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SURVEY OF BOUNDARY LAYER HEAT TRANSFER AT HIGH VELOCITIES AND HIGH TEMPERATURES

Ernst R. G. Eckert

University of Minnesota

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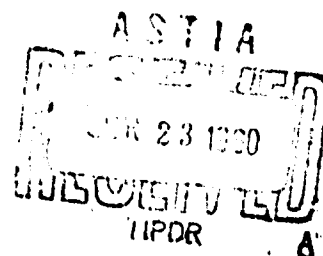WRIGHT AIR DEVELOPMENT CENTER
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FOREWORD

This report was prepared by Dr. Ernst R. G. Eckert, Professor
of Mechanical Engineering and Director of the Heat Transfer Laboratory,
University of Minnesota, under USAF Contract Nr AF 33(616)-5676.   The
contract was initiated and administered by the Fluid Dynamics Research
Branch of the Aeronautical Research Laboratory, Wright Air Development
Center, Wright-Patterson Air Force Base, Ohio, under Dr. R. Mills as
Branch Chief and Mr. E. Soehngen as Project Scientist, Project 7064 and
Task 70132.

ABSTRACT

Heat transfer in boundary layer flow at high velocities and high temperatures is discussed, and engineering relations for its calculation are presented. A similar discussion is the subject of WADC Technical Report 54-70, published in 1954. Aeronautical and space flight applications have, in the meantime, pushed the conditions under which heat transfer knowledge is important to much higher velocities and temperatures. This need for information created an intensive research effort, which is reflected in the large number of publications contained in appendix III of this report. By this research, the influence of very large temperature differences, of dissociation and ionization, of low densities, and of chemical reactions on heat transfer has been well established in its general features. In addition, new cooling methods, like transpiration, ablation, or film cooling have been studied. The reference enthalpy method, introduced in WADC TR 54-70, and well established in the meantime, is extended and generalized in this report to include the additional effects mentioned above. This leads to simple relations by which heat transfer can be calculated with an accuracy which should be sufficient for engineering purposes. These relations additionally have the advantage that most of them are applicable to any gas or gas mixture; therefore, not only to the reentry problem but also to heat transfer problems arising in rockets or other propulsion systems and in other applications.

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c_p specific heat at constant pressure

h   film heat transfer coefficient

i   enthalpy

k   heat conductivity

n   parameter for velocity variation

$\dot{m}$ coolant mass flux per unit area

q   heat flux per unit area

r   recovery factor

s   coordinate along curved surface

u   velocity component in x or s direction

v   velocity component in y direction

x   coordinate along plane surface

y   distance from surface

A   area

A,B,C   constants

F   function

I   total enthalpy

L   reference length

Q   heat flux

T   temperature

V   velocity

β fraction of liquid coolant, pressure gradient parameter

γ parameter for wall temperature variation

ϵ angle between surface normal and upstream flow direction

ϑ temperature potential

μ	viscosity
ψ	stream function
ρ	density
τ	shear stress

Dimensionless Parameters:

Nu	Nusselt number
St	Stanton number
c_f	friction factor
Pr	Prandtl number
M	Mach number
Sc	Schmidt number
Le	lewis number
r	recovery factor

Subscripts:

c	coolant
ch	chemical
co	convection
e	at outer edge of boundary layer
g	gas
i	in solid wall, enthalpy
o	initial, stagnation point
r	recovery
rad	radiation
s	solid
t	total
w	at wall surface
x	based on x
*	at reference condition

1. INTRODUCTION

Electronic computers have radically influenced the extent and kind of analytical work which is today performed in connection with engineering developments. Analyses which a few years ago were still considered much too tedious and time-consuming can be done today in a comparatively short time. However, regardless of the tremendous help which the computers offer to engineering designs, there still exists a definite need for simple relations which can be used in hand calculations. Such relations are needed for preliminary design work, or they are also useful for more extended calculations on electronic computers if the heat transfer is only a small part of the total program.

The purpose of the present report is to present a survey of the field of heat transfer from gases to surfaces at high velocities and high temperatures, situations as they occur in aeronautical engineering in the development of high speed vehicles like missiles and satellites or in space flight. Simple relationships are presented or developed with which heat transfer can be calculated with an accuracy which necessarily is limited, but which should be sufficient for normal engineering design purposes. The discussion of the field and the relations which will be presented are kept general so that they can be applied not only to external heat transfer on vehicles moving through the atmosphere of our earth, but also to other gases at other temperature and pressure conditions. In this way they can, for instance, be used to make calculations on heat transfer in atmospheres as they exist on other planets or to heat transfer in the nozzles of rockets through which combustion gases of various composition are exhausted.

a

$$-k\left(\frac{\partial T}{\partial y}\right)_w \qquad q_{rad}$$

w

$$q_w$$

b

$$-k\left(\frac{\partial T}{\partial y}\right)_w \qquad q_{rad} \qquad \dot{m}c_p T_{cw}$$

$$k_w\left(\frac{\partial T_w}{\partial y_b}\right)_w \qquad \dot{m}c_p T_{cw}$$

$$h_i\left(i_w - i_r\right) \qquad q_{rad} \qquad \dot{m}i_{cw}$$

w

$$q_{tr}$$

b.l.

$$\dot{m}i_{cb}$$

c

$$h_i\left(i_w - i_r\right) \qquad q_{rad} \qquad \dot{m}i_g$$

w

$$k_w\left(\frac{\partial T_w}{\partial y_b}\right)_w \qquad \dot{m}i_s$$

$$h_i\left(i_w - i_r\right) \qquad q_{rad} \qquad \dot{m}i_g$$

w

$$T_w$$

$$T_o$$

$$\dot{m}i_{so}$$
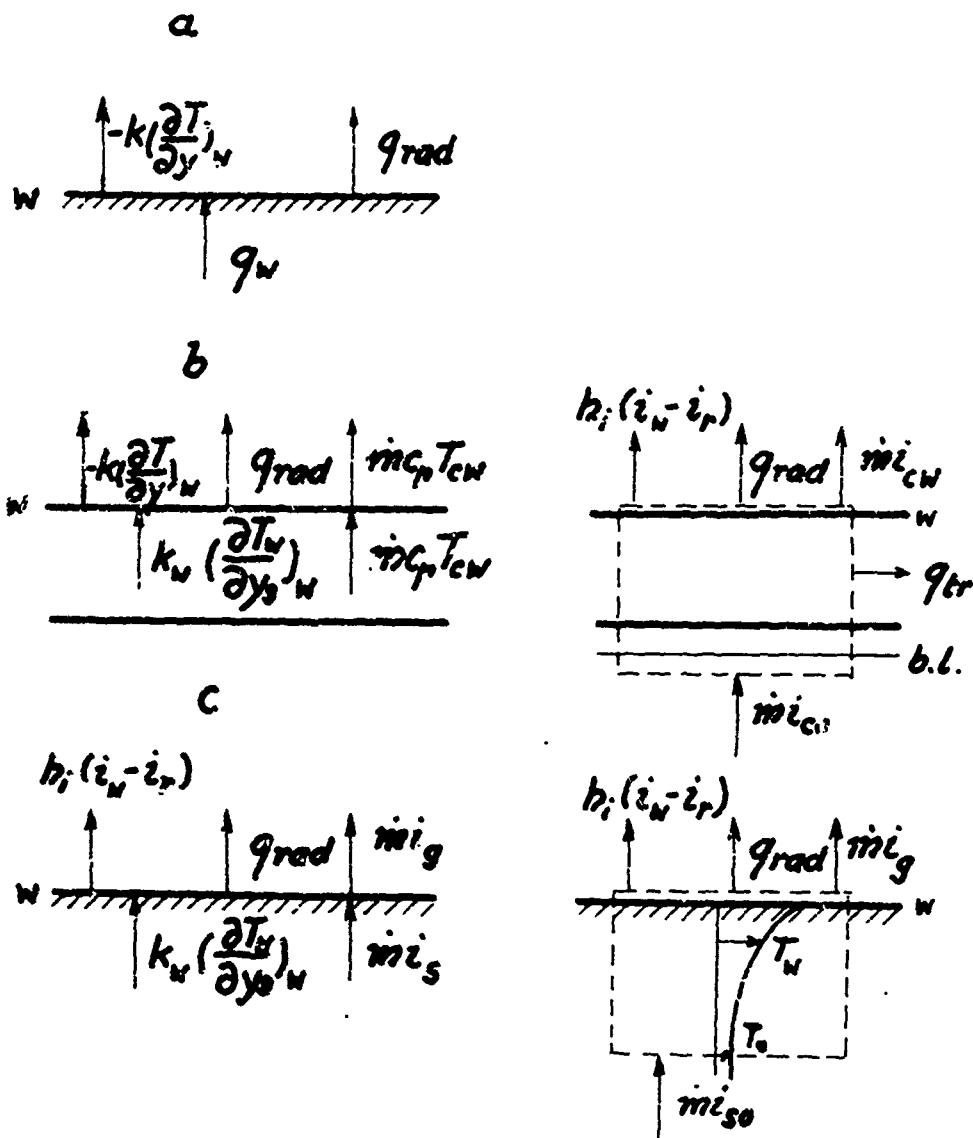
Fig. 1

Heat balances on surfaces with various cooling methods

A similar survey was made in 1954 and has been published under the title "Survey on Heat Transfer at High Speeds" as WADC TR 54-70. The present report can be considered as an extension of this one considering the changes in the conditions as they have occurred in new engineering development and of advances in our knowledge reported in the intervening years.

Research in the field of heat transfer as it is contained in this report has been very intensive during recent years. The Appendix to this report contains a collection of pertinent literature, and a special attempt was made to make this listing complete and up-to-date.

2. ENERGY BALANCES

Special cooling methods like mass transfer cooling, transpiration cooling, or ablation cooling are frequently used in recent engineering applications. Parameters which describe heat transfer to surfaces cooled by such methods, like recovery factors and heat transfer coefficients, are defined in various ways in the literature. It is therefore necessary to introduce in this chapter the parameters which will be used in this paper. The discussion will start with an energy balance on a solid surface, and then proceed to surfaces cooled by transpiration, film, or ablation cooling.

Figure 1a indicates a solid wall with a surface w. An energy balance can be set up for this surface expressing the fact that the amount of heat leaving the surface per unit time on the fluid side must be equal to the amount of heat entering the surface on the solid side. Heat will leave on the fluid side by conduction and by radiation. Heat

will enter on the solid side by conduction. An equation which states
this fact reads:

$$- k \left(\frac{\partial T}{\partial y} \right)_w + q_{rad} = q_w = k_w \left(\frac{\partial T_w}{\partial y_s} \right)_w \qquad (2-1)$$

In this equation, k indicates the heat conductivity on the gas side,
k_w the heat conductivity of the solid material. T is the gas temperature,
T_w the temperature of the solid wall material. q_{rad} indicates heat
transferred from the wall to the surrounding by radiation, and q_w the
heat conducted from the interior of the solid wall to the surface.
y indicates the direction normal to the wall surface on the fluid side
and y_1 on the solid side. Both are counted positive in the direction
away from the surface.

The temperature which the wall surface assumes when the heat
conduction into the interior and the heat loss by radiation are both
zero, is called its recovery temperature, T_r:

$$T_w = T_r, \quad i_w = i_r, \quad \text{when } q_w = 0, \quad q_{rad} = 0 \qquad (2-2)$$

i indicates the enthalpy of the gas. The heat flux by conduction in
the gas to the wall surface is conventionally expressed through a heat
transfer coefficient h, which is defined by the following equation:

$$q_{wco} = - k \left(\frac{\partial T}{\partial y} \right)_w = h(T_w - T_r) = h_i(i_w - i_r) \qquad (2-3)$$

In Reference 110 it has been demonstrated that it is advantageous to
re-define the heat transfer coefficient by referring it to enthalpies i
instead of to temperatures. This heat transfer coefficient is denoted
by h_i. In this way, simple relations which have been obtained for a
constant property fluid can be used to describe heat transfer with
good accuracy if the properties of the fluid, including its specific

heat, vary widely. Equation (2-3) indicates the definition of the heat transfer coefficient referred to enthalpies on the right hand side. The energy balance as contained in equation (2-1) may be re-written in enthalpies in the following way:

$$h_i(i_w - i_r) + q_{rad} = q_w \qquad (2-4)$$

We will now consider the transpiration cooling process. Figure 1b indicates a porous wall through which a coolant gas is flowing. A heat balance will again be made stating the fact that the sum of all heat fluxes leaving the surface w of the transpiration cooled wall must be equal to the sum of the heat fluxes travelling in the wall toward its surface. The following equation describes this heat balance:

$$- k \left(\frac{\partial T}{\partial y}\right)_w + q_{rad} + \dot{m}c_p T_{cw} = k_w \left(\frac{\partial T_w}{\partial y_w}\right)_w + \dot{m}c_p T_{cw} \qquad (2-5)$$

Two additional terms appear in this equation because a stream $\dot{m}$ of coolant gas continuously passes the wall surface w. Heat is carried in the wall by convection toward the surface, and on the other side heat leaves the surface by convection. The two corresponding terms are the last ones on both sides of equation (2-5). $\dot{m}$ is the mass velocity of the coolant gas, c_p is its specific heat, and T_{cw} is the temperature with which the coolant gas passes the surface w. It may be observed that the convective terms cancel on both sides of the equation, and that in this way the equation simplifies to:

$$h_i (i_w - i_r) + q_{rad} = q_w \qquad (2-6)$$

In this equation, a heat transfer coefficient h_i has been introduced to describe the heat flow in the gas to the wall by conduction. It is important to stress that this definition of the heat transfer coefficient

will be used in the present report. In the literature, the heat transfer coefficient is sometimes defined to describe not only the heat flow by conduction, but also the convective contribution according to the following equation:

$$h_i \ (i_w - i_r) = - k \ \left(\frac{\partial T}{\partial y}\right)_w + \dot{m} c_p \ T_{cw}$$

It is found, however, that the definition as used in equation (2-6) is advantageous because the heat transfer coefficient defined in this way depends on a smaller number of parameters. The heat flux by conduction can again be referred to an enthalpy difference instead of a temperature difference. The definition in equation (2-6) will, therefore, be used in this report.

For steady state conditions, it is often advantageous to make a heat balance not for the surface of the wall but for a control volume as indicated on the right hand side of Figure 1b. Also indicated in this sketch are the various heat fluxes which leave or enter the control volume. One surface of this volume may coincide with the wall surface w. On the coolant entry side, the surface is arranged so that it includes the boundary layer which also exists on this side of the wall surface. In this way, no heat transport by conduction will occur through this control surface, since it is located outside the boundary layer. The following equation states the heat balance:

$$h_i(i_w - i_r) + q_{rad} + \dot{m} i_{cw} + q_{tr} = \dot{m} i_{co} \qquad (2\text{-}7)$$

i_{cw} in this equation indicates the enthalpy of the coolant gas as it passes the wall surface w, and i_{co} indicates the enthalpy of the coolant gas on the entry side of the wall and outside of the boundary layer. q_{tr} indicates a heat flow which may leave the control volume in a

direction parallel to the wall surface w. Equation (2-7) has the advantage that the various terms appearing in it are relatively easily accessible to measurement.

In the ablation cooling process, the heat flux from the hot gas to the wall surface is such that the wall material sublimes and is gradually carried away with the gas stream. The surface of the wall in this way gradually recedes in the direction towards the wall interior. We consider a heat balance for the interface between the solid wall and the gas—this means for the surface which gradually recedes into the solid material. Accordingly a mass flow occurs through this surface whose magnitude per unit time and area may be $\dot{m}$. If the enthalpy of the subliming wall material in the gaseous state and at the temperature of the interface is indicated by i_g, and at the same temperature but in the solid state by i_s, then the heat balance for the heat fluxes through such a surface can be expressed by the equation:

$$h_i(i_w - i_r) + q_{rad} + \dot{m}i_g = k\left(\frac{\partial T_w}{\partial y_s}\right)_w + \dot{m}i_s \qquad (2\text{-}8)$$

The enthalpies i_w and i_r used in the definition of the heat transfer coefficient h_i will be discussed in more detail later on in the paper. The equation can be simplified to the following form if the heat of sublimation $i_{gs} = i_g - i_s$ is introduced:

$$h_i(i_w - i_r) + q_{rad} + \dot{m}i_{gs} = k\left(\frac{\partial T_w}{\partial y_s}\right)_w \qquad (2\text{-}9)$$

In some cases, the solid wall material is not sublimed but first melts and then evaporates into the gas stream. Equation (2-9) is still valid for this situation if i_{gs} is interpreted as the heat of evaporation,

and when the equation is used to describe the heat balance on the interface between the liquid film which covers the solid wall and the gas. The ablation process may also be affected by some chemical reaction between the gas and the solid material. In this case, the heat released by the chemical reaction has to be introduced for i_{gs}. This term must also include surface dissociation or surface recombination if it should occur in the actual process.

A quasi-steady condition often occurs in an ablation process in which the temperature of the interphase is constant and a temperature field which does not change its shape moves with constant velocity into the solid material. For such a condition, it is again advantageous to write an energy balance for a control volume as indicated on the right hand side of Figure 1c. One surface of this control volume coincides with the interface w of the ablating material; the other surface is parallel to this interface but at a sufficient distance so that the temperature increase occurring in the wall by conduction has not yet reached this point. The individual heat fluxes leaving this control volume are again indicated in the figure, and it is assumed that no heat will leave the volume in a direction parallel to the wall surface. The following equation then holds for this heat balance:

$$h_1(i_w - i_r) + q_{rad} + \dot{m}i_g = \dot{m}i_{so} \qquad (2\text{-}10)$$

i_{so} is the enthalpy of the solid wall material at temperature T_o. If the ablation process occurs as a melting and evaporation, and if

β indicates the fraction of the ablated material which leaves the control volume in liquid form, then the equation describing the heat balance is:

$$h_i(i_w - i_r) + q_{rad} + \beta \, \dot{m}(i_g - i_{so}) + (1 - \beta) \, \dot{m}(i_f - i_{so}) = 0 \quad (2\text{-}11)$$

i_f is the enthalpy of the liquid material leaving the control volume.

The balance equations contained in this section have to be used in connection with other relations, for instance, one which describes the heat conduction process in the solid material for the various engineering calculations. A prerequisite of their use is a knowledge of the heat transfer coefficient and of the recovery temperature or enthalpy. Their discussion is the subject of the following chapters of this paper.

3. HEAT TRANSFER TO SOLID SURFACES

31. SINGLE COMPONENT GAS

Heat transfer to a gas of high temperature is influenced by the fact that the gas may be partially or completely dissociated or ionized, and that the degree of dissociation changes throughout the boundary layer when the temperature of the surface exposed to the flow is different from the temperature of the gas. Such effects may also occur if a gas with a relatively low temperature flows with very high velocity over the surface, because the gas is then heated within the boundary layer by internal friction, and the increased temperature may again cause dissociation. The influence of such factors on heat transfer will be discussed in the next section of this report. In the present section, such effects will be neglected. It is therefore assumed that we deal with a single component gas, and that no dissociation or recombination in the gas occurs as a consequence of locally varying temperature.

314. Flat Plate

Heat transfer to the surface of an object exposed to a high
velocity flow depends on the shape of this object as well as on the flow
field to which it is exposed. One geometry, which has been investigated
very extensively because it can be treated theoretically in a simple
manner, is a situation conventionally referred to as "flat plate." We
define as a "flat plate" situation one under which a plane surface is
exposed to a flow field with locally uniform velocity and flow direction.
It is assumed that the flow velocity and, as a consequence, the pressure
are also constant along the outer edge of the boundary layer which exists
along the plane surface. It is further specified that the boundary
layer is so thin that the pressure is transmitted without change to
the surface of the plate. This flow condition can be set up experimen-
tally with good approximation in low velocity flow when the Reynolds
number is sufficiently high. In supersonic and hypersonic flow, the
boundary layers are usually thicker and, as a consequence, a shock
wave is generated by the boundary layer ahead of the plate. As a
consequence, the velocity and pressure vary along the outer edge of the
boundary layer and also along the plate surface. The temperature,
which is also constant along the outer edge of the boundary layer for
"flat plate" flow, varies locally where a shock is created by the
boundary layer. In the present discussion these effects will be
disregarded, and the condition which has been mentioned above of a
locally constant pressure and of a constant temperature or enthalpy
along the outer edge of the boundary layer will be specified. In this
way a standard case is created for which simple relationships describe
the heat transfer. These relations can also be used to approximate

conditions on slender objects in supersonic flow when the region near the leading edge is excluded, and when the actual velocity, pressure, and temperature at the outer edge of the boundary layer are introduced. These may be considerably different from the values in the free stream.

3111. Constant Surface Temperature. Even under the situation which has just been defined, heat transfer is not uniquely described. It still depends on the way in which the temperature on the plate surface varies locally. In this paragraph it will be assumed that the surface temperature is locally constant. The influence of a local variation of this temperature will be discussed in the following paragraph.

If the fluid to which the plate surface is exposed has constant properties (independent of temperature and pressure), then heat transfer as well as friction are described by simple relations which were derived for a laminar boundary layer a long time ago, and which have been experimentally verified to a high degree in the meantime. Similar relations from experimental results have been developed for a turbulent boundary layer. These relations are customarily expressed in dimensionless parameters defined in the following way. The recovery temperature in equation (2-2) is expressed by a dimensionless parameter r called temperature recovery factor and defined the following way:

$$ r = \frac{T_r - T_e}{c_p u_e^2 / 2} \tag{3-1} $$

u_e is the velocity and T_e the static temperature at the outer boundary layer edge. The heat transfer coefficient in equation (2-3) is expressed by the dimensionless Nusselt number

$$Nu = \frac{hx}{k} \tag{3-2}$$

where x is the distance from the leading edge measured along the plate surface, or alternately, by another one called Stanton number

$$St = \frac{h}{\rho\, c_p\, u_e} \tag{3-3}$$

where ρ is the density and c_p the specific heat at constant pressure of the fluid. In some situations the shear τ_w which the flow exerts at the plate surface is of interest as well as the heat transfer. The local shear is determined by a dimensionless parameter c_f called friction factor:

$$c_f = \frac{\tau_w}{\rho\, u_e^2/2} \tag{3-4}$$

For laminar flow conditions, the following relations express the temperature recovery factor, the Nusselt number, Stanton number, and friction factor:

$$r = \sqrt{Pr} \tag{3-5}$$

Pr is the Prandtl number of the fluid.

$$Nu = 0.332\,(Pr)^{1/3}\,\sqrt{Re} \tag{3-6}$$

$Re = \dfrac{\rho\, u_e x}{\mu}$ is the Reynolds number and μ the dynamic viscosity of the fluid.

$$St = 0.332/(Pr)^{2/3}\,\sqrt{Re} \tag{3-7}$$

$$c_f = 0.664/\sqrt{Re} \tag{3-8}$$

It can be seen that the following relation exists between the Stanton number and the friction factor:

$$St = (c_f/2)(Pr)^{-2/3} \tag{3-9}$$

For turbulent flow, the following relation was found from experiments to describe the temperature recovery factor:

$$r = \sqrt[3]{Pr} \qquad (3\text{-}10)$$

The local friction factor has been described by various analytic expressions. A simple one is, for instance, the equation developed by Schultz-Grunow:

$$c_f = \frac{0.370}{(\log_{10} Re)^{2.584}} \qquad (3\text{-}11)$$

It has also been found that the relation between Stanton number and friction factor in turbulent flow is practically the same as for laminar flow conditions, so that equation (3-9) holds for turbulent flow also. From this relation it follows that the Stanton number for a turbulent boundary layer on a flat plate is described by the following equation:

$$St = \frac{0.185}{(\log_{10} Re)^{2.584} (Pr)^{2/3}} \qquad (3\text{-}12)$$

For real gases, it has been found that the properties are not constant but vary with temperature. They can, however, be considered as independent of pressure as long as excessively high pressures and dissociation are excluded. Under this condition, and as long as the specific heat c_p can be considered constant, it has been established in reference 110 that the relations given in the preceeding paragraphs for a constant property fluid still describe the actual heat transfer and friction with good accuracy when the properties appearing in all of these relationships are introduced into those relations at a reference temperature described by the following equation:

$$T^* = T_e + 0.5(T_w - T_e) + 0.22(T_r - T_e) \qquad (3\text{-}13)$$

The second term in this equation can be expressed by the Mach number Ma_e existing in the flow at the outer edge of the boundary layer. The

equation then takes the following form which is sometimes more conven-
ient for an evaluation:

$$T^* = T_e + 0.5(T_w - T_e) + 0.22r \frac{\gamma - 1}{2} (Ma_e)^2 T_e \qquad (3-14)$$

γ is the ratio of specific heats at constant pressure and volume.

When the temperature variation within the boundary layer is so
large that the specific heat of the gas varies considerably, then the
better procedure is to base heat transfer calculations on enthalpies
instead of on temperatures. In this case the definitions, as given on
the right hand side of equations (2-2) and (2-3), are used. The heat
transfer coefficient h_i is expressed in dimensionless form as a Nusselt
number or a Stanton number:

$$Nu_i = \frac{c_p h_i x}{k} \quad , \quad St_i = \frac{h_i}{\rho u_e} \qquad (3-15)$$

The relations (3-5) to (3-12) can again be used to describe these new
parameters. The properties appearing in these equations are now
introduced at a reference enthalpy instead of a reference temperature.
The following equations describe the reference enthalpy:

$$i^* = i_e + 0.5(i_w - i_e) + 0.22(i_r - i_e) \qquad (3-16)$$

$$i^* = i_e + 0.5(i_w - i_e) + 0.22r \frac{\gamma_e - 1}{2} (Ma_e)^2 i_e \qquad (3-17)$$

The above procedure based on a reference enthalpy and on constant property
relations is today in widespread use and is usually referred to as reference
enthalpy method. Its accuracy has been checked by comparison with the
results obtained by boundary layer solutions for a laminar boundary
layer and for air as the flowing medium. Agreement within plus/minus
4% has been found for stream temperatures between 400 and 800°R, for
wall temperatures between 400 and 2000°R, and for Mach numbers up to
16 (Ref. 110). It will be seen in a later section that the method still
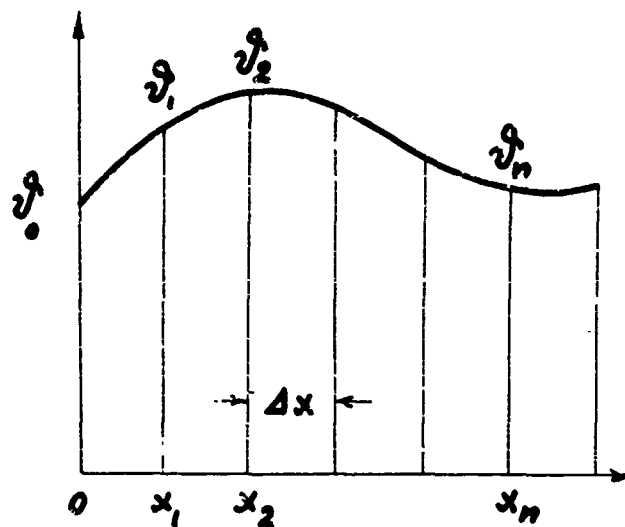
Fig. 2

Surface-temperature variation

is valid even if dissociation or chemical reactions occur within the boundary layer. For this reason, the discussion in this report will mainly be based on the reference enthalpy method. Agreement is also good with measurements on turbulent heat transfer in the Mach number range which have been covered up-to-date. It is also reasonable to expect that the relations hold with good accuracy for a gas different from air.

3112. Variable Surface Temperature. The various methods which have been developed to calculate heat transfer to a surface with locally varying temperature have already been discussed in reference 110. A simplified procedure which allows the calculation of heat transfer to a wall with prescribed varying temperature has been developed in the meantime (Ref. 113). The procedure approximates the actual temperature variation by a succession of straight lines and, in this way, simplifies the calculation to a summation process. It will be discussed in connection with Fig. 2. This figure indicates an arbitrary variation of the temperature potential $\vartheta = T_w - T_r$, which is determining the heat transfer according to equation (2-3). The surface of length x is subdivided into a number of partial lengths Δx of equal dimensions. The temperature potential at the positions x_0, x_1, x_2, $x_3 \ldots x_n$ is denoted by ϑ_0, ϑ_1, $\vartheta_2, \ldots \vartheta_n$. The following equation can be used to calculate the local heat flux $q_{w,co}$ per unit time and area at the position x_n, and a very similar equation allows calculation of the total heat $q_{w,co}$ transferred along the plate surface between location 0 and x_n:

$$q_{w,co} = h_{iso} \left\{ \vartheta_o + A(\vartheta_n - \vartheta_o) - B(\Delta x/x_n) \left[(2n - 1)\vartheta_n - \vartheta_o - 2\vartheta_1 - 2\vartheta_2 \ldots - 2\vartheta_{n-1} \right] \right\} \qquad (3\text{-}18)$$

$$q_{w,co} = \bar{h}_{iso} \, xb \left\{ \vartheta_o + A'(\vartheta_n - \vartheta_o) - B'(\Delta x/x_n) \left[(2n - 1)\vartheta_n - \vartheta_o - 2\vartheta_1 - 2\vartheta_2 - \ldots - 2\vartheta_{n-1} \right] \right\} \qquad (3\text{-}19)$$

h_{iso} is the local heat transfer coefficient and $\bar{h}_{iso}$ the average one for the same flow condition but for a constant wall temperature, b is the width of the plate normal to flow direction. The following equation can be used to calculate the isothermal heat transfer coefficient:

$$h_{iso} = C(k/x) \, Re_{x_n}^{\,n} \, Pr^{1/3}$$

The exponent n is equal 0.5 for laminar flow and equal 0.8 for turbulent flow. The constants, A, B, C, A', B', C' contained in these equations are listed in Table 1. The last equation represents the average heat transfer coefficient when C is replaced by C'.

TABLE 1

	m	A	B	C	A'	B'	C'
Laminar	0	0.895	0.690	0.332	0.696	0.432	0.664
	1/4	0.837	0.635	0.412			
	1/2	0.840	0.572	0.469			
	1	0.792	0.538	0.560			
Turbulent	0	0.991	0.117	0.0296	0.982	0.478	0.0370

The condition m = 0 applies to flat plate flow. The values for m parameters different from 0 apply to flow with pressure gradients and will be discussed later on in this report.

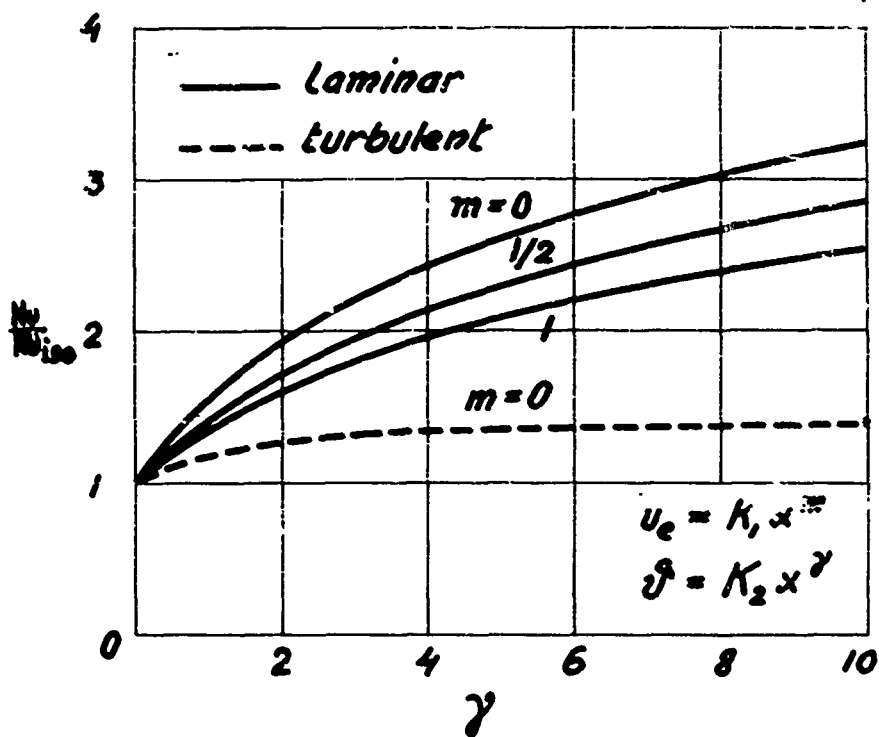Existing methods for the calculation of heat transfer with a

Fig. 3

Ratio of Nusselt number on surface with varying temperature
to Nusselt number on isothermal wall. (from Ref. 113)

locally varying surface temperature assume a fluid with constant
properties or a fluid for which the product, density times viscosity,
is a constant. The presented approximation, therefore, is based on the
same assumption. It checks the results of calculations performed with
the other methods within a few per cent.

For a fluid with variable properties, it is proposed to combine
the method presented here with the reference enthalpy method of the
preceding section by introducing properties at the reference enthalpy
into all of the relations appearing in the equations. Experiments
under conditions where the wall temperature varies widely, and where
the temperature differences in the boundary layer are sufficiently
large are not extensive enough to check the accuracy of a calculation
made with the proposal just mentioned.

The influence of a wall temperature variation on heat transfer
becomes significant only when the temperature variation is large relative
to the temperature potential. A first estimate of the influence of a
wall temperature variation and a decision whether it has to be considered
in a determination of the heat transfer can be made with the help of
Fig. 3. In this figure, the actual Nusselt number divided by the Nusselt
number at an isothermal surface with the same local temperature
difference is plotted over a parameter γ. The wall temperature
variation is assumed to follow an exponential law as indicated in the
figure with the value γ as exponent. The curves with the parameters
n = 0 apply to flat plate conditions and curves with a finite m value
to flows with pressure gradients which will be discussed later on. It
may be observed that the influence of a temperature variation is
considerably larger for laminar flow conditions than for turbulent flow.
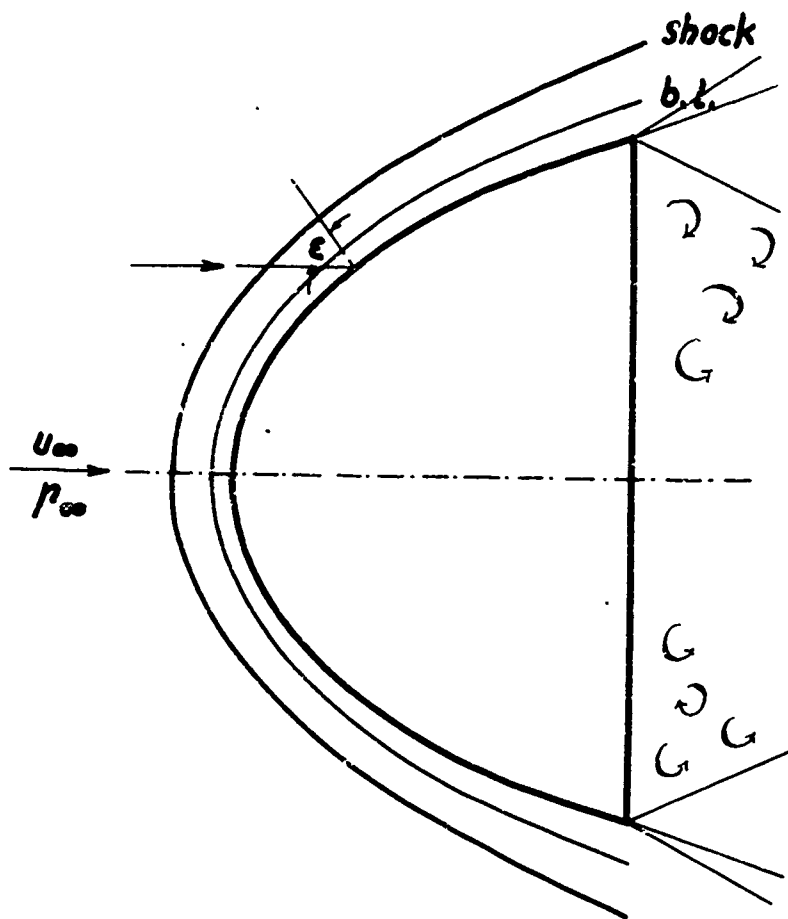
Fig. 4

Blunt object in supersonic flow

For turbulent flow it is only necessary in very rare cases to make a
correction for the temperature variation on isothermal heat transfer
coefficients.

312. Blunt Objects

Early developments in high speed flight were concerned with objects
of very slender shape. This was done in order to decrease the drag and
the power required to move such objects through the atmosphere. In
new developments and in astronautics, on the other hand, interest
usually centers around blunt objects, because drag is of minor importance.
Sometimes drag is even desired for a reduction of aerodynamic heating.
Such an object flying with hypersonic velocities finds conditions in its
surroundings as sketched in Fig. 4. A boundary layer envelopes the
object as a thin sheet. Outside of the boundary layer, a shock is
created. In the back of the object the flow usually separates, so that
the region behind the rearward surface is filled with a flow containing
strong irregular vortices. The density in this region is usually
quite low.

In moving through the shock wave, the air loses most of its
kinetic energy and converts it into internal energy. As a consequence,
the layer between the shock and the outer edge of the boundary layer
is very hot. The conditions at the outer edge of the boundary layer
which are required for a calculation of heat transfer to the surface
can be obtained with good accuracy from the so-called Newtonian Flow
approximation for Mach numbers greater than approximately 5. It is
found that in this case the shock wave is quite close to the surface
of the object. Correspondingly, the velocity component normal to the

shock wave and the plates all behind it. A momentum consideration then leads to the following relation between the pressure p_e behind the shock (equal to the pressure at the wall surface according to boundary layer theory) and the pressure p_∞ and velocity u_∞ in the upstream area (ahead of the shock):

$$p_e - p_\infty = \rho_\infty u_\infty^2 \cos^2 \epsilon \qquad (3\text{-}20)$$

ϵ is the angle between the surface normal and the upstream flow direction. The velocity and temperature along the outer edge of the boundary layer can then be found with the assumption that the flow expands outside the boundary layer isentropically from the pressure at stagnation point to the local pressure $\dot{p}_e$. This introduces an error which becomes larger with increasing distance from the stagnation point, because the air entering the boundary layer at some distance from the stagnation point actually crossed the shock wave at a location where the shock is inclined towards the upstream flow direction. This error is, however, usually small for blunt objects.

3121. Stagnation Flow. In a limited region close to the stagnation point, it is found that the flow velocity increases linearly with distance from the stagnation point. This region is referred to as "stagnation point region", and heat transfer in this region can be determined by exact solutions of the boundary layer equations. The flow in such a region is almost exclusively laminar. The dimensionless parameters describing heat transfer in this region are again described by the equations (3-15). The term x now indicates the distance from the stagnation point measured along the surface of the object in flow direction; u_e is the local velocity of the fluid at location x and just outside the boundary layer. The recovery enthalpy i in the

stagnation point region is practically equal to the total gas enthalpy
in the upstream. In stagnation point flow, one has to distinguish
between two-dimensional flow as it occurs, for instance, around the
front part of a wing and between rotationally symmetric flow as it
usually exists around the nose of a missile. The first situation will
be referred to as "plane stagnation flow" and the second as "rotationally
symmetric stagnation flow". For a constant property fluid and a
laminar boundary layer, the following relations describe heat transfer
for these flow conditions:

plane stagnation flow $\quad Nu_x = 0.570(Pr)^{0.4} \sqrt{Re_x}^2 \quad$ (3-21)

rot. symm. stagnation flow $\quad Nu_x = 0.763(Pr)^{0.4} \sqrt{Re_x} \quad$ (3-22)

The first one of these relations was obtained by Squire. The second
one follows from the first expression by Mangler's transformation. It
was also directly calculated by M. Sibulkin. For a fluid with
variable properties, the reference enthalpy method was again found to
describe heat transfer conditions with sufficient engineering accuracy.
This was pointed out by H. Romig (Ref. 296 and 297) by comparison of
results obtained with this method with exact boundary layer solutions.
It will be once more demonstrated here[x] by a comparison of the reference
enthalpy method with a relation which has been obtained by Fay and
Riddell (Ref. 124). The following relation is contained in Reference
124 for a constant Prandtl number:

$$\frac{Nu_w}{\sqrt{Re_w}} = 0.67 \left(\frac{\rho_e \, \mu_e}{\rho_w \, \mu_w} \right)^{0.4} \quad (3-23)$$

The index w in this equation indicates that properties are to be
introduced at the conditions as they exist at the wall. According to

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[x] According to R. Eichhorn
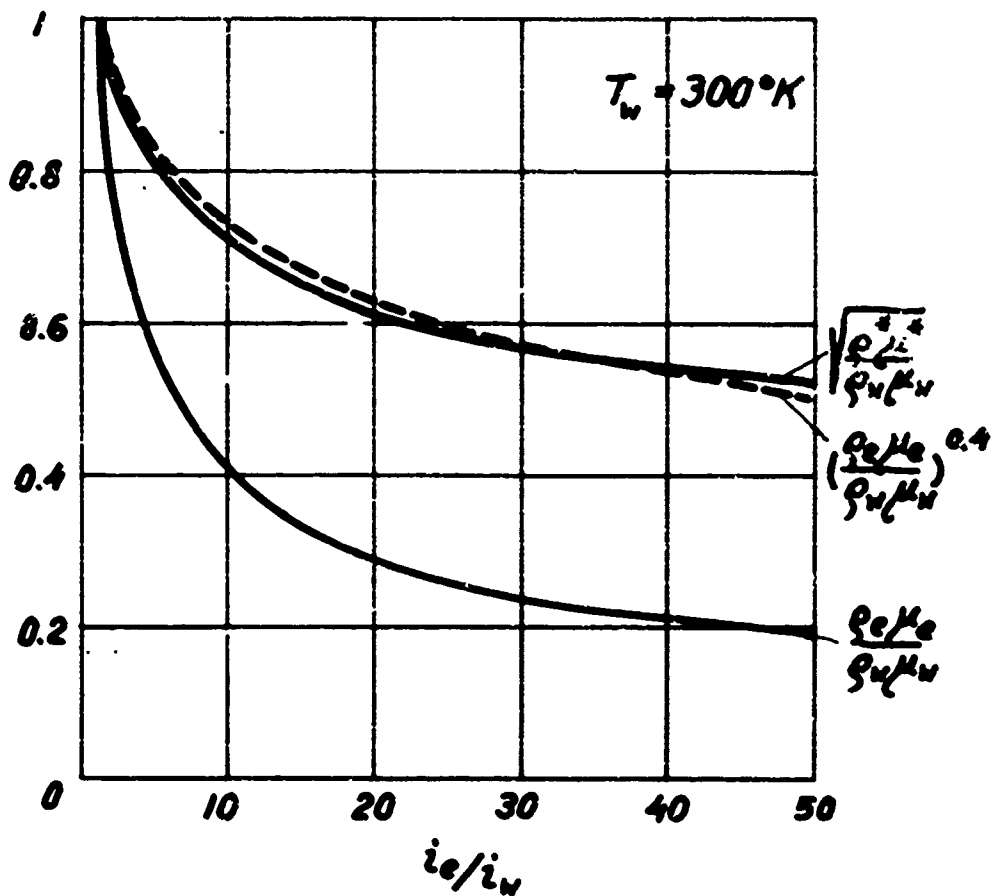
Fig. 5

Variation of property parameters
with enthalpy ratio

The figure shows curves of i_e/i_w on the horizontal axis (0 to 50) versus values 0 to 1 on the vertical axis, with $T_w = 300°K$ labeled. Curves are labeled:

$$\sqrt[4]{\frac{\rho_e \mu_e}{\rho_w \mu_w}}$$

$$\left(\frac{\rho_e \mu_e}{\rho_w \mu_w}\right)^{0.4}$$

$$\frac{\rho_e \mu_e}{\rho_w \mu_w}$$

the reference enthalpy method, the properties have to be introduced into the constant property relations at a reference condition. For either plane or rotationally symmetric stagnation flow the following relation exists:

$$\frac{Nu^*}{\sqrt{Re^*}} = C \, (Pr)^{0.4}$$

The constant C in this relation has different values for plane or rotationally symmetric flow. The reference enthalpy at which the properties have to be introduced is given by equation (3-16). In the neighborhood of the stagnation point the velocity is very small, and the difference between the recovery enthalpy i_r and the static enthalpy i_s is negligible. Correspondingly, equation (3-16) can be simplified to the following expression:

$$\frac{i^*}{i_e} = 0.5 \, (1 + \frac{i_w}{i_e})$$

The ratio of Nusselt to square root of Reynolds number expressed at reference condition can be written in the following way:

$$\frac{Nu^*}{\sqrt{Re^*}} = \frac{c_p^*}{k^*} \sqrt{\frac{\mu^*}{\rho^*}} \frac{h_i x}{\sqrt{u_e x}} = \frac{Pr}{\sqrt{\rho^* \mu^*}} \frac{h_i x}{\sqrt{u_e x}}$$

If both parameters are, on the other hand, determined at wall conditions, then the following condition holds:

$$\frac{Nu_w}{\sqrt{Re_w}} = \frac{Pr}{\sqrt{\rho_w \mu_w}} \frac{h_i x}{\sqrt{u_e x}} = \sqrt{\frac{\rho^* \mu^*}{\rho_w \mu_w}} \frac{Nu^*}{\sqrt{Re^*}} = \sqrt{\frac{\rho^* \mu^*}{\rho_w \mu_w}} \, B(Pr)^{0.4} \quad (3\text{-}24)$$

A comparison between the equations (3-23) and (3-24) is made in Fig. 5. The ratio of $\rho \mu$ at stream and at wall conditions, as used by Fay and Riddell, is shown in the lower curve of this figure. The expressions presented by the two upper curves are the ones appearing in the equations

(3-23) and (3-24). The fact that these two curves almo t coincide
indicates that the reference enthalpy method leads to practically the
same result as Fay - Riddell's equation.

3122. Swept Leading Edges. A considerable reduction of recovery
temperature and heat transfer on the stagnation line of a cylindrical
object occurs when the cylinder axis is inclined under an angle smaller
than 90 degrees toward the upstream direction. A shock surrounds the
front part of such a cylinder, when the upstream velocity is supersonic.
At the stagnation line, the velocity component normal to the cylinder
surface is reduced to zero. The velocity component parallel to the
cylinder axis, however, is maintained outside the boundary layer. Its
value may be u_p. This velocity component is maintained in the
"stagnation flow region", whereas the velocity component parallel to
the surface but normal to the cylinder axis increases again linearly
with distance x (measured now in a plane normal to the axis). Solutions
for such a flow situation have been reported in the literature. For a
constant property fluid, heat transfer coefficient and recovery factor
can be again calculated from equations (3-21) and (3-5) when the
recovery factor is defined in the following way:

$$r = \frac{T_r - T_{st}}{T_o - T_{st}}$$

T_o is the total temperature in the gas outside the boundary layer,
and T_{st} indicates the static temperature at the stagnation point outside
the boundary layer ($T_o - T_{st} = \frac{u_p^2}{2c_p}$). The heat transfer coefficient
is defined with the temperature difference $T_r - T_w$. For a single
component gas with variable properties, it is demonstrated in Ref. 417
that equation (3-23) (converted to plane stagnation flow) represents

the results of boundary layer solutions within 4% in the range of
parameters which is of practical interest, provided the heat transfer
coefficient and recovery factor are based on enthalpies instead of
temperatures. It is argued in Ref. 417 that this should apply also
when dissociation and recombination occur. From the comparison in
Fig. 5, it is evident that the reference enthalpy method and equations
(3-5) and (3-21) describe heat transfer to the stagnation region of a
yawed cylinder with the same accuracy.

3123. Arbitrary Shapes.

<u>Laminar Flow</u>. The flow over a blunt object is connected with pressure
gradients along the surface which influence the development of the
boundary layer and of the friction and heat transfer at the surface.
An exact solution of the differential equations describing boundary
layer flow is under such circumstances very tedious, because, even for
plane or rotationally symmetric flow, it means the solution of partial
differential equations in two independent variables. As a consequence,
many approximate procedures have been proposed which circumvent this
difficulty. These methods have to be evaluated according to the ease
with which they lead to results and according to their accuracy. One
which usually is especially simple is based on the assumption of
"local similarity". This method was proposed for the first time by
Falkner and Skan (V. K. Falkner and S. W. Skan: Phil. Mag. 12 (1931),
865, Aeron. Res. Comm. Rep. a. Mem. 1314 (1931)) for the calculation
of skin friction and by Eckert and Drewitz (E. Eckert and O. Drewitz:
Luftfahrtforschung, 19(1942), 189) for the calculation of heat transfer.
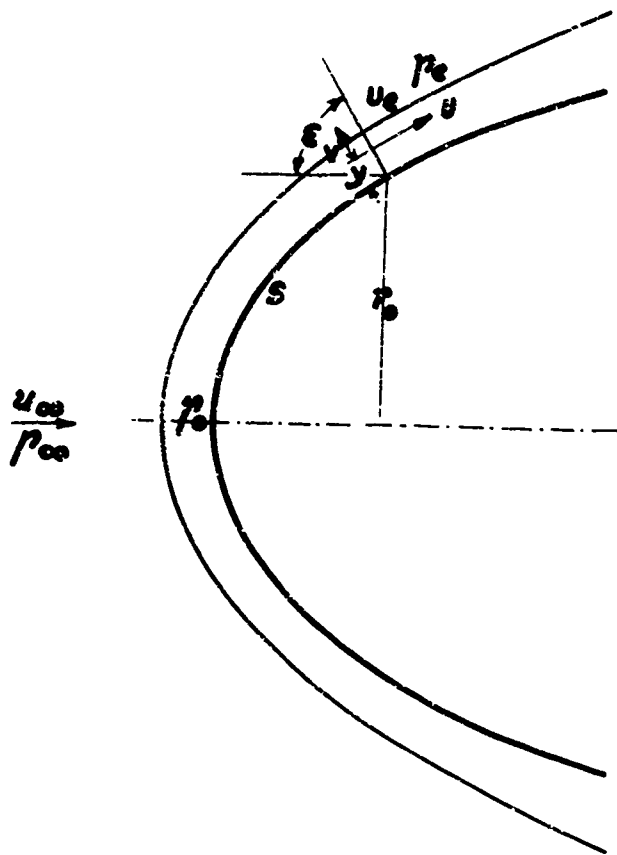Many extensions have been described since that time. The discussion

Fig. 6

Nomenclature for boundary layer equations

in this section will mainly follow a procedure suggested by L. Lees (Ref. 430), because it is relatively simple and can be applied for plane as well as for rotationally symmetric flow. The method will be somewhat modified to incorporate the reference enthalpy procedure.

The differential equations which describe steady laminar boundary layer flow of a medium with variable properties are:

$$\frac{\partial}{\partial s} \left(\rho\, u\, r_o^n \right) + \frac{\partial}{\partial y} \left(\rho\, v\, r_o^n \right) = 0 \tag{3-25}$$

$$\rho \left(u \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial p_e}{\partial s} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \tag{3-26}$$

$$\rho \left(u \frac{\partial I}{\partial s} + v \frac{\partial I}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\mu}{Pr} \frac{\partial I}{\partial y} \right) + \frac{\partial}{\partial y} \left[\mu \left(1 - \frac{1}{Pr} \right) \frac{\partial}{\partial y} \left(\frac{u^2}{2} \right) \right] \tag{3-27}$$

The equations are written in a coordinate system as indicated in Fig. 6. The symbol s indicates the distance measured in the flow direction along the surface from the stagnation point, y the distance from the surface, and r_o the distance of the surface point under consideration from the axis of rotation. The index e indicates conditions at the outer edge of the boundary layer. I is the symbol for the total enthalpy in the flow ($I_e = i_e + \frac{u_e^2}{2}$). The equations in the above form are applicable for plane as well as for rotationally symmetric flow. For the first situation, n has to be set equal to zero; for rotationally symmetric flow, n is equal 1. It has been shown by Dorodnitzyn, L. Howarth, C. R. Illingworth, and K. Stewartson that the equations can be brought by a transformation of coordinates into a form which closely resembles that for incompressible flow of a fluid with constant properties. In addition, W. Mangler demonstrated that the equations for rotationally symmetric flow can be made to assume the form of those for a plane flow

by another transformation. L. Lees combined both transformations by the choice of the following independent variables:

$$\bar{s} = \int_0^s \rho_e \mu_e \, r_0^{2n} \, ds \qquad (3\text{-}28)$$

$$\eta = \frac{\rho_e \, u_e r_0^{\,n}}{\sqrt{2\bar{s}}} \int_0^y \frac{\rho}{\rho_e} \, dy \quad . \qquad (3\text{-}29)$$

If, in addition the stream function which is introduced to eliminate equation (3-25) is written in the following form

$$\psi = \sqrt{2\bar{s}} \quad f(\eta)$$

then the above system of boundary layer equations transforms to:

$$(Cf'')' + ff'' + \beta \left[\frac{\rho_e}{\rho} - (f')^2 \right] = 0 \qquad (3\text{-}30)$$

$$\left(\frac{C}{Pr} \, g' \right)' + fg' + \frac{u_e^{\,2}}{2I_e} \left[2C(1 - \frac{1}{Pr}) f'f'' \right]' = 0 \qquad (3\text{-}31)$$

The following notations have been used in this equation:

$$g = \frac{I}{I_e} \quad , \quad C = \frac{\rho \, \mu}{\rho_e \mu_e} \quad , \quad \beta = \frac{2\bar{s}}{u_e} \frac{du_e}{d\bar{s}} \qquad (3\text{-}32)$$

A prime indicated differentiation towards η . The assumed form of the stream function ψ on which the derivation of the equations (3-30) and (3-31) were based restricts the boundary conditions to which this equation can be applied. A detailed discussion of this point is contained in the original paper by Lees (Ref. 438). Specifically, the method of local similarity postulates that the local flow conditions on any point of the surface of a body with arbitrary shape correspond with good approximation to those of a flow with β and $\frac{u_e^{\,2}}{2I_e}$ being constant and having values corresponding to the actual local condition. Solutions of the boundary layer equations for this condition are available for certain cases. For a vanishing Mach number which causes the
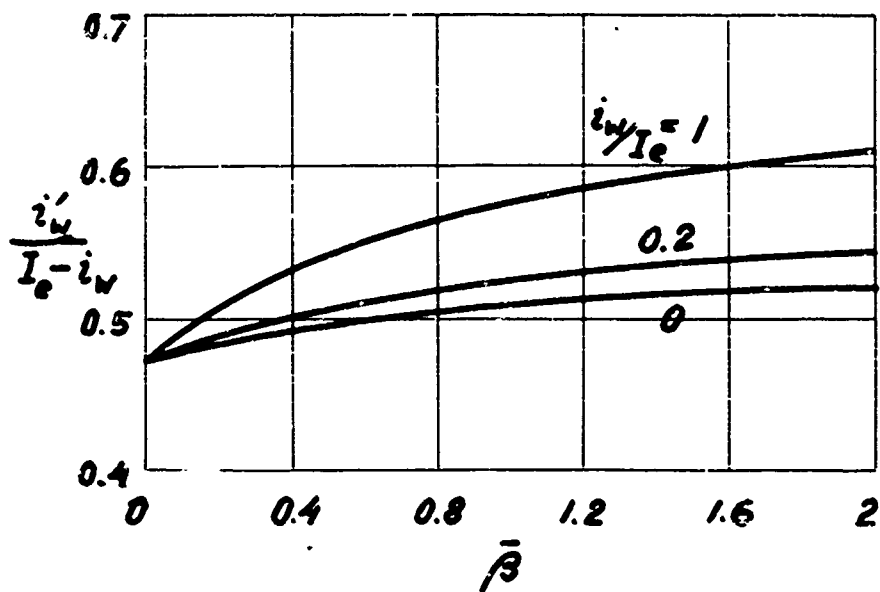
Fig. 7

Dimensionless enthalpy gradient on cooled
surfaces (from Ref. 213)

last term in equation (3-31) to disappear, they have been worked out by

S. Eckert and B. Brown and associates. For a gas with Prandtl = 1

and $\rho\mu$ = constant, solutions have been obtained by S. Levy (Journ.

Aeron. Sci., 21(1954), 459), as well as by Cohen and Reshotko (Ref. 66).

For $Pr = 1$ and $\rho\mu$ = constant, the equations in reference 66 become

identical to equations (3-30) and (3-31) when the parameter β is

based on Mach number instead of stream velocity corresponding to the

relation:

$$\bar{\beta} = \frac{2\bar{s}}{Ma_e} \frac{d(Ma_e)}{d\bar{s}} \qquad (3\text{-}32a)$$

The last term in Eq. (3-31) vanishes again because of the condition

$Pr = 1$. Fig. 7 has been taken from the solutions obtained by Cohen and

Reshotko and indicates the dimensionless enthalpy gradient at the wall

surface plotted over the pressure gradient parameter $\bar{\beta}$. The wall

temperature is assumed constant in these solutions. It can be observed

to be a characteristic feature of the dimensionless enthalpy gradient

that it varies only moderately with the pressure gradient parameter

$\bar{\beta}$. Especially for small values of the parameter i_w/I_e, the

variation becomes quite small. A small value of the parameter i_w/I_e

refers to a condition when the temperature of the surface is small

as compared to the temperature in the stream outside the boundary layer.

Such a condition is usually found in high velocity engineering

applications since high velocity flow is generally connected with high

temperatures, and since, on the other hand, the wall temperature

cannot exceed a limiting value determined by the strength characteristics

of the material. Consequently, Lees proposes to neglect the dependence

f the dimensionless enthalpy gradient on the pressure gradient for
such conditions and to use an average value equal 0.5.

The local heat flux per unit area into the wall surface can then
be expressed in the following way:

$$q_w = k_w \left(\frac{\partial T}{\partial y}\right)_w = \frac{k_w}{c_{pw}} \left(\frac{\partial i}{\partial y}\right)_w = \frac{k_w}{c_{pw}} (I_e - i_w) \frac{i_w'}{I_e - i_w} \left(\frac{d\eta}{ds}\right)_w$$

$$= \frac{k_w}{c_{pw}} (I_e - i_w) \frac{i_w'}{I_e - i_w} \frac{\rho_w u_e r_o^m}{\sqrt{2s}} = \frac{k_w}{c_{pw}} \frac{\rho_w}{\rho_e} (I_e - i_w) 0.50 \times$$

$$(Pr)^{1/3} \frac{\rho_e u_e r_o^m}{\sqrt{2s}} \qquad (3\text{-}33)$$

The factor $(Pr)^{1/3}$ was added in the last term to account for Prandtl
numbers somewhat different from one. The enthalpy difference
$I_e - i_w$ will be replaced by the difference between recovery and wall
enthalpies $i_r - i_w$ for the same reason. For a highly cooled wall,
where the wall temperature is much lower than the recovery temperature,
this is a sufficiently good approximation. With the definitions of a
heat transfer coefficient, Nusselt number, and a Reynolds number:

$$Nu_{wL} = \frac{c_{pw} h_w L}{k_w} \qquad\qquad Re_{oL} = \frac{\rho_o u_\infty L}{\mu_o} \qquad (3\text{-}34)$$

(L...reference length, o refers to condition at stagnation point outside
the boundary layer) the following equations are obtained from the above
relation for the wall heat flux:

$$Nu_{wL} = 0.35 \frac{\rho_w}{\rho_e} F(s) (Pr)^{1/3} \sqrt{Re_{oL}} \qquad (3\text{-}35)$$

$$F(s) = \frac{\frac{\rho_e}{\rho_o} \frac{u_e}{u_\infty} \left(\frac{r_o}{L}\right)^n}{\left[\int_0^{s/L} \frac{\rho_e}{\rho_o} \frac{\mu_e}{\mu_o} \frac{u_e}{u_\infty} \left(\frac{r_o}{L}\right)^{2n} d\left(\frac{s}{L}\right)\right]^{1/2}} \qquad (3\text{-}36)$$

With these equations the local heat flux along the surface of a blunt object can be calculated as soon as the variation of density, viscosity, and velocity outside the boundary layer and along the surface is known. How to obtain these values has been discussed in a previous section. Any arbitrary length can be selected as reference length--for instance, the largest dimension of the object.

The method as outlined up to now has the short-coming that it is based on Fig. 7 which holds only for a gas with the product, density times viscosity, equal to a constant. For a gas with other density and viscosity relations, errors may be introduced as will be demonstrated by calculating the heat transfer coefficient for the stagnation point region of a blunt object with equation (3-35). The following relations hold for rotationally symmetric stagnation flow:

$$n = 1 , \quad u_e = cx , \quad r_0 = s$$

In addition, the density and viscosity variation in the close neighborhood of a stagnation point can be neglected ($\rho_e = \rho_o$, $\mu_e = \mu$). Equation (3-36) can then easily be integrated, and the following result can be obtained:

$$F(s) = 2(\frac{L}{s})^{1/2} (\frac{u_s}{u_\infty})^{1/2}$$

Introducing this relation into the above equation for the Nusselt number results in:

$$\frac{c_{pw} h_i L}{k_w} = 0.70 \frac{\rho_w}{\rho_o} (\frac{L}{s})^{1/2} (\frac{u_e}{u_\infty})^{1/2} (Pr)^{1/3} \sqrt{\frac{\rho_o u_\infty L}{\mu_o}}$$

The equation can be re-written:

$$\frac{c_{pw} k_i \, s}{k_w} = 0.70(\text{Pr})^{1/3} \frac{\rho_w}{\rho_o} \sqrt{\frac{\rho_o u_e s}{\mu_c}} = 0.70(\text{Pr})^{1/3} \sqrt{\frac{\rho_w \mu_w}{\rho_o \mu_o}} \sqrt{\frac{\rho_w u_e s}{\mu_w}}$$

A comparison of this last expression with the equation obtained by Fay and Riddell for stagnation point flow and listed on page 23 indicates that equation (3-35) will not properly describe the heat transfer condition for a gas in which the product density times viscosity follows a relation which is different from a constant. Lees corrected this situation by multiplying equation (3-35) with $\dfrac{\rho_o \mu_o}{\rho_w \mu_w}$ which does not change the relation for a gas with $\rho \mu = \text{const.}$ In Ref. 437, it is proposed to modify the method by combining it with the reference enthalpy procedure. The equation (3-35) is replaced by the following equation

$$\text{Nu}^* = 0.35 \, F^*(s) \, (\text{Pr}^*)^{1/3} \sqrt{\text{Re}_{oL}} \tag{3-37}$$

in which Nusselt and Prandtl numbers as well as F(s) are referred to a reference condition as described by equations (3-16) or (3-17) and based on local conditions at the surface point under consideration. The function $F^*(s)$ is defined as

$$F^*(s) = \frac{\dfrac{\rho^*}{\rho_o} \dfrac{u_e}{u_\infty} \left(\dfrac{r_o}{L}\right)^n}{\left[\displaystyle\int_0^{s/L} \dfrac{\rho^* \mu^*}{\rho_o \mu_o} \dfrac{u_e}{u_\infty} \left(\dfrac{r_o}{L}\right)^{2n} d\left(\dfrac{s}{L}\right)\right]^{1/2}}$$

It is shown in Ref. 437 that the proposed method agrees very well with boundary layer solutions and experiments reported in Ref. 188.

A further extension of the method is tentatively proposed for situations for which the ratio i_w/I_e is not small as compared to 1. In this case a value which replaces the constant 0.35 in equation (3-37) can be read

off Fig. 7 for the appropriate value of the pressure gradient parameter β . This later parameter is obtained from equation (3-32a). Relations are developed in References 213, 125, and which are even easier to evaluate than equations (3-36) and (3-37). It is to be expected, however, that the method presented in this section can be extrapolated with more confidence to situations apart from the reentry problem.

Turbulent Boundary Layer.

Approximate methods similar to those used for laminar boundary layer flow have also been developed for the condition where the boundary layer has become turbulent. These methods have been discussed and recently compared in References 345, 419, and 435. Agreement among the various methods is still rather poor. It has, however, been found that the influence of the pressure gradient on heat transfer is less in a turbulent boundary layer than in a laminar one. Use of equation (3-12) describing heat transfer on a flat plate gives a fairly good approximation for a turbulent boundary layer on a blunt object when the velocity u_e appearing in the Stanton and Reynolds numbers is interpreted as the local value outside the boundary layer, and when the distance x in the Reynolds number is interpreted as the distance s of the surface point under the consideration from the stagnation point measured in flow direction along the body surface.

312g. Separated Flow. Heat transfer in separated flow regions like the one sketched in Fig. 4 is still very incompletely understood. This region is either more or less stagnant or filled with a regular circulating laminar flow or with a turbulent flow containing irregular vorticity and of an unsteady, fluctuating character. Which of these

flow situations occurs depends on the Reynolds number. In a constant
property fluid, heat transfer to the surface of an object bordering on
the separated region may be smaller or larger than the heat transfer
to the front portion which is covered by a boundary layer. The most
extensive information is available for heat transfer in flow normal to
a cylinder with circular cross-section. It has been found that for
Reynolds numbers up to 10 the local heat transfer is a minimum at the
rearward stagnation point of the cylinder. At higher Reynolds numbers,
a maximum appears at the rearward stagnation point which gradually
increases, and for Reynolds numbers beyond approximately 50,000 the
heat transfer coefficient on the rearward portion of the cylinder is
as large as the one on the front portion. With further increase of
Reynolds number the heat transfer becomes larger in the back portion
than in the front portion. For high velocity flow of a compressible
fluid, heat transfer in the separated region is influenced by another
factor which tends to decrease it relative to the values in an
incompressible fluid. It is known that heat transfer in forced con-
vection depends mainly on the product of density times velocity, and
in high velocity flow the density in separated regions is usually
considerably smaller than the density along the forward portion of
the object. Correspondingly, it was found that heat transfer coefficients
in separated regions of objects exposed to supersonic flow are quite
small as compared to heat transfer in the front portion. They are
usually of the same order of magnitude as local heat transfer
coefficients which exist on the object just upstream of the point of
separation.

For sufficiently small Reynolds numbers for which the flow in the

separated region exhibits a steady regular circulation of laminar character, heat transfer coefficients were recently obtained by analysis (References 51, 205). For higher Reynolds numbers where the flow exhibits the irregularly fluctuating turbulent character, heat transfer coefficients are only obtainable up to now by experiments and appear to be very sensitive to the particular shape of the object which is exposed to flow.

32. DISSOCIATION, IONIZATION

The discussion up to now was concerned with a "single component" gas; this means with a gas in which no changes in chemical composition occur. In reality, the temperature increase occurring in high velocity flow is often so large that the gas dissociates. This is especially true in the regions behind snock waves on objects like the one shown in Fig. 4. The surface temperatures of the objects are generally much lower, and therefore a temperature drop in the direction towards the surface occurs in the boundary layer. Accordingly, the dissociated atoms recombine again within the boundary layer. To which degree they can do this depends on the time which they have available. It is convenient to study two limiting cases which must bracket the situations as they actually occur. One limit is a condition in which the recombination rates are very fast compared with the time which the molecules or atoms need to change location in the boundary layer by diffusion and convection. Accordingly, thermodynamic equilibrium will be established at each point and the dissociation within the boundary layer is determined completely by the local temperatures and pressures. The other limiting situation is encountered when the dissociation rates are very slow

compared with the diffusion process. In such a case the molecules will not find sufficient time to recombine within the boundary layer itself. What concentration field of the atoms is established depends then on the condition at the wall. When the wall is catalytic, then the recombination of atoms occurs at the wall to such a degree that equilibrium is again established in the gas at that point. The recombined molecules diffuse away from the wall. On the other hand, the atoms have to diffuse towards the wall, and this diffusion process determines the concentration profile within the boundary layer. If, on the other hand, the wall is not catalytic, then the recombination rate will be slow even at the surface, and the atoms will maintain the concentration which they had outside the boundary layer through the whole region up to the surface of the object.

It might be expected that the heat transfer to the wall of an object in high speed flow is influenced strongly by this dissociation and recombination process. A closer inspection, however, will show that this is not the case. For this purpose, we will consider a gas consisting of atoms and molecules of the same species and at first determine the equations which govern the heat flow in such a dissociating and recombining gas. In this situation, two processes cause a heat flow: first, the usual thermal conduction process, and secondly the diffusion process of the atoms and molecules, because in this process the particles carry along their enthalpy. The equation describing the heat flux q in such a fluid with a gradient of temperature and concentration in y direction reads:

$$q = -k \frac{dt}{dy} - \rho D_A i_A \frac{dw_A}{dy} \qquad (3-38)$$

k in this equation denotes the heat conductivity (exactly the parameter determining the transport of energy according to the translational, rotational, and vibrational degrees of freedom). D_A is the diffusion coefficient for interdiffusion of the atoms and molecules, i_A is the energy of dissociation per unit mass of atoms, and w_A is the mass fraction of the atoms in the gas. The equation therefore is written in mass fraction rather than in concentration. The enthalpy of the molecule-atom mixture is described by the following equation:

$$di = c_p \, dT + i_A \, dw_A \qquad (3\text{-}39)$$

in which c_p again denotes the specific heat at constant pressure comprising the translational, rotational, and vibrational degrees of freedom, however, excluding chemical energy. Introducing this relation for the enthalpy into equation (3-38) gives for the heat flux the following relation:

$$q = - \frac{k}{c_p} \frac{di}{dy} - \left(\rho D - \frac{k}{c_p} \right) i_A \frac{dw_A}{dy}$$

This equation can be simplified by introduction of a dimensionless parameter called Lewis number and defined by:

$$Le = \frac{\rho D c_p}{k} \qquad (3\text{-}40)$$

With the Lewis number the equation for the heat flux reads:

$$q = - \frac{k}{c_p} \frac{di}{dy} - \rho D \left(1 - \frac{1}{Le} \right) i_A \frac{dw_A}{dy} \qquad (3\text{-}41)$$

The Lewis number for gases has a value which is not too far from one. Therefore, a good approximate information on heat flow under the influence of dissociation can be obtained when the condition in a gas with a Lewis number equal one is considered. For such a gas, the heat

flow equation simplifies to:

$$q = - \frac{k}{c_p} \frac{di}{dy} \qquad (3-42)$$

It can be seen that the equation for the heat flow is just as simple
as for a single component gas. The only difference is caused by the
fact that, in the enthalpy gradient describing the heat flow, the
chemical energy (energy of dissociation) has to be included. On the
other hand, it should be kept in mind that the heat conductivity and
specific heat do not include the chemical energy but only the pure
gas energies. One has to distinguish those properties from the ones
including the chemical energy which often are listed in reference works.
A more detailed discussion of this difference will be given in a
later section. Equation (3-42) indicates that the heat flux is
dependent on the enthalpy gradient in the gas alone, and that it does
not matter at all whether the transport in detail is caused by conduc-
tion or by diffusion. This already indicates that the heat flow does
not depend on the degree of dissociation in the gas. An example will
explain more clearly what is meant by this statement. Let us consider
for this purpose a gas layer with finite thickness b and let us
simplify the actual conditions by assuming that heat conductivity
and specific heats are constant. In this case the equation can
immediately be integrated over the thickness of the layer, and the
following equation results:

$$q = \frac{k}{c_p} \frac{(i_w - i_e)}{b}$$

i_w and i_e denote the enthalpies at the two borders of the layer. The
notation is used because the layer may be considered as a crude model

of a boundary layer. i_w then corresponds to the enthalpy of the gas
at the solid surface, and i_e to the enthalpy at the outer edge of the
boundary layer. This enthalpy i_e is prescribed through the concentra-
tion of atoms and through the temperature. The enthalpy i_w at the wall
surface, on the other hand, depends on the recombination conditions at
the surface. If the wall is catalytic, then the concentration of atoms
is equal to the equilibrium concentration which belongs to the
temperature and pressure of the wall surface. At a prescribed wall
temperature and pressure in the boundary layer, the enthalpy i_w is also
a fixed value, and this is so regardless of the fact whether the atoms
recombine within the layer itself or not -- in other words, whether
equilibrium or frozen state or any condition in between exists within
the layer. This then indicates that the heat flux to the wall at
prescribed wall temperature and prescribed conditions outside the
layer is a fixed value and independent of the ch.   omb' .tion
process.

If, on the other hand, the wall is non-catalytic, then the heat
flux depends on the condition for recombination within the layer.
For equilibrium state the concentration within the layer is everywhere
equal to the equilibrium concentration belonging to the local temperature.
Correspondingly, the equilibrium concentration is established at the
wall surface by the conditions in the gas itself, and the heat flow
will be again the same for a catalytic wall. If, on the other hand,
the recombination in the gas is negligible (frozen state), then the
concentrations will be uniform throughout the whole layer of the gas
and will also not be influenced by the presence of the wall. In this
case, no transport of energy by diffusion occurs, and only the first

term in equation (3-38) is active for the transport of energy. For a constant heat conductivity this term can again be integrated and gives:

$$q = k \frac{T_w - T_e}{b}$$

It is clear that the heat transport in this case will be less than when diffusion is present. The ratio of the heat flux q_n in a frozen state gas and at a non-catalytic wall to the heat flux q_c at a catalytic wall can be expressed in the following way:

$$\frac{q_n}{q_c} = \frac{c_p(T_w - T_e)}{i_w - i_e}$$

Note that c_p only contains the translational, rotational and vibrational energy, whereas i contains additionally chemical energy.

In a boundary layer, the energy transport by conduction and diffusion is still described by equation (3-38). To it, however, has to be added the transport by convection. Numerous calculations have been performed in which the laminar boundary layer equations have been solved for the determination of heat transfer in a high velocity air flow under the presence of dissociation and recombination. The corresponding reports are listed in the list of references. Fay and Riddell (Ref. 124) investigated the heat transfer in a rotationally symmetric stagnation flow of air and arrived at the statement that heat transfer at a catalytic surface is obtained from the corresponding equation in a single component gas by multiplication with the following factor:
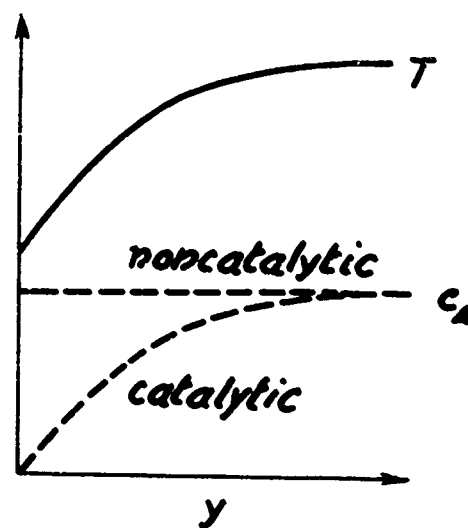
$$1 - (Le^n - 1) \frac{i_D}{i_o}$$

Fig. 8

Temperature and concentration profiles
on catalytic and noncatalytic surfaces

If our relation (3-37) is used then the following equation is obtained
which describes heat transfer in rotationally symmetric stagnation flow
of a dissociating air boundary layer:

$$\frac{Nu^*}{\sqrt{Re^*}} = 0.70 \ (Pr^*)^{1/3} \left[1 - (Le^n - 1) \ \frac{i_D}{i_o} \right] \qquad (3-43)$$

i_D indicates the energy of dissociation per unit mass of the gas
consisting of molecules and atoms, and i_o is the enthalpy of the
mixture at the stagnation point and outside the boundary layer. The
exponent n was determined to have the value 0.52 for equilibrium state
and 0.63 for frozen state within the boundary layer. For Couette flow
a value 0.5 had been found. It can therefore be assumed that the
exponent n depends only moderately on the specific flow situation and
that the above correction factor can generally be used to obtain heat
transfer in a dissociating or recombining gas. The Lewis number in air
for hypersonic flow conditions is expected to assume val. as between 1
and 1.5, and a consideration of diffusion and recombination may give
an increase of the heat transfer up to approximately 20 per cent. On
the other hand, for frozen state in the boundary layer and a non-
catalytic wall, a possible reduction of the heat transfer parameter
to one-third of the value which it would have if the wall were
catalytic was calculated by Fay and Riddell (Ref. 124). This
calculation was made for a flight condition as it may occur in the
re-entry of a missile. Frozen state in the boundary layer has been
assumed, and this requires that the flight occurs at very high altitude.
Fig. 8 gives a schematic sketch of the variation of temperature and
concentration through a boundary layer; once for a catalytic and
second for a non-catalytic surface. The temperature profile in the

boundary layer is essentially unchanged by the surface condition. On the other hand, the concentration is uniform for a non-catalytic surface and decreases to its equilibrium value according to the lower surface temperature for the catalytic surface. Energy transport is affected by the temperature gradient alone at the non-catalytic surface and by the concentration gradient and temperature gradient at the catalytic surface.

4. MASS TRANSFER COOLING

The general equations which describe the temperature condition in a solid wall and the heat flux to the surface of such a wall under the influence of mass transfer cooling have been discussed in chapter 2. In the present chapter we are concerned only with the process within the boundary layer itself, which determines the heat transfer coefficient. Correspondingly, the term mass transfer cooling will be interpreted here as incompassing all cooling methods by which a mass flow away from the surface and into the gas stream is generated, regardless of the specific method by which such a mass flow is achieved. It includes processes like transpiration cooling, sweat cooling, ablation cooling, liquid film cooling, and so on.[x] The coolant which in gaseous form moves away from the surface of the solid object is usually of a substance different from the fluid moving in the outside flow over the surface. Correspondingly, the mass transfer cooling process will, in

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[x] The interphase between liquid and gas in film cooling moves along the surface. The velocities of this shear flow in the liquid film are, however, so small that their effect on the gas boundary layer is negligible.

addition to the parameters occurring on a solid surface, depend on two
new parameters: on the amount of mass released at the surface and on
the nature of the coolant gas. Accordingly, it requires a much larger
number of calculations or experiments to obtain the required knowledge
on the mass transfer cooling process. This is the reason why our
present day information on mass transfer cooling is still scarce as
compared with that on heat transfer on a solid surface. Generally, it
is found that a mass flow away from the surface reduces heat transfer.
The procedure which appears best in the light of the remarks made above
and which today has been quite generally accepted is to calculate heat
transfer on a mass cooled wall in the following way: The heat transfer
on a solid surface under equivalent flow conditions is calculated and
this heat transfer coefficient is then multiplied with a correction
factor which describes the reduction obtained by the mass transfer
cooling process. This procedure will be used in the following discussion.

4.1. WITHOUT CHEMICAL REACTIONS

In many applications, the chemical species released at the surface
will react with the components in the outside gas flow. The discussion
in this section will assume that such reactions do not occur. In
addition, it will consider, on the one hand, the gas in the outside
flow and, on the other hand, the mass released from the surface as
single components. This is admissible as long as each of the two
components consists of species whose properties differ comparatively
little. We are therefore concerned with the mass movement by convection
and diffusion of two components in a gas mixture relative to each other.
It will be seen that this process has often a pronounced affect on

heat transfer. The discussion will again be carried out in the
following order. At first a mixture of two components will be considered
with properties which are constants (independent of pressure,
temperature, and composition). As a next step, the properties of the
mixture will be considered to depend on temperature but not on
composition. Such a situation is quite often approximated in actual
mass transfer cooling processes. In a third step, the properties will
be considered to depend on temperature and on composition, and in
addition to be widely different between the two components. In
recent investigations, it has been found that gases with low molecular
weight appear to offer special advantages as coolants for a mass
transfer cooling process. Since the properties depend essentially on
the molecular weight, such a situation has created interest in the
influence of large differences in properties between the coolant and
the outside gas on the mass transfer cooling process.

411. Constant Properties

The boundary layer equations for flow over a surface with mass
release are the same as those describing the velocity and temperature
fields on a solid wall. The only change occurs in the boundary
conditions where a finite velocity $v = \dot{m}/\rho$ normal to the surface is
now prescribed. Solutions of these equations for laminar and turbulent
boundary layers with mass transfer cooling have shown that Stanton
number or Nusselt number describing the local heat transfer coefficient
depend on the following parameter

$$\frac{\dot{m}}{\rho u_e} \, (Re)^n \qquad\qquad (4\text{-}1)$$

when the mass $\dot{m}$ released per unit time and surface area varies inversely proportional to the expression $x^n u_e^{n-1}$. The exponent n has the value 0.5 for a laminar boundary layer and 0.2 for a turbulent one. For this situation, the boundary layer profiles at different locations z are similar. Heat transfer coefficients obtained from these similarity solutions are often used as approximations for a determination of heat transfer even when the mass flow distribution over the surface is different from the one on which the calculation was based. This procedure is analogous to the method of "local similarity" which has been applied in Section 3123. For turbulent flow, it can be expected that the influence of a specific mass release distribution on heat transfer is comparatively small. For laminar boundary layers the influence may be considerable. However, little information is available from which its magnitude can be estimated.

The parameters may be brought to a form which is identical for laminar and turbulent flow when the Reynolds number is replaced by Stantons number

$$\frac{\dot{m}}{\rho u_e} \quad \frac{1}{St_s} \tag{4-2}$$

The Stanton number St_s describing heat transfer on a solid surface under the same boundary conditions (except $v = 0$) is used for this parameter in the present report. C. J. Scott demonstrated additionally that the results of calculations on heat transfer in laminar boundary layers with pressure gradients can be brought to near coincidence when the parameter (4-2) is changed to

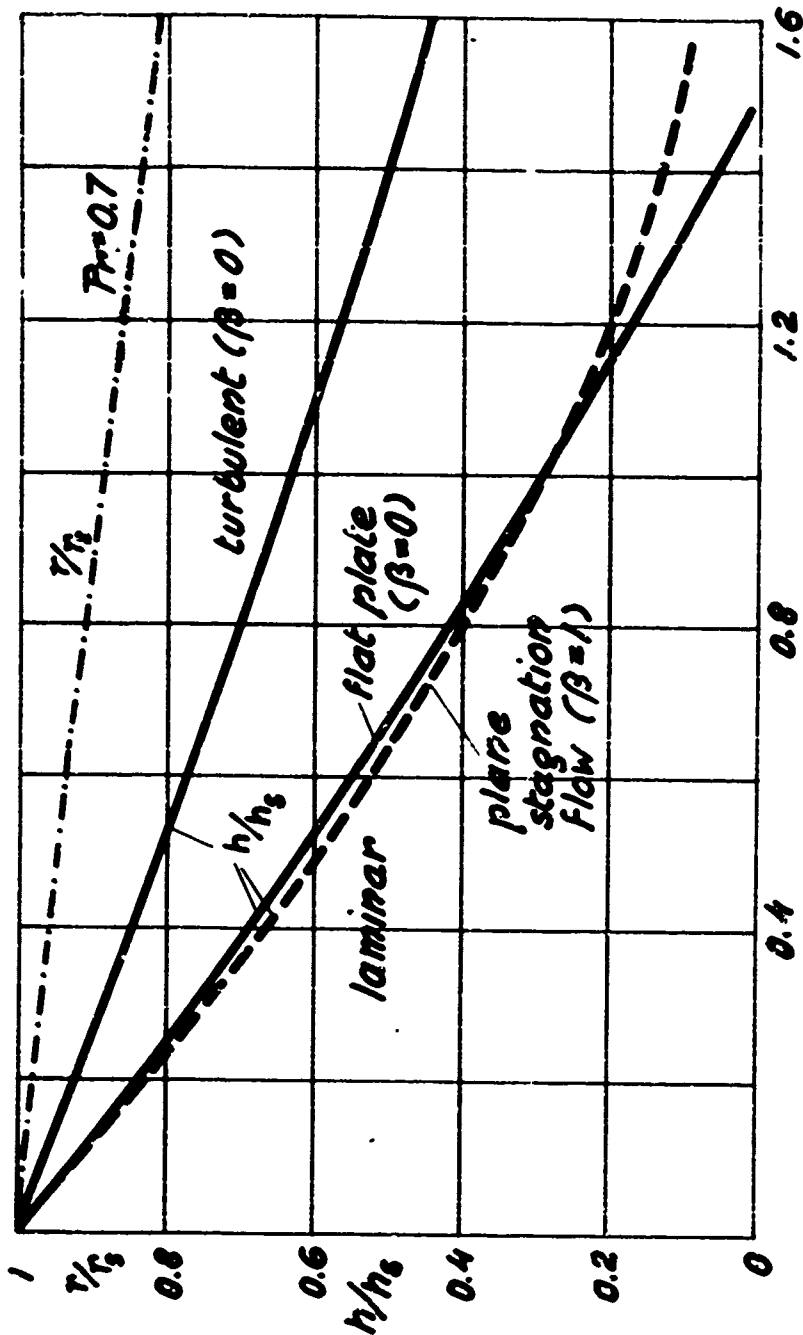$$\frac{\dot{m}}{\rho u_e} \quad \frac{\sqrt{1 - \beta/2}}{St_s} \tag{4-3}$$

Reduction of heat transfer and recovery factor by mass transfer cooling (constant property fluid)

Fig. 59

β is the pressure gradient parameter as used in Section 3123. It has the value 0 for flat plate flow, 1 for plane stagnation flow, and 0.5 for rotationally symmetric stagnation flow. For turbulent flow, $\beta = 0$ should be a good approximation even when pressure gradients are present.

Figure 9 presents the ratio h/h_s of the actual heat transfer coefficient to the one on a solid surface under the same outside flow conditions as a function of the mass release parameter (4-3) for a gas with Pr = 0.7. It may be observed that the values for flat plate flow and for plane stagnation flow agree quite closely in this present-ation. The figure also contains the ratio r/r_s of the actual temperature recovery factor to the one on a solid surface for laminar flat plate flow. Knowledge of the recovery factor is of minor importance in mass transfer cooling because the difference between total gas temperature and wall temperature is usually large compared with the difference between total gas temperature and recovery temperature. An error in the recovery factor then has little influence on the calculation of the heat flux.

412. Properties Dependent on Temperature

It has been demonstrated in Reference 425 that the results of numerous laminar boundary layer solutions for flow over a flat plate agree with the constant property relation presented in Fig. 9, when the blowing parameter is changed to the following form suggested by J. R. Baron:

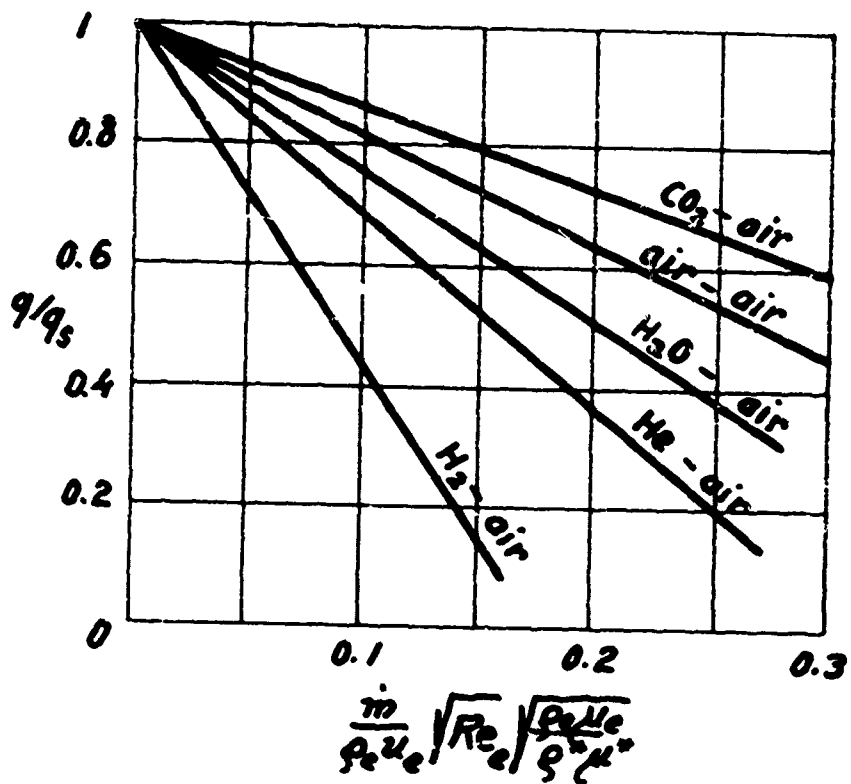$$\frac{\rho_w v_w}{\rho_e u_e} \sqrt{Re_w} \sqrt{\frac{\rho_e \mu_e}{\rho^* \mu^*}} \qquad (4-4)$$

Fig. 10

Reduction of heat flow by mass transfer cooling
(from Ref. 426)

This parameter can be transformed in the following way:

$$\frac{\rho_w v_w}{\rho_e u_e} \sqrt{\frac{\rho_w u_e x}{\mu_w}} \sqrt{\frac{\rho_e \mu_e}{\rho^* \mu^*}} = \frac{\dot{m}}{\rho^* u_e} \sqrt{\frac{\rho^* u_e x}{\mu^*}} = \frac{\dot{m}}{\rho^* u_e} \sqrt{Re^*}$$

This indicates that the rule proven so successful for heat transfer on solid surfaces, namely, that properties should be introduced at a reference temperature applied to the mass release parameter as well. The parameter can again be generalized by introduction of the Stanton number and by addition of the pressure gradient parameter. It can be expected that the parameter

$$\frac{\dot{m}}{\rho^* u_e} \frac{\sqrt{1 - \beta/2}}{St_s^*} \qquad (4\text{-}5)$$

correlates heat transfer coefficients for laminar boundary layers with pressure gradients and for turbulent boundary layers (setting $\beta = 0$). Fig. 9 can then be used to calculate heat transfer in fluid with temperature dependent properties when the properties in the mass release parameter are introduced at reference temperature or reference enthalpy. The reference state is calculated from equation (3-13) or (3-16).

413. Properties Dependent on Temperature and Composition

Fig. 10, taken from Reference 426, presents the ratio of the actual heat flux q at a mass transfer cooled wall to the heat flux q_s which the solid wall would have under the same flow conditions, plotted over the mass release parameter. It has been found that the results of various calculations correlate on these curves with reasonable accuracy. The notation on the various curves indicates the nature of the two component mixture. The first term gives the coolant gas, and

it can be observed that the outside flow over the surface was air in
all cases. The various curves represent with good accuracy the results
of boundary layer calculations on a flat plate with a large range of
Mach numbers, wall temperatures and free stream temperatures. It can
be observed that the curves arrange themselves according to the
molecular weight of the coolant. In this way it is suggested to
consider the molecular weight of the coolant as the main parameter
determining the coolant effect in the mass transfer process. In
Reference 426 , it has been found that the various curves in Fig. 10
can be expressed by the following equation:

$$q/q_s = 1-2.46(\rho_w v_w/\rho_e u_e) \sqrt{Re_e} \sqrt{\rho_e \mu_e/\rho^* \mu^*} (M_a/M_c)^{1/3} \qquad (4\text{-}6)$$

ρ^* and μ^* denote the density and viscosity of air at reference
temperature. M_a in this equation indicates the molecular weight of
air and M_c the molecular weight of the coolant gas. The same equation
describes the ratio of the actual wall shear on the transpiration
cooled wall to the shear which would exist on a solid surface under the
same flow condition when the constant is changed to a value of 2.08.
A similar relation was obtained in Reference 368 for the ratio of heat
transfer coefficients in a laminar and turbulent boundary layer on a flat
plate. The exponent on the molecular weight ratio in this reference
was found to be 0.4 for laminar flow and 0.6 for turbulent flow. Reference
425 gave 2/3 for the exponent 1/3 in turbulent flow. This result and the
generalized parameter (4-5) can now be used to obtain the relation

$$h/h_s = 1 - C \quad \frac{\dot{m}}{\rho^* u_e} \frac{\sqrt{1- \beta/2}}{St_s^*} (\frac{M_a}{M_c})^n \qquad (4\text{-}7)$$

C has the value 0.73 for laminar and 0.37 for turbulent flow.

The exponent n is 1/3 for laminar and 2/3 for turbulent flow. Properties for air at reference temperature have to be introduced for ς^* and in St_s^*. The ratio h/h_s of heat transfer coefficients is not quite identical to the ratio q/q_s of heat fluxes because the recovery temperature on a solid wall is somewhat different from the recovery temperature on a mass transfer cooled wall. In the boundary layer solutions on which equation (4-6) is based, this difference is very small. This has already been pointed out at the end of the preceeding section. Reference 426 contains information in Fig. 4 on the recovery factors of transpiration cooled surfaces.

Equation (4-7) is a relation which represents results of analyses and experiments available today with an accuracy which is sufficient for engineering purposes. However, it should be remembered that the range of information to date is still restricted, and that corrections to this relation may have to be made when more information becomes available in the future. There are, for instance, some indications from calculations presently performed at the Naval Ordnance Laboratory that the molecular weight is actually not the only parameter to describe the behavior of a specific coolant. It may well be that in the future either other properties have to be considered in the relation (4-7) or that a procedure will have to be used which considers all the properties of the coolant gas as well as of the gas in the outside flow as, for instance, a reference concentration procedure. Such a reference concentration procedure has been proposed in Reference 436. It should also be pointed out that equation (4-7) is based on analyses and experiments in all of which the surface was impermeable to the one component of the gas mixture, namely, to the gas in the outside

flow. A different situation is sometimes encountered, for instance, when a chemical reaction takes place at the surface. In this case, the oxygen necessary for the reaction has to be transported through the boundary layer toward the interface on which the reaction occurs, and the relation between the mass flow parameter and the local concentrations of the two components will be different from the one considered in the previous analyses. This may also have an effect on the reduction of the heat transfer coefficient.

42. WITH CHEMICAL REACTIONS

A mass transfer cooling process is often connected with chemical reactions when it occurs at high temperatures. These reactions change the temperature field and the properties, and influence the heat transfer coefficient and the heat flux to the surface. The influence of chemical reactions on heat transfer will at first be discussed on a simplified model which was considered in Reference 115 and 160. In this reference, heat flow to a surface in flat plate and in plane stagnation flow was considered when a steady, two-dimensional, laminar boundary layer exists, and when the flow velocities are sufficiently low to make dissipation negligible. It was assumed that the surface releases a mass $\dot{m}$ and that this mass (for instance, carbon or hydrogen) reacts with the oxygen of the air stream moving over the surface. The fluid properties are all postulated constant and having practically the same value for all components of which the air and the combustible material exists. Schmidt number and Prandtl number have the value 0.7 which means a Lewis number equal to one. The chemical reaction rates are assumed to occur very fast so that the reaction process is completely diffusion controlled. Chemical equilibrium may be close
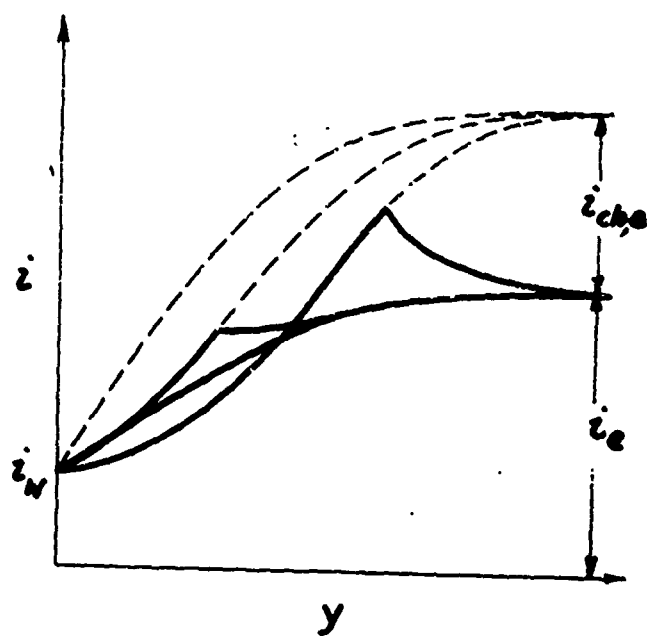
Fig. 11

Enthalpy profiles in a boundary layer
with combustion

to complete combustion. Fig. 11 shows as an example of the results of this calculation the enthalpy profiles plotted over the wall distance y. The wall temperature is assumed prescribed. The full lines indicate the enthalpy profiles for various mass release rates. The peak in the enthalpy curves coincides with the location of the flame front within which the combustion occurs. For very small mass release rates, the combustion occurs right at the surface, and the enthalpy profile is the lowest one indicated in the figure. Beyond a fairly small release rate, the combustion front lifts off the surface and moves with increasing release rate more and more into the interior of the boundary layer. The enthalpies presented by the full lines constitute the sum of the sensible heats of all the components of which the gas is composed. If the chemical energy which can be released by combustion of the local oxygen is added, then the dashed curves are obtained which indicate the total enthalpy profiles within the boundary layer. These total enthalpy profiles have exactly the same shape as the enthalpy profiles for a constant property fluid without combustion and with the proper mass release rate provided the enthalpy at the wall is equal i_w and the enthalpy at the outer edge of the boundary layer is equal to the total enthalpy at that location. Sensible heat and chemical energy of possible reactions have to be included in the latter value. From this statement it becomes obvious that the combustion process occurring in laminar boundary layer flow of a fluid with constant properties and Le = 1 is included in its effect on heat transfer when the heat flux is calculated with equation (2-3) in which the enthalpy at the outer edge of the boundary layer comprises chemical energy as well as sensible heat. The heat

transfer coefficient can be determined from equations (3-6) and (4-7).
This procedure applies to a situation in which the oxygen content at
the surface is zero. When the oxygen content at the surface is
different from zero, then the chemical energy corresponding to this
oxygen content must be included in the enthalpy of the gas at the
wall surface.

The explanation for this behavior can be obtained by a reconsider-
ation of Section 32 and specifically of the development which led to
equation (3-42). The discussion in that section was concerned with
dissociation. It may, however, be applied in the same way to any
chemical reaction within the boundary layer. Equation (3-42) then
indicates that the heat flux normal to the stream lines at any point
within the boundary layer is determined by the gradient of the total
enthalpy comprising sensible heat, kinetic energy, and chemical
energy, and that it is immaterial for this transport whether it occurs
as conduction of sensible heat or as diffusion of chemical enthalpy.
In Fig. 11, the transport of total enthalpy is purely by conduction
along the enthalpy curves in the region between the flame front and
the wall; it is by diffusion as well as by conduction along the dashed
part of the enthalpy curves in the outer region of the boundary layer
outside the flame front.

L. Lees developed in Reference 432 the laminar boundary layer
equations for plane and rotationally symmetric flow of a two component
gas mixture with chemical reactions and with mass transfer at the wall.
The continuity and momentum equations are the same as equations (3-25)
and (3-26). The energy equation is:

$$\rho \left(u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\mu}{Pr}\frac{\partial I}{\partial y}\right) + \frac{\partial}{\partial y}\left[\mu\left(1 - \frac{1}{Pr}\right)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right] +$$

$$\frac{\partial}{\partial y}\left[\rho\, D\left(1 - \frac{1}{Le}\right)\sum i_i \frac{\partial w_i}{\partial y}\right] \tag{4-8}$$

The total enthalpy I now contains chemical energy in addition to sensible heat and kinetic energy. D is the diffusion coefficient, w_i the mass fraction of the ith component (i running from 1 to 2), i_i is the enthalpy of the ith component. The last term in the equation disappears when the Lewis number is equal one. The fact that equations (3-29), (3-26), and (4-8) are not different there from those for a gas mixture without chemical reactions, indicates that the velocity field and the total enthalpy field are not influenced by any occurring reactions. This holds for a mixture with constant properties and also for a gas with $\rho\mu$ = constant and constant Pr when the pressure along the surface is constant. The last statement is easily verified by inspection of the equations (3-30) and (3-31) which are the transformed equations (3-25), (3-26), and (3-27). For $\rho\mu$ = constant, C has the value 1, and for constant pressure β is equal to 0. The equations are then the equations for a constant property fluid. The enthalpy gradient at the wall surface determines the heat flux into the wall according to equation (3-42).

The following rule for a calculation of heat transfer in mass transfer cooling and with chemical reactions follows from these considerations: One calculates the heat transfer coefficient from the relations in the preceding section, for instance, with equation (4-7). The heat flux to the surface is then calculated using the right hand expression in equation (2-3) and interpreting the enthalpies

as total enthalpies containing sensible heat, kinetic energy, and chemical energy corresponding to possible reactions within the boundary layer. This procedure holds strictly for gases with constant properties or with $\rho\mu$ = constant, Pr = 1, and constant pressure and in any case for Le = 1. It should be a good approximation when Pr and Le are not too different from one and for moderate pressure gradients, especially when properties are again introduced at reference enthalpy.

The mass release rate $\dot{m}$ which is required for this calculation has to be obtained from overall balances. In an evaporation or sublimation process, for instance, it is fixed by the energy balance in Chapter 2 together with the thermodynamic relation between evaporation or sublimation temperature and the pressure or partial pressure of the released substance at the surface.

5. LOW DENSITIES

In the discussion up to now, the fluid involved was considered to be a continuum. In reality, gases consist of individual molecules and this structure makes itself felt at low densities which, for instance, are obtained in the flight of an aircraft at very high altitude. The parameter which determines whether the molecular structure influences heat transfer and friction is the Knudsen number:

$$Kn = \frac{\lambda}{L} \qquad (5-1)$$

λ indicates the mean molecular path length and L denotes a character-istic dimension of the object involved. The flow in a boundary layer,

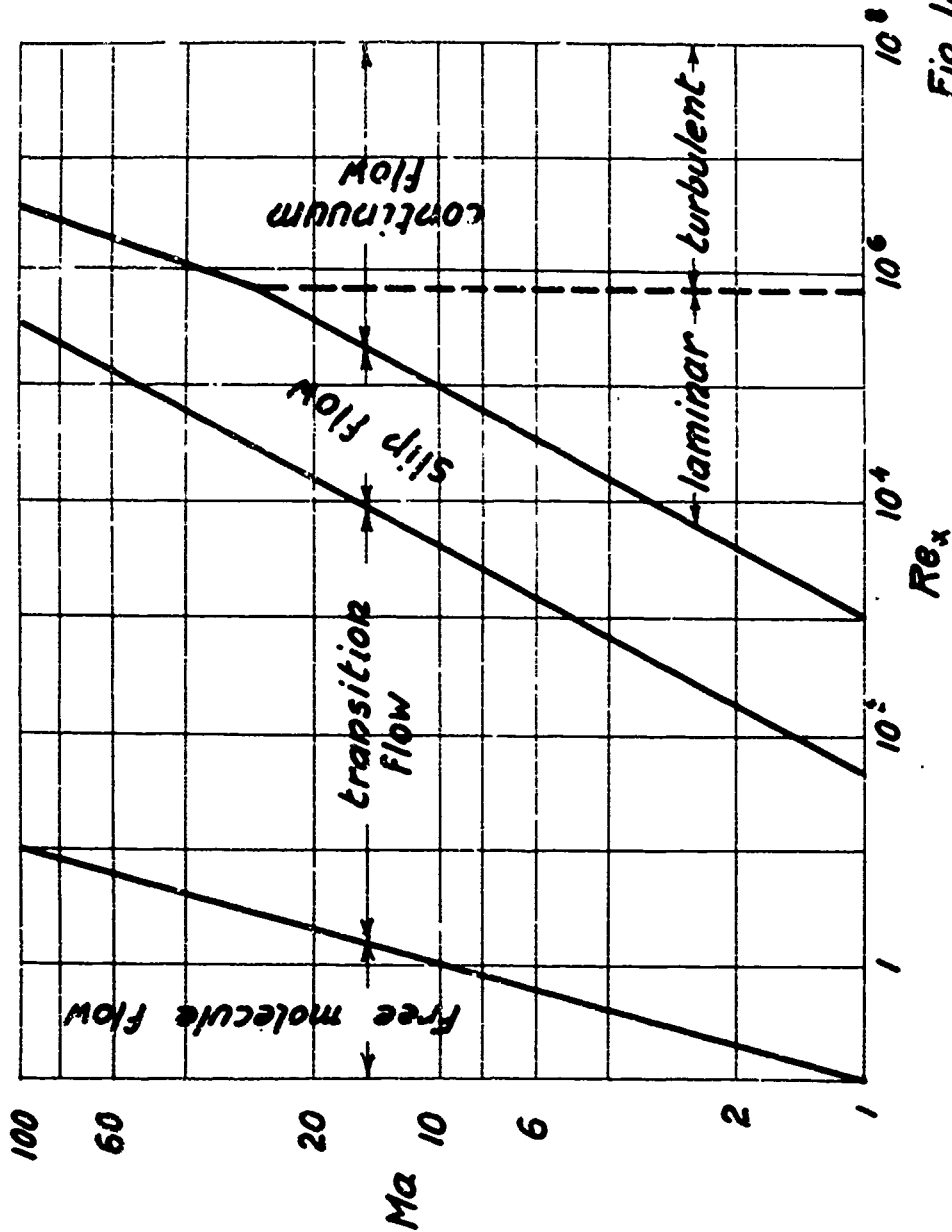Flow ranges in Mach-Reynolds diagram

Fig. 12

for instance, can be considered with good accuracy to be continuum

flow when the ratio of mean molecular path length to boundary layer

thickness has a value which is small as compared to one. With

increasing Knudsen number, the molecular structure makes itself felt

at first in the immediate neighborhood of the surface of the object

by the fact that the flow slips over the surface and that the

temperature profile in the boundary layer has a very sudden change near

the wall. This regime is referred to as slip flow regime. Simultan-

eously, an interaction between shock and boundary layer usually begins

to influence the flow and heat transfer. On the other end of the

scale of Knudsen numbers (large values) one encounters conditions

where the mean molecular path length is large compared to any body

dimension. This means that, after being reflected from the surface,

an approaching molecule has practically no chance to be reflected

back and to hit the surface a second time. This regime is referred

to as free molecular flow regime. In between this regime and the

slip flow regime is the transition regime in which the mean molecular

path length and the characteristic body dimension are of the same

order of magnitude. The Knudsen number is uniquely related to the

Reynolds and Mach numbers. As a consequence, the various flow regimes

can be indicated in a diagram in which the Reynolds number is used

as abscissa and the Mach number as ordinate. Such a diagram is

presented in Fig. 12 taken from Reference 422. The transition lines

between the regimes are to a certain degree arbitrary. Compared with

a similar diagram in Reference 110, the limits between continuum flow

and transition flow have been shifted to the left, because recent

experiments in low density wind tunnels indicated that the continuum regime extends to smaller Reynolds numbers than originally expected. It has also been pointed out in Reference 168 that the proper conditions existing within the boundary layer have to be introduced into Reynolds and Mach number in a determination whether the flow is in the continuum or slip flow regime. The best values to use in such a case are probably velocity outside the boundary layer and the properties at the pressure, temperature, and concentration as they exist within the boundary layer at the surface of the object. It has been found in Reference 168 that, in this way, practically all conditions under which cooling of missiles and satellites becomes critical are located in the continuum regime. This is very advantageous because heat transfer relations are much better established for this regime than for the slip or transition regime.

A very simple relation for the heat transfer is obtained in the free molecular flow regime for an object flying with a hypersonic velocity. For such a condition, the mean molecular velocity is small compared to the vehicle speed V, and the molecules can be considered as practically at rest. This allows calculation in a simple way of the number of molecules which strike the vehicle as it moves through the gas. If ρ_∞ indicates the undisturbed density of the gas, and M its molecular weight, then the number of molecules per unit volume is ρ_∞/M. The number of molecules hitting the object per unit time is then given by the following relation:

$$\frac{\rho_\infty}{M} VA$$

in which A indicates the area obtained when the vehicle is projected

onto a plane normal to the flight direction. Each molecule striking the surface transfers the following energy to the object:

$$a M \frac{V^2}{2}$$

The term a in this expression indicates what fraction of the initial kinetic energy the molecule transfers. This term is called accommodation coefficient. The numerical values for the accomodation coefficient have to be determined by measurements. They depend on the nature of the gas, on the nature and condition of the surface, and probably also on the velocity V. Only limited information is available. Experiments carried out up to now resulted in values between 0.3 and 1 for air and for various surface materials. From the two equations stated above the heat flow per unit area of projected surface and per unit time is:

$$q_{wc} = \rho_\infty a \frac{V^3}{2} \tag{5-2}$$

It is quite interesting to transform this equation into a Stanton number. The total enthalpy of the gas is practically only kinetic energy. Therefore, $I = V^2/2$. The enthalpy of the surface can be considered as very small compared to this value, so that the enthalpy difference is also $V^2/2$ and the heat transfer coefficient based on enthalpies is:

$$h_i = \rho_\infty a V$$

The Stanton number is therefore simply:

$$St_i = \frac{h_i}{\rho_\infty V} = a \tag{5-3}$$

The Stanton number is therefore simply equal to the accomodation coefficient if it is based on conditions in the undisturbed fluid and on projected area. Stanton numbers for continuum flow are usually

by two or three orders of magnitude smaller than this value. This
indicates the fact that a boundary layer serves as a kind of insulation
and decreases the energy transfer between the molecules in the free
stream and the object when it builds up around an object.

6. TRANSITION TO TURBULENCE

A prediction of the location on a body where the flow within the
boundary layer changes from laminar to turbulent state still has to
be based primarily on empirical information obtained in wind tunnel
experiments and in free flight tests. An extensive literature exists
according to the importance of the knowledge of the transition point.
The reader has to be referred to the literature, and only a few remarks
can be made in this report.

Stability theory predicted that the transition point would be
delayed by a decrease of the surface temperature so that very large
transition Reynolds numbers are expected on objects with cooled surfaces.
A figure indicating the results of such calculations is contained in
Reference 110. These predictions have generally been confirmed on
slender objects. Transition Reynolds numbers up to 10^8 have been
observed in wind tunnel experiments and free flight tests. On the
other hand, conclusions drawn from stability theory were completely
contrary to observation on blunt objects. A considerable favorable
pressure gradient usually exists on such objects over the region
which is covered by boundary layer flow, and from this fact one expects
the transition to be delayed. Experiments, however, showed that the

transition Reynolds number based on distance x is of the same order
of magnitude as on a flat plate under isothermal flow conditions.
Cooling of the surface was found to have very little effect on the
transition Reynolds number. Reference 367, for instance, reports the
results of experiments on sphere-cylinders and on elliptic cylinders
at a ratio of total enthalpy to wall enthalpy between 9 and 30 and
at a Mach number around 3. The experiments were performed in a shock
tube, and it was found that the transition occurred at a Reynolds
number based on momentum thickness between 225 and 325 (the last
figure for an enthalpy ratio equal 30). The transition Reynolds
number based on length x was between 500,000 and 10^6. From the above
figures it can be seen that cooling had no pronounced effect. The
shape influenced the transition Reynolds number somewhat.

Another observation for which a definite explanation is still
missing is the fact that on slender objects turbulent flow has been
observed at extremely strong cooling of the surface and under
conditions where the stability theory predicts completely laminar flow.

The momentum thickness as a function of the distance x has to be
known if Reynolds numbers based on momentum thickness are to be used
for a prediction of the location where the laminar boundary layer
changes into a turbulent one. Calculation of the momentum thickness
is considerably more tedious than that of heat transfer. In Reference
438, Lees proposes an iteration procedure by which the momentum
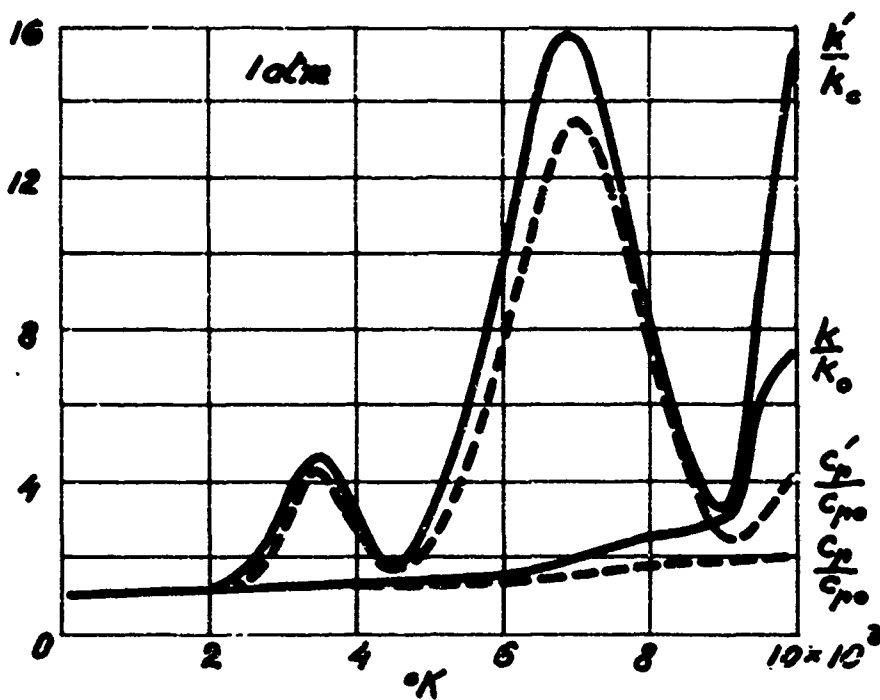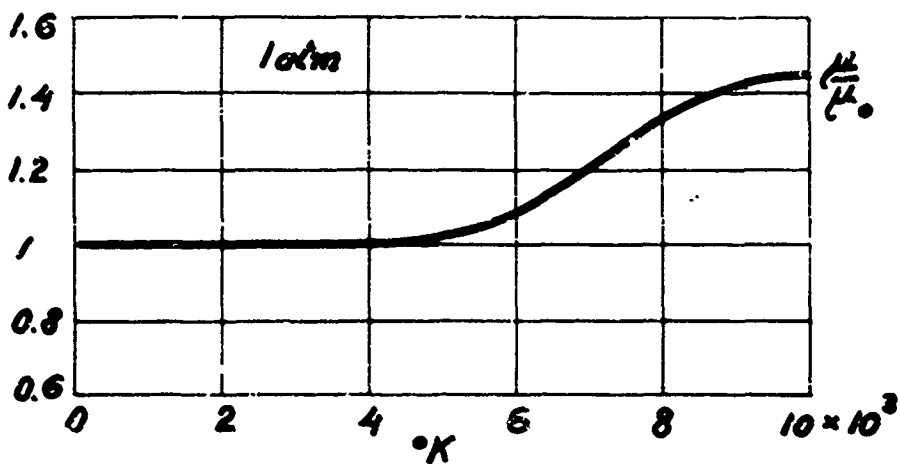thickness can be calculated for objects of arbitrary shape.

Fig. 13

Viscosity, eat conductivity and specific heat of air at
1 atm. pressure and in dissociated equilibrium (from Ref. 427)

7. PROPERTIES

A knowledge of the thermodynamic and transport properties is required for the gases participating in the flow when one wants to evaluate any of the relations contained in this report. For air, these properties have been calculated in a large pressure range and up to very high temperatures using the methods of statistical mechanics. Such information is, for instance, contained in References 48 and 427. Only indirect verification is available by the fact that heat transfer coefficients measured in shock tubes at very high temperatures agree quite well (within 10-20 %) with predictions based on the relations discussed in the previous sections and on the calculated air properties. Viscosity, heat conductivity, and specific heat of air under conditions close to thermodynamic equilibrium are presented in Fig. 13 for one atmosphere pressure. The values have been taken from Reference 427. Fig. 14 contains the Prandtl number and the Lewis number describing diffusion of dissociated atoms again for air at one atmosphere pressure. It has been pointed out before that two definitions are in use for the specific heat and for the heat conductivity. According to one definition, the specific heat (c_p') is based on the enthalpy which contains not only sensible heat but also the chemical energy. The second definition bases specific heat (c_p) on the sensible heat only. In a similar way, the heat conductivity can be defined by considering the transport by heat conduction alone (k), or by including the energy transport caused by diffusion (k'). The second part is uniquely related to the temperature and pressure since it is postulated that conditions near equilibrium exist in the gas. Fig. 13
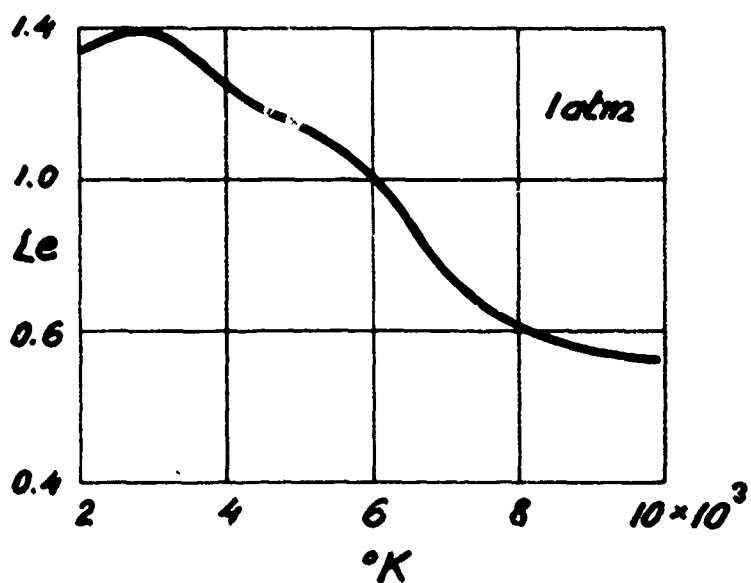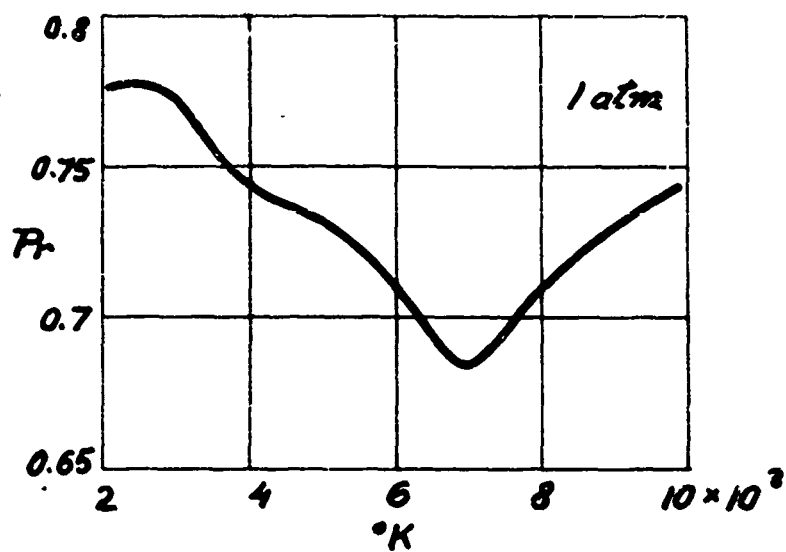
Fig. 14

Prandtl number and Lewis number of air at 1 atm. pressure
and in dissociated equilibrium (from Ref. 427)

shows that the difference between the values of specific heat and of heat conductivity according to the two definitions is very large. The values according to the definition which considers sensible heat only have to be used in the relations presented in this report. The Prandtl and Lewis numbers in Fig. 14 are based on the heat conductivity k and on the specific heat c_p.

In addition, the connection between enthalpy and temperature has to be known for calculations of heat transfer. For a gas in thermal equilibrium, this connection is best obtained from enthalpy entropy diagrams. (Mollier Diagram for Equilibrium Air, Avco Research Lab., Avco Manufacturing Corp., Everett, Mass., Jan 1957, Mollier Chart for Air in Dissociated Equilibrium, NAVORD Rep. 4446, Naval Ordnance Laboratory, Silver Springs, Md., 1957, see also Ref. 439).

The situation is considerably more involved in calculations on mass transfer cooling processes. The determination of the properties involved in this process is usually the most time-consuming prerequisite for such calculations, especially when chemical reactions occur within the boundary layer as they have been discussed in Section 42. Information on the properties of the various constituents involved in mass transfer cooling is usually quite restricted, and the calculation of the properties of the various mixtures which is possible by the methods of statistical mechanics is very involved. The reader has to be referred to the literature for the determination of such properties.

Use of temperatures or enthalpies in heat transfer calculations
has an influence on the energy balance describing heat exchange
between a wall and a mere component mixture. This will be demonstrated
in the following example, which considers flow over a mass transfer
cooled wall. The coolant may be denoted by 1, and it will be assumed
that the wall surface is impermeable to the gas 2 in the main flow.
Let us now fix attention to the mass flow of both components 1 and 2
through a plane a-a arranged in the gas just outside and parallel to
the surface. The mass release generates a convective velocity v
through this plane and a mass flux of component 1 equal $\rho_1 v_1$, where
ρ_1 is the partial density of 1. There is also a mass transport by
diffusion of amount $-\rho D(\frac{\partial w_1}{\partial y})_w$, if w_1 indicates the mass fraction
$w_1 = \frac{\rho_1}{\rho}$. The total transport of component 1 is:

$$\rho w_1 v - \rho D(\frac{\partial w_1}{\partial y})_w = \dot{m} \qquad (I-1)$$

For component 2, the mass flow through the surface and therefore also
through the control plane a-a is zero:

$$\rho w_2 v - \rho D(\frac{\partial w_2}{\partial y})_w = 0 \qquad (I-2)$$

Now we consider the energy transport through the control plane. The
energy flux per unit time and area may be denoted by q'. According
to equation (3-38), the energy flux is:

$$q' = -k(\frac{\partial T}{\partial y})_w + \rho w_1 v i_1 + \rho w_2 v i_2 - \rho D i_1 (\frac{\partial w_1}{\partial y})_w - \rho D i_2 (\frac{\partial w_2}{\partial y})_w \qquad (I-3)$$

This energy flux must be equal to the flux through a control plane b-b arranged in the solid wall just beyond the interphase. The interphase moves continuously towards the interior of the wall because of the mass release. A mass flow of solid material with the mass velocity $\dot{m}$ exists, therefore, through the plane b-b, which is considered fixed relative to the interphase. A corresponding energy flux by convection is $\dot{m}i_s$, where i_s indicates the enthalpy of the solid material. Another energy transport by conduction may be indicated by q_s. Conservation of energy requires:

$$q' = q_s + \dot{m}i_s \qquad (I-4)$$

Introduction of equation (I-1) into (I-3) results in:

$$q_s = - k(\frac{\partial T}{\partial y})_w + \dot{m}(i_1 - i_0) \qquad (I-5)$$

We have here obtained the result which was expressed in c'apter 2 by equations (2-3) and (2-9) with the exception that no radiation was considered in the present derivation.

The heat flux q_s may now be expressed by the enthalpy gradient in the gas mixture. With $\dot{m} = \rho v$ and equations (I-3) and (I-4) one obtains:

$$q_s = - \left[k(\frac{\partial T}{\partial y})_w + \rho D i_1 \frac{\partial w_1}{\partial y} + \rho D i_2 \frac{\partial w_2}{\partial y} \right] + \dot{m} \left[(w_1 i_1 + w_2 i_2) - i_s \right]$$

The first term in this equation is identical to equation (3-38), since in a two-component-mixture $\frac{\partial w_1}{\partial y} = - \frac{\partial w_2}{\partial y}$ and since $i_A = i_1 - i_2$. For a gas mixture with $Le = 1$, the first term takes the shape of equation (3-42), and the heat flux q_s is:

$$q_s = - \frac{k}{c_p} \cdot \frac{\partial i}{\partial y} + \dot{m}(i_w - i_s) \qquad (I-6)$$

i is the total enthalpy of the gas mixture, i_w indicates it value at the interphase, and i_s is the enthalpy of the ablating material at wall

surface temperature and in solid state.

In equation (I-5), on the other hand, the enthalpy i_1 of the ablating material in the gaseous state and at wall surface temperature takes the place of i_w in equation (I-6). The enthalpy i_1 can be more easily determined in most applications than i_w, and the form (I-5) of the heat flow equation is therefore preferable. In the use of heat transfer coefficients for mass transfer cooling, one has to watch carefully whether the coefficient was defined by equation (2-3) or by a definition which replaces the temperature gradient in this equation by an enthalpy gradient as suggested by equation (I-6). For a gas with properties depending on temperature only there is no difference between i_w and i_1, and both equations (I-5) and (I-6) become identical.

APPENDIX II

The mass fraction w_i of any of the components in a two-component mixture is determined by a diffusion equation:

$$\rho \left(u \frac{\partial w_i}{\partial s} + v \frac{\partial w_i}{\partial y} \right) = \frac{\partial}{\partial y} \left(\rho D_{12} \frac{\partial w_i}{\partial y} \right) + \omega_i \qquad (II-1)$$

ω_i denotes the production of the species i by the chemical reaction per unit time and volume.

The diffusion equations can also be written in "pseudo-mass-fractions" $\tilde{w}_i$. A pseudo-mass-fraction gives the mass fraction of an element in the mixture regardless in what compound it is contained. The pseudo-mass-fraction of oxygen, for instance, is the ratio of the oxygen mass to the total mass of the mixture regardless whether oxygen occurs as O, O_2, CO_2 or in any other compound. No element is destroyed or created in a chemical reaction. Therefore, the diffusion equation in pseudo-mass-fractions reads:

$$\rho \left(u \frac{\partial \tilde{w}_i}{\partial s} + v \frac{\partial \tilde{w}_i}{\partial y} \right) = \frac{\partial}{\partial y} \left(\rho D_{12} \frac{\partial \tilde{w}_i}{\partial y} \right) \qquad (II-2)$$

For a gas with $Pr = 1$ and $Le = 1$, equation (II-2) has the same form as the energy equation (4-8). This fact can be utilised to calculate the composition of the gas throughout the boundary layer and a' the wall surface. L. Lees carried this idea through in various examples (Ref. 432).

APPENDIX III

Literature

In this appendix, literature on the subject of heat transfer at high velocities is collected, which has been published since 1956. Previous literature is included only insofar as it is needed for reference.

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