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DESIGN THEORY OF OPTIMUM NEGATIVE-RESISTANCE AMPLIFIERS

by

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ABSTRACT

In this paper we consider a general amplifier obtained by imbedding a linear 1-port device in an arbitrary lossless reciprocal 3-port. The active device is assumed to have a representation of a negative conductance, $-G_D$, in parallel with a parasitic capacitance, C_D. We prove that for a given bandwidth ω_0 the transducer voltage gain is limited by

$$|S_{21}| < \frac{1}{2}\left(1 + e^{\frac{\pi G_D}{\omega_0 C_D}}\right)$$

A synthesis method to approach the optimum situation is presented.

Other amplifier configurations are next considered. In each case, the optimum gain-bandwidth formula, synthesis procedure and some useful design curves are given.

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I. INTRODUCTION

The recent development of new 1-port active devices has given amplifier designers a new challenge, i.e., the limitation of the given device in terms of maximum gain-bandwidth product and synthesis procedures for an optimum amplifier. It is obvious that an ideal negative resistance can provide infinite gain bandwidth. The limitation of practical active devices is due to parasitic elements. In the case of conventional amplifiers employing 2-port active devices, such as vacuum tubes, the maximum obtainable gain-bandwidth product was first derived by Bode.[1] Bode also gave synthesis methods for optimum passive networks which act as input, output or interstage coupling networks.

Many people have used tunnel diodes and degenerate parametric amplifiers as the active elements to design simple amplifiers.[2,3,4,5] However, no general theory has been found for the limitations of the devices and optimum synthesis procedures. In this paper, we assume that the 1-port active device has an equivalent representation of a negative conductance, $-G_D$, in parallel with a capacitance, C_D. We first consider the problem where the active device is imbedded in a 3-port lossless reciprocal network. The maximum obtainable gain bandwidth and a synthesis procedure for an optimum amplifier are presented. We then consider other amplifier configurations. In each case the maximum gain bandwidth and synthesis procedure are given. Finally, the noise performances of different configurations are summarized.

II. LOSSLESS, RECIPROCAL IMBEDDING

The most general amplifier employing a 1-port active device is illustrated in Fig. 1, where G_S and G_L are source and load conductances. For obvious reasons, we wish to restrict the passive 3-port to be reciprocal and lossless. The network is then redrawn as shown in Fig. 2. N_a is a lossless reciprocal 3-port described by its

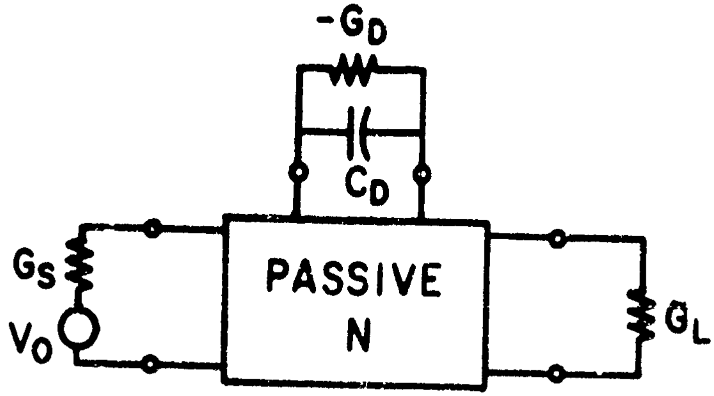


FIG. 1.

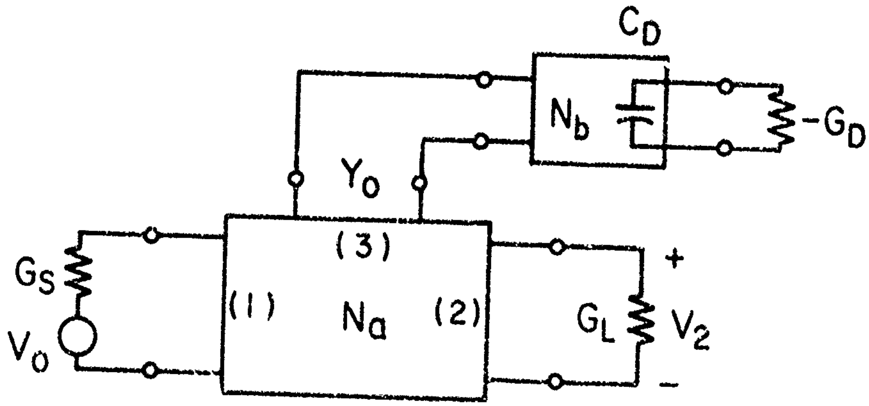


FIG. 2.

normalized scattering matrix with respect to a set of reference admittances: G_S, G_L, and Y_0, where Y_0 is arbitrary.[6] N_b is a lossless reciprocal 2-port with reference admittances, Y_0 and $-G_D$. The parasitic capacitance, C_D, is now included in N_b. With this representation, we can express the over-all transmission coefficient in terms of the scattering coefficients of N_a and N_b as follows:

$$S_{21} = \frac{2V_2}{V_0}\sqrt{\frac{G_L}{G_S}} = S_{21a} + \frac{S_{23a}S_{31a}S_{11b}}{1-S_{33a}S_{11b}} \tag{1}$$

Since the reference admittance, Y_0, is not involved in the expression, we can arbitrarily choose $Y_0 = Y_{33}$, where Y_{33} is the input admittance of N_a at port (3) with terminations G_S and G_L. Thus

$$S_{33a} = 0 \tag{2}$$

and Eq. (1) is simplified without losing any generality.

$$S_{21} = S_{21a} + S_{23a}S_{31a}S_{11b} \tag{3}$$

The physical interpretation of Eq. (3) is clear. The over-all transmission is equal to the sum of two parts. The first part S_{21a} represents the direct transmission of N_a from port (1) to port (2). The second part represents the product of three terms, namely: the direct transmissions from port (1) to port (3) and port (3) to port (2), and the reflection at port (3) due to mismatch of N_b. Since S_{21a}, S_{31a}, and S_{23a} are the scattering coefficients of a lossless network with passive reference admittances, their magnitudes cannot be larger than unity. The only term which could contribute gain to this expression is S_{11b}. In the following we will consider separately N_a and N_b and derive the optimum gain-bandwidth formula, which can be approached when both N_a and N_b are optimized.

N_a is a lossless reciprocal 3-port described by its normalized scattering matrix S_a with reference admittances G_S, G_L, and Y_0. Without losing any generality we can assume that $Y_0 = Y_{33}$ is real at one frequency. This is explained in Fig. 3, where a tuned circuit is

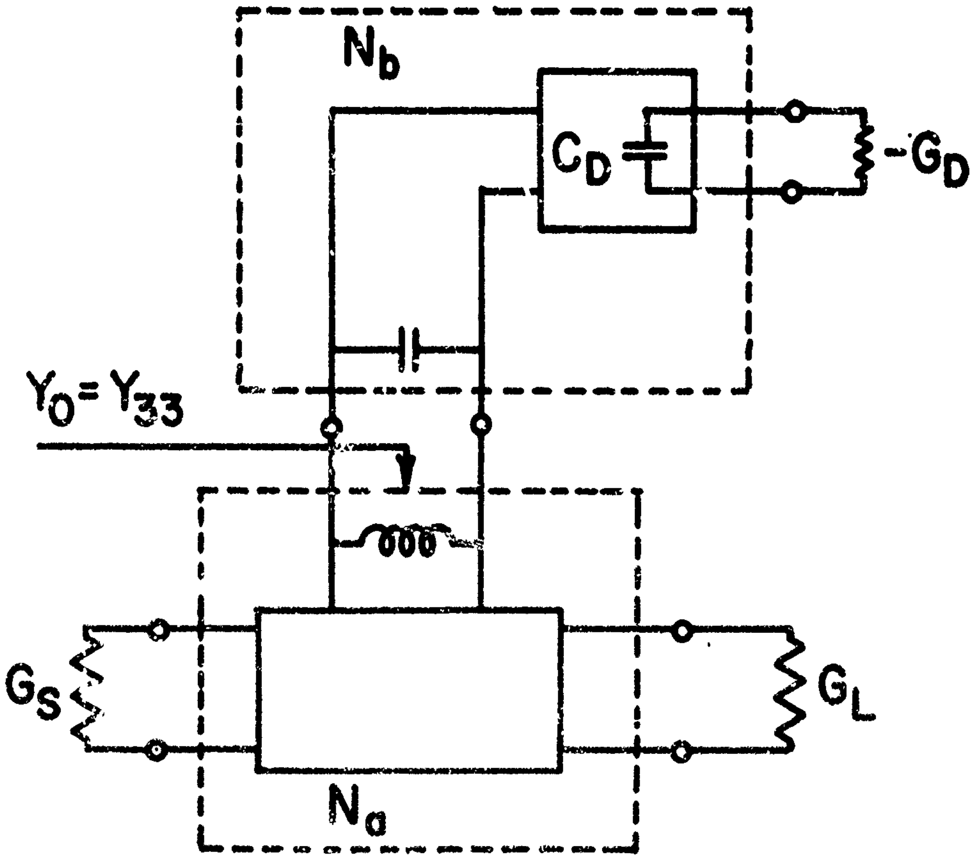


FIG. 3.

inserted between N_a and N_b. Clearly, at the resonant frequency, the complete network is not changed by this insertion, yet Y_{33} can now be made purely real.* With real reference admittances, S_a is unitary. In addition, since $S_{33a} = 0$, we have the following equations.

$$|S_{31a}|^2 + |S_{32a}|^2 = 1 \tag{4}$$

$$|S_{11a}|^2 + |S_{21a}|^2 + |S_{31a}|^2 = 1 \tag{5}$$

and

$$S_{31a}\bar{S}_{11a} + S_{32a}\bar{S}_{12a} = 0 \tag{6}$$

where the bars (—) designate the complex conjugate. Moreover, since N_a is reciprocal

$$S_{23a} = S_{32a} \qquad \text{and} \qquad S_{21a} = S_{12a} \tag{7}$$

Combining the above four equations and eliminating $|S_{11a}|$ we obtain the following useful result

$$|S_{21a}| = |S_{23a}S_{31a}| \tag{8}$$

Equation (8) is now substituted in Eq. (3), and the over-all transducer voltage gain is

$$|S_{21}| = |S_{23a}S_{31a}|\;|1 + S_{11b}\,e^{j\phi}| \tag{9}$$

where ϕ is a phase angle. Clearly we wish to maximize $|S_{23a}S_{31a}|$ to obtain the optimum $|S_{21}|$. From Eq. (4), it is seen that the maximum is realized if

$$|S_{23a}| = |S_{31a}| = \frac{1}{\sqrt{2}} \tag{10}$$

and the scattering matrix for an optimum N_a with all real coefficients

*Note that the real part of Y_{33} cannot be zero if S_{23a} and S_{31a} are finite. A lossless balanced bridge will illustrate this particular case.

is found as

$$S_a = \begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{bmatrix} \tag{11}$$

The complete network for $G_S = G_L = 1$ and $Y_0 = 2$ is shown in Fig. 4. From Eqs. (3) and (11), we conclude that

$$S_{21} = \frac{1}{2}(1 + S_{11b}) \tag{12}$$

The problem of finding the maximum gain-bandwidth product for given C_D and $-G_D$ is reduced to the problem of finding the optimum 2-port N_b such that S_{11b} is the maximum for a specified bandwidth. For this, we refer to Fig. 5, where N_b is shown with terminating conductances G_1 and G_2. Let ρ_1 be the reflection coefficient at the input for an output termination $G_2 = +G_D$ and ρ_1' be the reflection coefficient at the input for $G_2 = -G_D$. It is shown in Appendix A that

$$|S_{11b}| = |\rho_1'| = \frac{1}{|\rho_1|} \tag{13}$$

Thus the problem of finding the maximum $|S_{11b}|$ over a given band is reduced to the problem of finding minimum $|\rho_1|$ if the output termination is a positive conductance G_D. This is the familiar lossless matching problem and the solution is well known for a GC load. From Bode,

$$\int_{\omega_1}^{\omega_2} \log \frac{1}{|\rho_1|} \, d\omega \leq \pi \frac{G_D}{C_D} \tag{14}$$

or

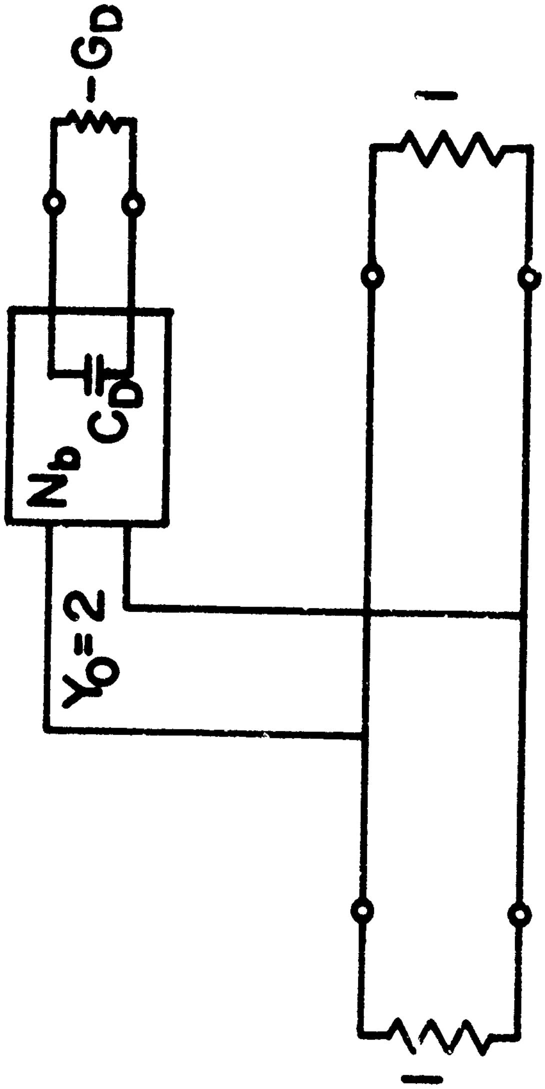


FIG. 4.

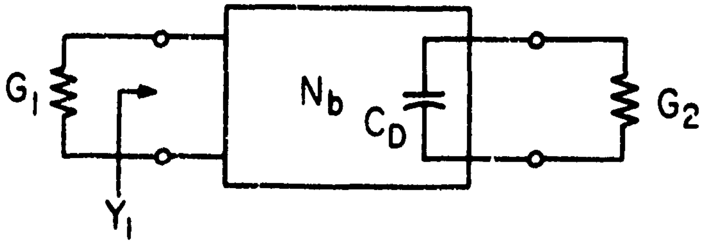


FIG. 5.

$$\int_{\omega_1}^{\omega_2} \log |S_{11b}| \, d\omega \leq \pi \frac{G_D}{C_D} \tag{15}$$

For a specified bandwidth, ω_0, the constant gain is limited by

$$|S_{11b}| \leq e^{\frac{\pi G_D}{\omega_0 C_D}} \tag{16}$$

Substituting in Eq. (12), we obtain the final result

$$|S_{21}| < \frac{1}{2} \quad 1 + e^{\frac{\pi G_D}{\omega_0 C_D}} \tag{17}$$

This expression gives the maximum gain obtainable for a given bandwidth by imbedding the active 1-port in a lossless reciprocal 3-port. Since the amplification is obtained through reflection, we refer to this as the reflection type amplifier. The synthesis procedure of the matching network and some design formulas are given in Sec. IV after we consider two other similar configurations in the next section.

III. THE CIRCULATOR AND HYBRID TYPE AMPLIFIERS

In this section, we analyze two familiar types of amplifier. The first one employs a nonreciprocal passive 3-port.[2,5] The second one uses two active 1-ports.[4] The gain-bandwidth performance is better than the reflection type discussed in the previous section.

A. THE CIRCULATOR TYPE AMPLIFIER

We again refer to Fig. 2 but remove the requirement of reciprocity on N_a. From Eq. (3) it is clear that in order to optimize S_{21}, the product $|S_{23a}S_{31a}|$ has to be maximized. An ideal circulator has the following normalized scattering matrix:

$$S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \tag{18}$$

Thus $S_{23a}S_{31a} = 1$, which is the maximum that can be obtained from a passive network. Hence the network as shown in Fig. 6 gives an

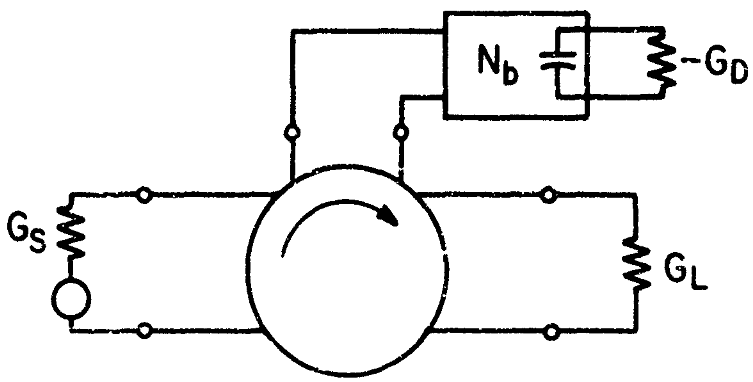


FIG. 6.

optimum amplifier configuration. The transmission coefficient is equal to the reflection coefficient of N_b.

$$S_{21} = S_{11b} \tag{19}$$

For a given ω_0,

$$|S_{21}| \leq e^{\frac{\pi G_D}{\omega_0 C_D}} \tag{20}$$

This is about 6 db better than the reflection type amplifier. However, the circulator type amplifier is a strictly one-way amplifier in contrast to the two-way amplifier of the reflection type.

B. THE HYBRID TYPE AMPLIFIER

To avoid the use of nonreciprocal elements, to provide two-way amplification and to achieve the gain level of Eq. (20), we need to use two active 1-ports. This is shown in Fig. 7. N_a is a lossless hybrid which has a scattering matrix

$$S_a = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \tag{21}$$

Networks N_b and N_c are lossless 2-ports; each is terminated by an active 1-port device. Let ρ_b and ρ_c be the reflection coefficients at the inputs of N_b and N_c as shown. The resulting scattering matrix of the over-all 2-port is

$$S = \frac{1}{2} \begin{bmatrix} \rho_b+\rho_c & \rho_b-\rho_c \\ \rho_b-\rho_c & \rho_b+\rho_c \end{bmatrix} \tag{22}$$

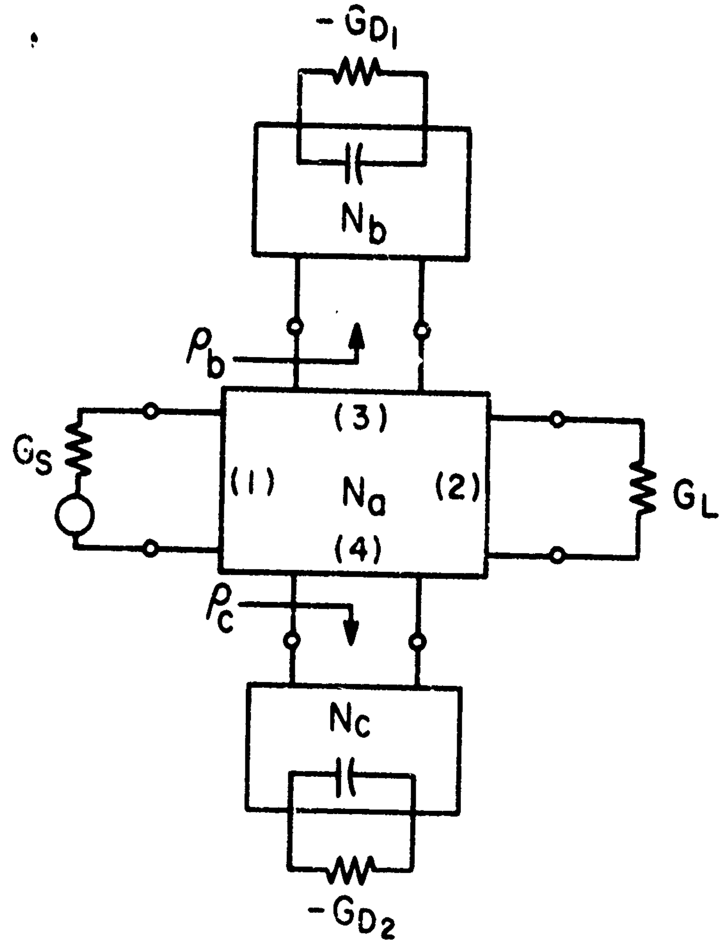


FIG. 7.

If we choose $\rho_c = -\rho_b$ (23)

then $S_{21} = S_{12} = \rho_b$ (24)

Thus we have a two-way amplifier with exactly the same gain-bandwidth limitation as the circulator type. The requirement of (23) indicates that the input admittance of N_b must be the reciprocal of that of N_c. Since the two 1-port active devices have the same equivalent circuit, the dual network concept cannot be used to obtain the reciprocal admittance. Practically, for a bandpass situation it is easier to obtain a 90° phase shift in a hybrid to approximately realize the requirement of Eq. (23).

IV. SYNTHESIS OF MATCHING NETWORKS

In order to achieve or approach the optimum gain bandwidth, we need to design an optimum matching network. The matching problem is a familiar one in network theory and has been solved by Bode, Fano, and others.[7] In our problem, since the load is parallel GC, exact synthesis procedure can be carried out to accomplish either a maximally-flat magnitude or an equal ripple match. The following simple derivation for the general maximally-flat match is presented along with some useful design formulas.

We refer again to Fig. 5. Let

$$G_1 = 1 \qquad \text{and} \qquad G_2 = \frac{1-\epsilon^n}{1+\epsilon^n} \tag{25}$$

where $\epsilon < 1$ is a positive design parameter. We denote t and ρ as the normalized transmission and reflection coefficients of the lossless 2-port with reference to G_1 and G_2. For a maximally-flat match,

$$|t|^2 = \frac{1-\epsilon^{2n}}{1+\omega^{2n}} \tag{26}$$

and

$$|\rho|^2 = \frac{\epsilon^{2n} + \omega^{2n}}{1 + \omega^{2n}} \tag{27}$$

Since the lossless ladder matching network is transparent at dc, $G_2 = (1 - \epsilon^n/1 + \epsilon^n)$ follows from Eqs. (26) and (27).

The transducer voltage gain of the over-all amplifier is equal to $1/|\rho|$ for the circulator and hybrid type and is equal to approximately $1/2|\rho|$ for the reflection type. In this section, we designate $|S_{21}|$ as the gain of the circulator or hybrid type, thus $S_{21} = 1/\rho$. From Eq. (27) the dc gain is

$$S_{21}(0) = \frac{1}{\epsilon^n} \tag{28}$$

and the 3-db bandwidth is found at $S_{21} = 1/\sqrt{2}\,\epsilon^n$ or

$$\omega_1 = \frac{\epsilon}{(1-2\epsilon^{2n})^{1/2n}} \tag{29}$$

The complete network can be obtained in the following way: we first determine the reflection coefficient from Eq. (27)

$$\rho(s)\,\rho(-s)\Big|_{s=j\omega} = |\rho(j\omega)|^2 = \frac{\omega^{2n} + \epsilon^{2n}}{1 + \omega^{2n}} \tag{30}$$

The poles of $\rho(s)$ are restricted to be in the left half plane for stability reasons. The zeros of $\rho(s)$ for maximum gain bandwidth are restricted to be in the right half plane.* Once $\rho(s)$ is determined, the input admittance, $Y_1(s)$, can be found.

$$Y_1 = \frac{1-\rho}{1+\rho} \tag{31}$$

A low pass ladder can be developed based on Cauer's continued fraction expansion. For the specified G_2 in Eq. (25) the last shunt

---

*From Ref. 1 the zeros of the reflection coefficient at the port where the capacitance appears should be in the left half plane. Thus zeros of $\rho(s)$, which is the reflection coefficient at the input port, should be in the right half plane.

capacitance is given by[8]

$$C_n = G_2 \frac{2 \sin (\pi/2n)}{1 - \epsilon} \tag{32}$$

From Eqs. (29) and (32) the actual 3-db bandwidth for a given set of C_D and G_D can be found by frequency and admittance denormalization.

$$\omega_{3\ db} = \frac{\omega_1 C_n}{C_D} \frac{G_D}{G_2} = \frac{\epsilon}{(1 - \epsilon^{2n})^{1/2n}} \frac{2 \sin (\pi/2n)}{1 - \epsilon} \frac{G_D}{C_D} \tag{33}$$

The asymptotic behavior of $\omega_{3\text{-db}}$ as $n \rightarrow \infty$ with a constant dc gain can be found by letting ϵ^n fixed and $\epsilon \rightarrow 1$,

$$\omega_{3\ db} \rightarrow \frac{\pi}{\ln S_{21}(0)} \frac{G_D}{C_D} \tag{34}$$

which is the optimum gain bandwidth given by Eq. (20). The network for n = 3 is shown in Fig. 8. For design convenience, the normalized

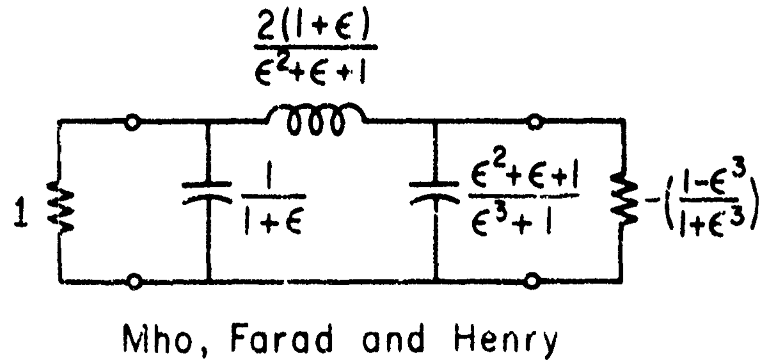


FIG. 8.

angular bandwidth $\omega/(G_D/C_D)$ is plotted vs the transducer gain in Fig. 9 for n = 1, 2, 3, 4 and ∞. The gain for the reflection type is about 6 db less. It should be pointed out that exact element values of the ladder network for both the maximally-flat and the equal-ripple matching networks can also be calculated from formulas given

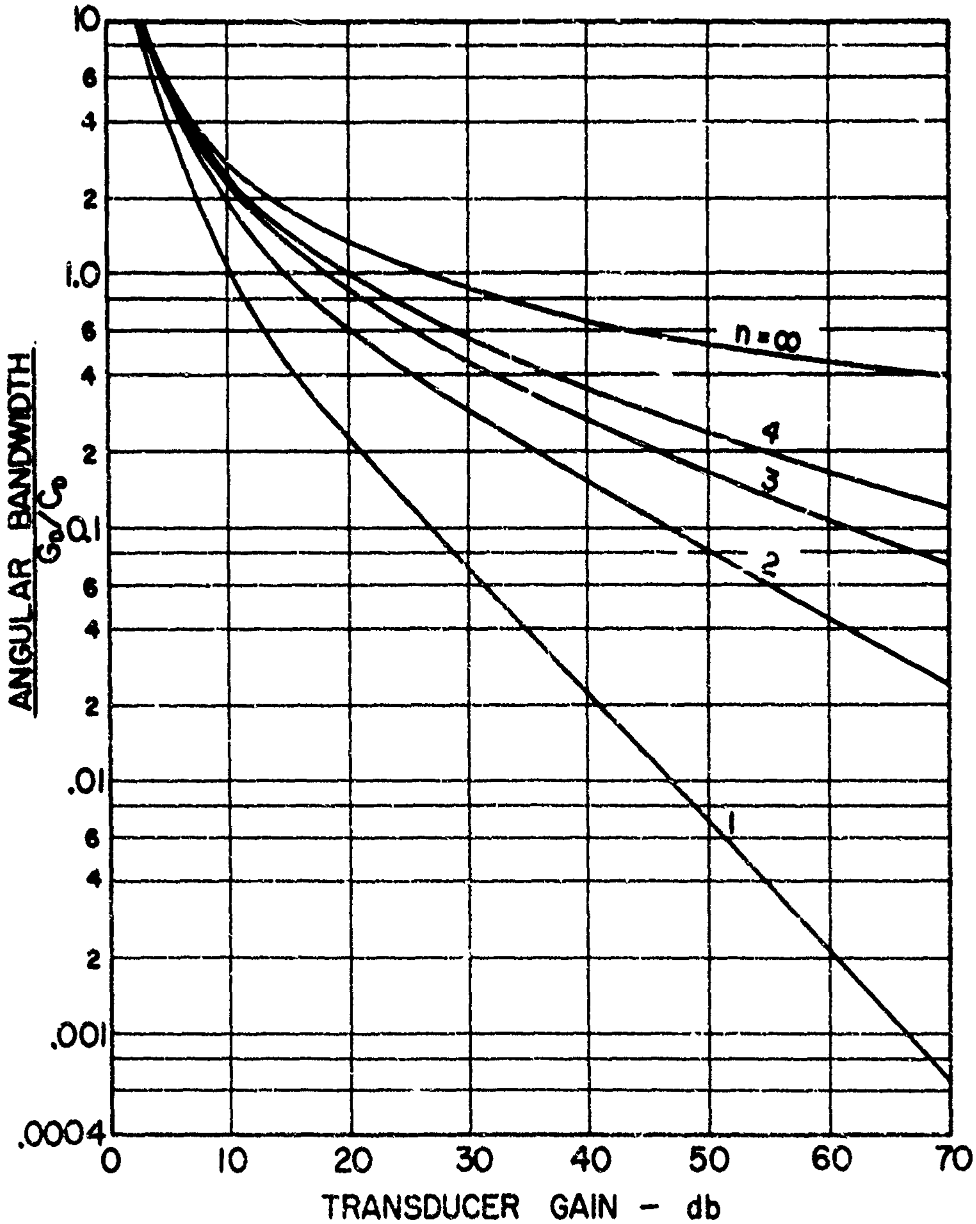

10
6
4
2
1.0
6
4
2
0.1
6
4
2
.01
6
4
2
.001
.0004
0
10
20
30
40
50
60
70
TRANSDUCER GAIN – db
ANGULAR BANDWIDTH
G_o/C_o
n = ∞
4
3
2
1

FIG. 9.

in Ref. 8. It is interesting to know that for a given n and for a fixed peak gain, the equal-ripple amplifier with a 3-db ripple has a smaller bandwidth than the maximally-flat case. This can be explained with Eq. (15), where the limit on the reflection coefficient is given in terms of an integral bandwidth.

V. OTHER AMPLIFIER CONFIGURATIONS

In this section, we consider three more amplifier configurations. For each configuration, we derive the optimum gain bandwidth and give the synthesis procedure.

A. THE DIRECT CONNECTED TYPE AMPLIFIER

The simplest amplifier configuration is shown in Fig. 10,

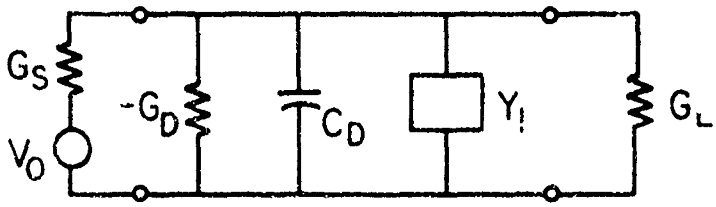


FIG. 10.

where Y_1 is a 1-port passive network. Chang uses a single inductance as the 1-port passive network in his tunnel diode amplifier.[3] The transmission coefficient can be expressed by

$$S_{21} = 2 \quad G_S G_L \quad Z \tag{35}$$

where

$$1/Z = G_S + C_L - G_D + sC_D + Y_1 \tag{36}$$

The problem of optimum gain bandwidth is similar to Bode's problem of the 2-terminal interstage. Using a similar approach, we find that for a given bandwidth ω_0, the maximum constant gain is given by

$$S_{21} = \frac{4\sqrt{G_S G_L}}{\omega_0 C_D} \frac{1}{\epsilon/\omega_0 C_D + \sqrt{1 + (\epsilon/\omega_0 C_D)^2}} \tag{37}$$

where

$$\epsilon = G_S + G_L - G_D$$

For $G_S = G_L = G_D/2$, $\epsilon = 0$, the maximum gain of the amplifier is given by

$$S_{21} = \frac{2G_D}{\omega_0 C_D} \tag{38}$$

The required Y_1 is the same as that given by Bode, that is, the input admittance of a constant-K type image filter. The circuit is shown in Fig. 11.

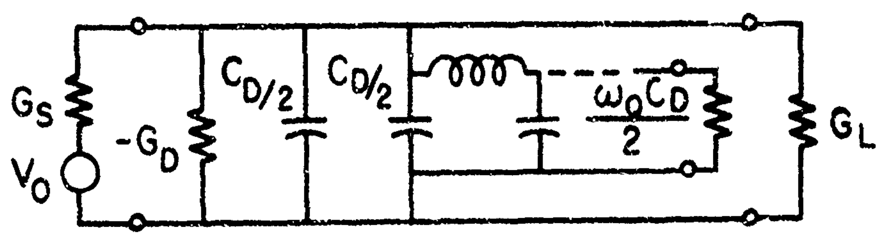


FIG. 11.

B. THE TRANSMISSION TYPE AMPLIFIER

This configuration was first used by Sard.[5] As shown in Fig. 12, the load and the source are separated by a coupling network,

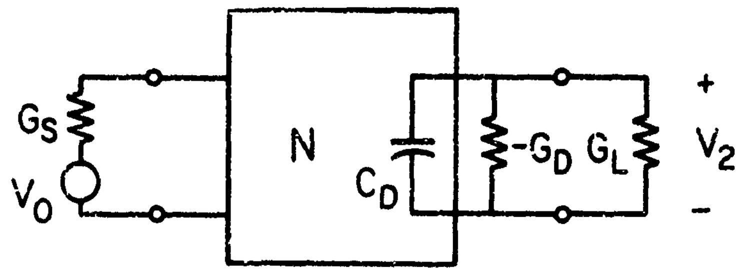


FIG. 12.

N, which contains the parasitic capacitance C_D as the final element. The other configuration, where the active 1-port is at the input end as shown in Fig 13 can be treated identically.

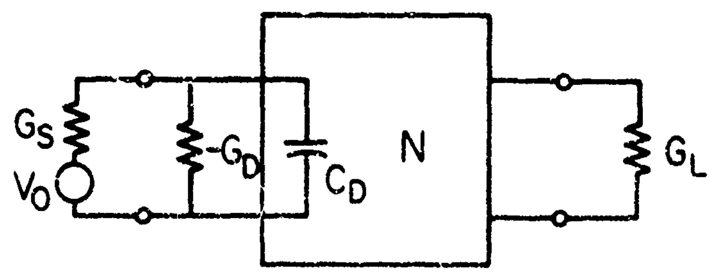


FIG. 13.

It is assumed that $G_L - G_D < 0$; hence, the over-all output conductance in Fig. 12 is negative. As shown in Appendix B, the transmission coefficient with load conductance, G_L can be written as

$$S_{21} = \sqrt{\frac{G_L}{G_D - G_L}}\,\frac{t}{\rho} \tag{39}$$

where t and ρ are, respectively, the transmission and reflection coefficients of the same network N with source and load conductances, G_S and $G_D - G_L$. Since

$$\frac{1}{|\rho|} \leq e^{\frac{\pi(G_D - G_L)}{\omega_0 C_D}} \tag{40}$$

and

$$|t|^2 = 1 - |\rho|^2 \tag{41}$$

it is found that for a given ω_0,

$$|S_{21}| \leq \sqrt{\frac{G_L}{G_D - G_L}} \left(e^{\frac{2\pi(G_D - G_L)}{\omega_0 C_D}} - 1 \right)^{\frac{1}{2}} \tag{42}$$

It is seen that the gain limitation is a function of the load conductance. For a maximally-flat magnitude design, let

$$|t|^2 = \frac{1 - \epsilon^{2n}}{1 + \omega^{2n}} \tag{43}$$

and

$$|\rho|^2 = \frac{\epsilon^{2n} + \omega^{2n}}{1 + \omega^{2n}} \tag{44}$$

The transducer power gain is

$$|S_{21}|^2 = \frac{G_L}{G_D - G_L} \; \frac{1 - \epsilon^{2n}}{\epsilon^{2n} + \omega^{2n}} \tag{45}$$

For the normalized conductances

$$G_S = 1 \qquad\qquad G_D - G_L = \frac{1 - \epsilon^n}{1 + \epsilon^n} \tag{46}$$

$$|S_{21}|^2 = G_L \frac{(1+\epsilon^n)^2}{\epsilon^{2n} + \omega^{2n}} \tag{47}$$

The dc gain and the 3-db bandwidth are given below

$$S_{21}(0) = \sqrt{G_L}\, \frac{1+\epsilon^n}{\epsilon^n} \tag{48}$$

$$\omega_{3\text{ db}} = \frac{2\epsilon \sin(\pi/2n)}{1-\epsilon}\, \frac{G_D - G_L}{C_D} \tag{49}$$

Thus the gain bandwidth is a function of G_L. Sard has shown that the optimum choice of G_L is given by

$$G_L = G_D \frac{1-\epsilon^{2n}}{2n(1-\epsilon)} \tag{50}$$

The design curves for such a load are shown in Fig. 14.

C. THE CASCADE TYPE AMPLIFIER

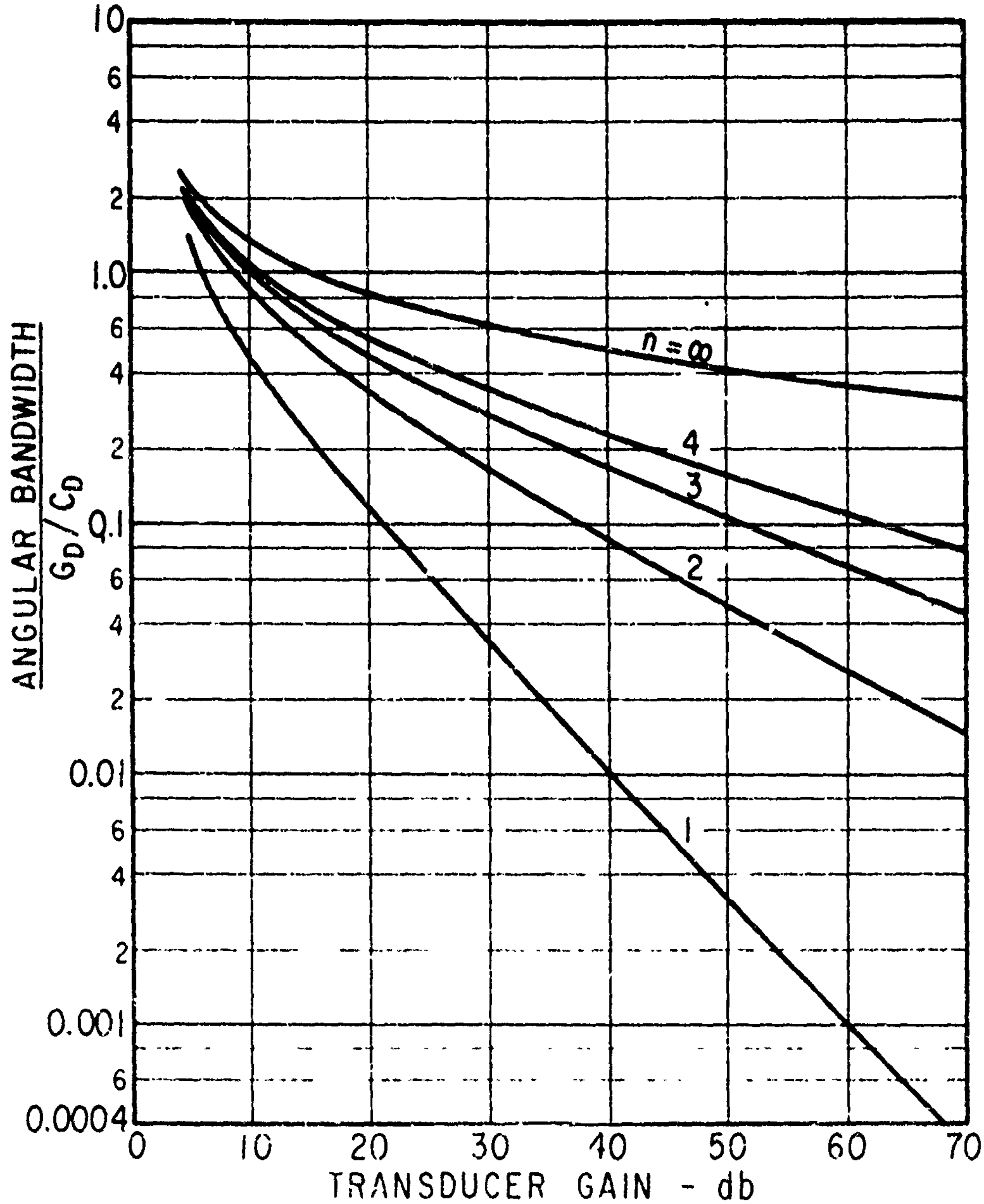
This configuration is shown in Fig. 15, where the negative conductance is separated from the source and the load by two coupling networks N_a and N_b. As shown in Appendix C, the over-all transmission coefficient can be expressed in terms of the normalized scattering parameters of N_a and N_b.

$$S_{21} = \frac{2\, S_{21a} S_{21b}}{2 - G - G(S_{22a} + S_{11b}) - (2+G)\, S_{22a} S_{11b}} \tag{51}$$

If we arbitrarily set $G = 2$ and $S_{22a} = -S_{11b}$, Eq. (51) becomes

$$S_{21} = -\frac{S_{21a} S_{21b}}{2\, S_{22a} S_{11b}} \tag{52}$$

The equation is similar to Eq. (39) of the previous case. The gain limitation is found to be



10
6
4
2
1.0
6
4
2
0.1
6
4
2
0.01
6
4
2
0.001
6
0.0004
0
10
20
30
40
50
60
70
TRANSDUCER GAIN - db
ANGULAR BANDWIDTH / G_D/C_D
$n = \infty$
4
3
2
1

FIG. 14.

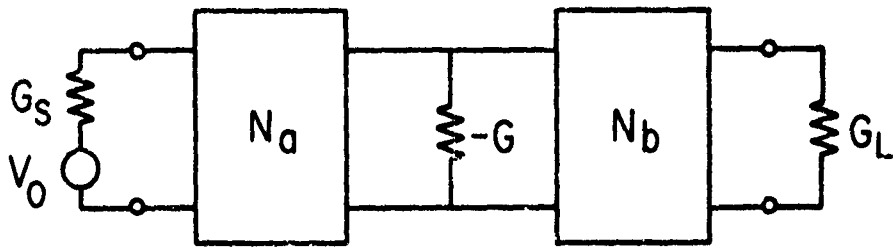


FIG. 15.

$$|S_{21}| \le \frac{1}{2}\left(e^{\frac{\pi G_D}{\omega_0 C_D}} - 1\right) \tag{53}$$

which approaches the optimum case for the reciprocal lossless 3-port imbedding as given by Eq. (17). The requirement of $S_{22a} = -S_{11b}$ indicates that N_a is the dual of N_b. For a maximally-flat design,

$$|S_{21}| = \frac{1}{2}\,\frac{1-\epsilon^{2n}}{\epsilon^{2n} + \omega^{2n}} \tag{54}$$

The dc gain and the 3-db bandwidth are

$$S_{21}(0) = \frac{1}{2}\,\frac{1-\epsilon^{2n}}{\epsilon^{2n}} \tag{55}$$

$$\omega_{3\,db} = (\sqrt{2}-1)^{1/2n}\,\frac{\epsilon}{1-\epsilon}\,\sin(\pi/2n)\,\frac{G_D}{C_D} \tag{56}$$

As n approaches infinity while $S_{21}(0)$ is fixed, Eq. (56) approaches Eq. (53), the limiting case. The network for n = 2 is shown in

Fig. 16. The design curves are given in Fig. 17.

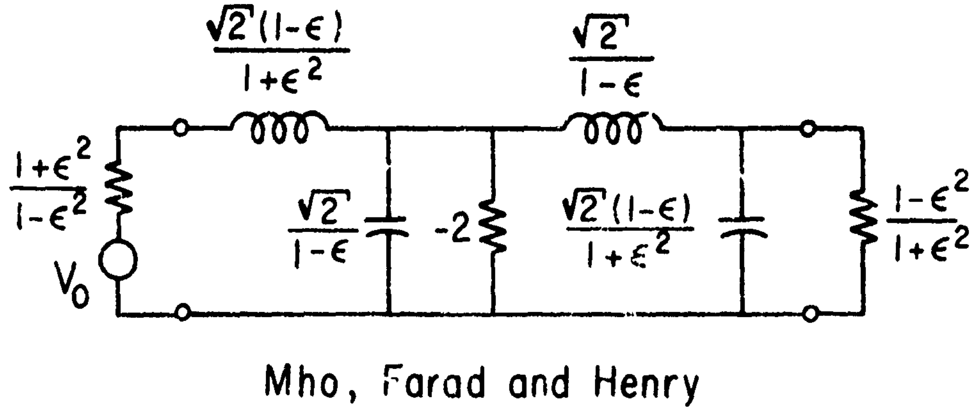


FIG. 16.

VI. NOISE PERFORMANCE

Let the shot-noise current generator of the active device be represented by its mean-square current

$$|i_D|^2 = 2\, q\, \Gamma\, I_0\, \Delta f \tag{57}$$

The noise figures of different configurations are shown in Table I.

If the noise measure is used, it is found that

$$M_e = \frac{-P_e}{kT\Delta f}, \qquad P_e = \frac{-|i_D|^2}{4G_D} \tag{58}$$

for the reflection, transmission, cascade and the circulator types.[9, 10] This, as pointed out by Penfield is the optimum value and is the same for negative resistance amplifiers that consist of a noisy negative resistance imbedded in a lossless 3-port. For the hybrid type

$$M_e = \frac{-(P_{e1} + P_{e2})}{kT\Delta f} \tag{59}$$

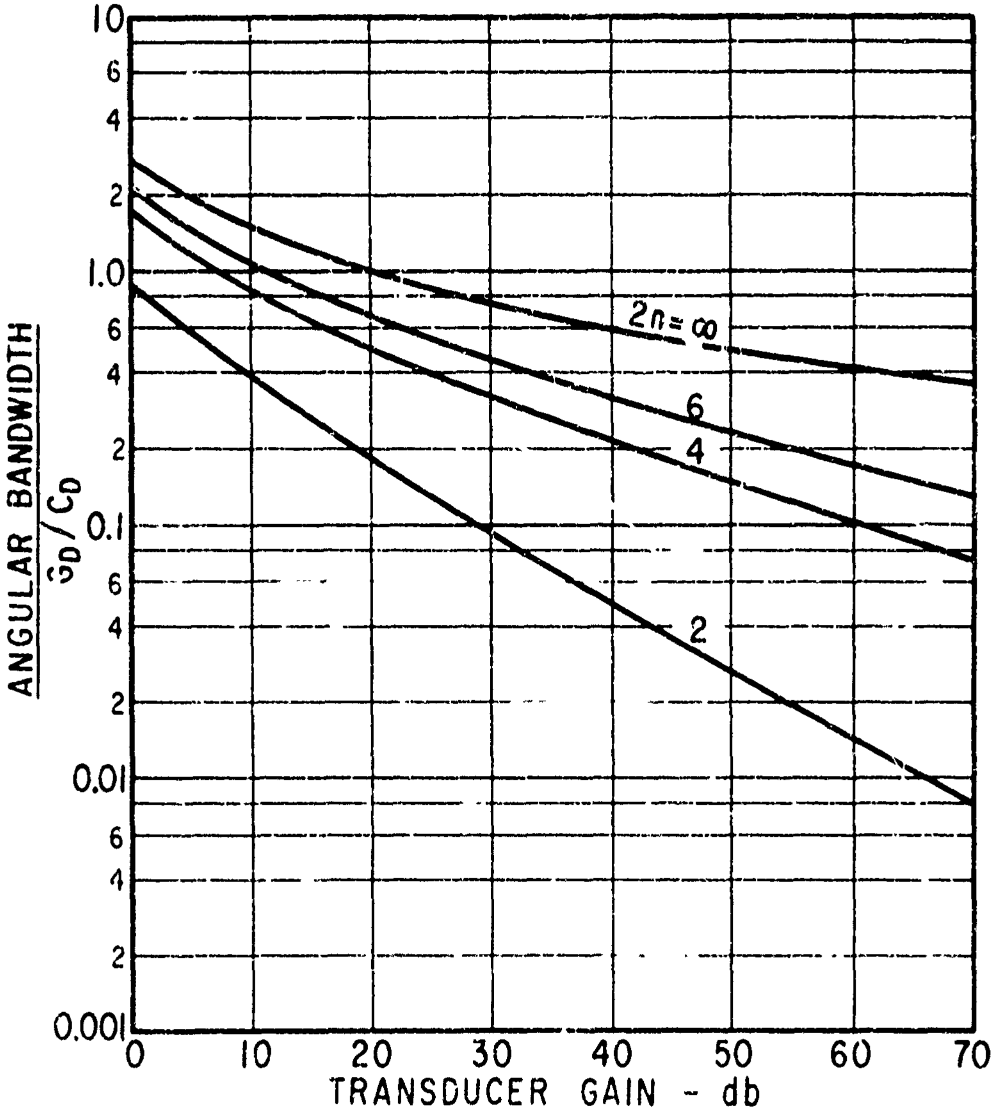

10
6
4
2
1.0
6
4
2
0.1
6
4
2
0.01
6
4
2
0.001
0
10
20
30
40
50
60
70
TRANSDUCER GAIN - db
ANGULAR BANDWIDTH
$\mathcal{G}_D / C_D$
$2n = \infty$
6
4
2

FIG. 17.

TABLE I

Amplifiers	Noise Figure
Reflection, transmission and cascade	$1 + \frac{\Gamma I_0}{2(kT/q)\, G_S}$
Circulator	$1 + \frac{2\Gamma I_0}{(kT/q)(1+G_D)^2}$
Hybrid	$1 + \frac{2\Gamma I_0}{kT/q} \frac{1+G_D^2}{(1+G_D)^2}$
Direct connection	$1 + \frac{Y_1(0)}{G_S} + \frac{\Gamma I_0}{2(kT/q)\, G_S}$

where P_{e1} and P_{e2} are the exchangeable noise power of the two negative resistances.

VII. CONCLUSION

The optimum gain-bandwidth performance of negative resistance amplifiers which consist of a 1-port ($-G_D$ and C_D) device imbedded in a lossless reciprocal 3-port is determined. Synthesis procedures to approach the optimum are presented along with useful design formulas.

Several other configurations have been analyzed with regard to the maximum gain bandwidth, design procedure, and noise figures.

A problem which is not considered in this paper is the reflection coefficients at the input and output of the amplifiers. Except for the matched circulator and hybrid types, all of the amplifiers seem to have large reflection coefficients.

APPENDIX A

With reference to Fig. 5, let Y_1 be the input admittance of N_b with $G_2 = +1$.

$$Y_1 = y_{11} \frac{1/z_{22} + G_2}{y_{22} + G_2} = \frac{m_1 + n_1}{m_2 + n_2} \tag{A-1}$$

where y_{11} and y_{22} are the short-circuit driving point admittances, z_{22} is the open-circuit driving point impedance at port (2), and m's and n's are, respectively, the even and odd polynomials of the complex frequency, s. The short-circuit admittances and the open-circuit impedance can be written in either the following forms:[11]

$$\left\{\begin{aligned} y_{11} &= \frac{m_1}{n_2} \\ y_{22} &= \frac{m_2}{n_2} \\ z_{22} &= \frac{m_1}{n_1} \end{aligned}\right. \qquad \text{or} \qquad \left\{\begin{aligned} y_{11} &= \frac{n_1}{m_2} \\ y_{22} &= \frac{n_2}{m_2} \\ z_{22} &= \frac{n_1}{m_1} \end{aligned}\right. \tag{A-2}$$

Let Y_1' be the input admittance of N_b with $G_2 = -1$, then

$$Y_1' = y_{11} \frac{1/z_{22} - G_2}{y_{22} - G_2} = \frac{m_1 - n_1}{n_2 - m_2} \tag{A-3}$$

Thus

$$\rho_1 = \frac{1 - Y_1}{1 + Y_1} = \frac{m_2 - m_1 + n_2 - n_1}{m_2 + m_1 + n_2 + n_1} \tag{A-4}$$

and

$$\rho_1' = \frac{1 - Y_1'}{1 + Y_1'} = \frac{-(m_2 + m_1) + n_1 + n_2}{m_1 - m_2 + n_2 - n_1} \tag{A-5}$$

Let $s = j\omega$

$$|\rho_1'|^2 = \frac{1}{|\rho|^2} \tag{A-6}$$

APPENDIX B

We refer to Appendix A and let t and t' be the transmission coefficients of N_b with $G_2 = +1$ and -1, respectively. In terms of the m's and n's

$$t = \frac{2\sqrt{m_1 m_2 - n_1 n_2}}{m_1 + m_2 + n_1 + n_2} \tag{B-1}$$

and

$$t' = \frac{2\sqrt{n_1 n_2 - m_1 m_2}}{m_1 - m_2 - n_1 + n_2} \tag{B-2}$$

Let $s = j\omega$

$$t'(j\omega) = j\frac{t(j\omega)}{\rho(j\omega)} \tag{B-3}$$

We therefore conclude from Fig. 12 that the transmission coefficient t' for the network with a load admittance $G_L - G_D$ can be expressed in terms of the transmission coefficient t and the reflection coefficient ρ with a load admittance $G_D - G_L$. Since the actual load is G_L,

$$S_{21} = \frac{2V_2}{V_0}\sqrt{\frac{G_L}{G_S}} = \frac{2V_2}{V_0}\sqrt{\frac{G_L - G_D}{G_S}}\sqrt{\frac{G_L}{G_L - G_D}}$$

$$= t'\sqrt{\frac{G_L}{G_L - G_D}} = \sqrt{\frac{G_L}{G_D - G_L}}\,\frac{t}{\rho} \tag{B-4}$$

APPENDIX C

Equation (51) can be derived most easily if Fig. 15 is redrawn, as shown in Fig. 18. The Thevenin's equivalent circuit can be used to determine the current I.

$$I = \frac{V_{oc}}{Z_{eq} + Z_L} \tag{C-1}$$

where

$$S_{21a} = \frac{2V_{oc}}{V_0} \frac{1}{\sqrt{G_S}} \tag{C-2}$$

$$Z_{eq} = \frac{1}{1 + Y_{22a}} \tag{C-3}$$

$$Z_L = \frac{1}{-1-G+Y_{11b}} \tag{C-4}$$

Next I' can be related to I by

$$I' = \frac{1 + Y_{11b}}{-1-G+Y_{11b}} \; I \tag{C-5}$$

V_2 can then be determined from I'

$$S_{21b} = \frac{2V_2}{I'} \sqrt{G_L} \tag{C-6}$$

Combining the above equations, we have

$$S_{21} = \frac{2V_2}{V_0}\sqrt{\frac{G_L}{G_S}} = \frac{S_{21a}\, S_{21b}\,(1 + Y_{22a})(1 + Y_{11b})}{2(-G + Y_{11b} + Y_{22a})} \tag{C-7}$$

Since

$$\frac{2}{1 + Y_{11b}} = 1 + S_{11b} \tag{C-8}$$

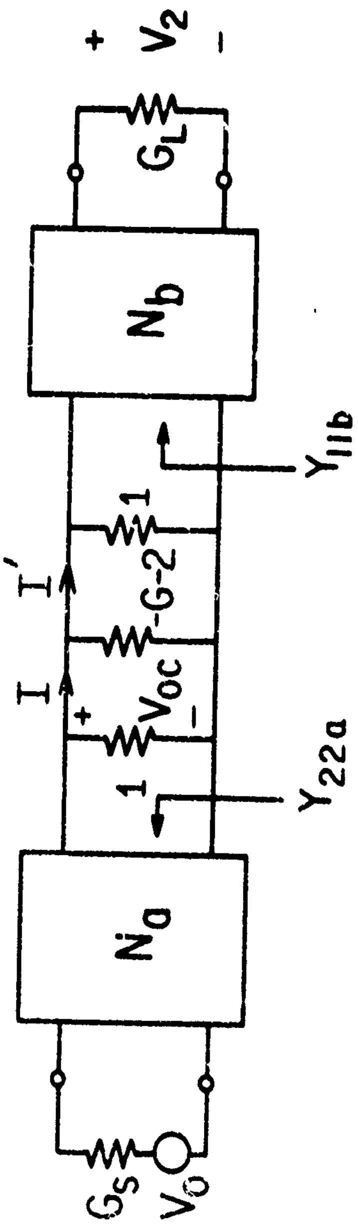


FIG. 18.

$$\frac{2}{1+Y_{22a}} = 1 + S_{22a} \tag{C-9}$$

We obtain

$$S_{21} = \frac{2\,S_{21a}\,S_{21b}}{2 - G - G(S_{22a} + S_{11b}) - (G+2)\,S_{22a}\,S_{11b}} \tag{C-10}$$

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