the calculation was made by means of the coding for torsion-bending modes, and, since in this case the terms coupling flexure and torsion were zero, the columns in Table A-2 for angular displacement ϕ and twisting moment T are both zero.

TABLE A-2

Data Furnished by Digital Computer for 2-Node Vertical Mode of GOPHER MARINER

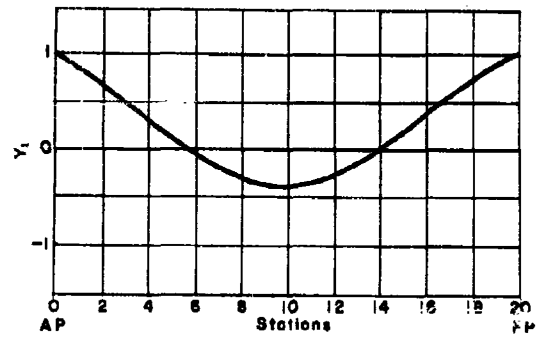
Station No.	*Y*	γ	ϕ	*V*	*M*	*T*
0	−1.00000	0.00641	0	0	0	0
1	−0.82620	0.00641	0	− 473	− 12,426	0
2	−0.64917	0.00635	0	−1203	− 44,026	0
3	−0.47141	0.00619	0	−2120	− 99,686	0
4	−0.29428	0.00590	0	−3276	−185,700	0
5	−0.12648	0.00544	0	−4329	−299,341	0
6	0.02725	0.00479	0	−4913	−428,329	0
7	0.15974	0.00393	0	−4794	−554,186	0
8	0.26270	0.00289	0	−4027	−659,905	0
9	0.32635	0.00170	0	−2606	−728,322	0
10	0.34308	0.00042	0	− 731	−747,518	0
11	0.30958	−0.00088	0	1333	−712,506	0
12	0.23148	−0.00213	0	2908	−636,170	0
13	0.11606	−0.00326	0	4017	−530,701	0
14	−0.02779	−0.00426	0	4510	−412,310	0
15	−0.19169	−0.00511	0	4421	−296,255	0
16	−0.36871	−0.00579	0	3922	−193,302	0
17	−0.55466	−0.00632	0	3320	−106,134	0
18	−0.74303	−0.00667	0	2386	− 43,485	0
19	−0.92899	−0.00683	0	1267	− 10,214	0
20	−1.11188	−0.00689	0	389	0	0
20A		−0.00689		0		0

The normal mode patterns of vertical displacement, angular displacement about a *Z*-axis, shear force, and bending moment, for the 2-node vertical flexural mode of GOPHER MARINER as determined by the digital calculation can then be plotted from Table A-2. The absolute values given in this table have no significance. Only the relative magnitudes in a normal mode pattern are indicated. It is to be noted, however, that the data were prepared in the foot-ton-second system of units. Hence, although a single amplitude of 1 foot at the after perpendicular would be an extremely large amplitude, the values of bending moment per foot amplitude at this station are given in the column for *M* provided the hull is not deformed beyond the elastic range. For smaller amplitudes the bending moments would be proportionately smaller.

The normal mode pattern for displacement given by Table A-2 is plotted in Figure A-7 (with sign reversed).

For further details on the preparation of data for a calculation of this type, see Reference A-1.

Figure A-7 – Normal Mode Pattern for 2-Node Vertical Flexural Mode of GOPHER MARINER as Computed by Digital Method for Heavy Displacement



REFERENCES

A-1. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

A-2. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

A-3. Russo, V.L. and Sullivan, E.K., "Design of the MARINER-Type Ship," Trans. SNAME, Vol. 61 (1953).

A-4. Prohaska, C.W., "The Vertical Vibration of Ships," Shipbuilder, Vol. 54 (Oct, Nov 1947).

APPENDIX B

ILLUSTRATION OF A CALCULATION OF TRANSIENT RESPONSE BY THE DIGITAL METHOD

As in Appendix A, the calculation presented in this Appendix serves to illustrate the basic principles involved, but it is not intended as a sample to be used in preparing requests for transient hull vibration calculations to be carried out by the Applied Mathematics Laboratory of the David Taylor Model Basin. As a matter of fact, the recoding of this particular problem with the addition of buoyancy forces is in progress at this time.

The analysis of transient vibration discussed in this book is confined to vibrations within the elastic range, as pointed out in Chapter 5. For a discussion of the general problem of ship slamming in a seaway, see Reference B-1. The digital method of transient analysis discussed in Chapter 5 was applied to GOPHER MARINER in the prototype example used in coding the initial treatment of this problem on the UNIVAC. Further details of the prototype calculation will be found in References B-2 and B-3.

The loading conditions for the calculation were as follows: a vertical force of 1 ton was instantaneously applied amidships, held constant for 1 sec, then instantaneously removed. No special significance is to be attached to the particular loading chosen for this example, at least for simulating the loading encountered by a ship in a seaway. Mathematically, however, this is a standard type of forcing function sometimes described as a "rectangular pulse load."

The reference axes were the same as indicated in Figure A-1, and an element of length Δx is shown in Figure 5-2. The hull data used for this calculation were the same as used in the calculation of the 2-node vertical normal mode given in Appendix A (same ship and same displacement in both cases) with the exception that the parameters I_{mz} (rotary inertia) and C (damping) were added. These additional parameters are given in Table B-1.

The rotary inertia values I_{mz} were estimated in this case by treating each lumped mass m (which includes a value for added mass of water) as though it were uniformly distributed throughout a rectangular parallelepiped bounded by the main deck, the bottom plating, and the hull side plating, and taking the mass moment of inertia of this parallelepiped with respect to a Z-axis passing through its centroid.

The damping values C were based on a value of 1 for the ratio c/μ. The experimental basis for this value is discussed in Chapter 8. In this calculation the time steps selected (Δt) were 0.02 sec. Hence, the load data was 1 ton at Station 10 for time steps 0 through 49 and zero at this station for all subsequent times. At all other stations the load was zero at all times.

The calculation was carried out for 250 time steps or for a total duration of 5 sec and the results were printed for each step.

TABLE B-1

Additional Parameters for GOPHER MARINER Used in the Calculation of Transient Response by the Digital Method

These parameters were not included in the normal mode calculation given in Appendix A.

n	I_{mz} ton-sec^2-ft	C ton-sec/ft
0	1,068	7.48
1	4,663	13.97
2	6,122	22.31
3	9,128	38.76
4	13,960	56.51
5	20,430	73.04
6	19,400	69.14
7	21,530	75.89
8	24,940	85.47
9	27,470	90.79
10	30,320	95.11
11	23,470	80.35
12	21,950	75.76
13	18,680	67.02
14	13,650	50.61
15	9,972	41.14
16	5,836	25.77
17	5,569	26.61
18	3,577	23.80
19	1,989	14.94
20	384	5.53

Figure B-1 shows the calculated displacement at Station 10 over the 5-sec interval.

Figure B-2 shows the instantaneous displacements and bending moments calculated for all stations of the hull at the instant when the peak bending moment was reached.

At the stage of the development of the transient hull vibration calculation at which this example was run, buoyancy and gravity forces were not included. Thus the hull was treated as free in space as in the hull normal mode calculations. As shown in Reference B-4, the coding was later modified to include buoyancy and gravity forces.

It must be realized that when the idealized nonuniform beam (with added mass and free in space) is subjected to a unidirectional force there will ensue, in general, not only vibratory elastic motions but also rigid body motions in both translation and rotation. Since the gravity and buoyancy forces are not included in the treatment discussed in this Appendix, the calculation will not be realistic if carried out long enough for large rigid body displacements to build up. The revised coding discussed in Reference B-4 will include gravity forces, buoyancy forces, an added mass that varies with heave and pitch, as well as provision for a forcing function varying arbitrarily in space and time. Plans are also underway for handling the hull slamming problem by means of an analog computer; see Reference B-5.

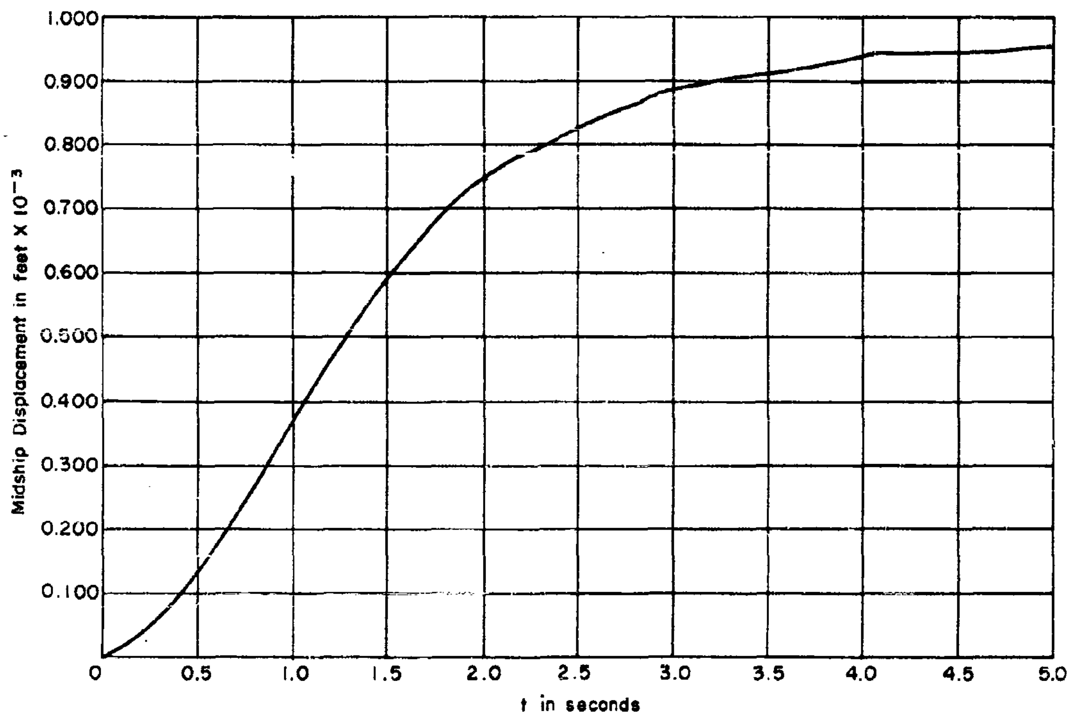


Figure B-1 – Transient Response of GOPHER MARINER with Displacement Calculated at Station 10

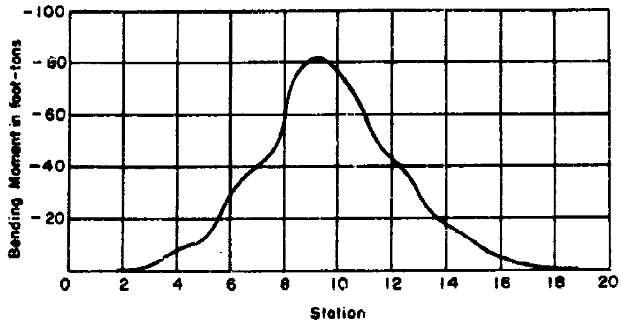


Figure B-2a – Bending Moment

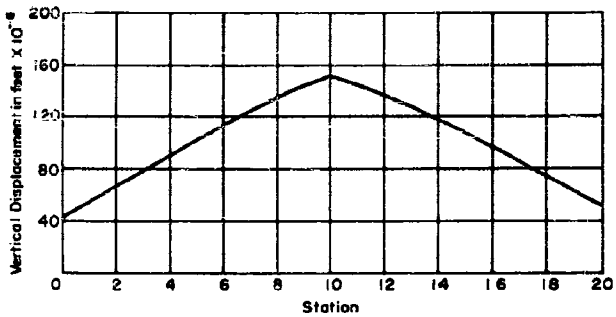


Figure B-2b – Vertical Displacement

Figure B-2 – Transient Response of GOPHER MARINER to 1-Ton Rectangular Pulse Load Amidships at Instant of Peak Bending Moment by Digital Calculation

REFERENCES

B-1. Jasper, N.H. and Birmingham, J.T., "Strains and Motions of USS ESSEX (CVA 9) during Storms near Cape Horn," TMB Report 1216 (Aug 1958).

B-2. McGoldrick, R.T., "Calculation of the Response of a Ship Hull to a Transient Load by a Digital Process," TMB Report 1119 (Mar 1957).

B-3. Polachek, H., "Calculation of Transient Excitation of Ship Hulls by Finite Difference Methods," TMB Report 1120 (Jul 1957).

B-4. Cuthill, E. and Olsten, E., "A Routine for the Calculation of the Response of a Ship to Transient Forces," TMB Report (in preparation).

B-5. Hoyt, E.D., "Simulation of the Sea and Analog Computation of the Forces on a Ship in Waves," Report to TMB on Contract Nobs 72376, T.O. 9, Project RR-1458-N by Reed Research, Inc (Nov 1959).

APPENDIX C

EMPIRICAL FORMULAS FOR HULL FREQUENCIES

A. FORMULAS FOR FREQUENCY OF 2-NODE VERTICAL MODE

It was pointed out in Chapter 10 and is emphasized in Reference C-1 that the designer is fortunate in having a very simple rough rule for the ratios of the frequencies of the higher vertical modes of vibration of a surface ship to the frequency of the fundamental mode. This rule is that the ratios follow the series 1, 2, 3, etc.

Some idea of the roughness of this rule may be obtained from Table C-1, based on experimental data obtained by the David Taylor Model Basin and summarized in Reference C-2. The average values in Table C-1 conform quite closely with the rule although several of the individual deviations are larger than 15 percent.

The availability of such a rule enhances the value of the empirical formulas for estimating the 2-node vertical frequency of ships since these, in effect, yield an estimate of the frequencies of the principal vertical modes.

The most widely known of the empirical formulas for the frequency of the 2-node vertical mode of a surface ship is the formula of O. Schlick:[C-3]

$$N = C\sqrt{\frac{I}{\Delta L^3}} \qquad [C\text{-}1]$$

TABLE C-1

Experimentally Determined Ratios of Frequencies of Higher Vertical Modes of Ships to the Frequency of the Fundamental Vertical Mode

Ship	Type	Ratios			
		1st Mode	2nd Mode	3rd Mode	4th Mode
NIAGARA	Transport	1	1.83	2.65	3.23
CHARLES R. WARE	Destroyer	1	2.08	3.30	4.55
C. A. PAUL	Ore Carrier	1	2.35	3.73	4.67
PERE MARQUETTE	Car Ferry	1	2.00	3.10	4.57
OLD COLONY MARINER	Dry Cargo	1	1.89	2.77	3.28
NORTHAMPTON	Cruiser	1	2.11	2.99	4.23
STATEN ISLAND	Ice Breaker	1	1.93	2.57	
Average		1	2.03	3.01	4.09

where N is frequency in vibrations per min,

C is Schlick's empirical "constant" ranging from 1.28×10^5 to 1.57×10^5,

I is the area moment of inertia of midship section in ft^2-in.^{2} units,

Δ is the displacement of ship in tons (2240 lb), and

L is the length of ship in ft.

Note that this formula is similar to the formula for the free-free uniform bar discussed in Chapter 3. The simplicity of the formula is naturally appealing but there remains the problem of selecting the best value of the empirical "constant." Table C-2, taken from Reference C-4, gives experimentally determined values of the Schlick constant for various types of ships. Such a table can be used by the designer as a guide in selecting an appropriate value of C to use in Equation [C-1]. It is recommended, however, that, if the designer can obtain more recent information on both the natural frequency and the design parameters of a ship of a type more nearly similar to the type he is designing, he use this information to estimate the fundamental vertical frequency of the new ship. Thus,

$$N_n = N_o \sqrt{\frac{I_n \, \Delta_o \, L_o^3}{I_o \, \Delta_n \, L_n^3}} \qquad [C\text{-}2]$$

where the symbols have the same meaning as in Equation [C-1] except that the subscript n applies to the new ship, and the subscript o applies to the old ship.

TABLE C-2

Empirical Values of Schlick Constants

Author and Reference	Type of Ship	Displacement tons	Overall Length ft	Moment of Inertia Amidships ft^2-in^2	$\sqrt{\frac{I}{DL^3}} \times 10^4$	Measured Frequency per min	Equiv. Schlick Constant $\times 10^{-5}$
Todd (C-5)	Tanker	15,190	440	476,000	6.08	78.9	1.30
Tobin (C-6)	"	8,300	350	233,890	8.11	112	1.38
Nicholls (C-7)	Destroyer	1,378	310	33,000	8.96	120	1.34
Schmidt (C-8)	Motorship	7,010	484	718,000	9.50	106	1.12
Schadlofsky (C-9)	Tanker	16,600	462	604,000	6.08	81	1.33
	Cable layer	834	181	15,600	17.8	207.5	1.17
	Tanker	8,160	366	234,000	7.65	112	1.47
	Freighter	8,360	371	264,000	7.88	105	1.33
Cole (C-10)	Tanker	8,151	350	233,890	8.19	112	1.37
EMB Report (C-11)	Destroyer	1,382	310	35,000	9.22	107	1.16
Roop (C-12)	Tanker	15,430	475	447,000	5.20	60.3	1.16
	Battleship	32,000	583	1,325,000	4.57	77.1	1.69

L. C. Burrill[C-13] proposed a formula of greater flexibility than the Schlick formula in that it includes the beam and the draft. Hence, while it also included an empirical constant, the latter was expected to be more stable than the Schlick constant. Burrill's formula is

$$N = \frac{\phi}{\sqrt{\left(1 + \frac{B}{2d}\right)(1 + r)}} \sqrt{\frac{I}{\Delta L^3}} \qquad \text{[C-3]}$$

where ϕ is an empirical coefficient given by Burrill as 24×10^5,
N is the fundamental vertical frequency in cpm,
I is the effective moment of inertia of the midship section area in ft^4,
Δ is the displacement in tons,
L is the length between perpendiculars in ft,
B is the beam in ft,
d is the draft in ft, and
r is J. Lockwood Taylor's shear correction factor

$$\left[r = \frac{3.5 D^2 (3a^3 + 9a^2 + 6a + 1.2)}{L^2 (3a + 1)} \right] \qquad \text{[C-4]}$$

where $a = \frac{B}{D}$ and D is the molded depth in ft. J. Lockwood Taylor's shear correction factor is discussed further in Reference C-14.

Some idea of the constancy of Burrill's coefficient can be obtained from Table C-3, derived from Reference C-15. Unfortunately, several of the values given in Table C-3 were based on tests in shallow water, a condition known to increase the water inertia effect.

Two other empirical formulas for the frequency of the 2-node vertical mode of vibration of ships are given here without further discussion; namely, the formulas of Prohaska, and Todd and Marwood.

Prohaska's formula (see Reference C-16) is

$$N = \frac{100 R}{\sqrt{q(1 + c)}} \sqrt{\frac{I}{\frac{\Delta}{1{,}000}\left(\frac{L}{100}\right)^3}} \qquad \text{[C-5]}$$

where N is the 2-node vertical frequency,
R is $r_1 r_2 r_3$,
r_1 is the correction for variable inertia,
r_2 is the correction for shearing force,
r_3 is the correction for transverse compression and dilatation,
q is the mass distribution coefficient,

TABLE C-3

Experimental Values of Burrill's Coefficient Derived from Vibration Generator Tests of Ships

Name of Ship	Type of Ship	Design Displacement tons	Test Displacement / Design Displacement	Depth of Water under Keel	Burrill's Coefficient
HAMILTON	Destroyer			16.3 feet	22.7×10^5
SOUTH DAKOTA	Battleship	42,500	0.90	Shallow	26.7×10^5
ALASKA	Battle Cruiser	31,600	0.90	Shallow	24.3×10^5
SHILOH	Tanker	21,800	0.27	Shallow	33.0×10^5
PHILIP SCHUYLER	Cargo	14,200 (full load)	0.36	14 feet	21.4×10^5
PONTCHARTRAIN	Coast Guard Cutter	1,970 (full load)	0.75	Shallow	27.7×10^5
MACKINAW	Icebreaker	5,090	0.81	80 feet 2 feet	27.5×10^5 21.0×10^5
NIAGARA	Transport	6,740 (full load)	0.82	127 feet	25.5×10^5

c is the added mass of water/displacement of ship,

I is the moment of inertia of midship section,

Δ is the displacement, and

L is the length of ship.

The choice of units in this formula and the evaluation of the r's are discussed in Reference C-16. It is noted here, however, that the formula, like the Schlick formula, is essentially the formula for the free-free uniform bar. Since c, q, r_1, r_2, r_3, and R are all dimensionless, one is at liberty to apply Prohaska's factor

$$\frac{R}{\sqrt{q(1+c)}}$$

to the uniform bar formula in any consistent set of units. The uniform bar formula is

$$\omega_1 = \frac{22.4}{l^2}\sqrt{\frac{EI}{\mu}} \qquad \text{[C-6]}$$

where ω_1 is the natural circular frequency of the 2-node flexural mode,

EI is the bending rigidity, and

μ is the mass per unit length.

This gives

$$N_1 = 3.58\sqrt{\frac{EI}{\mu l^4}} = 3.58\sqrt{\frac{EI}{ML^3}} \qquad [C\text{-}7]$$

where N_1 is the frequency of the uniform bar in cps,
E is Young's modulus in tons/ft^2,
I is the area moment of inertia in ft^4,
M is the total mass of the bar in ton-sec^2/ft units, and
L is the length of the bar in ft.

Hence these units may be retained in applying Prohaska's formula for the ship which becomes

$$N_1 = \frac{3.58R}{\sqrt{q(1+c)}}\sqrt{\frac{EI}{\frac{\Delta L^3}{g}}} \qquad [C\text{-}8]$$

where Δ is the displacement in tons,
g is the acceleration of gravity in ft/sec^2, and
R, q, and c are to be derived in nondimensional units from Prohaska's paper.[C-16]

The Todd and Marwood formula (see Reference C-17) is

$$N = \beta\sqrt{\frac{BD^3}{\Delta L^3}} \qquad [C\text{-}9]$$

where N_1 is the 2-node vertical frequency in cpm,
B is the moulded breadth in ft,
D is the moulded depth at side in ft,
Δ is the displacement in tons,
L is the length between perpendiculars in ft, and
β is an empirical coefficient.

In Reference C-18 Todd gives values of β ranging from 45,200 to 62,500.

B. FORMULA FOR RATIO OF HORIZONTAL TO VERTICAL FREQUENCIES

Experience has shown that the ratio of the 2-node horizontal frequency to the 2-node vertical frequency is roughly 1.5, as pointed out in Reference C-19. In Reference C-20 A. J. Johnson proposed the simple formula

$$\frac{N_H}{N_V} = \sqrt{\frac{I_H \times V_V}{I_V \times EV_H}} \qquad [C\text{-}10]$$

where N_H is the 2-node horizontal frequency,

N_V is the 2-node vertical frequency,

I_H is the moment of inertia of midship section area for bending in horizontal plane,

I_V is the moment of inertia of midship section area for bending in vertical plane,

V_V is the vertical virtual inertia factor =

$$1 + \frac{\text{added weight for vertical vibration}}{\text{displacement}}\text{, and}$$

EV_H is the equivalent horizontal virtual inertia factor =

$$1 + \frac{\text{effective added weight for horizontal vibration}}{\text{displacement}} .$$

Although the data available on horizontal modes are more scanty at this time than the data on vertical modes, the 1, 2, 3 rule for estimating the frequencies of the higher modes from the frequency of the 2-node mode still seems to have utility.

C. HORN'S FORMULA FOR THE 1-NODE TORSIONAL FREQUENCY

Horn's empirical formula for the fundamental torsional frequency of surface ships (see Reference C-21) is

$$N_e = 60\,k\sqrt{\frac{g\,GJ_{e0}}{\Delta(B^2 + D^2)\,L}} \qquad \text{[C-11]}$$

If English units are used in this formula, then

N_e is the natural frequency in cpm,

g is the acceleration of gravity in ft/sec^2,

J_{e0} is the effective polar moment of inertia of midship section area in ft^4,

Δ is the displacement in tons,

B is the beam in ft,

D is the depth in ft,

L is the length in ft,

G is the shear modulus of elasticity in tons/ft^2, and

k is the Horn's empirical constant.

For J_{e0}, Horn suggested using the formula for torsional rigidity proposed by L. Gumbel.[C-22]

$$J_{e0} = \frac{4F_0^{\,2}}{\Sigma \dfrac{\Delta s}{\delta}} \qquad \text{[C-12]}$$

where F_0 is the area enclosed by the shell plating of the midship section (not the area of the material),

δ is the plating thickness, and

Δs is a small distance along the shell plating in the plane of the section.

This formula follows from the shear flow concept and does not take account of the effect of inner decks and longitudinal bulkheads.

Horn assigned the value of 1.58 to k for a freighter (SS WASGENWALD). As in the case of the Schlick formula, the best procedure for the designer is to use Equation [C-11] in conjunction with data on the torsional frequency of a ship of the general type he is designing, if such data can be obtained. Moreover, in estimating J_{e0} he is not restricted to Equation [C-12] but may apply any method of estimating torsional rigidity of hulls; see, for example, Reference C-23.

In estimating the frequencies of higher torsional modes from the fundamental torsional frequency, little guidance can be offered at this time. It may be pointed out that Horn[C-21] found for the first three torsional frequencies of a freighter the ratios 1 : 1.6 : 1.9, whereas for GOPHER MARINER, if the apparent torsion bending modes are considered as flexure-free torsional modes, the corresponding ratios were found to be 1 : 1.6 : 2.2.

REFERENCES

C- 1. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol 63 (1955). Also TMB Report 1060 (Jul 1956).

C- 2. McGoldrick, R.T., "Comparison between Theoretically and Experimentally Determined Natural Frequencies and Modes of Vibration of Ships," TMB Report 906 (Aug 1954).

C- 3. Schlick, O., Series of Articles on Ship Vibration in TINA (1884, 1893, 1894, 1901, and 1911).

C- 4. McGoldrick, R.T., "A Study of Ship Hull Vibration," EMB Report 395 (Feb 1935).

C- 5. Todd, F.H., "Ship Vibration—A Comparison of Measured and Calculated Frequencies," NECI Trans (1933).

C- 6. Tobin, T.C., "A Method of Determining the Natural Periods of Vibration of Ships," TINA (1922).

C- 7. Nicholls, H.W., "Vibration of Ships," TINA (1924).

C- 8. Schmidt, F., "Die Berechnung von Schiffschwingungen," WRH, No. 6 (22 Mar 1930).

C- 9. Schadlofsky, E., "The Calculation and Measurement of Elastic Natural Frequencies of Ship Hulls," STG, Vol. 33 (1932). Also EMB Translation 7 (Jun 1934).

C-10. Cole, A.P., "The Natural Periods of Vibrations of Ships," Trans Inst of Engrs and Shipb Scotland, Vol. LXXII (1928-29).

C-11. McGoldrick, R.T., "Vibration Tests on USS HAMILTON at Washington Navy Yard," EMB Report 372 (Dec 1933).

C-12. Roop, W.P., "Natural Frequencies of Hull Vibration," Trans. SNAME (1930).

C-13. Burrill, L.C., "Ship Vibration: Simple Methods of Estimating Critical Frequencies," NECI Trans, Vol. 51 (1934-35).

C-14. Taylor, J.L., "Vibration of Ships," TINA (1930).

C-15. Hardy, V.S., "Vibration Studies of Ship Hulls by Means of Vibration Generators," TMB Report C-80 (Nov 1949) Unclassified.

C-16. Prohaska, C.W., "The Vertical Vibration of Ships," Shipbuilder, Vol. 54 (Oct, Nov 1947).

C-17. Todd, F.H. and Marwood, W.J., "Ship Vibration," NECI Trans, Vol. 64 (1948).

C-18. Todd, F.H., "The Fundamentals of Ship Vibration," Shipbuilder, Vol. 54 (May, Jun, Jul 1947).

C-19. Tachmindji, A.J. and McGoldrick, R.T., "Note on Propeller-Excited Hull Vibrations," JSR, Vol. 3, No. 1 (Jun 1959).

C-20. Johnson, A.J., "Vibration Tests on All-Welded and All-Riveted 10,000 Ton Dry Cargo Ships," NECI Trans, Vol. 67 (1951).

C-21. Horn, F., "Horizontal und Torsions–Schiffschwingungen auf Frachtschiffen," WRH, Vol. 6, No. 18 (1925).

C-22. Gumbel, L., "Torsional Vibrations of Elastic Shafts of Any Cross Section and Mass Distribution and Their Application to the Vibration of Ships," TINA, Vol. LIV (1912).

C-23. Vedeler, G., "On the Torsion of Ships," TINA, Vol. LXVI (1924).

APPENDIX D

EMPIRICAL FORMULAS FOR ESTIMATING THE LEVEL OF STERN VIBRATION

It is pointed out in Chapters 2 and 4 that the propeller-excited or blade-frequency vibration of ships is often concentrated in the stern and is practically imperceptible at points further forward than 25 percent of the length from the after perpendicular. This condition prevails when the operating blade frequencies are higher than the range of frequencies of significant hull modes.

An explanation of this phenomenon has been suggested in Chapter 4, and this Appendix is concerned only with the empirical formulas for estimating the amplitude of this stern vibration under such circumstances. The formulas presented here were derived from the concept of mechanical impedance which is the ratio of the driving force to the response in steady-state vibration. Obviously, impedance formulas are useful in predicting levels of service vibration only when the exciting forces themselves can first be predicted. The problem of estimating the exciting forces is discussed in Chapter 7.

The concept of mechanical impedance, although not an essentially new concept, has gained acceptance in naval architecture only quite recently. The preference of Committee S-2 (Mechanical Vibration and Shock) of the American Standards Association has been to define mechanical impedance in terms of vibratory velocity; see Reference D-1. This preference has been influenced by the concept of impedance in the field of acoustics in which driving pressures and particle velocities are important quantities. When so defined, mechanical impedance corresponds with electrical impedance in the classical analogy wherein electrical current is analogous to mechanical velocity, electrical voltage is analogous to mechanical force, and electrical impedance is analogous to mechanical impedance. Thus, the familiar electrical equation of alternating current theory is

$$I = \frac{E}{Z} \qquad \text{[D-1]}$$

where I is the current,

E is the voltage, and

Z is the impedance.

This equation corresponds to the equation in mechanical vibration

$$\dot{Y} = \frac{P_0}{Z} \qquad \text{[D-2]}$$

where $\dot{Y}$ is the single amplitude of the velocity,

P_0 is the single amplitude of the driving force, and

Z is the mechanical impedance.

In naval architecture, however, the designer is much more familiar with levels of vibration expressed in terms of displacement amplitude than in terms of vibratory velocity amplitude.

The formulas given here are based on experimental data on steady-state hull vibration in the stern produced by known mechanical exciting forces. They are written in terms of driving force and displacement amplitude and do not employ the mechanical impedance term implicitly. Their generality, however, does actually depend on the scaling of mechanical impedance. At this time it can only be said that, since hulls in general have been observed to follow the same general pattern of stern vibration (once the frequency rises above the range of significant hull natural frequencies) and, since at high frequencies mechanical impedance is chiefly inertial, there is some logic in expecting that the same empirical constant will find application to ships of different types.

Only when the designer has access to much more data than are now available will he be in a position to decide whether he can use a universal constant in the formula for stern vibration or whether he will have to establish separate constants for the various classes of ships.

The formula proposed in Reference D-2 for estimating vertical vibration at the after perpendicular under the conditions just stated is

$$Y \approx \frac{P_0}{3.4 \times 10^{-6} \times \Delta \times (\text{cpm})^2} \qquad \text{[D-3]}$$

where Y is the single amplitude in mils,

P_0 is the single amplitude of the vertical component of propeller-exciting force in lb (at blade frequency),

Δ is the displacement of the ship in long tons, and

cpm is the blade frequency in cycles per minute.

The empirical constant in this formula is the factor 3.4×10^{-6}. Should future experimental data show a wide variation, then, in using the formula, the designer should select his own factor from experimental data for the ship type nearest to the one he is designing.

It will be recalled that the rule for the number of significant vertical modes was adopted from Reference D-3; namely

$$N' \approx \frac{5L}{9D} \approx \frac{L}{B} \qquad \text{[D-4]}$$

where N' is the number of significant vertical modes,

L is the ship length,

D is the ship depth, and

B is the ship beam.

In considering the use of empirical formulas for horizontal and torsional vibration at the stern, the designer must recognize that, just as in the vertical case, such formulas are applicable only when the blade frequencies fall well above the range of significant hull mode frequencies. This means that there must be no significant hull horizontal or torsional natural frequencies in this range. It is also suggested that if in the approximate Equation [D-4], N' is 6 for vertical, the limit for horizontal is to be taken as 4, for torsional as 3. Thus, if in the case of a long slender hull the number of significant vertical modes is estimated to be greater than 6, the number of horizontal modes would be considered greater than 4, and the torsional modes greater than 3 in the same ratio.

There is, however, another element involved in dealing with horizontal and torsional stern vibration; namely, the possibility of coupling of these two motions. As pointed out in Chapter 7, the blade-frequency excitation at the stern is reduced for simplicity to a vertical force, a horizontal force considered to act at the center of twist, and a couple whose axis is parallel to the longitudinal axis of the ship. It does not necessarily follow, however, that if the couple is zero there will be no torsional vibration at the stern. Nor does it necessarily follow that, if the horizontal force is zero, there will be no horizontal vibration at the stern. This results from the fact that the effective center of mass may not fall on the axis of twist (as pointed out in Chapter 3), so that flexural vibration and torsional vibration are coupled in the hull.

At this stage of the development of ship vibration theory it is only feasible to propose empirical formulas for horizontal and torsional vibration at the stern on the assumption that this coupling effect is negligible. The possible coupling effects are discussed further in Reference D-4.

The empirical formula for horizontal stern vibration is then similar to that for vertical; namely, the approximate Equation [D-3]. The only difference is the factor to be used in the denominator. At this time, a factor based on only a single test is available. This factor is 1.9×10^{-6}, and is based on a vibration generator test on the USS DECATUR (DD 936) described in Reference D-5.

The similar empirical formula for torsional vibration in the stern is

$$\phi \approx \frac{T_0}{0.46 \times I \times (\text{cpm})^2} \qquad \text{[D-5]}$$

where ϕ is the single amplitude in radians,

T_0 is the single amplitude of the blade-frequency exciting couple in lb-ft,

I is the mass moment of inertia of the entire ship about the longitudinal axis through its center of gravity. This does not include any allowance for virtual mass moment of inertia of the surrounding water. It is expressed in ton-sec^2-ft units,

cpm is the blade frequency in cycles per minute.

Here again the factor 0.46 is based on a single test made on DECATUR, described in Reference D-5. The designer should inquire for later information if available in the future.

A final word of caution in the use of the formulas given in this Appendix is necessary. If any local stern structure or appendage has a natural frequency in the range of the driving blade frequencies to be specified by the designer then, even though the conditions for freedom from hull natural frequencies are met, the empirical formulas cannot forecast the stern amplitude under a given excitation, and the designer must either resort to the more detailed analyses discussed in Reference D-4 or assure himself that the exciting forces will be within safe limits.

REFERENCES

D-1. American Standards Association, "Proposed American Standard Nomenclature and Symbols for Mechanical Impedance of Structures," (pending).

D-2. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

D-3. Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single-Screw Vessels," Trans. SNAME, Vol. 60 (1952).

D-4. Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

D-5. Paladino, A.R., "Vibration Survey on USS DECATUR (DD 936)," TMB Report 1271 (Mar 1960).

Calibration

(a) Frequency response – For vibration of propeller blade frequency, a minimum frequency response of 3 cps is required. For shaft frequency (first order) a lower limit of 1 cps is required. This may be attained if the respective transducer or vibrograph is critically damped, producing a linear response at or below resonance.

(b) Amplitude response of ± ½ mil to ± 100 mils.

(c) Laboratory calibration check – Prior to each test, or at least every three months, the transducers and vibrographs should be calibrated. In the case of the electronic equipment, the complete system should be calibrated.

(d) Shipboard calibration check – At intervals throughout the test checks should be made on the electronic system by introducing known electrical signals. This will avoid error due to possible drift in the system.

Test Procedure

(a) Determine ship particulars:

(1) Ship dimensions.

(2) Propeller: type, dimensions, number, and number of blades.

(3) Propeller aperture clearance.

(4) After body configuration.

(5) Type of power plant.

(6) SHP, RPM, and speed.

(b) Determine test conditions:

(1) Displacement.

(2) Drafts forward and aft.

(3) Loading plan.

(4) Depth of water (120-ft minimum).

(5) Sea state (limit based on ship length).

(c) Take data during following conditions:

(1) Each 5-rpm increment from one-half to full power.

(2) Hard turn to port.

(3) Hard turn to starboard.

(4) Crashback – Full ahead to full astern.

(d) Data-taking procedure:

(1) Permit ship to steady on speed for constant speed runs.

(2) Take sufficient length of tape to permit collection of maximum and minimum values (about 30 sec for single-screw ships).

(3) For maneuvers, start recorder as throttle or wheel is moved. Allow to run until maximum vibration is noted.

Data Analysis and Reporting

(a) The following data are to be evaluated for all runs:

(1) Maximum overall values of amplitude.

(2) Maximum first-order amplitude.

(3) Maximum amplitude of blade frequency.

(4) Maximum amplitude of blade-frequency harmonics.

(b) Data presentation should include the following curves plotted on a basis of shaft rpm:

(1) Vertical hull vibration.

(2) Athwartship hull vibration.

(3) Vertical vibration at main thrust bearing.

(4) Athwartship vibration at main thrust bearing.

(5) Fore-and-aft vibration at main thrust bearing.

(6) Other curves as appropriate.

(c) Method of presentation of data:

(1) All curves should show single amplitude of displacement in mils plotted against rpm.

(2) Maximum amplitudes obtained during maneuvering runs should be presented in tabular form giving frequencies and amplitudes.

It will be noted that this draft of the Code makes no provision for measuring torsional vibration of the hull. The reason for this is that trouble with hull torsional vibration is so unusual that if a case develops a special investigation will probably be authorized.

REFERENCE

E-1. Noonan, Knopfle, and Feldman, a proposal for "A Code for Shipboard Hull Vibration Measurements," in preparation for SNAME.

APPENDIX F

LEVELS OF SERVICE VIBRATION

Even if hull vibration analysis were developed to the stage at which the designer could accurately predict the level of service vibration for a ship of given design, he would still be faced with the question of whether the level of vibration thus predicted was acceptable. This brings up the vital problem of vibration norms and the definition of such terms as "normal," "tolerable," "permissible," "acceptable," "severe," "intolerable," or "unacceptable," as applied to ship vibration.

Although at the present time it is safe to state that the naval architect is well aware of the need for standards of comparison in this field, no definite standards have as yet been established in the United States. As a matter of fact, there is a natural reluctance on the part of both the shipbuilder and the ship operator to collaborate in any program for establishing such standards because of adverse effect that publicity regarding the vibration levels on a particular ship might have on its earning capacity. Furthermore, in the case of naval ships, there is the added hazard of revealing classified information in presenting data on the levels of service vibration.

Accordingly, it seems feasible to include in this Appendix only certain data which have been accumulated in the course of the ship vibration research carried on by the David Taylor Model Basin, and which have already been disclosed elsewhere in unclassified reports or papers. Such information will serve to familiarize the reader with some of the levels of hull vibration encountered in practice. Much additional information on this phase of the subject will be found in the references listed in the general bibliography, and certainly much more will be available in the future.

The following quotation is taken from Reference F-1:

"While it would be of great assistance to the naval architect if tolerances of vibration amplitudes could be established over the entire range of frequencies encountered on ships below which the vibration could be assumed acceptable, it appears premature to propose such standards at present. Criteria based on vibration velocity, acceleration, and the rate of change of acceleration have all been proposed and these have been based on physiological effects as well as on engineering considerations. As a very rough indication of present conditions, it may be stated that single amplitudes of 50 mils at 100 cpm may be considered high as may also single amplitudes of 2 mils at 2000 cpm."

One of the chief points to keep in mind in connection with levels of service vibration of ships is the enormous difference between the levels of steady-state vibration due to propellers or machinery, and the levels of transient vibration due to slamming in a seaway. This point is brought out in the following quotation from Reference F-2 which deals with shock and vibration instrumentation for ships.

"If both hull and machinery are taken into account, instruments for measuring steady-state vibration in ships must be able to cover the range of frequencies from 80 to 10,000 cycles per minute (cpm) and the range of single amplitudes from 0.0001 inch to 1.0 inch. While desirable, it is not obligatory that these large ranges be covered by a single instrument. In the case of shock measurement, frequencies range from about 80 cpm to at least 50,000 cpm if account is taken of the elastic vibrations of the entire hull on the one hand, and the localized vibrations of component structures on the other. Shock amplitudes may range from 0.0001 inch to several feet, the larger amplitudes being due to whipping motions of the entire hull."

In connection with this quotation the reader is cautioned that the term "shock loading" here includes loading due to underwater explosion as well as to slamming in a seaway.

In Reference F-8 there are discussed criteria of acceptable levels of hull vibration proposed by the Boston Naval Shipyard as a result of many years of experience in making underway vibration surveys on naval ships. A criterion based on vibratory velocity was recommended. Figure F-1 shows three regions of displacement amplitude based on the Boston figures. As this is written, this is not a Navy-wide standard.

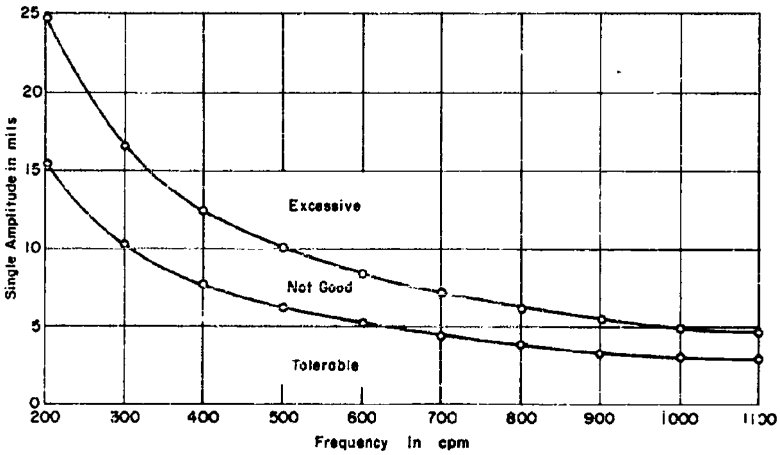


Figure F-1 – Range of Acceptable Amplitudes According to Boston Criteria (Velocity 0.32 to 0.52 Inch/Second)

Clearly, in the question of acceptable levels of steady-state hull vibration, physiological effects are a major consideration. The recognition of this has led to a number of recent investigations on the physiological effects of mechanical vibration. In 1959 one of the research panels of the Society of Naval Architects and Marine Engineers requested the preparation of a summary of such data by one of its members. This led to the preparation of a series of graphs by Mr. J.B. Montgomery of the Newport News Shipbuilding and Dry Dock Company in 1959. These graphs were based on various criteria depending on the original source. As an illustration, one of these charts, based initially on Reference F-4, is reproduced here as Figure F-2. It will be noted here that the criterion of comfort is acceleration regardless of frequency. Some investigators have considered the rate of change of acceleration or the third derivative of displacement with respect to time more significant than the acceleration itself.

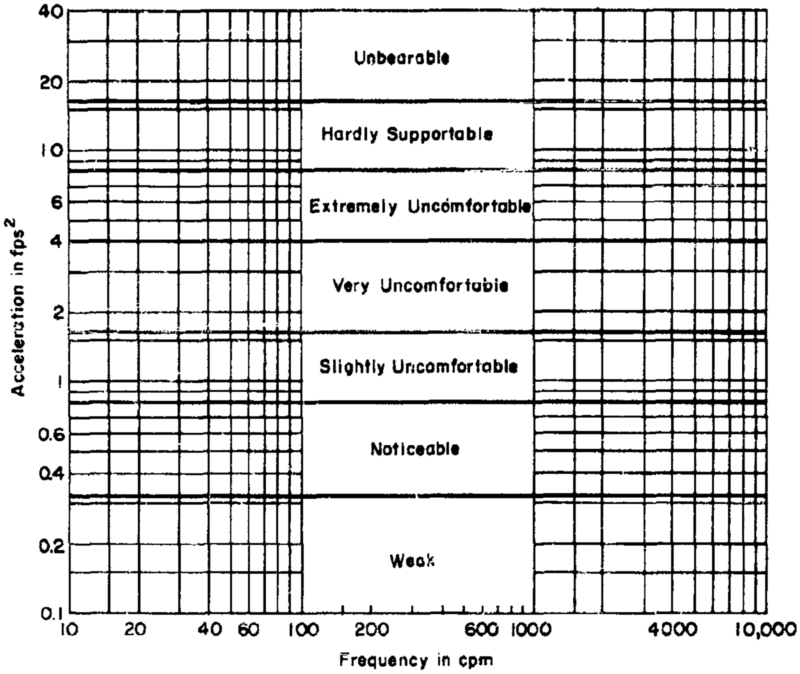


Figure F-2 – Vibration Standard Adopted by L'Institut de Recherches de la Construction Navale (Based on Physiological Effects)

The Navy's "Military Standard–Mechanical Vibrations of Ship Equipment" (Reference F-5) is intended as a guide for the vibration testing of equipment to be installed on board ship. Table F-1 taken from Reference F-5 gives the level of vibration at which endurance tests on equipment are to be run. Although this is based on accumulated data on service vibration of naval ships, it is not to be construed as a criterion of acceptable vibration levels for future designs.

TABLE F-1

Amplitudes of Vibration for Endurance Testing of Shipboard Equipment

Frequency Range cps	Table Amplitude single amplitude, inches
5 to 15	0.030 ± 0.006
16 to 25	0.020 ± 0.004
26 to 33	0.010 ± 0.002

REFERENCES

F-1. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

F-2. McGoldrick, R.T., "Shock and Vibration Instrumentation," Symposium at the 17th Applied Mechanics Division Conference of the ASME held at Penn State University (19–21 Jun 1952).

F-3. Gold, P.D., et al., "Vibration Engineering–Resume of Applications to Solution of Marine Operational Problems Encountered by Naval Vessels," Trans. SNAME, Vol. 61 (1953).

F-4. Dieudonné, J., "Vibration in Ships," Translation by W.C.S. Wigley, TINA (1958).

F-5. "Military Standard – Mechanical Vibrations of Shipboard Equipment," MIL-STD-167 (Ships) (20 Dec 1954).

APPENDIX G

SCALING CONSIDERATIONS IN MODEL VIBRATION EXPERIMENTS

In any attempt to determine the vibratory response characteristics of a ship by means of model experiments, the laws of similitude are of prime importance. It naturally occurs to the research worker that it is far simpler and less costly to conduct model experiments in the field of ship vibration than to conduct full-scale experiments. However, as pointed out in Chapter 7, aside from the difficulties of fabrication, it would not be feasible to run a self-propelled model of a ship to determine the amplitudes of service vibration by direct observation of the amplitude of the model.

The reason for this situation is that significant quantities affecting the vibratory response do not scale so as to give overall similitude in this case. This should not seem surprising, and, in fact, it is hardly to be expected that overall similitude regarding ship vibration could be obtained when it is not attainable even for the fundamental problem of ship resistance. The general subject of model experimentation and similitude is treated in Reference G-1 and in many other publications. In Reference G-1 it is shown how various dimensionless parameters can be derived from the total number of physical quantities affecting the phenomenon and the number of fundamental units involved (such as mass, length, and time; or force, length, and time). It turns out that dynamic similitude is attainable when the model is constructed to scale of the same material and when external loads are applied which are related to the full-scale loads both in magnitude and in time variation in accordance with Table G-1.

Thus, in accordance with Table G-1, any natural frequency of a solid body could be determined by making a scaled model of the same material and measuring the frequency of the desired mode of the model provided gravitational effects were negligible. The frequency of the prototype would then be λ^{-1} times the frequency of the model, where λ is a number determined by the first item of Table G-1. If the mode of oscillation is influenced by gravity, the relation between the model and full-scale frequencies will not conform to Table G-1. This relation could be fulfilled only if it were possible to modify the gravitational field surrounding the model in such a way that the acceleration of gravity satisfied the 15th item of Table G-1. A model of a simple pendulum would therefore not follow the rule for frequency given in Table G-1.

It is thus readily seen that the rules of similitude for vibration are not satisfied in self-propelled model tests of surface ships. In such tests the corresponding shaft speeds of model and prototype are related as follows:

$$\text{model shaft rpm} = \text{ship shaft rpm}\ \sqrt{\frac{L_{\text{ship}}}{L_{\text{model}}}}$$

TABLE G-1

Similitude Relations in Dynamic Model Testing for Equal Stresses

Measured Quantity	Prototype	Model
Length	L	λL
Angular Displacement	θ	θ
Area	A	$\lambda^2 A$
Area Moment of Inertia	I_a	$\lambda^4 I_a$
Volume	V	$\lambda^3 V$
Mass	m	$\lambda^3 m$
Mass Moment of Inertia	I	$\lambda^5 I$
Mass Density	ρ	ρ
Modulus of Elasticity	E	E
Stress	σ	σ
Time	t	λt
Natural Frequency	N	$\lambda^{-1} N$
Displacement (rectilinear)	d	λd
Velocity	v	v
Acceleration	a	$\lambda^{-1} a$
Force	F	$\lambda^2 F$
Torque	T	$\lambda^3 T$
Spring Constant (rectilinear)	k	λk
Spring Constant (angular)	k'	$\lambda^3 k'$
Damping Constant (rectilinear viscous)	c	$\lambda^2 c$
Damping Constant (angular viscous)	c'	$\lambda^4 c'$
Ratio of Damping to Critical Damping	c/c_c	c/c_c
Power	KW	$\lambda^2 KW$
Mechanical Impedance (based on velocity)	Z_v	$\lambda^2 Z_v$
Reduced Frequency (Strouhal number)	$\frac{b\omega}{v}$	$\frac{b\omega}{v}$

Since the blade frequencies would fall in the same ratio as the shaft speeds, the rule for frequencies in Table G-1 would not be satisfied.

It is important to note that, although the scaling law used in ship resistance model work (namely, Froude's law: $\frac{V^2}{Lg}$ = constant) does not conform with the rules for model testing involving elastic vibrations given in Table G-1, it does furnish a basis for studying rigid body oscillatory motions of ships. Thus, Froude's law is applicable to the determination of bending moments in ships in waves, and models may be devised (not true scale models in all respects) in which the frequencies of important modes of elastic vibration are made to conform with these rules. These rules are summarized in Table G-2.

TABLE G-2

Similitude Relations in Model Testing by Froude's Law

Measured Quantity	Prototype	Model
Length	L	λL
Velocity	v	$\lambda^{1/2} v$
Time	t	$\lambda^{1/2} t$
Acceleration (rectilinear)	a	a
Force	F	$\lambda^3 F$
Moment	M	$\lambda^4 M$
Pressure	p	p
Displacement (rectilinear)	d	λd
Angular Displacement	θ	θ
Mass Moment of Inertia	I	$\lambda^5 I$
Rigid Body Natural Frequencies	N	$\lambda^{-1/2} N$
Flexural Natural Frequency (required)	N	$\lambda^{-1/2} N$

Another important dimensionless quantity in model testing involving flow is the Reynolds number $\frac{Lv}{\nu}$ where L is a characteristic length, v is the fluid velocity, and ν is the kinematic viscosity. Where frictional resistance is predominant, as in the study of flow in pipes, the Reynolds number is used as a basis for similitude. Important similitude relations based on Reynolds number are given in Table G-3.

In Chapter 14 it was pointed out that in the aircraft field, model testing for flutter is a well-established practice. Here the dimensionless quantity on which the rules of similitude are based is the reduced frequency or Strouhal number $\frac{b\omega}{v}$ where b is the semichord of the airfoil, ω is the circular frequency of vibration, and v is the velocity of undisturbed air relative to the airfoil. The rules thus derived conform with those for dynamic model testing

TABLE G-3

Similitude Relations Based on Reynolds Number
(Assuming the Same Fluid for Both Cases)

Measured Quantity	Prototype	Model
Length	L	λL
Velocity	v	$\lambda^{-1} v$
Time	t	$\lambda^{2} t$
Acceleration (rectilinear)	a	$\lambda^{-3} t$
Force	F	F

for equal stresses given in Table G-1. In the case of wind-tunnel testing of aircraft flutter models, it is possible to reduce the discrepancies between the model and prototype Reynolds numbers by increasing the air density. In hydroelasticity this procedure is not feasible.

Although up to this time the extent of model work in hydroelasticity has been insignificant, the possibility of such developments should be noted. On the other hand, it must also be noted that in hydroelasticity the only craft comparable to the vehicle in the aircraft case is the submarine since the surface wave effect imposes the restriction that models of surface vessels be propelled at velocities conforming not to the Strouhal number but to the Froude number.

Further discussion of the use of models in vibration research will be found in References G-2 and G-3.

REFERENCES

G-1. Buckingham, E., "Model Experiments and the Forms of Empirical Equations," ASME Trans, Vol. 37 (1915).

G-2. Den Hartog, J.P., "Use of Models in Vibration Research," ASME Trans, Vol. 54 (1932).

G-3. Hermes, R.M. and Yen, C.S., "Dynamic Modeling for Stress Similitude," Contract Nonr-523, Dept of Appl Mech, Univ of Santa Clara, Santa Clara, Calif (Jul 1950).

APPENDIX H

MISCELLANEOUS INFORMATION ON VIBRATION OF SHIPS IN SERVICE

Certain unclassified data with regard to vibration of ships in service are presented in this Appendix without comment. A designer will often find such miscellaneous information helpful in his efforts to guard against the occurrence of hull vibration in a ship of new design. There is no attempt here to present a complete compendium of the information available at the time of this writing.

Table H-1 presents observed values of natural frequencies of hulls reported in technical literature. Table H-2 gives the frequencies of vertical flexural modes of hulls as determined in vibration generator tests conducted by the David Taylor Model Basin. Table H-3 gives the frequencies of the horizontal flexural modes (considered as torsion-free) for the ships listed in Table H-2.

Among the eight ships tested with a vibration generator (Tables H-2 and H-3), there were a few for which a 1-node torsional mode was either identified or appeared likely. The frequencies for these cases are given in Table H-4.

During and immediately after World War II the U.S. Navy conducted extensive investigations of shock effects on naval ships. In connection with these investigations, data on the natural frequencies of various local structures were obtained and summarized in an unclassified report; see Tables H-5 and H-6 taken from Reference H-15. Of special interest to the designer are the data on the natural frequencies of panels of side plating, deck plating, and transverse bulkheads which are reproduced here from that reference.

In connection with an investigation of environmental conditions at the location of radar equipment on naval ships, the David Taylor Model Basin obtained information in 1959 as to the relative magnitudes of various types of ship vibration in calm and rough seas; see Table H-7 taken from Reference H-16. While these data are based on statistical analysis and were an innovation at the time they were presented, they may serve to give the designer some idea of the augmentation of vibration levels which accompanies a change in the sea state.

The normal flexural modes of the free-free Euler-Bernoulli uniform beam are often used as assumed modes in starting calculations of natural frequencies and normal modes of hulls by the Stodola process. Values that may be used in plotting the first three modes are given in Table H-8.

A sample test schedule for a vibration-generator survey on a commercial cargo ship is given in Table H-9. The ship is GOPHER MARINER, which has been used as an example in many places in this book. The survey is discussed in detail in Reference H-17, and it should be noted that this included propeller-exciting force determination as well as hull-vibratory response characteristics.

TABLE [illegible]

Natural Frequencies of Flexural Modes of Hulls

(From Reference H-1)

Author and Reference	Type of Ship	Displacement tons	Type of Vibration	[illegible] Frequency [illegible]
Todd (H-2)	Cargo	7,9[illegible]	2-node [illegible]	[illegible]
	Tanker	15,100		[illegible]
		4,377		[illegible]
	Cargo	6,590		[illegible]
		3,835		[illegible]
	Tanker	4,1[illegible]		1[illegible]
		1,877		115
	Cargo	10,070		[illegible]00
	Tanker	5,18[illegible]		90–[illegible]
	Cargo	13,000		[illegible]8.[illegible]
	Tanker	11,475		98.[illegible]
		12,8[illegible]		95.[illegible]
		14,6[illegible]		[illegible]
[illegible] (H-3)	Tanker		[illegible] vert.	76
	Liner		[illegible]node [illegible]	[illegible]
			[illegible] vert.	
	Tanker	8,300	[illegible] vert.	11[illegible]
Nicholls (H-4)	Destroyer	1,378	[illegible] vert.	120
		1,378	2 node hor	[illegible]
		1,378	3-node vert.	170
		1,378	3-node hor.	260–390
		1,378	1-node tors.	630–700
Horn (H-5)	Liner	12,[illegible]	2-node hor.	125–130
Schadlofsky (H-6)	Tanker	16,5[illegible]	2-node vert.	61
	Cable layer	834		[illegible]–2[illegible]
	Destroyer	1,3[illegible]		170
	Tanker	8,160		112
	Freighter	8,[illegible]		105
Lovett (H-7)	Passenger		[illegible]node vert.	6[illegible]
				84
				67
				[illegible]
	Tanker			76
				112

Author and Reference	Type of Ship	Displacement tons	Type of Vibration	Measured Frequency cpm
[illegible] (H-8)	[illegible]ship	[illegible]	2-node vert	105–[illegible]
Taylor (H-9)		9,300		120.2
		4,100		148
		6,550		100
		9,050		89
		12,700		79
		6,850		[illegible]
		9,300	3-node vert	[illegible].5
		8,000		214
		12,700	2-node horiz	[illegible]5
		6,850		147
		8,000		137
Todd (H-10)	Cargo	7,540	2-node vert.	100(?)
	Tanker	15,190		78[illegible]
		4,377		[illegible]
	Cargo	6,590		[illegible]
		3,835		109
	Tanker	4,180		1[illegible]–5
		4,377		105–[illegible]
		1,877		115
	Cargo	10,070		10[illegible]
	Tanker	5,180		[illegible]
	Cargo	13,000		
	Tanker	11,475		
		1[illegible],632		
		14,635		
		6,400		
		8,200		
		1[illegible]		[illegible]–76
Roop (H-11)	Oiler	15,430	2-node v[illegible]	[illegible]
		7,9[illegible]		[illegible]
		12,6[illegible]		[illegible].1
		10,000		73.5
	Battleship	32,000		82
Cole (H-12)	Tanker	5,800	[illegible] vert.	112
Horn (H-13)	Freighter	13,280	[illegible]de vert.	83.1
		[illegible]	[illegible]node vert.	171.4

TABLE H-2

Frequencies of Vertical Flexural Modes of Hulls in CPM

(From Reference H-14)

Name of Ship	Type	Mode							
		1	2	3	4	5	6	7	8
NIAGARA	Transport	110	200	252	355	448			
CHARLES R. WARE	Destroyer	79	165	261	360				
E. J. KULAS	Ore Carrier		92	159	200	246	285	304	360
C. A. PAUL	Ore Carrier	45	106	168	210	312	354	432	
PERE MARQUETTE 21	Car Ferry	112	224	346	512				
OLD COLONY MARINER	Dry Cargo	82	155	227	270				
NORTHAMPTON	Cruiser	68	133	204	288	359	437	500	
STATEN ISLAND	Icebreaker	280	540	720					

TABLE H-3

Frequencies of Horizontal Flexural Modes of Hulls (Considered Torsion-Free) in CPM

(From Reference H-14)

Name of Ship	Type	Mode				
		1	2	3	4	5
NIAGARA	Transport	190	402	585		
CHARLES R. WARE	Destroyer	132	246			
E. J. KULAS	Ore Carrier		195*	320*	375*	
C. A. PAUL	Ore Carrier		180	300		
PERE MARQUETTE 21	Car Ferry	220	390			
OLD COLONY MARINER	Dry Cargo	118	280	350	435	
NORTHAMPTON	Cruiser	103	183	276	327†	392
STATEN ISLAND	Icebreaker	420				

*Experimental determination of number of nodes not made; tabulation made to yield best agreement with calculated values.

†Uncertainty as to whether this is a flexural or a torsional mode.

TABLE H-4

Frequencies of 1-Node Torsional Modes of Hulls (Assumed Flexure-Free) in CPM

(From Reference H-14)

Ship	Type	Frequency
NIAGARA	Transport	322
CHARLES R. WARE	Destroyer	310
E. J. KULAS	Ore Carrier	262*
NORTHAMPTON	Cruiser	346*

*Not positively identified as this mode.

TABLE H-5

Dominant Frequencies of Side Plating and Bulkheads of Various Naval Ships

(From Reference H-15)

Structural Units	Method of Obtaining Frequencies	Ranges of Dominant Frequencies, cps	Method of Exciting Vibration
Destroyer CAMERON			
Transverse bulkhead, Frame 29	Oscillograms	2[illegible]–77	Depth-charge firing
Transverse bulkhead, Frame 99	Oscillograms	42–116	Depth-charge firing
Transverse bulkhead, Frame 152	Oscillograms	25–177	Depth-charge firing
Heavy Cruiser CANBERRA (CA 70)			
Side plating at flag cabin and stateroom, main-deck level	Oscillograms	51– 91	Gun firing
Panel of side plating at flag cabin	Calculation	54–117	
Panel of side plating at flag stateroom	Calculation	38– 81	
Light Cruiser MIAMI (CL 89)			
Side plating and bulkhead at various locations	Vibrograph records	10–121	[illegible]ming
Destroyer SUMNER (DD 692)			
Side plating of after deckhouse	Oscillograms	29– 49	Gun firing
	Calculation	34– 80	
Side plating and bulkheads at various locations	Vibrograph records	52–108	Tapping
Submarine DRAGONET (SS 293)			
Periscope sheers	Vibrograph records	504, 536	Ramming
Conning-tower coaming	Vibrograph records	157	Ramming
Conning-tower fairing	Vibrograph records	134	Ramming
Back side starboard running light	Vibrograph records	60	Tapping
Bridge coaming	Vibrograph records	42	Tapping
Battleship MISSOURI (BB 63)			
Side plating at senior staff officer's cabin and stateroom	Oscillograms	43– 86	Gun firing
Side plating at senior staff officer's cabin and stateroom	Vibrograph records	61– 79	Tapping
Side plating at senior staff officer's cabin	Calculation	75–149	
Side plating at senior staff officer's stateroom	Calculation	33–1[illegible]	

TABLE H-6

Dominant Frequencies of Decks on Various Naval Ships

(From Reference H-15)

Structural Units	Method of Obtaining Frequencies	Ranges of Dominant Frequencies, cps	Method of Exciting Vibration
Light Cruiser MIAMI (CL 89)			
Decks at various locations	Vibrograph records	20–79, 93, 103	Ramming
Destroyer SUMNER (DD 692)			
Portion of main deck Portions of deck at various locations	Oscillograms Vibrograph records	26–46 49–315	Depth-charge firing Tapping
Submarine DRAGONET (SS 293)			
After torpedo room, over stiffener	Vibrograph records	296	Ramming

TABLE H-7

Factors for Converting Vibration Amplitudes in Calm Seas to Extreme Conditions

The vibration amplitude for calm sea operation is taken as A. (From Reference H-16)

Type of Vibration	Vibration Amplitude for Extreme Conditions	
	Aircraft Carrier	Destroyer
Propeller-excited	4A	4A
Excited by unbalance of propeller-shaft system	A	A
Transient vibration during maneuvers	2A	4A

TABLE H-8

Normal Mode Shapes of the Free-Free Euler-Bernoulli Beam

Mode	Station No.																				
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2-node	1.000	0.768	0.544	0.312	0.098	$\overline{0}.099$	0.272	$\overline{0}.414$	$\overline{0}.521$	$\overline{0}.586$	$\overline{0}.608$	$\overline{0}.586$	$\overline{0}.521$	$\overline{0}.414$	$\overline{0}.272$	$\overline{0}.099$	0.098	0.312	0.544	0.768	1.000
3-node	1.000	0.608	0.228	$\overline{0}.118$	$\overline{0}.397$	$\overline{0}.585$	$\overline{0}.66$[illegible]	$\overline{0}.625$	$\overline{0}.483$	$\overline{0}.263$	0.0	0.263	0.483	0.625	0.662	0.585	0.397	0.118	$\overline{0}.228$	$\overline{0}.608$	$\overline{1}.000$
4-node	1.000	0.454	$\overline{0}.052$	$\overline{0}.442$	$\overline{0}.643$	$\overline{0}.62$[illegible]	$\overline{0}.397$	$\overline{0}.045$	0.328	0.608	0.711	0.608	0.328	$\overline{0}.045$	$\overline{0}.397$	$\overline{0}.621$	$\overline{0}.643$	$\overline{0}.442$	$\overline{0}.052$	0.454	1.000

NOTE: The horizontal line above the decimal point indicates a negative value.

TABLE H-9

Schedule of Vibration Tests on GOPHER MARINER

Transverse Vibration
1st Day
6:00 a.m. – Check operation of equipment.
7:00 a.m. – Start testing immediately following the clearance of the 30-fathom line.
7:15 a.m. – Operate ship at 5 knots. Start running vibration generator. Make frequency survey to determine transverse natural modes.
10:15 a.m. – Continue frequency survey with increased eccentricity – check important frequencies.
2:15 p.m. – Finish frequency survey. Prepare for continuous acceleration.
2:30 p.m. – Build up from 5 knots to full power in period of ½ hour. Record transverse vibration.
3:00 p.m. – Finish continuous acceleration test. Prepare for propeller force measurements.
3:15 p.m. – Operate at 106 rpm. Hold rudder ± 2 degrees. Record transverse vibration.
3:30 p.m. – Operate at 100 rpm. Hold rudder ± 2 degrees. Record transverse vibration.
3:45 p.m. – Operate at 95 rpm. Hold rudder ± 2 degrees. Record transverse vibration.
4:00 p.m. – Operate at 90 rpm. Hold rudder ± 2 degrees. Record transverse vibration.
4:15 p.m. – Finish propeller forces. Build up to 95 rpm. Prepare for turning tests.
4:30 p.m. – Execute turning tests at 95 rpm. Record bending, torsional, and axial strains on rudder post. Accomplish 90-degree change of heading with following rudder angles: 5-degree left rudder 5-degree right rudder 10-degree left rudder 10-degree right rudder 15-degree left rudder 15-degree right rudder Time between change of heading 3 minutes.
5:30 p.m. – Finish turning tests. Build up to normal power. Prepare for hull calibration test.

TABLE H-9 (Continued)

Schedule of Vibration Tests on GOPHER MARINER

Transverse Vibration	
2nd Day	
6:00 a.m. –	Calibrate hull for transverse propeller forces. Operate at 106 rpm. Run vibrator at equivalent of 90 and 95 rpm.
8:00 a.m. –	Operate at 85 rpm. Run vibrator at equivalent of 100 and 106 rpm.
10:00 a.m. –	Finish hull vibration. Shift generator for producing vertical force. Operate at cruising speed.
Vertical Vibration	
2:00 p.m. –	Operate ship at 5 knots. Start running vibration generator. Make frequency survey to determine vertical natural modes.
4:00 p.m. –	Continue frequency survey with increased eccentricity–check important frequencies.
7:00 p.m. –	Finish frequency survey. Prepare for continuous acceleration.
7:15 p.m. –	Build up from 5 knots to full power in period of ½ hour. Record vertical vibration.
7:45 p.m. –	Finish acceleration run. Operate at cruising speed.
3rd Day	
6:00 a.m. –	Calibrate hull for vertical propeller forces. Operate at 106 rpm. Run vibrator at equivalent of 90 and 95 rpm.
8:00 a.m. –	Operate at 85 rpm. Run vibrator at equivalent of 100 and 106 rpm.
10:00 a.m. –	Finish hull calibration. Shift eccentrics for torque. Prepare for propeller force measurements. Record vertical vibration.
10:15 a.m. –	Operate at 106 rpm. Hold rudder ± 2 degrees.
10:30 a.m. –	Operate at 100 rpm. Hold rudder ± 2 degrees.
10:45 a.m. –	Operate at 95 rpm. Hold rudder ± 2 degrees.

TABLE H-9 (Continued)

Schedule of Vibration Tests on GOPHER MARINER

Vertical Vibration	
3rd Day	
11:00 a.m. –	Operate at 90 rpm. Hold rudder ± 2 degrees.
11:15 a.m. –	Finish propeller forces.
12: Noon –	Finish shift of eccentrics for torque. Operate ship at 5 knots. Start running vibration generator. Make frequency survey to determine torsional natural frequencies.
3:00 p.m. –	Continue frequency survey with increased eccentricity. Check important frequencies.
6:00 p.m. –	Finish frequency survey. Prepare for continuous acceleration.
6:15 p.m. –	Build up from 5 knots to full power in period of ½ hour. Record torsional vibration.
6:45 p.m. –	Finish continuous acceleration test. Build up to cruising speed.
4th Day	
6:00 a.m. –	Calibrate hull for torque. Operate at 106 rpm. Run vibrator at equivalent of 90 and 95 rpm.
8:00 a.m. –	Operate at 85 rpm. Run vibrator at equivalent of 100 and 106 rpm.
10:00 a.m. –	Finish hull calibration. Prepare for propeller force measurements.
10:15 a.m. –	Operate at 106 rpm. Hold rudder ± 2 degrees.
10:30 a.m. –	Operate at 100 rpm. Hold rudder ± 2 degrees.
10:45 a.m. –	Operate at 95 rpm. Hold rudder ± 2 degrees.
11:00 a.m. –	Operate at 90 rpm. Hold rudder ± 2 degrees.
11:15 a.m. –	Finish propeller forces. Prepare for anchor test.
11:45 a.m. –	Anchor test to occur in no less than 30 fathoms. Arrest anchor by brake after fall of about 1 fathom. Repeat anchor test.
12:45 p.m. –	Finish anchor test.

REFERENCES

H- 1. McGoldrick, R.T., "A Study of Ship Hull Vibration," EMB Report 395 (Feb 1935).

H- 2. Todd, F.H., "Ship Vibration–A Comparison of Measured and Calculated Frequencies," NECI Trans (1933).

H- 3. Tobin, T.C., "A Method of Determining the Natural Periods of Vibration of Ships," TINA (1922).

H- 4. Nicholls, H.W., "Vibration of Ships," TINA (1924).

H- 5. Horn, F., "Horizontal-und Torsions–Schiffschwingungen auf Frachtschiffen," WRH, Vol. 6, No. 18 (1925).

H- 6. Schadlofsky, E., "The Calculation and Measurement of Elastic Natural Frequencies of Ship Hulls," STG, Vol. 33 (1932). Also EMB Translation 7 (Jun 1934).

H- 7. Lovett, W.L., "The Vibration of Ships," Shipbuilding and Shipping Record (Jan 1927).

H- 8. Schmidt, F., "Die Berechnung von Schiffschwingungen," WRH, No. 6 (22 Mar 1930).

H- 9. Taylor, J.L., "Vibration of Ships," TINA (1930).

H-10. Todd, F.H., "Some Measurements of Ship Vibration," NECI Trans (Nov 1931).

H-11. Roop, W.P., "Natural Frequencies of Hull Vibration," Trans. SNAME (1930).

H-12. Cole, A.P., "The Natural Periods of Vibrations of Ships," Trans Inst of Engrs and Shipb Scotland, Vol. LXXII (1928-29).

H-13. Horn, F., "Hochseemessfahrt. Schwingungs-und Beschleunigungsmessungen," STG (1935).

H-14. McGoldrick, R.T., "Comparison between Theoretically and Experimentally Determined Natural Frequencies and Modes of Vibration of Ships," TMB Report 906 (Aug 1954).

H-15. Vane, F.F., "Natural Frequencies of Side Plating, Bulkheads, Decks, and Radar and Radio Supports, Measured Aboard Three Cruisers, Three Destroyers, a Submarine, and a Battleship," TMB Report 548 (Apr 1948).

H-16. Buchmann, E. and McConnel, J.D., "Environmental Conditions of Ship Motions and Vibrations for Design of Radar Systems on Destroyers and Aircraft Carriers," TMB Report 1298 (Jul 1959).

H-17. McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans. SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

GENERAL BIBLIOGRAPHY

Abramson, H.N. and Chu, W.H., "Problems of Hydroelasticity," JSR, Vol. 3, No. 1 (Jun 1959).

Abramson, H.N. and Chu, W.H., "A Discussion of the Flutter of Submerged Hydrofoils," JSR, Vol. 3, No. 2 (Oct 1959).

Adams, E.J., "The Steady-State Response of a Ship Hull to a Simple Harmonic Driving Force Computed by a Digital Process," TMB Report 715 (May 1950).

Allnutt, R.B., "Investigation of Hull Vibrations of USCGC PONTCHARTRAIN (WPG 70)," TMB Report R-294 (Aug 1946).

Allnutt, R.B. and Mintz, F., "Instruments at the David Taylor Model Basin for Measuring Vibration and Shock on Ship Structures and Machinery," TMB Report 563 (Jul 1948).

American Standards Association, "Proposed American Standard Nomenclature and Symbols for Mechanical Impedance of Structures," (pending).

Arkenbout Schokker, J.C., et al., "Design of Merchant Ships," N.V. De Technische Vitgerei, H. Stam-Haarlem-Holland (1953).

Babaev, N.N., "Contemporary Status of Development of the Problem of Ship Vibration (Sovremennoe Sostoianie I Razvitie Voprosa O Vibratsii Sudov)," Sudostroenie, No. 3 (161) (Mar 1956), pp. 4–10. TMB Translation 291 (Jul 1959).

Baier, L.A. and Ormondroyd, J., "Vibration at the Stern of Single-Screw Vessels," Trans SNAME, Vol. 60 (1952).

Bisplinghoff, R.L., et al., "Aeroelasticity," Addison-Wesley Publishing Co. (1955).

Boyd, J.E., "Strength of Materials," McGraw-Hill Book Co., New York (1924).

Breslin, J.P., "The Pressure Field near a Ship Propeller," JSR, Vol. 1, No. 4 (Mar 1958).

Breslin, J.P., "Ship Vibration. Part I – Propeller-Generated Excitations," Appl Mech Rev, Vol. 13, No. 7 (Jul 1960).

Breslin, J.P. and Tsakonas, S., "Marine Propeller Pressure Field Including Effects of Loading and Thickness," Trans SNAME, Vol. 67 (1959).

Buchmann, E. and McConnell, J.D., "Environmental Conditions of Ship Motions and Vibrations for Design of Radar Systems on Destroyers and Aircraft Carriers," TMB Report 1298 (Jul 1959).

Buckingham, E., "Model Experiments and the Forms of Empirical Equations," Trans ASME, Vol. 37 (1915).

Bunyan, T.W., "Practical Approach to Some Vibration and Machinery Problems in Ships," Paper presented to Inst of Mar Engin and Inst of Nav Arch (Feb 1955).

Burrill, L.C., "Ship Vibration: Simple Methods of Estimating Critical Frequencies," NECI Trans, Vol. 51 (1934–35).

Burrill, L.C., "Calculation of Marine Propeller Characteristics," NECI Trans, Vol. 60 (1943–44).

Cole, A.P., "The Natural Periods of Vibrations of Ships," Trans Inst of Engrs and Shipb Scotland, Vol. LXXII (1928–29).

Crede, C.E., "Vibration and Shock Isolation," John Wiley & Sons, Inc, New York (1951).

Csupor, D., "Methods for Calculating the Free Vibrations of a Ship's Hull (Methoden zur Berechnung der Freien Schwingungen des Schiffskörpers)," Jahrb. d. STG, Vol. 50 (1956). TMB Translation 288 (May 1959).

Cuthill, E. and Olsten, E., "A Routine for the Calculation of the Response of a Ship to Transient Forces," TMB Report (in preparation).

Den Hartog, J.P., "Mechanical Vibrations," McGraw-Hill Book Co., New York, Fourth Edition (1956).

Den Hartog, J.P., "Use of Models in Vibration Research," ASME Trans, Vol. 54 (1932).

Description of Schelde Synchronizing Gear in Private Communication to SNAME (26 Aug 1957).

Dieudonné, J., "Vibration in Ships," Translation by W.C.S. Wigley, TINA (1958).

Feldman, S., "Dynamic Balancing for Noise Reduction," Dept of Navy, Bureau of Ships (Code 371), R & D Report 371-V-24 (Apr 1955).

Flax, A.H., "Aero-Hydro-Elasticity," Struc Mech, Proceedings of First Symposium on Nav Struc Mech, Pergamon Press, New York (1960).

Fothergill, A.E., "Vibrations in Marine Engineering," Trans Inst of Engrs and Shipb Scotland, Vol. 95, Part 4 (1951–52).

Francis, J.J. and Antkowiak, E.T., "Marine Vibration Diagnosis," Paper presented at New England Section Meeting, SNAME (Oct 1960).

Fung, Y.C., "An Introduction to the Theory of Aeroelasticity," John Wiley & Sons, Inc, New York (1955).

Gariboldi, R.J., "Procedure for Torsional Vibration Analysis of Multimass Systems," Dept of Navy, Bureau of Ships (Code 371), R & D Report 371-V-19 (Dec 1953).

Gold, P.D., et al., "Vibration Engineering – Resumé of Applications to Solution of Marine Operational Problems Engountered by Naval Vessels," Trans SNAME, Vol. 61 (1953).

Gumbel, L., "Torsional Vibrations of Elastic Shafts of Any Cross Section and Mass Distribution and Their Application to the Vibration of Ships," TINA, Vol. LIV (1912).

Hardy, V.S., "Vibration Studies of Ship Hulls by Means of Vibration Generators," TMB Report C-80 (Nov 1949) Unclassified.

Harrison, M., et al., "Wave Effects in Isolation Mounts," TMB Report 766 (Jan 1952).

Heller, S.R., Jr., and Abramson, H.N., "Hydroelasticity: A New Naval Science," Journal ASNE (May 1959).

Henry, C.J., et al., "Aeroelastic Stability of Lifting Surfaces in High-Density Fluids," JSR (Mar 1959).

Hermes, R.M. and Yen, C.S., "Dynamic Modeling for Stress Similitude," Contract N8onr-523, Dept of Applied Mechanics, Univ of Santa Clara, Santa Clara, Calif. (Jul 1950).

Hirowatari, T., "On the Natural Frequencies of the Vertical Flexural Vibrations of a Ship's Hull (Part 3)," J Soc Nav Arch, Japan, Vol. 97 (Aug 1955).

Horn, F., "Horizontal-und Torsions–Schiffsschwingungen auf Frachtschiffen," WRH, Vol. 6, No. 18 (1925).

Horn, F., "Tilgung Elastischer Schwingungen des Schiffskörpers," Schiffbau, Vol. 43 (1942).

Horn, F., "Hochseemessfahrt. Schwingungs-und Beschleunigungsmessungen," STG, Vol. 37 (1936).

Hoyt, E.D., "Simulation of the Sea and Analog Computation of the Forces on a Ship in Waves," Report to TMB on Contract Nobs 72376, T.O. 9, Project RR-1458-N by Reed Research, Inc. (Nov 1959).

Inglis, C.E., "A Suggested Method of Minimizing Vibration in Ships," TINA (1933).

Jacklin, H.M., "Human Reactions to Vibration," SAE Journal, Vol. 39 (1936).

Jaeger, H.E., "The Determination of the Natural Frequency of Ship Vibrations," Journal of BSRA, Vol. 5 (1950).

Jasper, N.H., "Structural Vibration Problems of Ships – A Study of the DD 692-Class of Destroyers," TMB Report C-36 (Feb 1950) Unclassified.

Jasper, N.H., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," Trans SNAME, Vol. 64 (1956). Also TMB Report 921 (Oct 1957).

Jasper, N.H., "A Design Approach to the Problem of Critical Whirling Speeds of Shaft-Disk Systems," TMB Report 890 (Dec 1954).

Jasper, N. H. and Birmingham, J.T., "Strains and Motions of USS ESSEX (CVA 9) during Storms near Cape Horn," TMB Report 1216 (Aug 1958).

Jasper, N.H. and Rupp, L.A., "An Experimental and Theoretical Investigation of Propeller Shaft Failures," Trans SNAME (1952).

Jewell, D.A., "A Note on Hydroelasticity," JSR, Vol. 3, No. 4 (Mar 1960).

Johnson, A.J., "Vibration Tests on All-Welded and All-Riveted 10,000 Ton Dry Cargo Ships," NECI Trans, Vol. 67 (1951).

Johnson, A.J. and Ayling, P.W., "Graphical Presentation of Hull Frequency Data and the Influence of Deck-Houses on Frequency Predictions," NECI Trans, Vol. 73 (1956–57).

Kapiloff, E., "Calculation of Normal Modes and Natural Frequencies of Ship Hulls by Means of the Electrical Analog," TMB Report 742 (Jul 1954).

Kaplan, P., "A Study of the Virtual Mass Associated with the Vertical Vibration of Ships in Water," SIT, Davidson Lab Report 734 (Dec 1959).

Kane, J.R. and McGoldrick, R.T., "Longitudinal Vibrations of Marine Propulsion-Shafting Systems," Trans SNAME, Vol. 57 (1949). Also TMB Report 1088 (Nov 1956).

Kennard, E.H., "Forced Vibrations of Beams and the Effect of Sprung Masses," TMB Report 955 (Jul 1955).

Kenny, J.H. and Leibowitz, R.C., "Design Details and Operating Procedure for the TMB Network Analyzer," TMB Report 1272 (Apr 1959).

Kjaer, V.A., "Vertical Vibrations in Cargo and Passenger Ships," Acta Polytechnica Scandinavica, A.P. 244/1948; Mech Engin Series 2, Copenhagen (1958).

Koch, J.J. "Eine Experimentelle Methode zur Bestimmung der Reduzierten Masse des Mitschwingenden Wassers bei Schiffschwingungen," Ingenieur-Archiv, Vol. IV, No. 2 (1933).

Krivtsov, Y.V. and Pernik, A.J., "The Singing of Propellers (Penie Grebnykh Vintov)," Sudostroeniye, No. 10 (Oct 1957). TMB Translation 281 (Oct 1958).

Krylov, A.N., "Vibration of Ships," in Russian (1936).

Kurdiumov, A.A., "Ship Vibration," in Russian (1953).

Kumai, T., "Some Notes on the Local Vibrations of Ships," Reports of Research Inst for Applied Mechanics, Kyushu Univ, Japan, Vol. III, No. 12 (1955).

Kumai, T., "Estimation of Natural Frequencies of Torsional Vibration of Ships," Reports of Research Inst for Applied Mechanics, Kyushu Univ, Japan, Vol. IV, No. 13 (1955).

Kumai, T., "Shearing Vibration of Ships," Reports of Research Inst for Applied Mechanics, Kyushu Univ, Japan, Vol. IV, No. 15 (1955). In English.

Kumai, T., "Coupled Torsional-Horizontal Bending Vibrations in Ships," European Shipbldg, Vol. V, No. 4 (1956).

Kumai, T., "Damping Factors in the Higher Modes of Ship Vibrations," European Shipbldg, Vol. VII, No. 1 (1958).

Kumai, T., "Some Measurements of Hull Vibration and Human Sensitivity to Vibration," European Shipbldg, Vol. VI, No. 3 (1957).

Lamb, H., "Hydrodynamics," Dover Publications, New York (1945).

Landweber, L. and de Macagno, M.C., "Added Mass of Two-Dimensional Forms Oscillating in a Free Surface," JSR (Nov 1957).

Leibowitz, R.C., "Natural Modes and Frequencies of Vertical Vibration of a Beam with an Attached Sprung Mass," TMB Report 1215 (Sep 1958).

Leibowitz, R.C., "Modes of Vibration of Rudder of USS ALBACORE (AGSS 569)," TMB Report C-952 (Feb 1959) CONFIDENTIAL.

Leibowitz, R.C. and Kennard, E.H., "Theory of Freely Vibrating Nonuniform Beams Including Methods of Solution and Application to Ships," TMB Report 1317 (May 1961).

Lewis, F.M., "The Inertia of Water Surrounding a Vibrating Ship," Trans SNAME, Vol. 37 (1929).

Lewis, F.M., "Propeller Vibration," Trans SNAME, Vol. 43 (1935).

Lewis, F.M., "Propeller Vibration," Trans SNAME, Vol. 44 (1936).

Lewis, F.M., "Dynamic Effects," Chapter 2, Vol. 2, Marine Engin, edited by H.L. Seward, published by SNAME (1944).

Lewis, F.M., "Vibration and Engine Balance in Diesel Ships," Trans SNAME (1927).

Lewis, F.M. and Auslaender, J., "Virtual Inertia of Propellers," JSR (Mar 1960).

Lewis, F.M. and Tachmindji, A., "Propeller Forces Exciting Hull Vibration," Trans SNAME, Vol. 62 (1954).

Little, R.S., "Bibliography on the Longitudinal Strength of Ships and Related Subjects," Tech and Res Bulletin No. 2–3, SNAME (Sep 1958).

Loser, U., "Vibration Neutralizer for Ships (Neutralizzatore di vibrazioni per navi)," Ric. di Ingegn., Vol. 12, No. 1 (Jan-Feb 1934). EMB Translation 12 (Jan 1936). *

*EMB stands for U.S. Experimental Model Basin. These reports are available on request to David Taylor Model Basin (Code 141).

Lovett, W.L., "The Vibration of Ships," Shipbldg and Shipping Record (Jan 1927).

MacNaught, D.F., Discussion of Paper "Hull Vibration Investigation on SS GOPHER MARINER," by R.T. McGoldrick and V.L. Russo, Trans SNAME, Vol. 63 (1955).

Mathewson, A.W., "Preparation of Data for Computation of Vertical Flexural Modes of Hull Vibration by Digital Process," TMB Report 632 (Sep 1949).

McCann, G.D. and MacNeal, R.H., "Beam-Vibration Analysis with the Electric-Analog Computer," Journal of Appl Mech, Vol. 17, No. 1 (Mar 1950).

McGoldrick, R.T., "A Study of Ship Hull Vibration," EMB Report 395 (Feb 1935).

McGoldrick, R.T., "Experiments with Vibration Neutralizers," EMB Report 449 (May 1938).

McGoldrick, R.T., "Vibration Tests on USS HAMILTON at Washington Navy Yard," EMB Report 372 (Dec 1933).

McGoldrick, R.T., "Comparison between Theoretically and Experimentally Determined Natural Frequencies and Modes of Vibration of Ships," TMB Report 906 (Aug 1954).

McGoldrick, R.T., "A Vibration Manual for Engineers," TMB Report R-189, Second Edition (Dec 1957).

McGoldrick, R.T., "Calculation of the Response of a Ship Hull to a Transient Load by a Digital Process," TMB Report 1119 (Mar 1957).

McGoldrick, R.T., "Calculation of Natural Frequencies and Normal Modes of Vibration for a Compound Isolation Mounting System," TMB Report 1420 (Jul 1960).

McGoldrick, R.T., "Rudder-Excited Hull Vibration on USS FORREST SHERMAN (DD 931) – A Problem in Hydroelasticity," Trans SNAME (1959). Also TMB Report 1431 (Jun 1960).

McGoldrick, R.T., "Shock and Vibration Instrumentation," Symposium at the 17th Appl Mech Div Conf of the ASME held at Penn State Univ (19–21 Jun 1952).

McGoldrick, R.T., "Buoyancy Effect on Natural Frequency of Vertical Modes of Hull Vibration," JSR (Jul 1957).

McGoldrick, R.T. and Jewell, D.A., "A Control-Surface Flutter Study in the Field of Naval Architecture," TMB Report 1222 (Sep 1959).

McGoldrick, R.T. and Russo, V.L., "Hull Vibration Investigation on SS GOPHER MARINER," Trans SNAME, Vol. 63 (1955). Also TMB Report 1060 (Jul 1956).

McGoldrick, R.T., et al., "Recent Developments in the Theory of Ship Vibration," TMB Report 739, Revised Edition (Oct 1953).

Michel, R., "Interim Design Data Sheet DDS 4301, Propulsion Shafting," Dept of Navy, Bureau of Ships, S1/1–2 (415), DDS 4301 (1 May 1957).

"Military Standard – Mechanical Vibrations of Shipboard Equipment," MIL-STD-167 (Ships) (20 Dec 1954).

Motora, S., "On the Effect of the Free Surfaces upon the Virtual Mass of Submerged Bodies and Ships," J Soc Nav Arch, Japan, Vol. 86 (Mar 1954).

Neifert, H.R. and Robinson, J.H., "Further Results from the Society's Investigation of Tailshaft Failures," Trans SNAME (1955).

Noonan, Knopfle, and Feldman, A Proposal for "A Code for Shipboard Hull Vibration Measurements," in preparation for SNAME.

Noonan, E.F., et al., "Vibration Measurements on Ship S6-2 during Builder's Trials," prepared for Hull Structure Committee of SNAME by Noonan, Knopfle, and Feldman, Professional Engineers (Nov 1959).

Nicholls, H.W., "Vibration of Ships," TINA (1924).

Okabe, T., et al., "Vibration Measurements of 32,000 DW Super Tanker," Reports of Research Inst for Applied Mechanics, Kyushu Univ, Japan, Vol. IV, No. 13 (1955). In English.

Ormondroyd, J., et al., "Dynamics of a Ship's Structure," Engin Res Inst, Univ of Michigan, Ann Arbor (Mar 1949). Out of print.

Paladino, A.R., "Vibration Survey of USS DECATUR (DD 936)," TMB Report 1271 (Mar 1960).

Panagopulos, E.P., Discussion of Paper "Longitudinal Vibrations of Marine Propulsion-Shafting Systems," by J.R. Kane and R.T. McGoldrick, Trans SNAME, Vol. 57 (1949).

Panagopulos, E.P., "Design-Stage Calculations of Torsional, Axial, and Lateral Vibrations of Marine Shafting," Trans SNAME, Vol. 58 (1950).

Pavlenko, G.E., "A Method of Calculating Ship Vibration Frequencies," Engineering (25 Jun 1926).

Pien, P.C. and Ficken, N.L., "The Measurement of Propeller-Induced Vibratory Forces on Scale Ship Models," Paper presented at ATTC Conference (Sep 1959).

Pipes, L.A., "Applied Mathematics for Engineers and Physicists," McGraw-Hill Book Co (1946).

Pohl, K.H., "Unstationary Pressure Field in the Neighborhood of a Ship Propeller and the Periodic Forces Produced in Its Neighborhood" (in German), Schiffstechnik, 6, 32 (Jun 1959).

Polachek, H., "Calculation of Transient Excitation of Ship Hulls by Finite Difference Methods," TMB Report 1120 (Jul 1957).

Pollard, J. and Dudebout, A., "Theorie du Navire," Treizieme Partie, "Vibrations des Coques des Navires a Helice," Vol. IV, Chapter LXIX, "Causes et Periodes des Vibrations," Paris (1894).

Prohaska, C.W., "The Vertical Vibration of Ships," Shipbuilder, Vol. 54 (Oct, Nov 1947).

Ramsay, J.W., "Aspects of Ship Vibration Induced by Twin Propellers," TINA (1956).

Rayleigh, Lord, "Theory of Sound," Second Edition, Dover Publications, New York (1945).

Reed, F.E. and Bradshaw, R.T., "Ship Hull Vibrations. Part II – The Distribution of Exciting Forces Generated by Propellers," CONESCO Report F-101-2, Contract Nobs 77150 (Jun 1960).

Richards, J.E., "An Analysis of Ship Vibration Using Basic Functions," NECI Trans (1951).

Robinson, Q.R., "Vibration Machines at the David W. Taylor Model Basin," TMB Report 821 (Jul 1952).

Roop, W.P., "Natural Frequencies of Hull Vibration," Trans SNAME (1930).

Rossell, H.E. and Chapman, L.B., Editors, "Principles of Naval Architecture," published by SNAME, Vols. 1 and 2 (1939).

Russo, V.L. and Sullivan, E.K., "Design of the MARINER-Type Ship," Trans SNAME, Vol. 61 (1953).

Ryder, F.L. and Zaid, M., "Direct Electrical Analogs of Double 6-Degree-of-Freedom Systems," Report on Contract Nonr-2952 (00) (X), Scientific Research Staff, Republic Aviation Corp, Farmingdale, New York (Feb 1960).

Saunders, H.E., "Hydrodynamics in Ship Design," published by SNAME (1957).

Scanlan, R.H. and Rosenbaum, R., "Introduction to the Study of Aircraft Vibration and Flutter," Macmillan Co, New York (1951).

Schade, H.A., Discussion of Paper "Hull Vibration Investigation on SS GOPHER MARINER," by R.T. McGoldrick and V.L. Russo, Trans SNAME, Vol. 63 (1955).

Schadlofsky, E., "The Calculation and Measurement of Elastic Natural Frequencies of Ship Hulls (Uber Rechnung und Messung der elastischen Eigenschwingungen von Schiffskorpern)," Jahrb. d. STG, Vol. 33 (1932). EMB Translation 7 (Jun 1934) Supplement (Nov 1935).

Schlick, O., Series of Articles on Ship Vibration in TINA (1884, 1893, 1894, 1901, 1911).

Schmidt, F., "Die Berechnung von Schiffschwingungen," WRH, No. 6 (22 Mar 1930).

Schmitz, M.A. and Simons, A.K., "Man's Response to Low Frequency Vibration," ASME Paper 59-A-200, presented at Annual Meeting (1959).

Sezawa, K., "The Effect of the Distribution of Damping Resistance in a Ship on Her Vibration," J Soc Nav Arch, Japan, Vol. 66 (Jun 1940).

Spath, W., "Theory and Practice of Vibration Testing Machines (Theorie und Praxis der Schwingungsprüfmaschinen)," Julius Springer, Berlin (1934). EMB Translation 51 (Mar 1938).

Stodola, A., "Steam and Gas Turbines" (Dampf-und Gasturbinen), Translated by Dr. Lewis C. Lowenstein, P. Smith, New York (1945).

Stuntz, G.R., et al., "Series 60 – The Effect of Variations in Afterbody Shape upon Resistance, Power, Wake Distribution, and Propeller-Excited Vibratory Forces," Paper prepared for Annual Meeting of SNAME (1960).

Suetsugu, I., "A Contribution to the Vibration of the Stern of Single-Screw Vessels," Internatl Shipbldg Progress, Vol. 2, No. 10 (1955).

Tachmindji, A.J. and McGoldrick, R.T., "Note on Propeller-Excited Hull Vibrations," JSR, Vol. 3, No. 1 (Jun 1959).

Taylor, J. Lockwood, "Propeller Blade Vibration," TINA (1943).

Taylor, J.L., "Vibration of Ships," TINA (1930).

Taylor, J.L., "Ship Vibration Periods," NECI (1927–28).

Taylor, K.V., "Vibration of Ships," Journal of BSRA, Vol. 6 (1951).

Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," NACA Report 496 (1934).

Thornton, D.L., "Mechanics Applied to Vibration and Balancing," Chapman & Hall, Ltd, London (1951).

Timoshenko, S., "Vibration Problems in Engineering," D. Van Nostrand Co, New York (1955).

Tobin, T.C., "A Method of Determining the Natural Periods of Vibration of Ships," TINA (1922).

Todd, F.H., "Some Measurements of Ship Vibration," NECI (Nov 1931).

Todd, F.H., "Ship Vibration – A Comparison of Measured with Calculated Frequencies," NECI (1933).

Todd, F.H., "The Fundamentals of Ship Vibration," Shipbuilder, Vol. 54 (May, Jun, Jul 1947).

Todd, F.H. and Marwood, W.J., "Ship Vibration," NECI Trans, Vol. 64 (1948).

Tomalin, P.G., "Marine Engineering as Applied to Small Vessels," Trans SNAME, Vol. 61 (1953).

Vane, F.F., "A Guide for the Selection and Application of Resilient Mountings to Shipboard Equipment," TMB Report 880, Revised Edition (Feb 1958).

Vane, F.F., "Shock and Vibration in Ships," Handbook of Shock and Vibration Control, Harris, C.M. and Crede, C.E., Editors, McGraw-Hill Book Co. (to be published in 1961).

Vane, F.F., "Natural Frequencies of Side Plating, Bulkheads, Decks, and Radar and Radio Supports, Measured Aboard Three Cruisers, Three Destroyers, a Submarine, and a Battleship," TMB Report 548 (Apr 1948).

Vedeler, G., "On the Torsion of Ships," TINA, Vol. LXVI (1924).

von Karman, T. and Biot, M.A., "Mathematical Methods in Engineering," McGraw-Hill Book Co., New York (1940).

Voight, H., "Recent Findings and Empirical Data Obtained in the Field of Ship Vibrations (Einige neuere Erkenntnisse und Erfahrungen bei Schiffsvibrationen)," Jahrb. d. STG, Vol. 47 (Feb 1953). TMB Translation 268 (Feb 1958).

Waas, H., "Federnde Lagerung von Kolbenmaschinene auf Schiffen," ZVDI, Vol. 18, No. 26 (1937).

Waas, H., "Erfahrungen bei der Untersuchung und Bekampfung von Schiffschwingungen," Forschungshefte fur Schiffstechnik, No. 3 (1953).

Waas, H. and Walter, H., "Technische Neuerungen auf dem Vermessungsschiff 'Nord' der Wasser-und Schiffahrtsverwaltung," Schiffstechnik, Vol. 3, No. 12/13 (Nov 1955).

Wendel, K., "Hydrodynamic Masses and Hydrodynamic Moments of Inertia (Hydrodynamische Massen und Hydrodynamische Massentragheitsmomente)," Jahrb. d. STG, Vol. 44 (1950). TMB Translation 260 (Jul 1956).

Weinblum, G., "Uber Hydrodynamische Massen," Schiff und Hafen, Vol. 3, No. 12 (1951).

Whitaker, J., "Vibration in Small Ships," Paper read before the FAO International Fishing Boat Congress, Paris (Oct 1952).

Wilson, W.K., "Practical Solution of Torsional Vibration Problems," 2 Vols., John Wiley & Sons, Inc, New York (1956).

Wilson, W.K., "A Review of Ship Vibration Problems," Mar Engr & Nav Arch (Jul, Aug, Oct, Nov 1955). Also published in ASNE Journal (May 1954).

Wright, E.A., "New Research Resources at the David Taylor Model Basin," Paper presented at Spring Meeting of SNAME (Jun 1958). Also TMB Report 1292 (Jan 1959).

Wylie, C.R., "Advanced Engineering Mathematics," McGraw-Hill Book Co, New York (1951).

Yeh, H.Y., "Thrust and Torque Fluctuations for APA 249, TMB Model 4414," TMB Report 1364 (Jan 1960).

Yoshiki, M., "A Simple Method for Estimating the Natural Frequencies of Flexural Vibration of Ships," J Soc Nav Arch, Japan, Vol. 73 (Jul 1951).

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