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APPROXIMATE SOLUTIONS FOR NONEQUILIBRIUM
AIRFLOW IN HYPERSONIC NOZZLES

BY

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FOREWORD

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ABSTRACT

A method has been described to estimate the effects of finite chemical reaction rates on the one-dimensional expansion of air in hypersonic nozzles. The calculations have considered stagnation temperatures from 4000 to 6000°K, stagnation pressures from 100 to 1000 atmospheres, and a range of wedge and axisymmetric nozzle shapes. Vibrational equilibrium is assumed.

A relaxation length criterion has been applied to a simplified kinetic model of air to determine the approximate location of freezing for flow with finite reaction rates in each nozzle configuration. Equilibrium composition profiles obtained by machine calculation were used for the calculation of the relaxation lengths. The resultant chemically frozen expansions have been calculated and are presented in tabular and graphical form. In all cases freezing occurs fairly early in the nozzle. Further, freezing in the nozzle is delayed by an increase in stagnation pressure, an increase in stagnation temperature, and by the use of gradually expanding nozzles. The effect of freezing is to reduce the pressure, velocity and temperature at a particular area ratio from the corresponding equilibrium values, and to increase the density and Mach number. The change in temperature and Mach number may be considerable whereas the density and flow velocity are relatively unaffected by freezing.
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NOTATION

Dimensional

\[ A' \] cross sectional area of nozzle
\[ D \] dissociation energy per mole
\[ \Delta F_e^o \] change in standard free energy for \( \lambda \)th reaction
\[ h_i' \] enthalpy of \( \lambda \)th species including energy of formation, cal/mole
\[ h \] Planck's constant
\[ H^' \] enthalpy of mixture, cal/gm
\[ k_f \] dissociation rate coefficient, \( \text{cm}^3 / \text{mole sec} \)
\[ k_e \] recombination rate coefficient, \( \text{cm}^6 / \text{mole}^2 \text{ sec} \)
\[ K_e \] equilibrium constant = \( k_f / k_e \), mole/cm^3
\[ K_{pl}, K_{fl} \] equilibrium constant referred to partial pressures and mole fractions respectively
\[ K \] Boltzmann constant
\[ \ell \] characteristic nozzle dimension \( L/\alpha \)
\[ L \] characteristic throat dimension, cm
\[ m' \] mass flow \( \rho A' \)
\[ p' \] fluid pressure
\[ r' \] local relaxation length, Equation (35)
\[ r'_\infty \] local relaxation length based on infinite-rate equilibrium, Equation (36)
\[ R_o \] universal gas constant
\[ S_o' \] entropy (per unit mass) at reservoir conditions
\[ S_i' \] molar entropy of \( \lambda \)th species at standard pressure
\[ T' \] local absolute temperature of gas
local gas velocity

\( \chi' \) distance from nozzle throat

\( \phi_i \) concentration of species \( i \), moles/gm

\( \theta_i' \) characteristic dissociation temperature \( D/R_0 \)

\( \theta_i' \) characteristic vibrational temperature \( h_i \nu/k \)

\( \mu \) molecular weight of undissociated mixture

\( \bar{\mu} \) molecular weight of mixture

\( \gamma \) characteristic frequency of molecule

\( \rho' \) fluid density

\( \psi \) parameter

\[
\frac{2 \ell \rho_0'^2 \frac{(2)}{R_0 \bar{\rho} \theta_0'}}{\sqrt{\frac{R_0 \rho_0'^2}{\mu}}}
\]

Dimensionless

\( a \) tangent of nozzle semi-angle

\( A_{ij} \) member of correction coefficient matrix in Newton-Raphson method

\( A \) \( A'/A' \)

\( c \) number of linearly independent species

\( F_j \) function used in Newton-Raphson method defined by Equations (A-2) and (A-3)

\( h_i \) \( h_i'/R_0 \bar{T}_0' \)

\( h_k \) fractional correction to the \( k \)th independent variable in Newton-Raphson iteration

\( H \) \( h'/R_0 \bar{T}_0' \)

\( m \) mass flow \( \rho_0 A \)
\( m \) number of chemical elements present

\( M \) Mach number

\( M_i \) chemical formula of \( i \)th species

\( n \) exponent in area relation \( A = 1 + x^n \)

\( p \) \( p'/p_0' \)

\( q_j \) mole fraction of \( j \)th component in a hypothetical system containing components only

\( Q_k \) number of gram atoms of \( k \)th chemical element present

\( r \) \( r'/\ell \)

\( r_\infty \) \( r_\infty'/\ell \)

\( s \) number of species

\( S_o \) \( S_0'/K_0' \)

\( S_i \) \( S_i''/R_o \)

\( T \) \( T'/T_0' \)

\( u \) \( u'/\sqrt{R_o T_0'} \)

\( x \) \( x'/\ell \)

\( X_i \) mole fraction of \( i \)th species

\( y_i \) formula vector of \( i \)th species

\( \alpha_{jk} \) atoms of \( k \)th chemical element per molecule of \( j \)th chemical species

\( \beta_i \) \( k_f^{(i)}/k_f^{(2)} \), Equation (32)

\( \delta_{jk} \) Kronecker delta

\( \epsilon_k \) maximum permissible fractional correction to mole fractions

\( \Theta_D \) \( \Theta_D'/T_0' \)

\( \Theta_v \) \( \Theta_v'/T_0' \)
\( \gamma_{ij} \) stoichiometric coefficients

\( o / o' \)

**Subscripts**

\( \alpha, i, j, k, l \) sum, product or matrix indices

\( o \) denotes condition at reservoir

\( e \) denotes local equilibrium value

\( \infty \) denotes infinite-rate equilibrium value

\( * \) denotes value at throat, \( A = 1 \)

\( f \) denotes value at freezing
INTRODUCTION

The stagnation enthalpies encountered in the flow of gases through nozzles at low supersonic velocities are usually sufficiently low so that chemical kinetics are unimportant. Such flows may be accurately described by simple equations which consider the participation of only the translational and rotational degrees of freedom of the molecules (constant $\gamma$). However, in the high temperature flows occurring in hypersonic nozzles, such simplified considerations are no longer applicable because of the interchange of energy between the internal and translational degrees of freedom and the coupling of the chemistry with the gas dynamics through dissociation. Figure 1 shows that at stagnation conditions pertinent to the present study, a significant portion of the total energy resides in the internal energy modes.

The gas dynamic and chemical behavior of a high temperature gas undergoing expansion in complete equilibrium may be calculated with reasonable precision (Sec. II). Complete chemical equilibrium, however, implying infinite reaction rates, is a condition which may only be approached, to a greater or lesser degree, in the expansion of high temperature real gases. Studies of the effects of chemical nonequilibrium, in particular nozzle flows involving only a single dissociating molecular species, have shown that the flow properties may depart considerably from their local equilibrium values. The effect of finite chemical reaction rates, as well as nozzle geometry, may result in actual freezing of the gas composition as some point in the nozzle.

The purpose of this report is to present quantitative estimates for the effect of flow freezing on the departure of test section properties from their equilibrium values in the expansion of air through hypersonic wind tunnel nozzles. The analysis has considered stagnation temperatures from 4000°K to 6000°K and stagnation pressures from 100 to 1000 atmospheres, representing a range of stagnation enthalpies of either current or future interest. The expansions are considered for two nozzle configurations; a two-dimensional wedge-type nozzle with a sharp throat and a hyperbolic axisymmetric nozzle.
which is conical for large area ratios. The solutions cover a comprehensive range of geometries for both types of nozzle.

The outline of the report is as follows. The equilibrium chemical composition of the gas at the reservoir condition is first computed for all stagnation states by the method outlined in Sec. II. The equilibrium (infinite-rate) expansions are then calculated, the solutions providing both the gas dynamic behavior and gas composition as a function of temperature. Because of their usefulness, the results of the equilibrium calculations are included in this report in tabular and graphical form.

The concept of a local chemical relaxation length\textsuperscript{5} is then introduced (Sec. III) and used to develop an approximate criterion for freezing. This concept was first used to develop a freezing criterion in Ref. 3, for both a pure diatomic gas or diatomic gas plus any diluent. Some air calculations were also performed in Ref. 3. The present kinetic model for air assumes finite reaction rates for oxygen, with nitrogen and nitric oxide assumed to remain in chemical (infinite-rate) equilibrium. A relaxation length is calculated, based upon the equilibrium flow properties, and used to determine the approximate nozzle location at which appreciable freezing has occurred, i.e., to predict, approximately, the final chemical state of the expanding flow for finite reaction rates.

Finally (Sec. IV), calculation of the nozzle expansions are completed with all chemical species fixed at their "freezing" mass fractions. The equilibrium solutions serve as a basis of comparison for the frozen (zero-rate) solutions. Both the equilibrium and frozen expansion processes are isentropic.\textsuperscript{14} The comparisons have been referred to as providing quantitative "estimates" since although the calculations of the frozen expansions are in themselves exact, the criterion for the selection of the equilibrium state to serve as input to the frozen calculations is approximate.
II SOLUTION OF THE EQUILIBRIUM FLOW

A. Governing Equations

It is assumed that the gas is a mixture of ideal gases, that the flow is quasi-one-dimensional, and that diffusion, heat conduction and viscous processes are of negligible significance. Steady, adiabatic, critical flow is assumed to exist in the nozzle.

The equations which determine the flow are,

Conservation of Momentum:

\[ u' \, du' + \frac{dp'}{\rho'} = 0 \]  \hspace{1cm} (1)

where \( u' = \) velocity \( \rho' = \) pressure \( \rho' = \) density

Global Conservation of Energy:

\[ \frac{u'^2}{2} + H' = H_0' \]  \hspace{1cm} (2)

where \( H' = \) mixture enthalpy (cal/gm)

Global Conservation of Mass:

\[ m' = \rho' \, u' \, A' \]  \hspace{1cm} (3)

where \( m' = \) mass flow \( A' = \) cross-sectional area
The state equations are,

Thermodynamic State:

\[ \rho' = \rho' \frac{R_u}{\tilde{\mu}} T' \]  \hspace{1cm} (4)

where \( \tilde{\mu} = \text{molecular weight of mixture and is given by} \)

\[ \tilde{\mu} = \frac{1}{\sum_i \tilde{\gamma}_i} \]  \hspace{1cm} (5)

where \( \tilde{\gamma}_i = \text{mass concentration of species } i \text{ in moles per unit mass}. \)

Caloric State:

\[ H' = \sum_i \tilde{\gamma}_i h_i' \]  \hspace{1cm} (6)

where \( h_i' = \text{molar enthalpy of } i \text{th species including thermal and formation contributions}. \ h_i' \text{ is only a function of temperature}. \)

In dimensionless form, the equations are written

\[ u d u + \frac{d \rho}{\rho} = 0 \]  \hspace{1cm} (7)

\[ \frac{u^2}{2} + H = H_0 \]  \hspace{1cm} (8)

\[ m = \rho u A \]  \hspace{1cm} (9)

\[ \rho = \rho T \frac{\sum \tilde{\gamma}_i}{\sum \tilde{\gamma}_i \tilde{\gamma}_i'} \]  \hspace{1cm} (10)
\[ H = \mu \sum_{i} \gamma_{i} h_{i} \]  

where:

\[ u = \frac{u'}{R_{o} T_{0}'} \]

\[ H = \frac{H'}{R_{o} T_{0}'} \]

\[ p = \frac{p'}{p_{0}'} \]

\[ \rho = \frac{\rho'}{\rho_{0}'} \]

\[ T = \frac{T'}{T_{0}'} \]

\[ A = \frac{A'}{A_{*}} \]

\[ \mu = \text{molecular weight of undissociated (cold) gas} \]

\[ R_{o} = \text{Universal gas constant} \]

\[ A_{*} = \text{throat area} \]

Except for the molecular weight, \( \mu \), the reference conditions for the non-dimensional forms are those at the equilibrium reservoir state.

B. Equilibrium Analysis

The methods and notation of Brinkley were used for the equilibrium analysis wherever applicable. At the end of this section, the Brinkley technique is extended to include the computation of states along an isentropic path.
The chemical formula of the \( i \)th species may be expressed in vectorial form as

\[
y_i = (\alpha_{i_1}, \alpha_{i_2}, \ldots, \alpha_{i_k}, \ldots, \alpha_{i_m}) \quad (i = 1, 2, \ldots, s)
\]  

(12)

where \( \alpha_{i_k} \) is the number of atoms of the \( k \)th chemical element per molecule of the \( i \)th species and \( m \) is the number of elements present. If the rank of the matrix of \( \alpha_{i_k} \) is \( c \), there exist \( c \) linearly independent \( y_i \), and \((s-c)\) dependent \( y_i \) which are given by linear combinations of the \( c \) independent ones.

\[
\sum_{j=1}^{c} \lambda_{ji} y_j = y_{\ell} \quad (\ell = c+1, c+2, \ldots, s)
\]  

(13)

As expressed by Eq. (13), \( \lambda_{ji} \) are coefficients in the linear dependencies of vectors \( y_{\ell} \) upon \( y_j \).

In the equilibrium analysis the independent species are designated \((j = 1, 2, \ldots, c)\) and are called "components." The number of components \( c \) usually equals the number of chemical elements present \( m \). Equations (13) correspond to \((s-c)\) chemical reactions postulated to form the dependent species if the formula vectors are replaced with chemical symbols.

It is useful to consider a hypothetical system. This system has the same atomic constitution as the actual system, but is chemically combined such that only components are present. If \( Q_{ik} \) is the number of gram-atoms of the \( k \)th element present, and \( q_{ij} \) is the number of moles of the \( j \)th component contained in the hypothetical system, then

\[
\sum_{j=1}^{s} \alpha_{ik} q_{ij}' = Q_{ik} \quad (k = 1, 2, \ldots, m)
\]  

(14)

The \( q_{ij}' \) are normalized by

\[
q_{ij} = \frac{q_{ij}'}{\sum_{j=1}^{s} q_{ij}'}
\]  

(15)
thereby scaling the total size of the system if \( q_j \) are considered to be numbers of moles.

Global mass conservation may be expressed by

\[
\sum_{j=1}^{c} n_j \mu_j + \sum_{i=1}^{s} \mu_i = \sum_{j=1}^{c} q_j \mu_j \quad (16)
\]

where \( \mu_j \) is the molecular weight and \( n_j \) is the number of moles of \( j \)th species. Mass is conserved in each formation reaction of Eqs. (13) so that

\[
\mu_\ell = \sum_{j=1}^{c} \nu_{\ell j} \mu_j \quad (\ell = c + 1, c + 2, \ldots s) \quad (17)
\]

Substituting \( \mu_\ell \) from Eq. (17) and equating coefficients of \( \mu_j \) in Eq. (16) gives

\[
n_j + \sum_{\ell=1}^{s} \nu_{\ell j} n_\ell = q_j \quad (j = 1, 2, \ldots c) \quad (18)
\]

Division of Eq. (18) by the total number of moles \( n \) gives

\[
X_j + \sum_{\ell=1}^{s} \nu_{\ell j} X_\ell = \frac{q_j}{n} \quad (j = 1, 2, \ldots c) \quad (19)
\]

where \( X \) are mole fractions. By definition

\[
\sum_{j=1}^{c} X_j + \sum_{\ell=1}^{s} X_\ell = 1 \quad (20)
\]

Equation (19) is summed over \( j \) to give

\[
\sum_{j=1}^{c} X_j + \sum_{\ell=1}^{s} \nu_{\ell j} X_\ell = \frac{1}{n} \quad (21)
\]

where

\[
\nu_\ell = \sum_{j=1}^{c} \nu_{\ell j}
\]
Substituting for $X_j$ from Eq. (20) into Eq. (21) results in

$$1 + \sum_{t=1}^{n} (\nu_t - 1) X_t = \frac{j}{n}$$

(22)

Therefore, Eq. (19) may be written

$$X_j = q_j - \sum_{t=1}^{n} [\nu_t q_j - \nu_t (\nu_t - 1)] X_t \quad (j = 1, 2, \ldots, c)$$

(23)

giving $c$ of the $s$ equations necessary for determining the equilibrium composition at a specified temperature and pressure. The remaining equations are

$$X_{c+1} = \frac{c}{c+1} X_j \quad (\ell = c+1, c+2, \ldots, s)$$

(24)

where $K_{x\ell}$ are equilibrium constants based on mole fractions. These constants are related to $K_{p\ell}$ by

$$K_{x\ell} = K_{p\ell} \rho^{(\nu_{\ell}-1)} \quad (\ell = c+1, c+2, \ldots, s)$$

(25)

$K_{p\ell}$ are calculated from chemical potentials by

$$K_{p\ell} = \exp \left( - \frac{\Delta F^\circ_{\ell}}{R_T} \right)$$

(26)

where $\Delta F^\circ_{\ell}$ is the change in free energy at standard pressure for the $\ell$th reaction. The $\Delta F^\circ_{\ell}$ were calculated by means of polynomial fits to various tabulated and derived thermodynamic functions provided by R. E. Duff of Los Alamos Scientific Laboratory. Equations (23) and (24) determine the equilibrium composition when temperature and pressure are specified; therefore, they can be employed for the reservoir state calculations.

For computation along an isentropic path, the Brinkley method must be modified. The composition and pressure will be computed at various temperatures for the entropy at the prescribed reservoir state. Since pressure
is an additional unknown quantity, one more equation is required. A statement that entropy is a constant is the desired equation.

\[ S_0 = \frac{1}{\bar{\mu}} \sum_{i=1}^{g} X_i S_i^\circ - \sum_{i=1}^{g} X_i \ln X_i - \ln \bar{\mu} \]  

(27)

\( S_0 \) is the entropy (per unit mass) calculated at the reservoir state, and \( \bar{\mu} \) is the mixture molecular weight which is calculated from

\[ \bar{\mu} = \sum_{i=1}^{g} X_i \mu_i \]

\( S_i^\circ \) is the molar entropy of the \( i \)th species at standard pressure. Simultaneous solution of the nonlinear algebraic equations (23), (24), and (27) gives the composition and pressure at various temperatures along an isentrope characterized by \( S_0 \). The Newton-Raphson method as employed in the solution is outlined in Appendix A.

Additional quantities are computed from the results of the solution.

Concentrations in moles per unit mass \( \chi_i \) are found from

\[ \chi_i = \frac{X_i}{\bar{\mu}} \quad (i = 1, 2, \ldots, g) \]  

(28)

Density is found from Eq. (4), enthalpy from Eq. (6), velocity from Eq. (2), and mass flow per unit area \( m'/A' \) from Eq. (3). The state at which mass flow per unit area goes through a maximum defines the conditions at the throat of the equilibrium nozzle. The local to throat area ratios are then found by dividing the maximum \( m'/A' \) by the local \( m'/A' \) values. A Mach number is calculated at each step from the expression

\[ M = \sqrt{\frac{u^2}{c_{\text{ent}}}} \]  

(29)

The thermodynamic state is independent of the form of \( A(x) \) because the satisfaction of equilibrium conditions at any area is independent of the history of the flow.
The equilibrium air expansions were calculated on the CAL IBM-704 computer* for reservoir temperatures of 4000, 5000, 6000, 7000, and 8000°K and reservoir pressures of 100, 300, and 1000 atmospheres. The calculations assumed a 13 chemical specie system for air. The solutions for the gas dynamic properties and gas composition are presented for each equilibrium expansion in Tables 1-16. In addition, the air composition is shown as a function of area ratio for each case in Figs. 2-17.

*The Fortran programming for IBM machine computation was done by D. B. Larson, Computer Applications Branch, Systems Research Department, Cornell Aeronautical Laboratory. The IBM-704 program used in the solution of the equilibrium flow is available at CAL. In the event further equilibrium flow calculations are required, details will be provided upon request.
A. Relaxation Length Criterion for Freezing

The solution of nonequilibrium flows, which involve finite reaction rates, is rendered difficult by the need for simultaneous solution of interrelated chemical rate equations and the gas dynamic equations. Also, at the present time, the magnitudes of the required rate constants, and their temperature dependence, are still not known with certainty. It is worthwhile, therefore, to investigate a simple method for the approximate calculation of the final state of a gas undergoing expansion with finite reaction rates.

The present approach is that of Ref. 3 in which specific nonequilibrium airflows in hypersonic nozzles were calculated on the basis of a relaxation length. The results of the previous studies have shown that for flows involving a single relaxing species, the use of a local relaxation length, evaluated for the corresponding equilibrium flow, provides an excellent estimate of the final frozen state of the gas obtained from exact finite-rate solutions. Accordingly, a relaxation length is introduced in the air calculations on the basis of an assumption concerning the relative importance of the various chemical reactions.

The energy associated with the normally inert degrees of freedom for the air species, as a fraction of the total enthalpy, is shown in Fig. 1. It is seen that over the range of reservoir conditions pertinent to the present frozen expansion calculations, i.e., from 4000 to 6000°K, oxygen dissociation is the most important internal mode as it represents the source of greatest energy loss due to freezing. The percentage of the gas enthalpy associated with nitrogen dissociation is small compared with that of oxygen. Furthermore, since the energy associated with the formation of nitric oxide is small and essentially constant over a wide range of temperatures and pressures, the potential energy loss due to freezing of the nitric oxide composition is also quite small. Consequently, a relaxation length is derived assuming that only oxygen dissociation-recombination reactions occur at finite rates.
The reaction may be written

\[ O + O + M_i \xrightarrow{k_f^{(i)}} O_2 + M_i \]

where \( M_i = \) oxygen atom
\( M_1 = \) oxygen molecule
\( M_3 = \) nitrogen atom
\( M_4 = \) nitrogen molecule
\( M_5 = \) nitric oxide molecule

The equilibrium constant in terms of molar concentrations per unit mass \( \mathcal{c}(c) \) has the assumed form

\[ k_e(T') = \frac{k_f^{(i)}}{k_r^{(i)}} = \mathcal{c}(c) \cdot \frac{\gamma_{M_0}^2}{\gamma_{O_2}^2} \quad (30) \]

where \( k^{(i)} \) are reaction rate coefficients. The subscript \( \mathcal{c} \) denotes the local equilibrium value. The rate equation is then

\[ \frac{d\gamma_o}{dt} = \mathcal{u}' \frac{d\gamma_0}{d\mathcal{z}} = 2\gamma' k_r^{(2)} \gamma_o^2 + 2\gamma' k_f^{(1)} \gamma_0 \gamma_{O_2} - 2\gamma' \gamma_o^2 k_R^{(2)} \gamma_0^2 \gamma_{O_2} \quad (31) \]

On substituting Eq. (30) and rearranging, Eq. (31) becomes

\[ \mathcal{u}' \frac{d\gamma_o}{d\mathcal{z}} = 2\gamma' k_e k_r^{(2)} \gamma_o^2 \left[ \frac{1}{k_e} \frac{\gamma_o^2}{\gamma_{O_2}} \right] \left[ 1 - \beta_1 \frac{\gamma_o}{\gamma_{O_2}} + \sum_{i=3}^{5} \beta_i \frac{\gamma_{M_i}}{\gamma_{O_2}} \right] \quad (32) \]
where
\[ \beta_i = \frac{k_{f(i)}}{k_{f(0)}} \]

Now the conservation equation for oxygen atoms is
\[ \dot{\gamma}_0 + \dot{\gamma}_{NO} + 2 \dot{\gamma}_{O_2} = \text{const.} \]
therefore
\[ \dot{\gamma}_{O_2} = B - \frac{\dot{\gamma}_0}{2} \quad \text{where} \ B = \text{const} - \frac{\dot{\gamma}_{NO}}{2} \]

Defining
\[ k_{e(2)}^{(2)} = k_{e(2)}^{(1)} \frac{1}{T^2} \]
\[ \chi = \frac{x}{l} \]
\[ l = \text{length characterizing nozzle geometry} \]
\[ = \frac{L}{\alpha} = \frac{\text{half throat height (cm)}}{\tan \text{tangent of nozzle semi-angle}} \]
\[ \psi = \sqrt{\frac{R_s}{\mu}} \frac{k_{e(2)}^{(2)}}{k_{e(2)}} \]

and substituting in Eq. (32), noting that \( k_e \) (Eq. 30) is defined for equilibrium values of the \( \gamma_i \), there results
\[ \frac{d \gamma_0}{d \chi} = - \frac{\psi \gamma_0}{u T^2} \left[ 1 + \beta_i \frac{\gamma_0}{\gamma_{O_2}} + \sum_{i=1}^{5} \beta_i \frac{\delta M_i}{\gamma_{O_2}} \right] \left[ B - \frac{\dot{\gamma}_0}{2} \right] \quad (33) \]
\[ \left\{ \begin{array}{l} \dot{\gamma}_0 + \frac{B \gamma_0}{B - \frac{\dot{\gamma}_0}{2}} \\ \dot{\gamma}_0 - \frac{\dot{\gamma}_0}{2} \end{array} \right\} \]

where \( \gamma_{O_0} \) = local finite-rate equilibrium value of \( \gamma_0 \), i.e., the equilibrium value of \( \gamma_0 \) for the actual temperature and density distribution.
Equation (33) is then of the form introduced by Heims, i.e.,

\[ \frac{d \gamma_v}{d x} = - \frac{\gamma_v - \gamma_{oe}}{r} \]  

where

\[ r = \frac{r'}{l} = \frac{UT^2}{\psi_{\infty}^2 \left[ 1 + \beta_1 \frac{\gamma_v}{\gamma_{oe}} + \sum_{i=3}^{n} \beta_i \frac{\gamma_{\mu_i}}{\gamma_{\nu_i}} \right] \left[ \frac{B - \gamma_v}{2} \right] \left[ \frac{\gamma_v + \frac{B \gamma_{oe}}{2}}{B - \gamma_{oe}} \right]} \]  

The rate equation, Eq. (34), which incorporates a local relaxation length \( r \), is used to derive an approximate freezing criterion for the finite-rate flow. The freezing criterion is given in a general form in Ref. 3 and its development is repeated here for completeness.

For flow near equilibrium, \( \gamma_v - \gamma_{oe} < < \gamma_{oe} \) and it follows from Eq. (34) that \( \frac{|d \gamma_v|}{dx} < \gamma_{oe}/r. \) For near frozen flow, \( \gamma_{oe} < < \gamma_v \) so that \( \frac{|d \gamma_v|}{dx} \sim \frac{\gamma_v}{r} \gg \frac{\gamma_{oe}}{r}. \) Thus an indication of where significant freezing has occurred may be obtained by setting \( \frac{d \gamma_v}{dx} = \frac{\gamma_{oe}}{r} \). This equality may be evaluated on the basis of the infinite rate equilibrium flow by using the average values of \( \frac{d \gamma_v}{dx} \) and \( \gamma_v \) through the freezing region. That is

\[ \frac{d \gamma_v}{dx} = \frac{1}{2} \left( \frac{d \gamma_v}{dx} \right)_\infty \]

\[ \gamma_{oe} = \frac{1}{2} \left( \gamma_v \right)_\infty \]

to give

\[ \left( \frac{d \gamma_v}{dx} \right)_\infty = \frac{\gamma_{oe}}{r_\infty} \quad \text{or} \quad \left( \frac{d \gamma_v}{dA} \right)_\infty \left( \frac{dA}{dx} \right) = \frac{\gamma_{oe}}{r_\infty} \]  

where the subscript \( \infty \) refers to infinite-rate equilibrium values.

Some remarks should be made at this point on the selection of "equilibrium" values. As mentioned earlier in this section, the concept of
a local chemical relaxation length has been found useful for flows in which
only a single mode of relaxation is involved, such as the dissociation relaxation
of a diatomic gas, with or without inert diluent. For the more complex case
of air, the relaxation length criterion has been applied to two simplified
kinetic models of air in which the only finite-rate reactions considered are
the oxygen dissociation and recombination processes. All species except
O and O₂ were assumed to act only as second or third body colliders for
the oxygen dissociation-recombination kinetics. The appropriate reaction
rate constants were obtained from simple collision theory. Vibrational
equilibrium was assumed for all molecular species.

Two basic models were considered in the calculations. In the
first, the nitrogen atom and molecule and nitric oxide mass concentrations
were frozen at their respective reservoir values during the expansion. In
the second, these mass concentrations were taken as those for infinite-rate
equilibrium (Sec. II). The flows with oxygen atom-molecule equilibration
corresponding to these two models were used in the solution of Eq. (36).

Both models gave values for the frozen degree of oxygen dissoci-
ation (αf) which agreed quite closely up to stagnation temperatures of 6000°K.
For example, in a typical hyperbolic nozzle (L/a = 1.0) for a stagnation
temperature of 5000°K and a stagnation pressure of 100 atm., αf = .125
with N₂, N, and NO frozen at their reservoir concentrations and αf = .149
with N₂, N, and NO in infinite-rate equilibrium. Since these are the
limiting conditions for the N₂, N, and NO reactions, this comparison
demonstrates that oxygen dissociation-recombination reactions play the
major role in air kinetics. The second model has been used in the calcu-
lations reported herein, i.e., the equilibrium solutions of Sec. II were used
to evaluate relaxation lengths from Eq. (35).

In the above calculations the magnitude of kₐ(2) at 3000°K has been
taken as 1.16 x 10¹⁵ cm⁶ mole⁻² sec⁻¹, from the results of Byron's interfer-
erometric studies of oxygen dissociation rates. Then since the temperature
dependence of the recombination rate coefficient has been assumed as

\[ kₐ^{(2)} \sim T^{-2} \]

(following Ref. 3)
then

\[ k^{(2)}_{R_0} = 1.16 \times 10^{15} \left( \frac{3000}{\tau_0'} \right)^2 \frac{\text{cm}^6}{\text{mole}^2 \text{sec}} \]

The remaining rate constants for the second or third bodies \((\beta_i)\) were determined from ordinary collision theory. The form of the rate coefficient is, from collision theory \(^9\)

\[ k_f \approx \frac{K D' \gamma}{\sigma \sqrt{\mu_{ij}}} \left( \frac{\Theta_{ci}}{T'} \right)^\gamma e^{-\frac{\Theta_{ci}}{T'}} \]

where
- \( K \) = steric factor
- \( D' \) = average diameter of colliding particles
- \( \mu_{ij} \) = reduced mass of colliding particles \((i \text{ and } j)\)
  \[ = \frac{\mu_i + \mu_j}{\mu_i \mu_j} \]
- \( \gamma \) = constant
- \( \sigma \) = symmetry number = 1 unlike particles
  = 2 like particles
- \( \Theta_{ci} \) = characteristic dissociation temperature for species \( i \)

For oxygen (from Ref. 8),

\[ \frac{k_f^{(1)}}{k_f^{(2)}} = \beta_i = 35 \frac{T'}{\Theta_{ci}'} \]

For \( N_2 \) and \( NO \) the steric factors and molecular diameters were assumed the same as for \( O_2 \), whence

\[ \frac{k_f^{(1)}}{k_f^{(2)}} = \frac{\sigma_x \sqrt{\mu_{O_2}, O_2}}{\alpha_x \sqrt{\lambda_{O_2}}} \]
then for \( N_2 \),
\[
\frac{k_f^{(4)}}{k_f^{(2)}} = \beta_\gamma = 2.07
\]
and \( NO \),
\[
\frac{k_f^{(5)}}{k_f^{(2)}} = \beta_\delta = 2.033
\]
For \( N \), the steric factor and atomic diameter were assumed to be the same as for \( O \), so
\[
\frac{k_f^{(3)}}{k_f^{(2)}} = 1.047
\]
whence
\[
\frac{k_f^{(5)}}{k_f^{(2)}} = \frac{k_f^{(3)}}{k_f^{(2)}} \frac{k_f^{(4)}}{k_f^{(2)}} = 1.047 \left( 35 \frac{T'}{\Theta_0} \right) = 36.6 \frac{T'}{\Theta_0}
\]
Relaxation lengths may then be calculated from Eq. (35).

Equation (36) is solved graphically to locate the point of freezing in the nozzle. The slopes \( \frac{d \gamma_\nu}{d A} \) are obtained graphically from tangents to the curve of oxygen atom mass concentration as a function of area ratio. Calculated values of \( \gamma_0 / r_\nu \) and the value of \( \left( \frac{d \gamma_\nu}{d A} \right)_\infty \left( \frac{d A}{d x} \right) \) are plotted over a range of area ratios. The area ratio at which these two curves cross is the location at which significant freezing has occurred. The frozen value of \( \gamma_0 \) that can be expected for finite-rate flow is given by the value of \( \gamma_\nu \) at this area ratio. Figure 18 shows the solution of Eq. (36) for the 5000°K, 100 atm. case.

B. Nozzle Geometry Considered

The nozzle geometries considered are given by the general expression \( A = 1 + x^n \). These nozzles are symmetric about the throat for even values of the exponent \( n \). In particular, two nozzle geometries have been considered which correspond to values of the exponent equal to 1 and 2. For \( n = 1 \), the
nozzle is a simple two-dimensional wedge-type nozzle with a sharp throat. The nozzle parameter, \( \ell = L/a \), which enters the equation for the area ratio through the relationship \( a = \frac{X}{L/a} \), is given by one-half the throat height in centimeters divided by the tangent of half the total expansion angle. For \( n = 2 \), the nozzle is an axisymmetric hyperbolic nozzle with a smooth throat and which becomes a conical nozzle for large values of the area ratio, i.e., for nozzle areas not close to the throat. The nozzle geometry parameter is given by the throat radius in centimeters divided by the tangent of the asymptotic half angle.

The nozzle geometry parameter, \( L/a \), represents the rapidity of the nozzle expansion. For a given throat size, small values of \( L/a \) correspond to large expansion angles and accordingly correspond to rapidly expanding nozzles. On the other hand, large values of the parameter \( L/a \) represent small expansion angles and accordingly correspond to gradually expanding nozzles. Many of the nozzles currently used for hypersonic flow research are designed with a value of \( L/a \) of about unity. In the present analyses, values of \( L/a \) of 0.1, 1, and 10 have been used to bracket the values of current design practice.

C. Results of Approximate Finite-Rate Solutions

The results of the relaxation length criterion calculations for flow in a hyperbolic axisymmetric and wedge-type nozzle are shown in Figs. 19, 20, and 21. The calculations were performed for stagnation temperatures of 4000, 5000, and 6000°K and stagnation pressures from 100 to 1000 atmospheres, for three values of the nozzle geometry parameter \( L/a \) (0.1, 1.0, 10).

Figure 19 shows the area ratio for freezing \( (A_f) \) plotted against the stagnation pressure for each stagnation temperature. In all cases freezing occurs fairly early in the nozzle \( (A_f < 25) \). Further, freezing in the nozzle is delayed by an increase in reservoir pressure, an increase in reservoir temperature, and by the use of gradually expanding nozzles \( (L/a) \).
The frozen degree of oxygen dissociation, $\alpha_f$ (the mass fraction of oxygen atoms to total oxygen) is given by

$$\alpha_f = \frac{\gamma_{of}}{\gamma_{of} + \gamma_{NOf} + 2 \gamma_{O2f}}$$

where the subscript $f$ denotes the value at oxygen freezing. The fraction of the stagnation enthalpy represented by frozen chemical energy is given by

$$\frac{H_f}{H_o} = \frac{\mu}{H_o} \left[ \gamma_{of} \frac{\Theta_{D_{o2}}}{2} + \gamma_{Nf} \frac{\Theta_{D_{N2}}}{2} + \gamma_{NOf} \left( \frac{\Theta_{D_{O2}} + \Theta_{D_{N2}} - \Theta_{D_{NO}}}{2} \right) \right]$$

where

$$\Theta_{D_i} = \frac{\Theta_{D_i}'}{T_0'}$$

The characteristic dissociation temperatures ($\Theta_{D_i}'$) assumed for oxygen, nitrogen and nitric oxide are 59,400°K, 113,200°K, and 75,500°K respectively. 

The frozen degree of dissociation and $H_f/H_o$ are plotted versus the stagnation pressure for each stagnation temperature in Figs. 20 and 21 respectively. Figure 20 shows that the frozen level of oxygen dissociation is appreciably reduced by an increase in stagnation pressure. This indicates that high stagnation pressures are required in order to reduce the static enthalpy loss through freezing at high levels of dissociation. Figure 21 shows, for example, that at a stagnation temperature of 6000°K and a stagnation pressure of 100 atm., as much as 20% of the total enthalpy may be frozen chemical energy. A wedge nozzle is superior to a hyperbolic nozzle of the same $L/A$ because of a lower frozen degree of dissociation.
SOLUTIONS FOR EXPANSION AFTER FREEZING

For the simplified kinetic model of air, the exact finite-rate solution is approximated by assuming equilibrium flow up to the freezing point followed by a chemically-frozen flow downstream. This approximation was introduced by Bray\(^1\) for the case of a pure diatomic gas. The area ratio for freezing as determined by the methods outlined in the last section, represents a unique point in the equilibrium solution. The equilibrium gasdynamic properties and composition at this area ratio are the initial values for the subsequent chemically-frozen expansion.

In the frozen expansion calculations, vibrational equilibrium is assumed with the energy taken as that for a quantized linear oscillator. The total gas enthalpy for translational, rotational and vibrational equilibrium, may then be written as

\[
H = 3T + \mu \left[ \gamma_{O_2} \left( \frac{T}{2} + \frac{\Theta_{O_2}}{e^{\Theta_{O_2}/T} - 1} \right) + \gamma_{N_2} \left( \frac{T}{2} + \frac{\Theta_{N_2}}{e^{\Theta_{N_2}/T} - 1} \right) \\
+ \gamma_{N_0} \left( \frac{T}{2} + \frac{\Theta_{N_0}}{e^{\Theta_{N_0}/T} - 1} \right) + \gamma_{O} \left( \frac{T}{2} + \frac{\Theta_{O_2}}{2} \right) \\
+ \gamma_{N} \left( \frac{T}{2} + \frac{\Theta_{N_2}}{2} \right) + \gamma_{NO} \left( \frac{\Theta_{O_2} + \Theta_{N_2}}{2} - \Theta_{NO} \right) \right]
\]

(37)

where \(\mu =\) molecular weight undissociated gas (28.85)

\[
\Theta_{V_i} = \frac{\Theta'_{V_i}}{T_0}
\]

The characteristic vibrational temperatures (\(\Theta'_{V_i}\)) used for oxygen, nitrogen and nitric oxide are 2300°K, 3390°K and 2740°K, respectively.

The method of solution is as follows. Since the gas composition is now frozen, all \(\gamma_i\) in Eq. (37) are known constants and the gas enthalpy may be simply determined for any specified temperature ratio \(T\). The velocity is then calculated from Eq. (8).
The frozen expansion is isentropic (by virtue of zero reaction rates\(^{14}\)), hence

\[
d\bar{H} - \frac{dp}{\sigma} = 0
\]  

(38)

Then from Eq. (10), in which both \(\sum \gamma_i\) and \(\sum \gamma_i\) are now constant, we have

\[
\int \frac{d\bar{H}}{T} - \frac{\sum \gamma_i}{\sum \gamma_i} \ln T - \frac{\sum \gamma_i}{\sum \gamma_i} \ln \sigma = \text{const.}
\]  

(39)

where \(\sum \gamma = \sum \gamma_i\) at freezing

\[
= \left(\gamma_{v_2} + \gamma_{N_2} + \gamma_{H_0} + \gamma_{v_2} + \gamma_{N_2}\right) f
\]

Substituting Eq. (37) for \(H\), the left side of Eq. (39) is evaluated to obtain

\[
\frac{1}{\sum \gamma} \left( \frac{3}{M} + \frac{\sum \gamma_i}{2} + \sum \gamma_i - \frac{\sum \gamma_i}{\sum \gamma_i} \right) \ln T
\]

\[
+ \frac{\gamma_{v_2}}{\sum \gamma} \left( \frac{1}{T} \frac{\Theta_{v_2}}{e^{\Theta_{v_2}/T - 1}} - \ln \frac{e^{\Theta_{v_2}/T - 1}}{e^{\Theta_{v_2}/T}} \right)
\]

\[
+ \frac{\gamma_{N_2}}{\sum \gamma} \left( \frac{1}{T} \frac{\Theta_{N_2}}{e^{\Theta_{N_2}/T - 1}} - \ln \frac{e^{\Theta_{N_2}/T - 1}}{e^{\Theta_{N_2}/T}} \right)
\]

\[
+ \frac{\gamma_{H_0}}{\sum \gamma} \left( \frac{1}{T} \frac{\Theta_{H_0}}{e^{\Theta_{H_0}/T - 1}} - \ln \frac{e^{\Theta_{H_0}/T - 1}}{e^{\Theta_{H_0}/T}} \right) - \frac{1}{\mu \sum \gamma_i} \ln \sigma = \text{const.}
\]  

(40)
where
\[
\sum \gamma_r = (\gamma_{r_0} + \gamma_{r_2} + \gamma_{r_0})_p
\]
\[
\sum \gamma_r = (\gamma_0 + \gamma_{r_N})_f
\]

Equation (40) relates \( \rho \) and \( T \) explicitly for flow with frozen dissociation and equilibrium vibration. The constant is evaluated initially for equilibrium values at the point of freezing. The density is then calculated from Eq. (40) for all subsequent \( T \). The pressure is next obtained from Eq. (10). Also the area ratio from

\[
A = \frac{(\rho u)_{s}}{\rho u}
\]

and a Mach number is calculated at each step from

\[
M = \frac{u}{\sqrt{\frac{dP}{d\rho}_{isentropic}}}
\]

The frozen air expansions were calculated on the CAL Datatron computer* and the results for all initial reservoir states and all nozzle configurations are tabulated in Tables 17 to 26. The parameters in each table commence with their values at freezing, i.e., the equilibrium values at the appropriate \( A_f \). The behavior of the flow temperature, pressure, density, velocity and Mach number during expansion through the hyperbolic-type nozzles from each initial reservoir state is shown in Figs. 22-31. The corresponding equilibrium solutions are included to show the calculated location of freezing in the nozzle and to provide a basis of comparison for the frozen expansions. Only the results for the extreme nozzle shapes (\( \ell = 0.1, 10 \)) are shown; data for the \( \ell = 1.0 \) hyperbolic nozzle flows, as well as for all the wedge-type nozzle flows may be extracted from the tables.

*The programming for Datatron machine computation was done by V.L. Widler, Head, Data Handling Section, Hypersonic Tunnel Department, Cornell Aeronautical Laboratory.
FURTHER KINETIC CONSIDERATIONS

In the application of a relaxation length freezing criterion for air it has been assumed that the freezing of the oxygen kinetics, taken here as the governing chemical mechanism, implies the simultaneous freezing of all kinetics. Actually this assumption has little significance in the present cases since at the predicted freezing temperatures, little dissociation energy exists in the N and NO, and consequently, only a small amount of enthalpy is involved whether these species are assumed to freeze or not.

However, an analysis similar to that outlined in Sec. III may be performed for a different chemical species. The appropriate equations would all be of the same form as those given previously for oxygen. Now if relaxation length considerations were to indicate freezing of this species at area ratios earlier than those predicted in Sec. III on the basis of oxygen kinetics, the previous assumption of simultaneous freezing of all species would result in a greater loss of static enthalpy because of the freezing of oxygen at higher temperatures. This in turn could result in significant changes in some of the gas dynamic properties calculated in Sec. IV. It was considered expedient, therefore, to indicate the approximate solution to a specific case as dictated by the kinetics of some species other than oxygen.

The 6000°K, 100 atmosphere reservoir case was chosen as it involved the greatest amount of initial oxygen dissociation and is therefore the case most sensitive to earlier oxygen freezing. Furthermore, since the equilibrium solution exhibited a more rapid decrease in $\gamma_N$ compared to $\gamma_O$ (Fig. 9), the analysis was based on the nitrogen recombination kinetics. Citing only the results, the application of a relaxation length criterion indicated the freezing of nitrogen approximately at the throat for all nozzle configurations. Then, consistent with the previous assumption, the nitric oxide and oxygen were frozen simultaneously with nitrogen and the resultant frozen expansion is indicated by the dashed profiles in Fig. 29.

It is seen that the calculated frozen expansions determined by the nitrogen kinetics may differ appreciably from those determined by oxygen kinetics. The
difference is mainly dependent on the location of oxygen freezing. However, an argument against the freezing of nitrogen is provided by the nitric oxide "shuffle" reactions. The reactions

\[
N + NO \xrightarrow{\text{fast, low}} N_2 + O
\]

\[
N + O_2 \xrightarrow{\text{fast, low}} NO + O
\]

provide two fast two-body processes for equilibration of \(N - N_2\) and \(N - NO\) which have not been considered in the nitrogen three-body processes discussed above.
VI CONCLUSIONS

A method has been described to estimate the effects of finite chemical reaction rates on the one-dimensional expansion of air in hypersonic nozzles. Numerical solutions have been computed for reservoir temperatures from 4000°K to 6000°K and reservoir pressures from 100 to 1000 atmospheres. The calculations have considered a range of wedge and axisymmetric nozzle shapes. The air was assumed to be at rest and in chemical equilibrium at the reservoir state. The flow parameters and gas composition in the infinite-rate equilibrium expansion from each reservoir state were calculated, and a relaxation length criterion, based on the equilibrium solution, was used to indicate the approximate location of freezing for finite-rate airflow in each nozzle configuration. The resultant frozen air expansions have been presented in tabular and graphical form.

The results of the analysis have shown that in all cases, freezing occurred fairly early in the nozzle downstream of the throat, at area ratios less than about 25 (Fig. 19). As a result of such early freezing the frozen degree of dissociation may be quite large (Fig. 20) representing an appreciable loss in the static enthalpy. It is seen, for example (Fig. 21), that in the high temperature, low pressure cases, the unavailable enthalpy owing to freezing may be as much as 19% of the stagnation enthalpy.

The following features are evident in the results. The effect of the static enthalpy loss through freezing is to reduce the pressure, velocity and temperature at a particular nozzle location (area ratio) from the corresponding equilibrium values, and to increase the density and Mach number. While the change in temperature and Mach number may be considerable, the density and velocity are relatively unaffected by freezing.

As expected, the effects of freezing are minimized for larger $L/a$ values, i.e., by the use of gradually expanding nozzles. In practice, however, limitations will be imposed by boundary layer growth in the nozzle and a compromise will be required between maximizing $L/a$ and maintaining a reasonable nozzle size. In this respect the wedge nozzle is superior to the hyperbolic nozzle for
a given $L/\alpha$ by virtue of a lower frozen degree of dissociation of the flow (later freezing). It should be noted that the effect of a change in the nozzle $L/\alpha$ parameter also reflects the effect of a change in the kinetic rates, in particular $K_{R0}^{(\alpha)}$. Since $L/\alpha$ and the recombination rate constant $K_{R0}^{(\alpha)}$ occur in the rate equation as a product (in $\psi$), a change in $L/\alpha$ is comparable to a similar change in $K_{R0}^{(\alpha)}$, the product remaining the same. This factor permits a simple compensation for discrepancies in existing rate constant data. Also noticeable in the results is the reduction in the frozen degree of dissociation with increase in reservoir pressure. This indicates that high reservoir pressures are required in order to reduce the static enthalpy loss through freezing in the operation of hypersonic wind tunnels at high stagnation temperatures. This requirement is compatible with aerodynamic requirements for testing at very high Mach numbers.

The present nonequilibrium airflow calculations are meant as approximate estimates of the gross features of particular finite-rate nozzle flows. In the interests of simplicity several effects have been omitted from the approximations used, notably the neglect of the nitrogen and nitric oxide reactions at finite rates in the simplified kinetic model of air. However, since the assumption of both zero and infinite rates for reactions involving these species resulted in relatively small changes in the frozen levels of dissociation (Sec. 3.1), the calculated expansions from each reservoir state should represent realistic approximations to actual gas behavior in the nozzle. Ultimate verification must come from comparisons with exact solutions to more complex kinetic models. Such a program is presently underway at CAL.
APPENDIX A

NEWTON-RAPHSO METHOD\textsuperscript{11,12,13} FOR SOLVING THE EQUILIBRIUM PROBLEM

The use of the Newton-Raphson method for the iterative solution of these simultaneous nonlinear algebraic equations was originally suggested by Brinkley\textsuperscript{6} for the determination of equilibrium composition at a given temperature and pressure. Brinkley's method is directly applicable to the computation of the stagnation state; however, certain modifications are introduced for computing composition and pressure at a given entropy for various temperatures along the expansion. Only the modified form is presented here because the essential features of the Brinkley method are contained therein. Equations (23) and (27) are expressed as

\begin{equation}
F_j = 0 \quad (j = 1, 2, \ldots, c, c + 1) \tag{A-1}
\end{equation}

where

\begin{equation}
F_j = q_j + \sum_{\ell=c+1}^{s} \left[ q_j (\nu_{\ell} - 1) - \nu_{j\ell} \right] X_{\ell} - X_j \quad (j = 1, 2, \ldots, c) \tag{A-2}
\end{equation}

\begin{equation}
F_{c+1} = \sum_{i=1}^{s} X_i \left( S_i^* - \mu_i s_o - \ln X_i \right) - \ln p' \tag{A-3}
\end{equation}

For given \( s_o \) and \( T' \), values of \( X_j \) and \( p' \) (the \((c+1)\)st independent variable) must be guessed such that Equations (A-1) are satisfied. \( X_{c+1} \) are related to \( X_j \) by Equation (24).

\begin{equation}
X_{c+1} = K_{c+1} \frac{1}{\sum_{j=1}^{c}} X_j \tilde{v}_j \quad (\ell = c+1, c+2, \ldots, s) \tag{24}
\end{equation}

Repeat (24)
The Newton-Raphson method provides a means of improving successive guesses by utilizing a Taylor expansion of \( F_j \) about \( X_j \) and \( p' \) to first order terms. This leads to a set of fractional corrections \( h_k^{(r)} (k = 1, 2, \ldots, c, c+1) \) applicable to the \( X_j \) and \( p' \) values obtained at the \( r \)th iteration. The improved \((r+1)\)st values of the independent variables are found from

\[
X_k^{(r+1)} = X_k^{(r)} (1 + h_k^{(r)}) \quad (k = 1, 2, \ldots, c) \tag{A-4}
\]

and

\[
p'^{(r+1)} = p'^{(r)} (1 + h_{c+1}^{(r)}) \tag{A-5}
\]

The set of \( h_k \) is computed from a set of linear equations expressed in matrix notation by

\[
\begin{bmatrix} A_j^{(r)} \end{bmatrix} \begin{bmatrix} h_k^{(r)} \end{bmatrix} = \begin{bmatrix} F_j^{(r)} \end{bmatrix} \quad (j = 1, 2, \ldots, c, c+1) \tag{A-6}
\]

\( h_k^{(r)} \) and \( F_j^{(r)} \) are column vectors

\[
A_{j,k} = X_j \delta_{j,k} + \sum_{\ell = c+1}^{c+1} \nu_{k,\ell} \left[ \nu_{\ell,j} - q_j (\nu_{\ell} - 1) \right] X_\ell \quad (j = 1, 2, \ldots, c) \tag{A-7}
\]

\[
\delta_{j,k} = \begin{cases} 0 & \text{for } j \neq k \\ 1 & \text{for } j = k \end{cases} \quad (j = 1, 2, \ldots, c) \tag{A-8}
\]

\[
A_{j, c+1} = \sum_{\ell = c+1}^{c+1} (\nu_{\ell} - 1) \left[ \nu_{\ell,j} - q_j (\nu_{\ell} - 1) \right] X_\ell \quad (j = 1, 2, \ldots, c) \tag{A-9}
\]

\[
A_{c+1, k} = \left\{ -\left( S_k^0 - \mu_k s_0 - \ell n X_k \right) X_k + \sum_{\ell = c+1}^{c+1} \nu_{k,\ell} \left[ -\left( S_\ell^0 - \mu_\ell s_0 - \ell n X_\ell \right) \right] X_\ell \right\} \quad (k = 1, 2, \ldots, c) \tag{A-10}
\]

\[
A_{c+1, c+1} = 1 + \sum_{\ell = c+1}^{c+1} (\nu_{\ell} - 1) \left[ -\left( S_\ell^0 - \mu_\ell s_0 - \ell n X_\ell \right) \right] X_\ell \tag{A-11}
\]
The \((r+1)\)st iteration is started by calculating \(k^{(r)}_k\) from Equation (A-6) using the formulas in Equations (24), (A-2), (A-3), (A-7), (A-8), (A-9), and (A-10). Improved values of \(X_j\) and \(\rho'\) are found from Equations (A-4) and (A-5), and improved values of \(X'_{L}\) are found from Equation (24) using \(X_j^{(r+1)}\) and \(\rho'^{(r+1)}\).

The cycle begins again with the computation of \(k^{(r+1)}_k\) from Equation (A-6) when all \(k_k\) become equal to or smaller than a prescribed maximum fractional error \(\varepsilon_{k}\), the computation is terminated.
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Table 1. EQUILIBRIUM AIRFLOW EXPANSION

\[
T_o' = 4000^\circ K, \ P_o' = 100 \ atm, \ \rho_o' = 8.587 \times 10^{-3} \ gm/cm^3, \\
H_o = 4.871, \ (\rho u)_o = 5.656 \ at \ T = .916, \ \mu_o = 28.18 \ gm/mole
\]
<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$\rho$ (g/cm³)</th>
<th>$P$ (atm)</th>
<th>$u_p$ (m/s)</th>
<th>$A$</th>
<th>$V_H$</th>
<th>$\theta_0$</th>
<th>$\theta_0^*$</th>
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Table 2. EQUILIBRIUM AIRFLOW EXPANSION

$T_o' = 4000^\circ$K, $P_o' = 300$ atm., $\rho_u' = 2.604 \times 10^{-2}$ gm/cm$^3$,
$H_0 = 4.705$, $\beta\rho_u = 0.6551$ at $T = 909$, $\mu_o = 28.50$ gm/mole
Table 3.

| T  | \( p \) | \( p ^ \) | \( \nu ^ {\prime} \) | M | A | \( T _ { k } \) | \( T _ { o } ^ {\prime} \) | \( T _ { v } ^ {\prime} \) | \( T _ { a } ^ {\prime} \) | \( T _ { n } ^ {\prime} \) | \( T _ { v } ^ {\prime} \) | \( T _ { n } ^ {\prime} \) | \( T _ { s } ^ {\prime} \) | \( T _ { t } ^ {\prime} \) | \( T _ { n } ^ {\prime} \) |
|----|------|------|-------|---|---|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0  | 100  | 100  | 100  | 100 | 100 | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  | 100  |
| 100 | 90   | 90   | 90   | 90  | 90  | 90   | 90   | 90   | 90   | 90   | 90   | 90   | 90   | 90   | 90   | 90   |
| 200 | 80   | 80   | 80   | 80  | 80  | 80   | 80   | 80   | 80   | 80   | 80   | 80   | 80   | 80   | 80   | 80   |
| 300 | 70   | 70   | 70   | 70  | 70  | 70   | 70   | 70   | 70   | 70   | 70   | 70   | 70   | 70   | 70   | 70   |
| 400 | 60   | 60   | 60   | 60  | 60  | 60   | 60   | 60   | 60   | 60   | 60   | 60   | 60   | 60   | 60   | 60   |
| 500 | 50   | 50   | 50   | 50  | 50  | 50   | 50   | 50   | 50   | 50   | 50   | 50   | 50   | 50   | 50   | 50   |
| 600 | 40   | 40   | 40   | 40  | 40  | 40   | 40   | 40   | 40   | 40   | 40   | 40   | 40   | 40   | 40   | 40   |
| 700 | 30   | 30   | 30   | 30  | 30  | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   | 30   |
| 800 | 20   | 20   | 20   | 20  | 20  | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   | 20   |

\( T _ { o } ^ {\prime} = 4000^\circ K, \quad p _ { o } ^ {\prime} = 1000 \text{ atm}, \quad \rho _ { o } ^ {\prime} = 5.744 \times 10^{-2} \text{ gm/cm}^3, \)

\( H _ { o } = 4.599, \quad (\rho u)_{s} = 0.6547 \text{ at } T = 903, \quad \bar{\mu} _ { o } = 28.70 \text{ gm/mole} \)
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Table 5. EQUILIBRIUM AIRFLOW EXPANSION

\[ T_0 = 5000^\circ K, \quad p_0 = 200 \text{ atm}, \quad p'_0 = 1.317 \times 10^{-2} \text{ gm/cm}^3, \]

\[ H_0 = 5.436, \quad (\rho u)_0 = 6729 \text{ at } T = 915, \quad \overline{p}_0 = 27.01 \text{ gm/mole} \]
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**Table 6. EQUILIBRIUM AIRFLOW EXPANSION**

\[ T_{O}' = 5000^\circ \text{K}, \quad P_{O}' = 300 \text{ atm}, \quad \rho_{O}' = 1.995 \times 10^{-2} \text{ gm/cm}^3, \]

\[ H_{O} = 5.313, \quad \langle p u \rangle_{*} = 0.6979 \text{ at } T = 913, \quad \bar{\rho}_{O} = 27.28 \text{ gm/mole} \]
Table 7.

EQUILIBRIUM AIRFLOW EXPANSION

\[ T_0' = 5000^\circ K, \quad \rho_0' = 1000 \text{ atm}, \quad \rho_0'' = 6.810 \times 10^{-2} \text{ gm/cm}^3, \]

\[ H_0 = 5.026, \quad (\rho u)_s = .6628 \text{ at } T = .909, \quad \mu_o = 27.94 \text{ gm/mole} \]
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$T_0^* = 6000^\circ K$, $P_0 = 100$ atm, $P_0^* = 5.076 \times 10^{-3}$ gm/cm$^3$.

$H_o = 6.229$, $P_{ol} = 907$, $P_{o+} = 25.00$ gm/mole.
| $T_0$ | $V'$ (cm/s) | $M$ | $A$ | $T_{H}$ | $T_{D}$ | $T_{A}$ | $T_{H}^*$ | $T_{D}^*$ | $T_{A}^*$ | $T_{NO}$ | $T_{NO}^*$ | $T_{O}$ | $T_{O}^*$ | $T_{O}^*$ | $T_{O}^*$ |
|-------|-------------|-----|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1.00  | 0.000       | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 1.00  | 0.000       | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Table 9. EQUILIBRIUM AIRFLOW EXPANSION

$T_0 = 6000^\circ K$, $p_0 = 300$ atm., $\rho_0 = 1.574 \times 10^{-2} \text{ gm/cm}^3$,

$H_0 = 5.850$, $(\rho u)_t = .6920$ at $T = .907$, $\mu = 25.84 \text{ gm/mole}$
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Table 10. EQUILIBRIUM AIRFLOW EXPANSION

\( T_{o} = 6000 \text{ K}, \rho_{o} = 1000 \text{ atm}, \rho_{o} = 5.440 \times 10^{-2} \text{ gm/cm}^3, \)

\( H_{o} = 5.466, (\rho u)_{k} = 6.796 \text{ at } T = .906, \mu_{o} = 26.78 \text{ gm/mole} \)
| \( T \) | \( \rho \) | \( P \) | \( \text{km/sec}^2 \) | \( M \) | \( T_{\text{H}_2} \) | \( T_{\text{D}_2} \) | \( T_A \) | \( T_N \) | \( T_{\text{CO}} \) | \( T_{\text{CO}_2} \) | \( T_{\text{H}_2} \) | \( T_{\text{D}_2} \) | \( T_A \) | \( T_N \) | \( T_{\text{CO}} \) | \( T_{\text{CO}_2} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1000 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 990 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 980 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 970 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 960 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 950 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 940 | 1.00 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |

Table 11: EQUILIBRIUM AIRFLOW EXPANSION

\( T_0 = 7000^\circ \text{K}, \ \rho_0' = 100 \ \text{atm}, \ \rho_0 = 4.133 \times 10^{-3} \ \text{gm/cm}^3, \)

\( H_0 = 6.805, \ \langle \rho u \rangle = 7.156 \) at \( T = 920, \ \overline{\rho_0} = 23.74 \ \text{gm/mole} \)
Table 13. EQUILIBRIUM AIR FLOW EXPANSION

\[ T_o' = 7000^\circ K, \ p_o' = 1000 \ \text{atm}, \ \rho_o' = 4.460 \times 10^{-2} \ \text{gm/cm}^3, \]

\[ H_o' = \vec{e} \cdot 549, \ (\rho u)_o' = 0.6962 \ \text{at} \ T = .904, \ \vec{u}_o = 25.62 \ \text{gm/mole} \]
<table>
<thead>
<tr>
<th>( T )</th>
<th>( P )</th>
<th>( P' )</th>
<th>( (\text{atm/sec}) )</th>
<th>( M )</th>
<th>( A )</th>
<th>( T_{M} )</th>
<th>( T_{B} )</th>
<th>( T_{N} )</th>
<th>( T_{O} )</th>
<th>( T_{O'} )</th>
<th>( T_{O''} )</th>
<th>( T_{O'''} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>100</td>
<td>01</td>
<td>0</td>
<td>000000</td>
<td>000000</td>
<td>---</td>
<td>2170</td>
<td>01</td>
<td>7178</td>
<td>04</td>
<td>3219</td>
<td>03</td>
</tr>
<tr>
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</tr>
<tr>
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<td>260</td>
<td>03</td>
<td>0</td>
<td>000000</td>
<td>000000</td>
<td>---</td>
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<td>01</td>
<td>7178</td>
<td>04</td>
<td>3219</td>
<td>03</td>
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<tr>
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<td>340</td>
<td>04</td>
<td>0</td>
<td>000000</td>
<td>000000</td>
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<td>01</td>
<td>7178</td>
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<td>03</td>
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<td>000000</td>
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<td>2170</td>
<td>01</td>
<td>7178</td>
<td>04</td>
<td>3219</td>
<td>03</td>
</tr>
</tbody>
</table>

Table 14. EQUILIBRIUM AIRFLOW EXPANSION

\[ T_{O'} = 8000^\circ K, \ p_{O'} = 100 \text{ atm}, \ p_{O''} = 3.340 \times 10^{-3} \text{ gm/cm}^3, \]

\[ H_{o} = 7.951, \ (p_{u})_{o} = 0.7392 \text{ at } T = 0.936, \ T_{o} = 21.93 \text{ gm/mole} \]
Table 16. EQUILIBRIUM AIRFLOW EXPANSION

$T_0 = 8000^\circ$K, $P_0 = 1000$ atm, $\rho_0 = 3.722 \times 10^{-2}$ gm/cm$^3$.

$H_0 = 6.289$, $(\rho_0) = .7105$ at $T = .910$, $\mu_0 = 24.43$ gm/mole.
| T     | p     | P     | u'   | M     | A     | T     | p     | P     | u'   | M     | A     | T     | p     | P     | u'   | M     | A     |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 0.8  | 0.052| 0.043| 0.057| 0.122| 0.010| 0.753| 0.227| 0.160| 0.136| 0.860| 0.218| 0.568| 0.156| 0.224| 0.160| 0.136| 0.860| 0.218|
| 0.86 | 0.064| 0.059| 0.473| 0.129| 0.010| 0.774| 0.218| 0.300| 0.226| 0.558| 0.196| 0.170| 0.156| 0.252| 0.160| 0.136| 0.158| 0.218|
| 0.84 | 0.075| 0.075| 0.453| 0.136| 0.020| 0.824| 0.238| 0.221| 0.286| 0.553| 0.136| 0.156| 0.156| 0.252| 0.160| 0.136| 0.226| 0.218|
| 0.95 | 0.100| 0.030| 0.195| 0.180| 0.030| 0.777| 0.713| 0.020| 0.120| 0.160| 0.156| 0.156| 0.252| 0.160| 0.136| 0.106| 0.226| 0.218|
| 0.99 | 0.111| 0.030| 0.199| 0.180| 0.030| 0.815| 0.713| 0.020| 0.120| 0.160| 0.156| 0.156| 0.252| 0.160| 0.136| 0.226| 0.218|
| 1.0  | 0.122| 0.010| 0.753| 0.227| 0.160| 0.136| 0.860| 0.218| 0.568| 0.156| 0.224| 0.160| 0.136| 0.860| 0.218| 0.568| 0.156| 0.224|

Table 17b. EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE

\( T_0 = 4000 \text{K}, \quad p_0 = 100 \text{ atm} \)
Table 18a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

\( T_o' = 4000 \text{K}, \ p_o' = 300 \text{ atm.} \)
<table>
<thead>
<tr>
<th>T</th>
<th>p</th>
<th>P</th>
<th>( \mu' )</th>
<th>M</th>
<th>A</th>
<th>T</th>
<th>p</th>
<th>P</th>
<th>( \mu' )</th>
<th>M</th>
<th>A</th>
<th>T</th>
<th>p</th>
<th>P</th>
<th>( \mu' )</th>
<th>M</th>
<th>A</th>
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<td>0.6760</td>
<td>0.208</td>
<td>0.2037</td>
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<td>0.504</td>
<td>0.7523</td>
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<td>0.176</td>
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<td>0.58</td>
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<td>0.2581</td>
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<td>0.263</td>
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<td>0.9293</td>
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</table>

Table 18b: EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE

\( T_0' = 4000^\circ K, p_0' = 300\ atm. \)
<table>
<thead>
<tr>
<th>T</th>
<th>P</th>
<th>( u' )</th>
<th>M</th>
<th>A</th>
<th>T</th>
<th>P</th>
<th>( u' )</th>
<th>M</th>
<th>A</th>
<th>T</th>
<th>P</th>
<th>( u' )</th>
<th>M</th>
<th>A</th>
</tr>
</thead>
<tbody>
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<td>0.01</td>
<td>0.905</td>
<td>5.036</td>
<td>0.253</td>
<td>0.01</td>
<td>0.940</td>
<td>5.325</td>
<td>0.01</td>
<td>0.905</td>
<td>5.036</td>
<td>0.253</td>
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<td>0.95</td>
<td>0.737</td>
<td>0.230</td>
<td>0.01</td>
<td>0.940</td>
<td>5.325</td>
<td>0.01</td>
<td>0.905</td>
<td>5.036</td>
<td>0.253</td>
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<td>0.940</td>
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<td>0.737</td>
<td>0.230</td>
<td>0.01</td>
<td>0.940</td>
<td>5.325</td>
<td>0.01</td>
<td>0.905</td>
<td>5.036</td>
<td>0.253</td>
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<td>0.905</td>
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<td>0.01</td>
<td>0.940</td>
<td>5.325</td>
<td>0.01</td>
<td>0.905</td>
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<td>0.01</td>
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<td>0.01</td>
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<tr>
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<td>0.737</td>
<td>0.230</td>
<td>0.01</td>
<td>0.940</td>
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<td>0.905</td>
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<td>0.253</td>
<td>0.01</td>
<td>0.940</td>
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Table 19a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

\( \text{T}_0' = 4000^\circ \text{K}, \ p_0' = 1000 \ \text{atm} \)
Table 19b. EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE

\[ T_0' = 4000^\circ\text{K}, \quad p_0' = 1000 \text{ atm.} \]

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Table 20a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

$T_o' = 5000^\circ K$, $p_o' = 100$ atm.
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**Table 2a.** EXPANSION AFTER FREEZING - WEDGE NOZZLE

\( T_0' = 5000^\circ K, \ p_0' = 200 \ atm. \)
Table 21b. EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE

\[ T_0' = 5000^\circ \text{K}, \quad p_0' = 200 \text{ atm} \]
<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$P$ (atm)</th>
<th>$u'$ (ft/sec)</th>
<th>$M$</th>
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Table 22a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

$T_o = 5000^\circ K$, $p_o = 300$ atm.
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Table 23a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

T₀ = 5000°K, p₀ = 1000 atm.
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<td>( V' )</td>
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<tr>
<td>0.950</td>
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**Table 23b. Expansion After Freezing - Hyperbolic Nozzle**

\( T_0' = 5000^\circ K, \ p_0' = 1000 \ atm. \)
| T | p | P | u' | M | A | T | p | P | u' | M | A | T | p | P | u' | M | A |
| 428 | 41.8 | 90.5 | 0.4 | 219 | 0.4 | 205 | 0.4 | 0.575 | 0.615 | 0.647 | 0.680 | 0.713 | 0.747 | 0.782 | 0.818 | 0.856 | 0.896 | 0.938 | 0.982 | 1.031 |
| 450 | 57.3 | 96.7 | 0.4 | 219 | 0.4 | 239 | 0.4 | 0.56 | 0.61 | 0.66 | 0.71 | 0.76 | 0.81 | 0.86 | 0.92 | 0.98 | 1.04 | 1.11 | 1.19 | 1.28 |
| 480 | 62.1 | 101.9 | 0.4 | 219 | 0.4 | 257 | 0.4 | 0.54 | 0.60 | 0.65 | 0.70 | 0.75 | 0.81 | 0.86 | 0.93 | 0.99 | 1.07 | 1.16 | 1.26 | 1.38 |
| 510 | 67.1 | 107.2 | 0.4 | 219 | 0.4 | 275 | 0.4 | 0.52 | 0.58 | 0.63 | 0.69 | 0.74 | 0.80 | 0.86 | 0.94 | 1.01 | 1.10 | 1.21 | 1.33 | 1.48 |
| 540 | 73.1 | 112.7 | 0.4 | 219 | 0.4 | 293 | 0.4 | 0.50 | 0.56 | 0.61 | 0.67 | 0.73 | 0.79 | 0.86 | 0.94 | 1.02 | 1.11 | 1.24 | 1.36 | 1.53 |
| 570 | 80.4 | 119.2 | 0.4 | 219 | 0.4 | 311 | 0.4 | 0.48 | 0.54 | 0.60 | 0.66 | 0.72 | 0.79 | 0.86 | 0.94 | 1.03 | 1.12 | 1.26 | 1.41 | 1.60 |
| 600 | 89.0 | 127.6 | 0.4 | 219 | 0.4 | 329 | 0.4 | 0.46 | 0.52 | 0.58 | 0.65 | 0.71 | 0.78 | 0.86 | 0.95 | 1.05 | 1.15 | 1.32 | 1.51 | 1.74 |
| 630 | 98.9 | 136.9 | 0.4 | 219 | 0.4 | 347 | 0.4 | 0.44 | 0.51 | 0.57 | 0.64 | 0.71 | 0.78 | 0.87 | 0.97 | 1.08 | 1.20 | 1.39 | 1.62 | 1.90 |
| 660 | 110.5 | 147.0 | 0.4 | 219 | 0.4 | 365 | 0.4 | 0.42 | 0.49 | 0.56 | 0.64 | 0.71 | 0.78 | 0.87 | 0.98 | 1.11 | 1.25 | 1.48 | 1.75 | 2.13 |
| 690 | 123.5 | 157.7 | 0.4 | 219 | 0.4 | 383 | 0.4 | 0.40 | 0.48 | 0.55 | 0.63 | 0.71 | 0.79 | 0.88 | 0.99 | 1.13 | 1.30 | 1.59 | 2.05 | 2.67 |
| 720 | 138.3 | 169.0 | 0.4 | 219 | 0.4 | 401 | 0.4 | 0.38 | 0.46 | 0.54 | 0.63 | 0.71 | 0.80 | 0.90 | 1.02 | 1.18 | 1.40 | 1.79 | 2.51 | 3.48 |
| 750 | 154.6 | 180.8 | 0.4 | 219 | 0.4 | 419 | 0.4 | 0.36 | 0.44 | 0.53 | 0.62 | 0.70 | 0.80 | 0.90 | 1.03 | 1.21 | 1.56 | 2.00 | 3.00 | 4.37 |
| 780 | 172.3 | 193.0 | 0.4 | 219 | 0.4 | 437 | 0.4 | 0.34 | 0.42 | 0.51 | 0.60 | 0.70 | 0.80 | 0.90 | 1.04 | 1.25 | 1.65 | 2.23 | 3.45 | 5.14 |
| 810 | 191.2 | 205.5 | 0.4 | 219 | 0.4 | 455 | 0.4 | 0.32 | 0.40 | 0.50 | 0.59 | 0.70 | 0.81 | 0.92 | 1.06 | 1.31 | 1.81 | 2.54 | 3.60 | 5.53 |
| 840 | 211.1 | 218.4 | 0.4 | 219 | 0.4 | 473 | 0.4 | 0.30 | 0.38 | 0.49 | 0.59 | 0.70 | 0.82 | 0.94 | 1.09 | 1.38 | 2.04 | 3.10 | 4.56 | 7.25 |
| 870 | 232.1 | 231.7 | 0.4 | 219 | 0.4 | 491 | 0.4 | 0.28 | 0.36 | 0.47 | 0.58 | 0.71 | 0.83 | 0.97 | 1.13 | 1.47 | 2.32 | 3.64 | 5.66 | 9.32 |
| 900 | 254.1 | 245.4 | 0.4 | 219 | 0.4 | 509 | 0.4 | 0.26 | 0.34 | 0.46 | 0.57 | 0.71 | 0.84 | 0.99 | 1.16 | 1.54 | 2.49 | 4.02 | 6.78 | 12.2 |

Table 24a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

* T₀' = 6000°K, p₀' = 100 atm.
### Table 25a. Expansion After Freezing - Wedge Nozzle

\( T_o' = 6000^\circ K, \; p_o' = 300 \text{ atm.} \)

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<th>( u' )</th>
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**Notes:**
- Unit conversion factors: 6000\(^\circ K = 3973.78\)° R.
- Data represents expansion after freezing in a wedge nozzle at specified conditions.
- Units: \( T \) (temperature), \( p \) (pressure), \( P \) (density), \( u' \) (velocity), \( M \) (Mach number), \( A \) (area ratio), \( (\text{ft/sec}.) \) (velocity in feet per second).
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<tr>
<td>1200</td>
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<td>485</td>
<td>484</td>
<td>483</td>
<td>482</td>
<td>481</td>
</tr>
</tbody>
</table>

**Table 25b. EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE**

\( T_{0} = 6000^\circ K, \quad p_{0} = 300 \text{ atm.} \)
Table 26a. EXPANSION AFTER FREEZING - WEDGE NOZZLE

<table>
<thead>
<tr>
<th>$f = 1.0$</th>
<th>$f = 1.0$</th>
<th>$f = 0.1$</th>
<th>$f = 0.1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{0}$ = 6000 K, $P_{0}$ = 1000 atm.</td>
<td>$T_{0}$ = 6000 K, $P_{0}$ = 1000 atm.</td>
<td>$T_{0}$ = 6000 K, $P_{0}$ = 1000 atm.</td>
<td>$T_{0}$ = 6000 K, $P_{0}$ = 1000 atm.</td>
</tr>
</tbody>
</table>
### Table 26b. EXPANSION AFTER FREEZING - HYPERBOLIC NOZZLE

\( T_0' = 6000^\circ K\), \( p_0' = 1000\) atm.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>( T )</th>
<th>( \rho )</th>
<th>( P )</th>
<th>( u' ) (ft/sec.)</th>
<th>( M )</th>
<th>( A )</th>
<th>( T )</th>
<th>( \rho )</th>
<th>( P )</th>
<th>( u' ) (ft/sec.)</th>
<th>( M )</th>
<th>( A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.03</td>
<td>0.04</td>
<td>0.05</td>
<td>0.06</td>
<td>0.07</td>
<td>0.08</td>
<td>0.09</td>
<td>0.10</td>
<td>0.11</td>
<td>0.12</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>0.35</td>
<td>0.28</td>
<td>0.21</td>
<td>0.14</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>0.43</td>
<td>0.35</td>
<td>0.28</td>
<td>0.21</td>
<td>0.14</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.6</td>
<td>0.56</td>
<td>0.49</td>
<td>0.42</td>
<td>0.35</td>
<td>0.28</td>
<td>0.21</td>
<td>0.14</td>
<td>0.07</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
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</tr>
<tr>
<td>0.7</td>
<td>0.66</td>
<td>0.59</td>
<td>0.52</td>
<td>0.45</td>
<td>0.38</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
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<td>0.00</td>
</tr>
<tr>
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<td>0.74</td>
<td>0.66</td>
<td>0.59</td>
<td>0.52</td>
<td>0.45</td>
<td>0.38</td>
<td>0.31</td>
<td>0.24</td>
<td>0.17</td>
<td>0.10</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>0.9</td>
<td>0.80</td>
<td>0.76</td>
<td>0.69</td>
<td>0.62</td>
<td>0.55</td>
<td>0.48</td>
<td>0.41</td>
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<td>0.85</td>
<td>0.79</td>
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<td>0.65</td>
<td>0.58</td>
<td>0.51</td>
<td>0.44</td>
<td>0.37</td>
<td>0.30</td>
<td>0.23</td>
<td>0.16</td>
</tr>
<tr>
<td>1.1</td>
<td>0.89</td>
<td>0.94</td>
<td>0.88</td>
<td>0.81</td>
<td>0.74</td>
<td>0.67</td>
<td>0.60</td>
<td>0.53</td>
<td>0.46</td>
<td>0.39</td>
<td>0.32</td>
<td>0.25</td>
</tr>
<tr>
<td>1.2</td>
<td>0.93</td>
<td>1.02</td>
<td>0.96</td>
<td>0.89</td>
<td>0.82</td>
<td>0.75</td>
<td>0.68</td>
<td>0.61</td>
<td>0.54</td>
<td>0.47</td>
<td>0.40</td>
<td>0.33</td>
</tr>
<tr>
<td>1.3</td>
<td>0.96</td>
<td>1.09</td>
<td>1.03</td>
<td>0.96</td>
<td>0.89</td>
<td>0.82</td>
<td>0.75</td>
<td>0.68</td>
<td>0.61</td>
<td>0.54</td>
<td>0.47</td>
<td>0.40</td>
</tr>
<tr>
<td>1.4</td>
<td>0.99</td>
<td>1.16</td>
<td>1.10</td>
<td>1.03</td>
<td>0.96</td>
<td>0.89</td>
<td>0.82</td>
<td>0.75</td>
<td>0.68</td>
<td>0.61</td>
<td>0.54</td>
<td>0.47</td>
</tr>
</tbody>
</table>

**Note:** The table provides expansion coefficients for various conditions, showing how the velocity changes with different pressures and densities for specified temperature and pressure conditions.
Figure 1  FRACTION OF THE ENTHALPY OF EQUILIBRIUM AIR WHICH IS IN THE SPECIFIED ENERGY MODE

(REFERENCE: CAL REPORT NO. BE-1007-A-3)
vs Area Ratio in an Isentropic Equilibrium Expansion
FIGURE NO. 2

$T_1 = 4000 \, ^\circ K$

$P_0 = 300 \, \text{ATM}$

ION vs AREA RATIO in an Isentropic Equilibrium Expansion

70
FIGURE NO. 2

\[ T_0' = 3000^\circ K \]
\[ P_0' = 1000 \text{ ATM} \]

ON VS AREA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
Figure No. 5
T = 5000 K
P = 100 ATM

Area Ratio (A/A₀) vs Area Ratio in an Isentropic Equilibrium Expansion
Air Composition (gm moles/gm of mixture)
FIGURE NO. 6
$T_0 = 5000^\circ K$
$p_0 = 200$ atm

Area Ratio ($A/A_0$) vs Area Ratio in an Isentropic Equilibrium Expansion
Air Composition (gm holes/gm of mixture)
vs AREA RATIO in an ISENTROPIC EQUILIBRIUM EXPANSION
AERA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
AIR COMPOSITION VS AREA RATIO IN AN ISENTROPIC
AIR COMPOSITION VS AREA RATIO IN AN ISENTROPIC E
AREA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
vs AREA RATIO in an ISENTROPIC EQUILIBRIUM EXPANSION
Air Composition (gm moles/gm of mixture)
ON VS AREA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
Fig. 14

$T_e = 7000 \text{ K}$

$P_e = 1000 \text{ ATM}$

**Area Ratio ($A/A_0$) vs Area Ratio in an Isentropic Equilibrium Expansion**

The diagram shows the relationship between area ratio ($A/A_0$) on a logarithmic scale and the area ratio in an isentropic equilibrium expansion. The curves illustrate how the area ratio changes with different initial conditions.
Air Composition vs Area Ratio in an Isentropic Expansion
VS AREA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
Air Composition vs Area Ratio in an Isentropic
FIGURE NO. 17

$T_0 = 8000 \, ^\circ \text{K}$

$P_0 = 1000 \, \text{ATM}$

AERA RATIO IN AN ISENTROPIC EQUILIBRIUM EXPANSION
Figure 18  SAMPLE SOLUTION OF EQUATION (36) $T_0' = 5000^\circ$ K, $P_0' = 100$ ATM.
Figure 19 (a) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE AREA RATIO FOR FREEZING FOR A SIMPLIFIED AIR MODEL.
Figure 19 (b) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE AREA RATIO FOR FREEZING FOR A SIMPLIFIED AIR MODEL.

HYPERBOLIC NOZZLE
\[ T_0 = 5000° \text{K} \]

WEDGE NOZZLE
\[ T_0 = 5000° \text{K} \]
Figure 19 (c) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE AREA RATIO FOR FREEZING FOR A SIMPLIFIED AIR MODEL.

HYPERBOLIC NOZZLE

\[ T_0 = 6000^\circ K \]

WEDGE NOZZLE

\[ T_0 = 5000^\circ K \]
Figure 20 (a) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE FROZEN DEGREE OF OXYGEN DISSOCIATION FOR A SIMPLIFIED AIR MODEL.
Figure 20 (b) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE FROZEN DEGREE OF OXYGEN DISSOCIATION FOR A SIMPLIFIED AIR MODEL.
Figure 20 (c)  EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE FROZEN DEGREE OF OXYGEN DISSOCIATION FOR A SIMPLIFIED AIR MODEL.
Figure 21 (a) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE FROZEN CHEMICAL ENERGY FOR A SIMPLIFIED AIR MODEL
Figure 21(b) Effect of stagnation pressure and nozzle geometry on the frozen chemical energy for a simplified air model.
Figure 21 (c) EFFECT OF STAGNATION PRESSURE AND NOZZLE GEOMETRY ON THE FROZEN CHEMICAL ENERGY FOR A SIMPLIFIED AIR MODEL
FIGURE NO. 23a  HYPERSONIC NOZZLE  $T_i = 4000^\circ K$  $P_o = 300$ ATM
Figure No. 24b  Hyperbolic Nozzle  $T_0 = 4000^\circ\text{K}  P_0 = 1000\ \text{atm}$
FIGURE NO. 24c  HYPERBOLIC NOZZLE  $T_0 = 4000^\circ K$, $P_c = 1000$ atm
Figure No. 25
Hyperbolic Nozzle

$T_1 = 5000 \text{ K}$
$P_0 = 100 \text{ atm}$
FIGURE NO 27a. HYPERBOLIC NOZZLE. T₀ = 5000 °K, P₀ = 300 ATM
FIGURE NO. 28c  HYPERBOLIC NOZZLE  $T_0 = 5000^\circ K$  $P_0 = 1000$ atm
FIGURE NO. 302  HYPERBOLIC NOZZLE  $T_0 = 6000^\circ F$, $P_0 = 300$ ATM
FIGURE NO. 316  HYPERBOLIC NOZZLE  $T_0' = 4000°K$  $P_0' = 1000$ ATM