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ELECTRO-THERMAL EQUATIONS FOR ELECTRO-EXPLOSIVE DEVICES (U)

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ELECTRO-THERMAL EQUATIONS FOR ELECTRO-EXPLOSIVE DEVICES

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ABSTRACT: A basic equation that relates the wire bridge temperature of an electro-explosive device to the applied power is introduced. Techniques for measuring the important thermal parameters are presented together with experimental observations. The equations which describe the temperature rise as a function of time for constant current, condenser discharge, and constant voltage firing are developed.

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The work reported on here has been carried out as part of the Naval Ordnance Laboratory's participation in the HERO (Hazards of Electromagnetic Radiation to Ordnance) program supported by Tasks 506-925/56015/07040, 506-925/56035/01073, and NOL-443. The objective of the HERO effort at NOL is generally to characterize the response of electro-explosive devices to electric and electromagnetic energies. This report presents the basic electro-thermal equations describing the thermal response of electro-explosive devices to different types of electrical energy and various methods for observing the thermal-time characteristics of electro-explosive devices are discussed.

This work should be of interest not only to the HERO project but also the broad field of electro-explosive device design, development, manufacture and use.

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INTRODUCTION

1. An electro-explosive device (EED) is essentially a converter of electrical energy into heat which with its accompanying temperature rise triggers off the explosive reaction. This report is concerned with the thermal and electrical parameters which can describe the energy conversion and the temperature rise. The theroetical and experimental work reported on here has been concerned with wire bridge devices, but it is possible to extend the presented concepts to other types of EED's.

2. When power is dissipated in a wire bridge, part of the input energy goes into heating the wire and part goes into heat loss from the wire. If there is a temperature rise in the wire then the power transfer will be affected giving rise to some form of nonlinear equation. The solution of this equation will describe the temperature vs. time characteristics of the wire bridge. Since the solution depends on the type of power-forcing or driving function supplying energy to the wire bridge, many variations to the problem are possible.

3. In this report the basic thermal equation representing temperature of the wire bridge system as a function of time will be described together with procedures and techniques for determining the important equation parameters. Then solutions for energy input functions representing certain popular EED firing wave forms will be

discussed. Experimental results or calculations using Squib Mk 1 wire bridge units as typical EED items will be presented to amplify the discussion.

THE BASIC THERMAL EQUATION

4. If the wire bridge is considered as a lumped system then the temperature gradients along the wire and the non-uniform heat loss are replaced by mean or nominal values. The resulting simplified equation describing the thermal behavior of the wire bridge is

$$C_p \frac{d\theta}{dt} + \gamma\theta = P(t), \tag{1}$$

where C_p is the heat capacity of the system (power-time/$^{\circ}$temperature change)

γ is the heat loss factor (power/$^{\circ}$temperature change)

θ is the temperature rise above ambient

and $P(t)$ is the input power function.

Every term in the equation has the dimensions of power. The factor γ includes several parts. Heat is lost to the explosive mixture in contact with the wire bridge and through conduction to the wire leads and surroundings. (Significant radiation losses are not expected for the temperature excursions nominally encountered in EED's.) The heat capacity includes not only the mass times the specific heat of the wire, but also coupled effects due to heating of the explosive mixture. Since power depends on the two-terminal resistance of the device, which in turn is sensitive to temperature rise, it is always expected that P(t) is actually more complicated than the simple time variations in the driving function. However, it is expected that the simplified equation will prove to be sufficiently accurate to describe the thermal behavior of the wire bridge system when experimentally evaluated thermal parameters are used for the coefficients.

5. In addition to γ and C_p, there is the ratio $\tau = C_p/\gamma$ which has the dimensions of time and is the thermal time constant of the unit. Practically it is the ability of the thermal system to follow input power fluctuations. A long thermal time constant is indicative of slow response or good integrating characteristics for input energies.

6. The thermal parameters can be physically described by operational types of definitions:

If, for example, adiabatic conditions are assumed ($\gamma = 0$), then the solution for Equation (1) becomes

$$\theta = \frac{\int P(t)\,dt}{C_p}, \qquad (2)$$

or the temperature rise is directly proportional to the energy input (integral of power with respect to time) and inversely proportional to the heat capacity. Another variation is to write Equation (1) as

$$\frac{d\theta}{dt} + \frac{\theta}{\tau} = P(t)/C_p, \qquad (1a)$$

and if the time constant τ is large compared to the duration of P(t) then the same integrating action takes place as shown in Equation (2). This corresponds to ballistically dumping energy into the wire bridge before cooling can enter the picture.

After some temperature θ_0 is reached and the P(t) function has become zero, cooling takes place in accordance with the differential equation

$$C_p \frac{d\theta}{dt} + \gamma\theta = 0. \tag{3}$$

The solution for this equation is

$$\theta = \theta_o e^{-t/\tau}, \tag{4}$$

where τ is the time constant which describes the cooling curve. Exponential cooling is well known and is often experimentally observed.

If a steady power level is applied to the wire bridge and temperature equilibrium is reached (i. e. $d\theta/dt = 0$) then Equation (1) describes the steady state temperature rise as

$$\theta = \frac{P(t)}{\gamma} \tag{5}$$

The value of θ can also be determined by using the known temperature coefficient of resistivity, α*, in the relationship

$$R = R_o(1+\alpha\theta). \tag{6}$$

Thus the wire bridge can be used as its own resistance thermometer.

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* α has the dimensions of ohms/ohm-degree, or reciprocal temperature.

Solving Equations (5) and (6) simultaneously, the result is

$$R = R_o\left(1 + \frac{\alpha}{\gamma} P(t)\right). \tag{6a}$$

If a curve of resistance vs. power dissipation is experimentally obtained then the slope of this curve expressed as

$$\frac{(\Delta R/R_o)}{\Delta P(t)} = \frac{\alpha}{\gamma} = \frac{\text{per unit charge in resistance}}{\text{watt of dissipation}}$$

leads directly to a determination of γ, providing α is known. The term $(\Delta R/Ro)/P\Delta(t)$ is often referred to as a power sensitivity in describing bolometers.

7. Having operational descriptions of the important thermal parameters, a series of experimental procedures can be developed for determining them. In certain cases it will be necessary to develop special test equipment. In general, the purpose of the test equipment is to provide a controllable and predictable P(t) function and to measure the temperature variation.

MEASUREMENT OF PARAMETERS

8. Several experimental techniques will be described which lead to the evaluation of EED parameters.

A. Self-Balancing Bridge Applications

9. The self-balancing bridge can be employed to determine the temperature coefficient of resistivity, α, and the heat loss factor, γ, with convenience and accuracy. A self-balancing bridge is a device which

puts audio-frequency power into a thermally sensitive arm of a Wheatstone bridge and automatically brings the bridge to balance. The bridge can operate very close to balance (i. e. 1 % or better) at all power levels.

10. Consider the circuit shown in Figure 1 in which a tuned (selective) amplifier has a positive feedback network in the form of a resistive, nonlinear bridge connected between the output and input. The bridge is nonlinear because as the amplitude of the feedback voltage increases, the resistance of the wire bridge changes and changes the degree of unbalance of the bridge. For example, consider the case when the wire bridge is cold. Its resistance is low, and the bridge is therefore unbalanced. The circuit will start oscillating due to the positive feedback, and the amplitude of oscillation will build up. With the build-up of the amplitude of oscillation, the power dissipated in the wire bridge goes up and its resistance increases in a direction to balance the bridge[1]. Balance cannot be reached since then feedback would cease, but the amplitude of oscillation seeks a stable level so that the losses in the bridge feedback network, β , just equal the amplifier gain, A. Mathematically this condition can be described as:

$$A\beta = 1 . \tag{7}$$

As the amplifier gain becomes larger, the bridge seeks a condition closer to balance. This balance condition will keep the resistance of the wire bridge constant and if the

1 - L. A. Rosenthal and J. L. Potter, "A Self-Balancing Microwave Power Measuring Bridge" Proc. of I.R.E. Vol. 39, August 1951.

ambient temperature were to increase, or if external power (i.e. microwave) were to be absorbed by the element, less self-balancing power would be required.

11. The circuit of the self-balancing bridge that has been used in the experiments discussed here is shown in Figure 2. A conventional power supply employing a Sola regulating transformer supplies 250 volts at 50 milliamperes. (A degenerative regulator could have been used for still better regulation performance.) A twin triode amplifier stage provides voltage gain. The second triode employs a tuned circuit as the plate load in order to provide some selectivity. The power output stage is a 6V6 pentode driving a conventional 5-watt output transformer. Bridge power is derived from the 3-ohm output winding. Since about 3.0 watts of undistorted power is available, this is more than sufficient to drive any conventional wire bridge EED to burn-out. The actual measured gain of the circuit shown is 450 and the balance is sufficiently good so that the error voltage contains a large third harmonic. This thermally generated harmonic cannot be balanced by the bridge but appears as a residual, limiting the degree of balance that can practically be achieved. The operating frequency has been 1500 cps but could be set at any convenient point since the tuning inductor is adjustable. A General Radio 0-10K decade resistor provides the reference resistance, R, and since the actual resistance value of the wire bridge is close to (within 1%) R/1000, good resolution can be obtained. For example, in a typical cold resistance determination a break-in point for R was 1138 ohms and a drop-out point was 1134 ohms indicating that 0.004 ohms is the degree of resolution.

12. As previously indicated a useful application of the self-balancing bridge is for the determination of power sensitivity curves. These curves lead directly to the determination of γ . A power sensitivity curve is a relation between the resistance and the power dissipated in the wire bridge. The self-balancing bridge is made to operate the wire at higher and higher resistance values while the power dissipated is computed as V^2/R. Four such curves are shown in Figure 3 for two loaded and two unloaded Squib Mk 1 wire bridge units. In all cases the resistance is initially linearly related to power dissipation. The slopes can be computed as $\Delta R/R_o$ per unit power. The following slopes were observed:

	Squib Mk 1 Wire Bridge Unit	R/R_o per milliwatt
Inert Loaded	No. 92	0.00137
	No. 94	0.00159
Unloaded	No. 70	0.00398
	No. 78	0.00435

As expected no loading, and hence no heat loss due to conduction, yields more sensitive wire bridges, i.e. the per unit resistance change is greater per milliwatt of power dissipation for the unloaded units. If the power sensitivity slope is divided into the temperature coefficient of resistivity ($\alpha = 870 \times 10^{-6}/^{o}C$) then the quotient is γ, the heat loss factor.

13. For the bridge of Figure 1, the applied voltage (e_{in}) is across the 1-ohm comparison resistor and the wire bridge (R_B). The upper two arms are 1000 times as large to conserve self-balancing power and provide good resolution in the resistance determinations. For the numerical values shown

$$e_{out} = e_{in} \left[\frac{1}{1 + R_B} - \frac{1}{\frac{R}{1000} + 1} \right] . \tag{8}$$

If $R_B + \Delta R_B = R/1000$ then

$R_B = (R/1000) - \Delta R_B$,

where ΔR_B is the amount of resistance necessary in the R_B arm to exactly balance the bridge ($e_{out} = 0$) and R_B is the actual operating resistance. Equation (8) can be rewritten as

$$\frac{e_{out}}{e_{in}} = \beta = \left[\frac{1}{1 + \frac{R}{1000} - \Delta R_B} - \frac{1}{1 + \frac{R}{1000}} \right] . \tag{8a}$$

Factoring out the $(1 + R/1000)$ term and expanding the first term by the binominal expansion, but using only the first two terms of the expansion, since ΔR_B is small

$$\beta = \frac{\Delta R_B}{\left(1 + \frac{R}{1000}\right)^2} . \tag{8b}$$

Thus the actual operating resistance can be determined if R is established and the amplification $A (= 1/\beta)$ is known. A typical value of R is 1200 ohms and if A = 400, then

$$\Delta R_B = \frac{1}{400} \times (2.2)^2 = 0.0121 \text{ ohms.}$$

The wire bridge is operating at 0.0121 ohms below the exact balance value of 1.2 ohms or 1% low. For nominal 1-ohm wire bridges it turns out that a gain of 400 will result in the bridge operating at 99% of the balance value (R/1000). For all practical purposes, the bridge is balanced. This resistance will be kept constant as long as the self-balancing amplifier is not over-driven. Using average values for the power sensitivity, the following values for γ are computed:

Type of Squib Mk 1 Bridge Wire	γ (milliwatts/$^{\circ}$C)
Loaded	0.59
Unloaded	0.209

Loading has increased γ by a factor of 2.8.

14. Another interesting observation is that all the wire bridge units, loaded or unloaded, deviate from a straight line power sensitivity relationship when the resistance has increased by 15%. This corresponds to 173°C rise above ambient. As temperature increases, γ will also increase (although assumed constant in the analysis) but not significantly enough to disturb the analyses to date. These self-balancing bridge data certainly are directly related to the intimacy of thermal contact between the wire proper and the explosive mixture. The cold resistance data are also of value. It is to be noted that these tests can be run safely with explosively loaded EED's since the temperature rise can be kept at a safe level.

15. The self-balancing bridge can also be used to determine α , the temperature coefficient of resistivity. The resistance was measured by a self-balancing bridge since, by using a self-balancing bridge and observing the resistance at which the circuit breaks into self oscillation, it is possible to measure wire bridge resistance with no power dissipation in the wire. Any conventional Wheatstone bridge will dissipate significant powers into the wire bridge; especially at low resistance values. With a self-balancing bridge resistance measurements to better than three significant figures can be made at power levels of less than 5 milliwatts. Four wire bridges, of which three were inert loaded, were placed in a temperature controlled box and their resistances were measured at various temperatures. The observed data are plotted in Figure 4 for the temperature range of 25°C to 100°C. The curves are linear and of virtually the same slope with or without loading. An average value for all four samples was $870 \times 10^{-6}/^{\circ}C$. Since this value seemed unexpectedly low, a fine-wire manufacturer was contacted[2] who furnished the data plotted in Figure 5. These data show that for a platinum-iridium wire as used in the Squib Mk 1, α is very sensitive to small percentages of iridium. For 15% iridium the temperature coefficient is $900 \times 10^{-6}/^{\circ}C$, indicating that a measurement of $870 \times 10^{-6}/^{\circ}C$ was correct for the typical Squib Mk 1 wire bridge.

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2 - Telephone conversation with Mr. Richard Cohn of Sigmund Cohn Corporation, Mount Vernon, New York.

B. Constant Current Firing Measurements

16. The self-balancing bridge has yielded the measurement of γ and α. By employing a constant current burst it is possible to determine C_p, the heat capacity. The real advantage in using constant current is that the P(t) function is then well defined mathematically. For example, it is possible to take any known short burst of energy ($\int$ P(t) dt) and then by use of Equation (2) and the maximum temperature reached, establish the value of C_p. However, determining $\int$ P(t) dt can be difficult except in simple cases, such as constant current firing. The equations will indicate that constant current firing results in a simple analytic interpretation.

17. The details of a system for delivering a constant current pulse or burst of energy into an EED under study will not be discussed. At present there is available at NOL two generator designs. A generator should be capable of delivering currents up to several amperes for adjustable time durations. Although more than sufficient energy should be available to fire the EED, the measurements described can be performed at a nondestructive level. The traces observed, for the voltage drop across the EED, can be correlated directly to the heat capacity, C_p, for the device as indicated below.

18. The sketch shown in Figure 6(a) is typical of observed traces. Initially a constant current pulse (e.g. 4 amperes) is passed into a 1-ohm resistor for a period of time (300 microseconds in the present experiments). The observed trace is rectangular in shape following the path O - A - B - C. After this calibrating trace is obtained, the standardizing resistor is replaced by a

wire bridge Squib Mk 1. When the system is fired again, the observed trace climbs linearly from O to D. In this procedure, the linear rise is attributed to the resistance increase with temperature as the wire bridge ballistically integrates the incoming energy. At the point D the wire burns itself out and opens the circuit.

19. If a constant current I is passed into the wire bridge then the equation describing the temperature rise is

$$C_p \frac{d\theta}{dt} + \gamma\theta = I^2 R (1 + \alpha\theta). \tag{9}$$

Here it is assumed that the wire has a linear temperature coefficient of resistivity. Collecting the θ terms on the left side of the equation gives

$$C_p \frac{d\theta}{dt} + \theta(\gamma - I^2 R \alpha) = I^2 R . \tag{9a}$$

The apparent heat loss factor is now

$$\gamma' = \gamma - I^2 R \alpha, \tag{10}$$

and the apparent time constant is

$$\tau' = \frac{C_p}{\gamma - I^2 R \alpha} . \tag{10a}$$

Solving Equation (9a) for the temperature rise yields

$$\theta = \left(\frac{I^2 R}{\gamma - I^2 R \alpha}\right)\left(1 - e^{-t/\tau'}\right), \tag{11}$$

and the steady state temperature (if ever reached before burn-out) is

$$\Theta_{max} = \frac{I^2 R}{\gamma - I^2 R \alpha} . \tag{12}$$

If $\alpha = 900 \times 10^{-6}/^{o}C$, I = 4 amperes, and R = 1 ohm, then the term $I^2R\alpha = 14.4$ milliwatts/^{o}C. Its importance depends on the size of γ , but it is conceivable that if I is large enough the apparent time constant can be extremely small yielding a sensitive wire bridge. If γ' were to go to zero it would mean that the increased energy input due to the wire bridge heating up is cancelling the heat lost due to the wire cooling. Under these conditions the wire temperature increases linearly with time in accordance with

$$\theta = \frac{I^2R}{C_p} \ t.$$

If t (the time over which the observation is made) is much smaller than τ' then by expanding Equation (11)

$$\Theta = \left(\frac{I^2R}{\gamma - I^2R\alpha}\right)\left(\frac{t}{\tau}\right) = \frac{I^2Rt}{C_p} . \tag{13}$$

22. Voltage traces observed experimentally with constant current pulses have been found to be linear, indicative of the temperature rise being linear. The voltage then should follow the relationship:

$$V = IR\left(1 + \frac{\alpha I^2 Rt}{C_p}\right) \tag{14}$$

or

$$\frac{dV}{dt} = \frac{IR(I^2R)\alpha}{C_p} \tag{14a}$$

The following was observed for two different loaded Squib Mk 1 wire bridges (R = 1 ohm).

dV/dt (volts/sec.)	I (amperes)	C_p computed (microwatt-sec./^oC)
$0.02 \times 10^{+6}$	4	2.88
$0.0027 \times 10^{+6}$	2	2.68

Considering the inaccuracies in reading the oscillographs, this is a good check between two different loaded EEDs. Assuming a round figure of C_p = 2.7 microwatt-sec./^oC then if the time constant of a loaded unit is 4000 microseconds[3], a computed value of γ is 0.68 milliwatts/^oC. This value compares favorably with the 0.59 milliwatts/^oC value for γ previously determined by power sensitivity curves.

21. The following thermal constants can be considered typical for the loaded Squib Mk 1:

C_p = 2.7 x 10^{-6} watt-seconds/^oC

γ = 0.68 milliwatts/^oC

τ = 4000 microseconds.

22. If the apparent heat loss factor is reexamined as

$$\gamma' = \gamma - I^2R\alpha.$$

then for a 4-ampere pulse (R = 1 ohm nominal)

$$\gamma' = 0.68 \times 10^{-3} - 14.4 \times 10^{-3}$$
$$= -13.72 \text{ milliwatts}/^oC.$$

3 - Based on direct thermal time constant measurements.

The meaning of a negative heat loss factor (and time constant) is unique to constant current firing. It is due to an extremely rapid heat build-up which far outweighs the heat loss. It does not show up in the constant-current firing trace since the heat loss factor does not appear in the initial temperature rise characteristic (Equation (14a)). A negative value of γ' would result in a trace as shown in Figure 6(b) if burnout did not occur and the temperature rise would follow the equation

$$\theta = \frac{I^2 R}{\gamma'}\left(e^{\frac{+\gamma' t}{c_p}} - 1\right) \tag{15}$$

(The negative sign of γ' is absorbed in the equation.) The resistance change follows the temperature rise and appears as a voltage drop in Figure 6(b). As a point of interest, if γ' were to be made zero then a constant current of about

$$I = \sqrt{\frac{\gamma}{\alpha R}} = \sqrt{\frac{0.68 \times 10^{-3}}{900 \times 10^{-6}}} = 0.87 \text{ amperes}$$

would yield the most straight firing trace. This can be illustrated as in Figure 6(c), where the voltage drop is normalized and plotted as a function of time. All curves branch out from the initial straight section. Burnout is generally experienced in the region A, so that the effects of a modified heat loss factor, γ', are generally not experienced.

23. Constant current firing not only provides information for the determination of C_p but also verifies the basic heat equations. By using short enough current pulses, firing can be avoided and the test performed in a non-destructive manner. For example, if a change in resistance of 20% can be resolved on an oscilloscope then the temperature rise would be

$$R = R_o(1 + \alpha\Theta),$$

$$\Theta = \left(\frac{R}{R_o} - 1\right)/\alpha = \frac{0.2}{0.9} \times 10^3 = 220^\circ C.$$

This might be dangerously close to the ignition level of the explosive. Reducing the desired change in resistance to 10% would correspond to a temperature of 110°C.

C. Direct Thermal Time Constant Measurements

24. Procedures for determining γ and C_p have been outlined. Actually the ratio C_p/γ is the thermal time constant. It is also possible to determine τ directly. A novel method based on the harmonics generated when a sinusoidal current is passing through an EED has been developed in NavOrd Report 6691*. Briefly, if a sinusoidal current is passing through the wire the heating takes place at the second harmonic of the power frequency. This results in a resistance modulation of the wire bridge at a second harmonic frequency depending on how well the wire bridge can follow the cyclic power variations. If the resistance, with its dynamic variation, is multiplied by the sinusoidal current, then a third harmonic voltage appears across the EED. This third harmonic is indicative of the degree of follow or thermal time constant of the unit.

* NavOrd Report 6691, The Harmonic Generation Technique for the Determination of Thermal Characteristics of Wire Bridges Used in Electro-Explosive Devices, by Louis A. Rosenthal

By examining the variation of this harmonic generated with frequency of the applied current, a drop in the level by $1/\sqrt{2}$ corresponds to

$$\tau = \frac{1}{2\omega_0}, \tag{16}$$

where ω_0 is the $1/\sqrt{2}$ frequency.

25. Since the derivations for this technique are lengthy, another report has been devoted to this measurement. A less accurate procedure based on cooling curve observations is, however, presented here. Figure 7(a) shows a circuit diagram of a test set-up that has been employed to measure this time constant. The wire bridge of a Squib Mk 1 of approximately 1-ohm resistance was made to pass a current of 250 milliamperes continuously. The voltage drop across the element was monitored on a Tektronix Oscilloscope Model 535. As shown, the system is stable and the current is insufficient to cause any aging in the wire bridge. A capacitor, C, (1-microfarad) when charged to a voltage of 45 volts is then discharged into the bridge. This energy of approximately 10,000 ergs is discharged ballistically into the bridge which heats rapidly and then cools in an exponential manner. The temperature drop, measured as a linear function of the voltage drop across the wire bridge carrying the constant, small current, is shown in Figure 7(b). (This method for determining the time constant is based on the assumption of a linear increase of bridge wire resistance with increase of its temperature.) By noting the time required for the voltage V_0 to drop to $1/e$ (approximately 38%) of its initial value, the time constant can be determined. The following values were observed on Squib Mk 1 bridge wires:

Bridge Wire	Time Constant (microseconds)
Unloaded	10,000
Loaded with Plaster of Paris & Graphite	6,000

Loading the squib resulted in a shorter time constant, since the heat loss factor, γ , was probably increased. The voltage change across the bridge was an increase of approximately 50 millivolts above the initial level of 250 millivolts. Thus the effective resistance increased by 20% because of the energy burst.

26. This time constant would nominally be considered large, and hence the Squib Mk 1 is an efficient energy integrator. For example, radar pulses at a repetition rate of 4000 per second would have only 250 microseconds between pulses. These pulses would be integrated efficiently since much longer times are needed for the bridge to cool from an elevated temperature down to near ambient temperature. The first pulse would raise the temperature as would the second, third, and so on. Twenty-four pulses would be integrated before significant cooling came into the picture.

27. This technique has certain limitations. The steady currents through the wire bridge during the test should be limited to a safe level at the sacrifice of sensitivity. The values given for the test set-up employed here are too high to perform the test on loaded EED's. In addition, it is difficult to accurately resolve the size of the first initial rise to θ_0. The capacitor discharge pulse is superimposed on the steep rise so that the maximum displacement is somewhat ambiguous. In addition, certain errors are possible when the 38% ($1/e$)

point is to be located. However, the test is of value and the time constant can surely be measured to about ± 10%. An oscilloscopic photograph from which the critical points can be measured is certainly the best procedure to follow. When the thermal time constants of eight Squib Mk 1 wire bridges were measured using the harmonic generation technique discussed in NavOrd Report 6691 the following values for the time constant were obtained:

Unloaded	7,700 microseconds
Loaded (Inert)	4,030 microseconds.

It is not correct to compare these values with the single measurement values of 10,000 and 6,000 microseconds obtained by the cooling curve technique, since in any group of EED's it has been observed that there is a spread in the data. It is noted that loading reduces the time constant by a factor of 47% (40% from cooling curve data).

28. Assuming that the time constants are 7,700 and 4,030 microseconds for unloaded and loaded Squib Mk 1 wire bridges, respectively, it is interesting to check with the time constant that would be computed from the determination of γ and C_p for the Squib Mk 1 wire bridge that was discussed earlier. From constant current firing C_p for loaded units was determined to be about 2.7 microwatt-sec./$^\circ$C. The value of γ for loaded units was determined as 0.59 milliwatts/$^\circ$C. A computed time constant is

$$\tau = \frac{C_p}{\gamma} = \frac{2.7}{0.59} \times 10^{-3} \text{ sec.} = 4580 \text{ microseconds}$$

compared to 4030 microseconds. This is a good check considering the limited number of samples tested, the fact that a complete set of measurements was not made on individual units, and the assumption that the plaster of paris loading is equivalent to explosive loading. A tabulation of observed parameters is presented below for a Squib Mk 1.

	C_p (microwatt-sec./$^\circ$C)	γ (milliwatts/$^\circ$C)	τ computed (micro-seconds)	τ experimental (microseconds)
Loaded	2.7	0.59	4580	4030
Unloaded	*	0.209		7700

* not experimentally evaluated. Computed as 1.6.

29. It appears that loading increases the γ by a factor of 2.82 but the time constant has decreased only by a factor of 0.52. This would indicate that C_p has also increased by a factor of 1.47 due to loading. Thus, not only has the proximity of the explosive mixture increased the cooling rate but it has increased the effective, lumped heat capacity.

30. The data presented are sketchy but are given as an example of what can be determined and the accuracy that might be expected in the measurements. The time parameters could be computed only after many measurements are made on identical units. Trends are definitely established and the change in thermal parameters between the unloaded and loaded states is a measure of the intimacy of thermal contact.

31. All three measurements of γ, C_p, and τ could be made with relatively simple equipment and there is a cross check of the results by comparison of τ determined by two independent techniques. Once the parameters are established for an EED then they can be used directly in the basic electro-thermal equations to predict the maximum temperature that will be reached, and at what time this occurs. This will be illustrated in the following section.

SOLUTIONS OF THE ELECTRO THERMAL EQUATIONS

32. It is apparent that the solution of Equation (1) can describe the mechanism of heating for a wire bridge in an EED. If the temperature at which the explosive is triggered is known then the firing point, if it exists, can also be determined. For example, in the discussions of constant current firing the equations were solved to yield a procedure for measuring C_p. It was also shown that the temperature rise becomes limited under certain circumstances to

$$\theta_{max} = \frac{I^2 R}{\gamma - I^2 R \alpha}, \tag{12}$$

where R is the cold resistance of the wire bridge. Two other popular cases will be considered; capacitor discharge firing and constant voltage firing.

A. Capacitor Discharge Firing

33. Starting again with the general thermal equation for a wire bridge

$$C_p \frac{d\theta}{dt} + \gamma\theta = P(t) \tag{1}$$

the power function P(t) describes the discharge or firing power as a function of time. For example, a discharging capacitor of capacitance C has an initial energy $E = Q^2/2C$ and the power function is

$$P(t) = \frac{dE}{dt} = \frac{1}{C}\frac{dq}{dt} \cdot q \tag{17}$$

where q is the instantaneous charge on the capacitor. Since $i = \frac{dq}{dt}$, where i is the discharging current

$$P(t) = \frac{iq}{C} = iv , \tag{18}$$

where v is the instantaneous voltage.

It is necessary to know the relationship between the current and charge (or voltage) before P(t) can be evaluated. Consider the case where the resistance, into which the capacitor discharges, is a function of temperature. Then the discharge equation for the capacitor is

$$\frac{q}{C} + iR(1 + \alpha\Theta) = 0. \tag{19}$$

If Θ is eliminated between Equations (1) and (19), the following equation results:

$$\left(\frac{dq}{dt}\right)^2 (a+1) + \frac{q}{RC}\cdot\frac{dq}{dt} - aq\frac{d^2q}{dt^2} - \frac{\alpha q}{\gamma C}\left(\frac{dq}{dt}\right)^3 = 0. \tag{20}$$

In this equation, $a = C_p/\gamma RC$, a ratio of two time constants as previously defined. The third degree, second order nonlinear differential equation appears to require a formidable solution by ordinary procedures. Once q and $i = \frac{dq}{dt}$ are made available, Equation (19) can be re-employed to determine the actual temperature rise, Θ . The expected solution to Equation (20) is a modified discharge relationship. A capacitor having an initial charge Q when discharging into a linear resistor (i.e. R) has an instantaneous charge given by

$$q = Qe^{-\frac{t}{RC}}. \tag{21}$$

If the resistance of the discharge resistor increases with time, then the decay curve will be drawn out or the discharge will take place at a slower rate with time. It is expected that the increase in resistance will be less than 100% and this will not severely distort the basic discharge curve. The most significant distortion will result if γ is assumed equal to zero in the general thermal equation, Equation (1), in which case the wire bridge will just heat up without cooling. This case can be solved. The thermal equation is

$$C_p \frac{d\theta}{dt} = P(t) \tag{22}$$

and

$$\theta = \int_0 \frac{P(t)}{C_p} dt = \frac{Q^2 - q^2}{2C_pC} \tag{22a}$$

In addition

$$R(1+\alpha\theta)\frac{dq}{dt} + \frac{q}{C} = 0, \tag{19}$$

and if θ is eliminated the result is

$$\int_Q^q \left[\frac{1+\delta(Q^2-q^2)}{q}\right] dq = -\int_0^t \frac{dt}{RC}, \text{ where } \delta = \frac{\alpha}{2C_pC}.$$

Integrating, there results

$$\ln\frac{q}{Q} + \delta Q^2 \ln\frac{q}{Q} - \frac{\delta}{2}(q^2 - Q^2) = -\frac{t}{RC}.$$

Rearranging terms the result is

$$q = Q\left[e^{\frac{-\frac{t}{RC} + \frac{\delta}{2}(Q^2 - q^2)}{1+\delta Q^2}} \right]. \tag{23}$$

This equation, which describes the charge decay for a capacitor into a resistor with a positive temperature coefficient (and no cooling), should be compared to Equation (21). If α (or δ) equals zero, then Equations (23) and (21) are identical. Once q (as a function of t) is determined, it is possible to return to Equation (22a) and determine the temperature rise $\theta(t)$. The charge q is implicitly related to time in Equation (23) since the right side unfortunately contains q(t).

34. The final resistance of the wire bridge is $(1 + \delta Q^2)$ times its cold resistance and, since the quantity δQ^2 can be computed and interpreted as a resistance change, it is known that the capacitor discharge must fall between two idealized exponential decays. As an example, consider the case of a 4-mfd. capacitor, charged to 40 volts discharging into a Squib Mk 1 wire bridge with an initial resistance of 1 ohm. Taking $Q = 160 \times 10^{-6}$ coulombs, $\alpha = 900 \times 10^{-6}/^{\circ}C$ and $C_p = 2.61$ microwatt-sec./°C, $\delta Q^2 = 1.1$. Thus the wire bridge with an initial resistance of 1 ohm reaches a final resistance of 2.1 ohms (an increase of 110%). Figure 8 is a calculated plot of several discharges. Curve A is for a 1-ohm resistance, Curve C is for a 2.1-ohm resistance, and Curve B is the actual case based on Equation (23). Recalling that this idealized Case B is for no heat loss, the decay is an exponential that starts out as though the discharge resistance is 1 ohm and ends up approaching the 2.1-ohm resistance curve. If γ is introduced into the equations,

then the shape of the discharge will exhibit some inflection as the resistance varies from the cold value to the hot value and then back to the cold level. However, since the Squib Mk 1 has a thermal time constant of 4000 microseconds, cooling hardly has an opportunity to enter the discharge curve relationship. For a discharge time constant of 4 microseconds Curve B of Figure 8 is probably an accurate representation and cooling would start to become effective at about 400 microseconds.

35. To work directly with Equation (23) would be feasible but difficult. Certainly it would be of no value where the capacitor discharge is at a slow rate. To investigate the general problem of capacitor discharge firing, it appears wise to assume that the variation in EED resistance is a second order effect. Based on the data presented in Figure 8, the deviation from an exponential decay appears to be small. Perhaps modifying the resistance from the cold values to say 150% of the cold value would offer more accurate results. The analysis to follow is based on a constant resistance value for the wire bridge.

36. When Equation (1) is applied to a discharging capacitor, it becomes

$$C_P \frac{d\theta}{dt} + \gamma\theta = \frac{V^2}{R} e^{-\frac{2t}{RC}} . \tag{24}$$

This equation can be integrated by the integrating factor technique to yield

$$\theta = \frac{V^2}{RC_P\left(\frac{\gamma}{C_P} - \frac{2}{RC}\right)} \left(e^{-\frac{2t}{RC}} - e^{-\frac{\gamma t}{C_P}} \right). \tag{25}$$

Note that the time constants, RC for the electrical circuit and C_p/γ for the thermal circuit, are the major constants. Another form is

$$\theta = \frac{CV^2}{2C_p\left(\frac{\gamma RC}{2C_p} - 1\right)}\left(e^{-\frac{2t}{RC}} - e^{-\frac{\gamma t}{C_p}}\right), \tag{25a}$$

where the factor $CV^2/2C_p$ is the maximum temperature reached for the case of no heat loss. The temperature rise described by Equations (25) and (25a) reaches a maximum, as determined by differentiation, at

$$t_{max} = \frac{1}{\left(\frac{2}{RC} - \frac{\gamma}{C_p}\right)} \cdot \ln\left(\frac{2}{RC} \cdot \frac{C_p}{\gamma}\right) \tag{26}$$

and this maximum temperature is

$$\theta_m = \frac{V^2C}{2C_p}\, e^{-\frac{\gamma t_{max}}{C_p}}. \tag{27}$$

Several other simplifications are possible but not warranted in this discussion.

37. It is of interest to insert into these equations the constants for the Squib Mk 1 and study the resulting plots. In all cases a unit with a wire bridge resistance of 1 ohm is considered. Figure 9 gives a plot of Equation (25) for an input energy of 32,000 ergs. The temperature rise as a function of time is plotted for two discharge time constants, but the same energy level. The 4-microsecond RC time discharge brings the wire up to 1235°C while the 400-microsecond RC time discharge brings the wire up to 1050°C and at a later time. With the longer discharge time, there is a delay in temperature build-up. In both cases cooling is essentially along the same curve.

These curves indicate significant trends. A slow discharge is less efficient than a fast discharge. Note also that the fast discharge keeps the wire at a higher temperature (above 1000°C, for example) for a longer time. The curve corresponding to the 400-microsecond RC time discharge case is replotted in Figure 10 with a linear time base to show the rapid temperature rise and the slow cooling. This curve is again based on Equation (25) using Squib Mk 1 parameters. If the 4-microsecond RC time discharge case were plotted, only the cooling curve would be apparent; the temperature rise would be too rapid to be observed. Based on these plots the following questions arose:

(a) What is the relationship between the maximum temperature location and the discharge time constant -- the lag effect?

(b) What is the maximum temperature achieved (at constant energy input) as a function of discharge time constant?

(c) To reach 700°C, as an example, how much energy is required if the capacitor size, or discharge time constant, is varied?

Figure 11 shows a plot of Equation (26) in which for a given discharge time constant, RC, the time of maximum temperature rise is determined. The thermal time constant of the Squib Mk 1 is fixed at 4000 microseconds. It is observed that the temperature peak always occurs after a time corresponding to 1 time constant (1RC). This lag in temperature rise can result in an apparent delay in firing the squib, if the temperature rise is significant. It turns out that the greater the delay or lag, the lower the maximum temperature since cooling has entered the picture.

38. Figure 12 shows the maximum temperature reached as a function of discharge time constant for a given fixed energy input delivered to a Squib Mk 1 wire bridge. A basic energy of 32,000 ergs is considered. That energy can be delivered rapidly using a capacitor of small capacitance or slowly using a capacitor of large capacitance. As shown for a discharge with a 10-microsecond discharge time constant the maximum temperature is 1225°C, and it progressively falls off as the discharge time is increased. Thus supplying energy into the wire bridge ballistically is the most efficient procedure for firing. Slow discharges result in low maximum temperatures. In each case, the typical temperature rise vs. time curve of Figure 10 results.

39. Extending these concepts, it is of interest to reverse the procedure and compute the amount of energy required to bring a Squib Mk 1 wire bridge to 700°C as a function of discharge time constant. The temperature of 700°C was chosen because constant current firing traces indicated that the Squib Mk 1 would fire at that temperature. This curve might be experimentally determined using capacitor discharge testing procedures. Figure 13 shows the energy required to reach 700°C in a Squib Mk 1 wire bridge as a function of capacitor size. It is apparent that there is a rapid increase in required energy as large discharge time constants, indicated by large capacitor sizes, are approached. This is observed experimentally to be a correct variation.

40. By combining capacitor discharge equations with thermal equations for a wire bridge, it is possible to analytically describe the heating of the wire bridge. Insight as to the importance of certain parameters is obtained. Although several simplifications have been made, the trends described are essentially true. Certainly

the thermal parameters such as C_p and γ and their variations among wire bridges contribute to the spread observed in firing sensitivities.

B. Constant Voltage Firing

44. Constant voltage firing of an EED corresponds to the case when a battery of low internal impedance is placed across the EED wire bridge. The equations describing the temperature build-up will be derived. Again the heat equation

$$C_p \frac{d\theta}{dt} + \gamma\theta = P(t) \tag{1}$$

is the starting point with the power input function simply

$$P(t) = \frac{V^2}{R(1+\alpha\theta)},$$

where R is the cold resistance and all other parameters have been previously described. The battery voltage is V. If $V^2/R = P_o$, the initial power level in the EED, then the heat equation can be rewritten as

$$\frac{d\theta}{dt} = \frac{P_o}{C_p(1+\alpha\theta)} - \frac{\gamma\theta}{C_p},$$

and the variables have been separated so that integration is possible. The resulting form is

$$\int_0^\theta \left(\frac{1}{\frac{P_o}{C_p(1+\alpha\theta)} - \frac{\gamma\theta}{C_p}} \right) d\theta = t \tag{28}$$

If the thermal time constant is extracted ($\tau = \frac{C_p}{\gamma}$) then

$$\int_0^\theta \left(\frac{1}{\frac{P_o}{\gamma(1+\alpha\theta)} - \theta} \right) d\theta = \frac{t}{\tau} . \tag{28a}$$

The term P_o/γ is the final steady state temperature that would be achieved if α were equal to zero and can be represented as

$$\theta_o = \frac{P_o}{\gamma} . \tag{29}$$

Substituting for Po into Equation (28a), the heat equation is reduced to

$$\int_0^{\theta} \left(\frac{1}{\theta - \frac{\theta_o}{1+\alpha\theta}} \right) d\theta = \frac{t}{\tau} , \tag{28b}$$

a form ready for integration.

42. Certain aspects of constant voltage firing are worthy of noting. As the EED heats up and its resistance increases, the power dissipation decreases. Assuming no failure, the final temperature of the wire(θm) occurs when $d\theta/dt = 0$ or

$$\theta_m = \frac{\theta_o}{1 + \alpha \theta_m} , \tag{30}$$

Only if $\alpha = 0$, can θm approach θ_o. (If α is negative then temperatures greater than θm can be reached.) The value of θm is

$$\theta_m = -\frac{1}{2\alpha} \pm \sqrt{\left(\frac{1}{2\alpha}\right)^2 + \frac{\theta_o}{\alpha}} \tag{31}$$

and the + sign in front of the radical must be used (physically significant). Further simplification yields

$$\theta_m = \frac{1}{2\alpha}\left(\sqrt{1 + 4\theta_o\alpha} - 1\right), \tag{31a}$$

43. Another interesting variation of the describing equations results when the heat loss term γ is considered negligible. The basic power equation is then

$$\frac{d\theta}{dt} = \frac{P_o}{C_p(1+\alpha\theta)}, \tag{32}$$

which has the solution

$$\theta + \frac{\alpha\theta^2}{2} = \frac{P_o}{C_p}t. \tag{32a}$$

This is a simple parabola as shown in Figure 14(a) which monatonically increases depending on the value of α if the value of P_o/C_p, a scaling factor, is kept constant. With cooling it is expected that the curves indicated will bend over at a faster rate. If the complete equation is solved, assuming no simplifications, then the result is

$$\ln\left(\frac{\theta_o-\theta-\alpha\theta^2}{\theta_o}\right)+\frac{1}{\sqrt{4\alpha\theta_o+1}}\ln\left\{\frac{\dfrac{2\alpha\theta}{1-\sqrt{4\alpha\theta_o+1}}+1}{\dfrac{2\alpha\theta}{1+\sqrt{4\alpha\theta_o+1}}+1}\right\}=-\frac{2t}{T} \tag{33}$$

This equation is awkward to work with and is practically useless unless certain simplifications are valid. For example if α is small then the second term is close to zero and the equation simplifies to

$$\theta_o\left(1-e^{-\frac{2t}{T}}\right)=\theta+\alpha\theta^2. \tag{34}$$

If the time constant T is large (i.e. $t<T$) then the rise characteristics follow the equation

$$\frac{2\theta_o t}{T} = \theta+\alpha\theta^2. \tag{34a}$$

The important constants in this equation are α and θ_o. The latter value is determined by the initial size of the power pulse and γ, and can be thought of as an asymptotic temperature (see Equations (29) and (30)).

44. Another approach to the solution is to assume that

$$\frac{1}{1+\alpha\theta} \simeq 1-\alpha\theta.$$

Then the basic heat equation can be written as

$$C_p\frac{d\theta}{dt}+\left(\gamma+\alpha\frac{V^2}{R}\right)\theta=\frac{V^2}{R}, \tag{35}$$

and it appears as though the heat loss factor has been modified to account for the change in absorbed power with temperature rise. The resulting solution for this quasi-linearization is

$$\theta=\frac{V^2}{R\left(\gamma+\frac{\alpha V^2}{R}\right)}\left(1-e^{-\left(\gamma+\frac{\alpha V^2}{R}\right)\frac{t}{C_p}}\right). \tag{36}$$

The limiting temperature in this case occurs when $t \rightarrow \infty$ yielding

$$\theta_m=\frac{\frac{V^2}{R}}{\gamma+\frac{\alpha V^2}{R}}=\frac{\theta_o}{1+\alpha\gamma\theta_o} \tag{37}$$

The value of θm is, of course, only approximate compared to the true value previously derived, Equation (30), but if α is small, or better, if $\alpha\theta_o$ is small the two answers coincide. Several sketches based on different values of $\gamma+\frac{\alpha V^2}{R}$ are shown in Figure 14 (b).

47. It appears that for constant voltage firing a small α is desirable. Also, a large Po will yield the highest temperature in the shortest possible time. If the determined parameters are inserted in the equations developed, certain magnitudes can be established and the temperature vs. time relationship can be accurately established.

CONCLUSIONS

48. The basic electro-thermal equations for an EED have been introduced. Techniques for experimentally determining the thermal parameters have been developed. Having the parameters and the equations it is now possible to describe the temperature-time characteristic for the device. It follows that if a critical temperature is necessary for an explosion, the case may exist where that temperature cannot be achieved. If it can be obtained, then there will be some finite time lag. Having an analytic description of this mechanism will add to the basic understanding and design of electro-explosive devices.

ACKNOWLEDGEMENT

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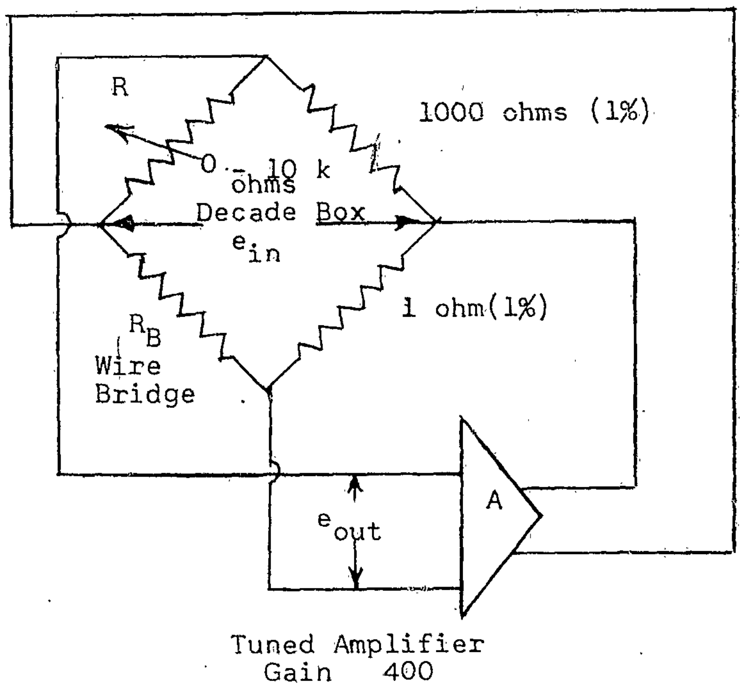


Figure 1. Self-Balancing Bridge — Block Diagram

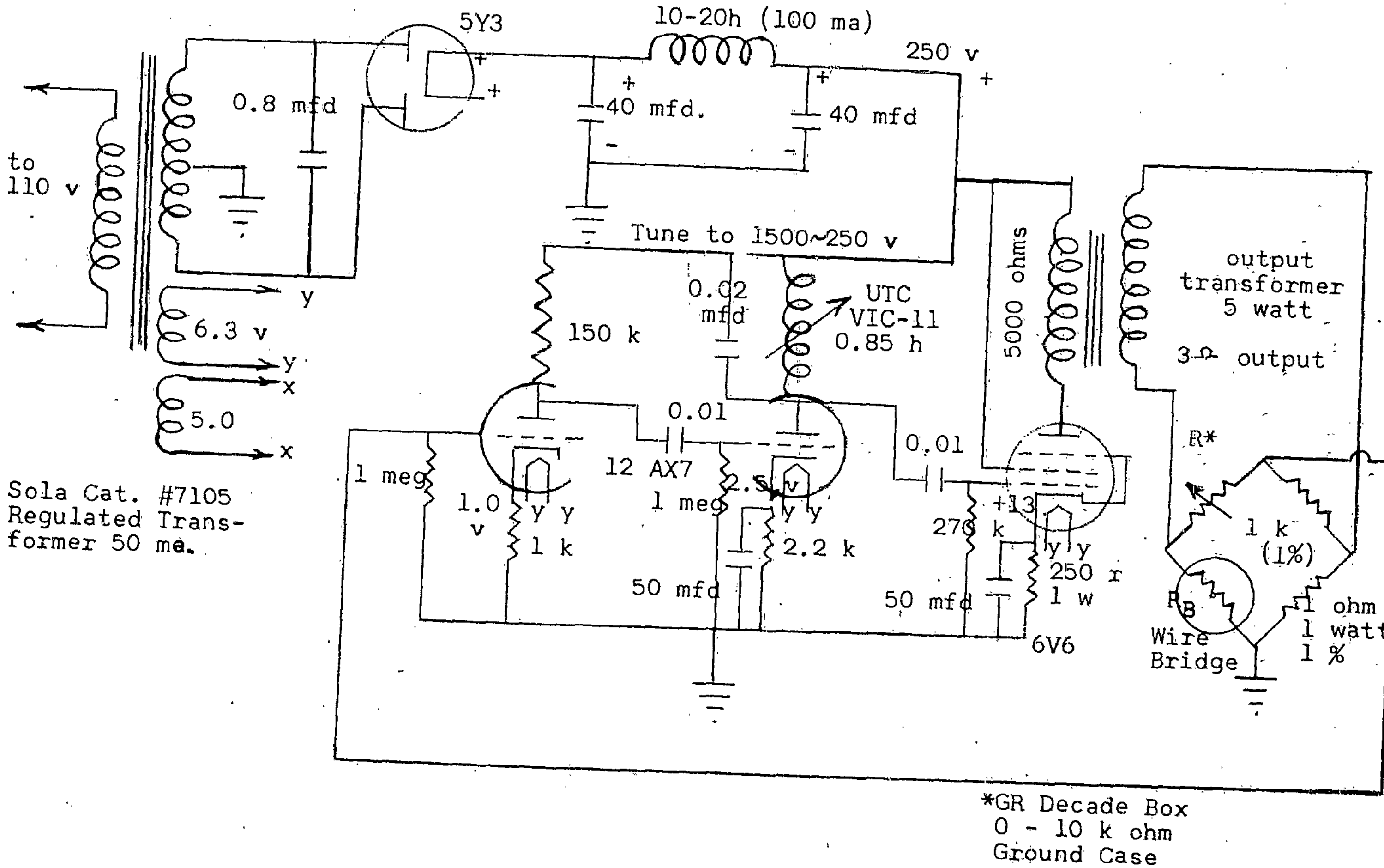


Figure 2. Circuit of Self-Balancing Bridge for Wire Bridge Studies

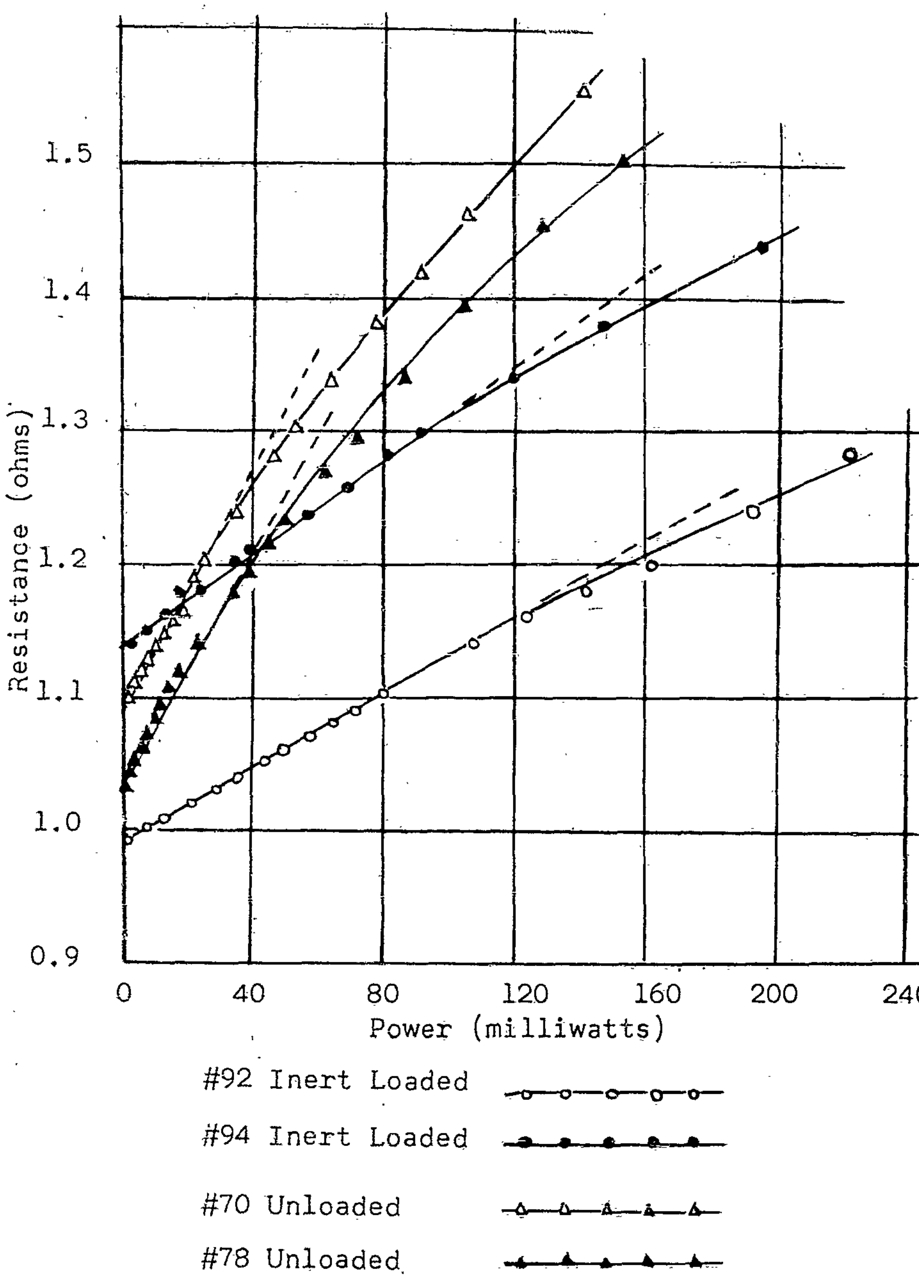


Figure 3. Power Sensitivity Curves for Squib Mk 1 Bridge Wire Units

Temperature Coefficients of Resistivity

Unit	α (ohms/ohm-$^{\circ}$C)
94	853×10^{-6}
93	835×10^{-6}
90	895×10^{-6}
83*	895×10^{-6}
Ave.	870×10^{-6}

* Only unloaded unit, all others loaded.

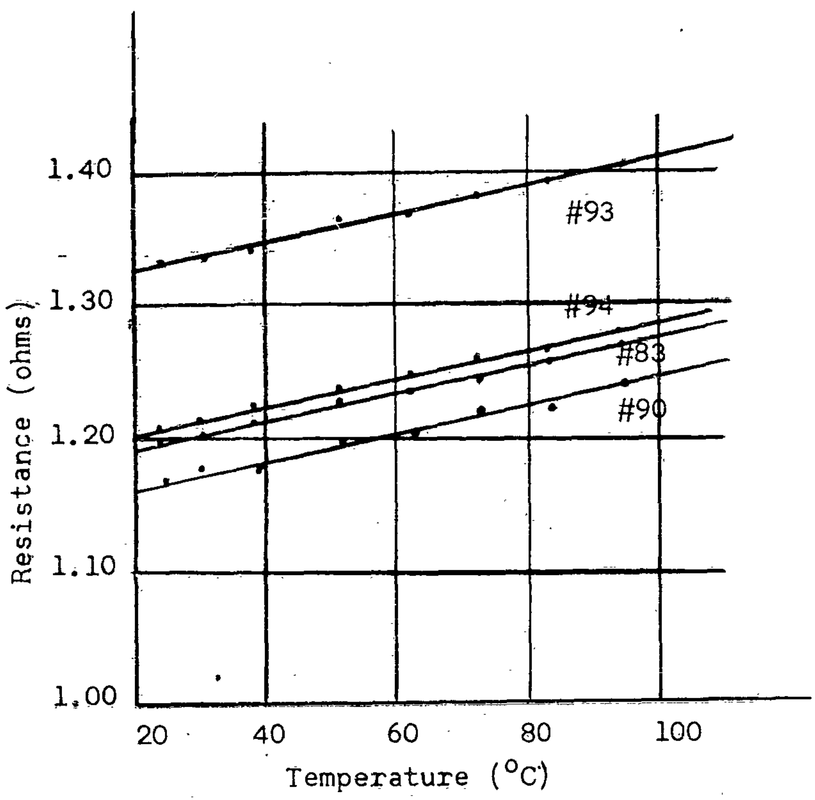
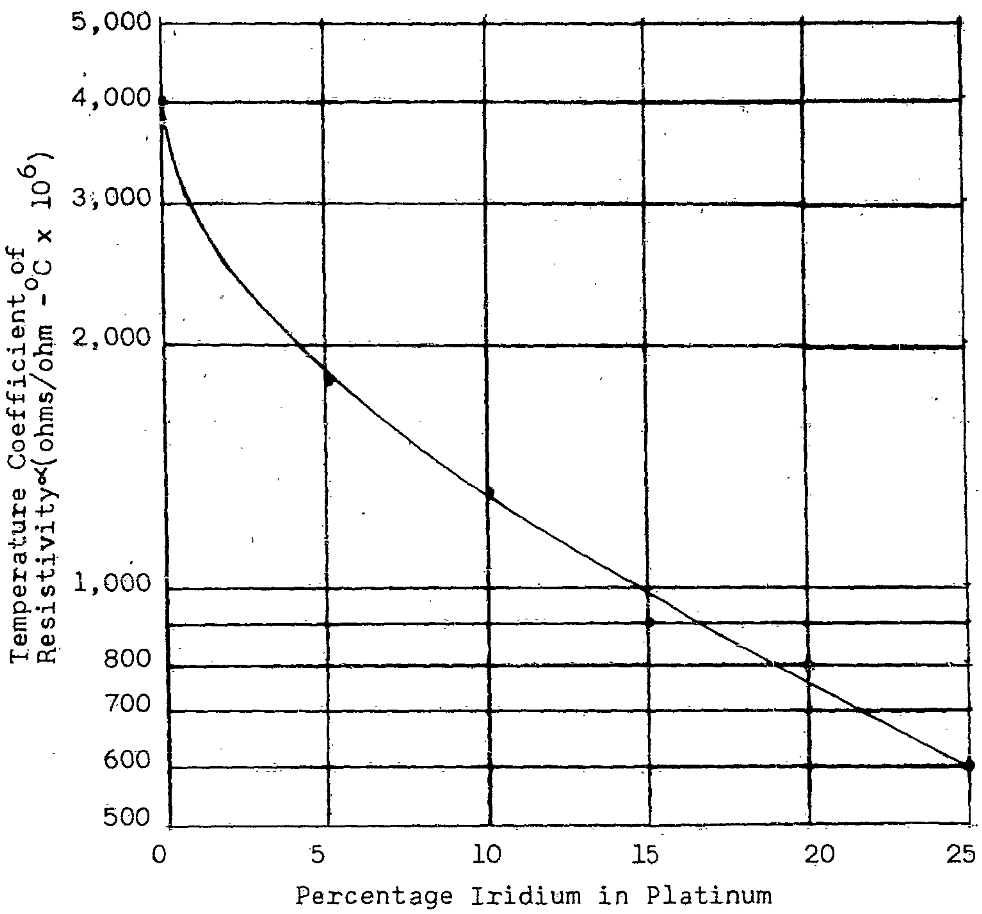
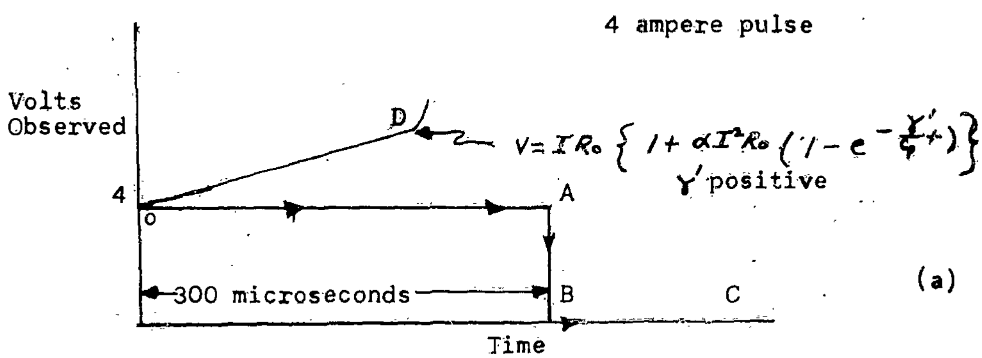


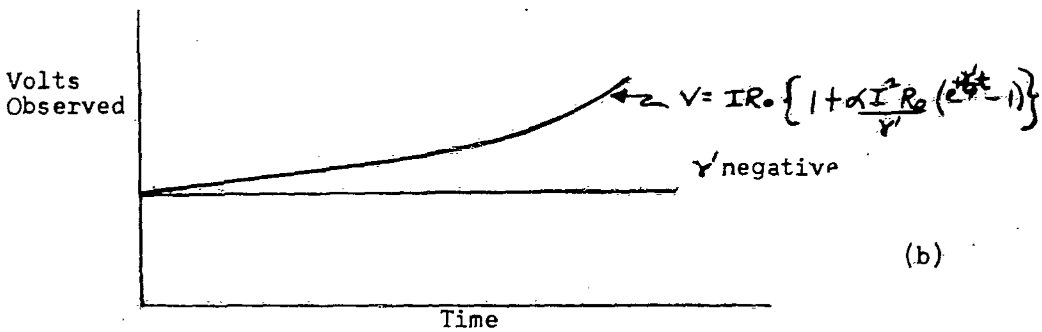
Figure 4. Resistance - Temperature Characteristics for Squibs Mk 1



Based on data from
E. M. Wise "Platinum Metals"
Publication of International
Nickel Co.

Figure 5. Temperature Coefficient of Resistivity vs. Percentage of Iridium in Platinum





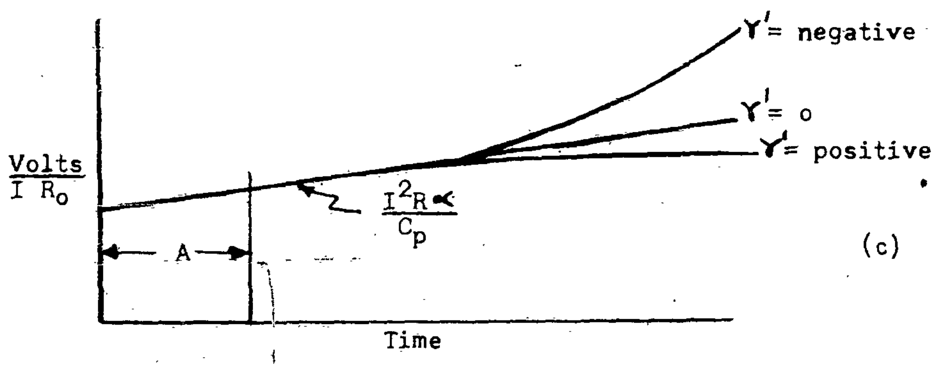
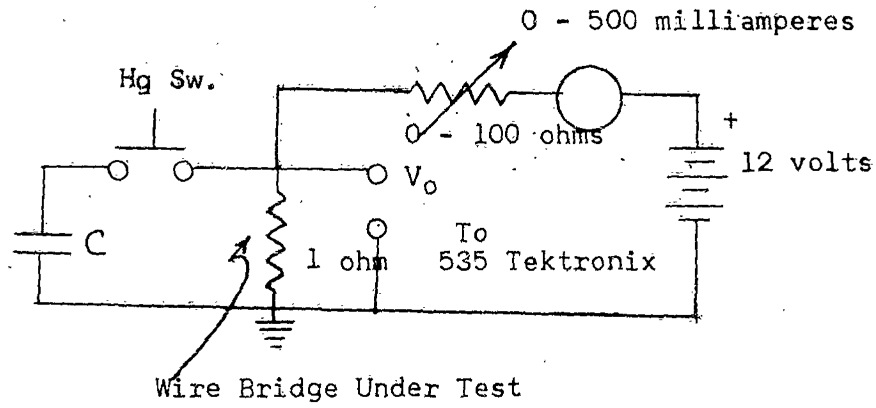
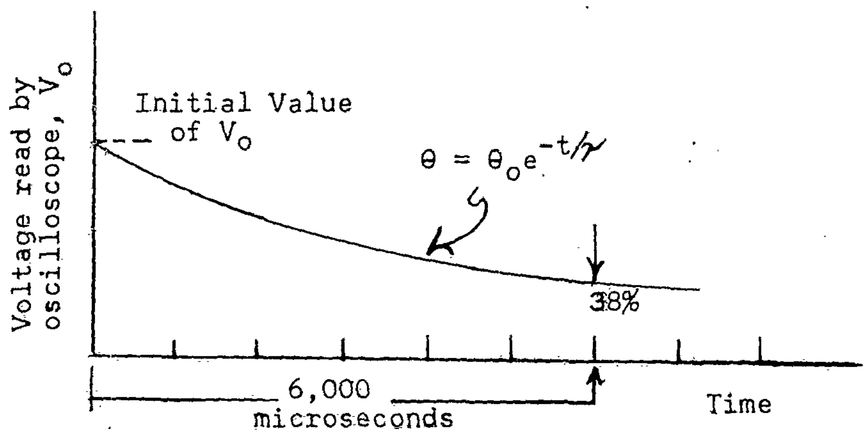


Figure 6. Constant Current Firing Traces



(a) Circuit Diagram



(b) Typical Cooling Curve for a Squib Mk I bridge

Figure 7. Cooling Curve Circuit and Display of Typical Record.

v = 40 volts   c = 4 microfarads
Assuming γ = 0, No Heat Loss
$(RC)_A$ = 4 microseconds, R = 1 ohm
$(RC)_B$ = 8.4 microseconds,
R = 2.1 ohms

A) $q/Q = e^{-t/(RC)_A}$

B) $q/Q = e^{-\frac{\left(t/RC + \frac{\delta}{2}Q^2\left\{1-(q/Q)^2\right\}\right)}{1+\delta Q^2}}$

C) $q/Q = e^{-t/2(RC)_C}$

$\delta = \frac{\alpha}{2cC_p}$

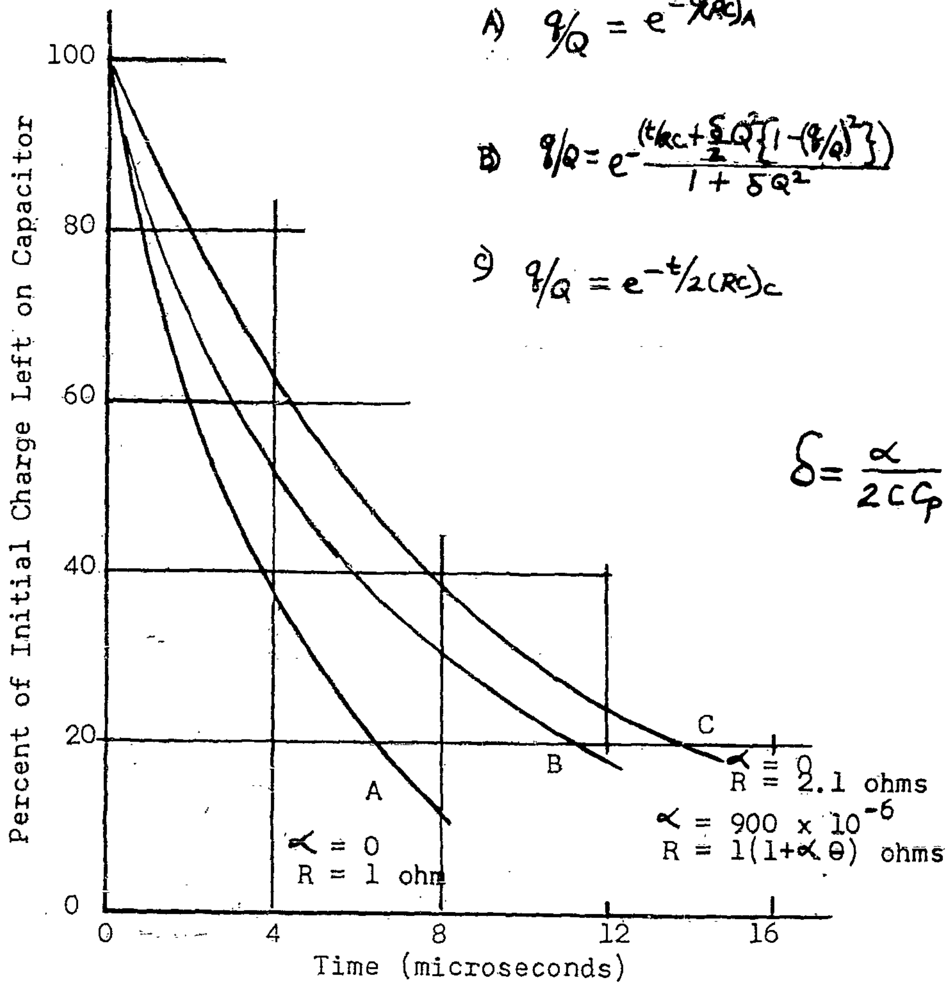


Figure 8. Capacitor Discharge into a Wire Bridge

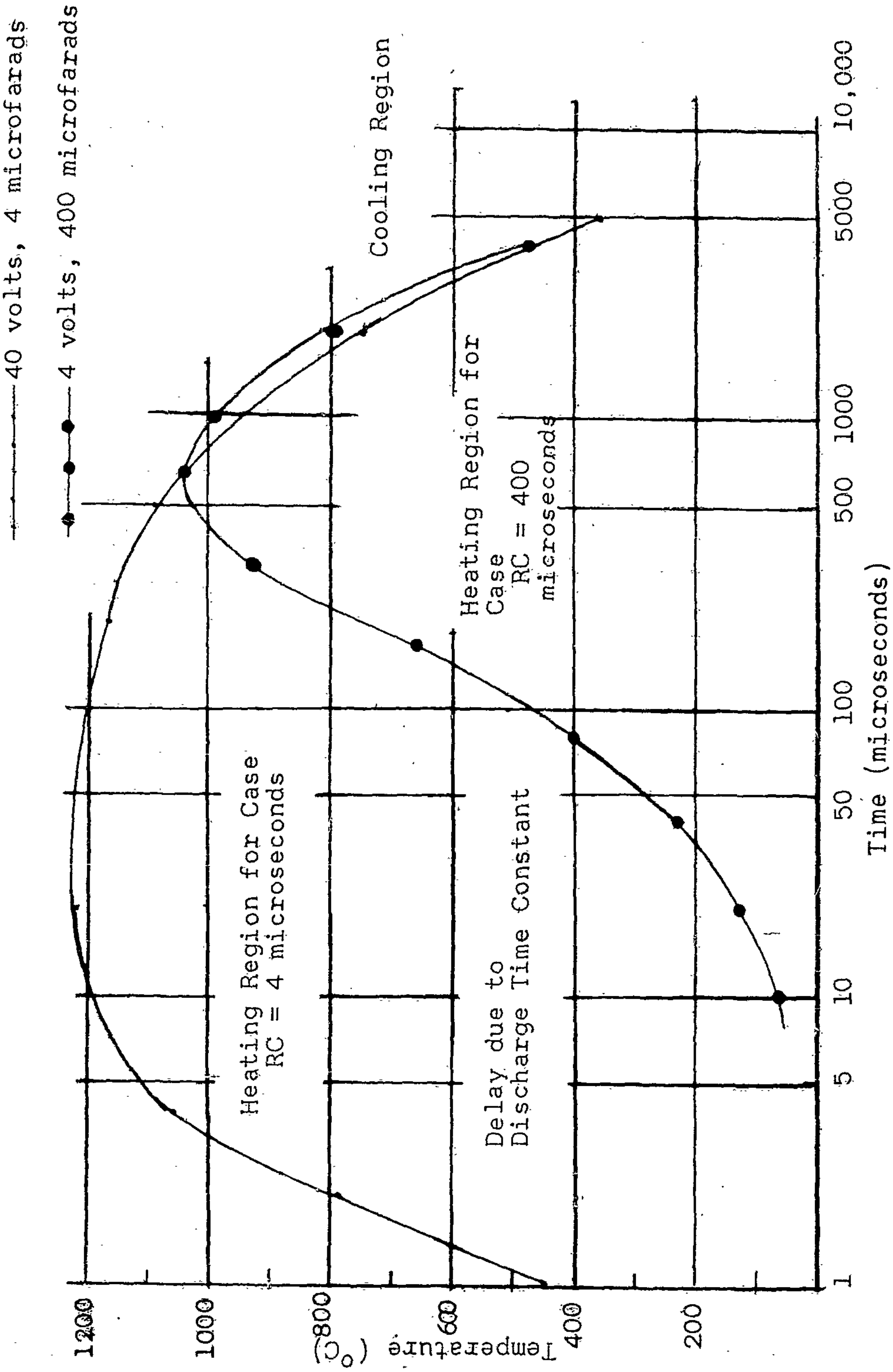


Figure 9. Temperature-Time Curve for Capacitor Discharge into a Squib Mk 1

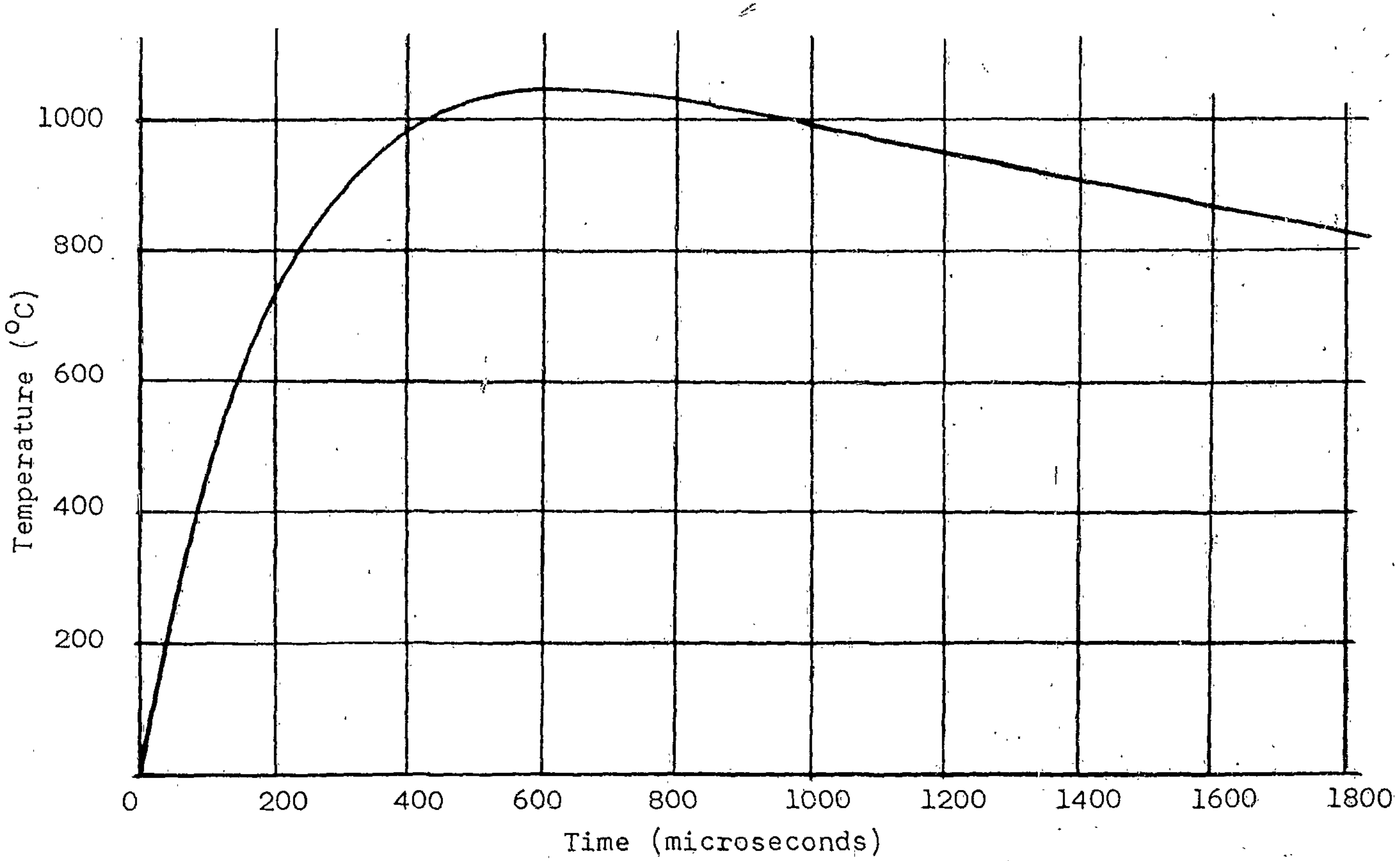


Figure 10. Temperature - Time Curve for Capacitor Discharge for a Squib Mk 1(400 microfarads at 4 volts)

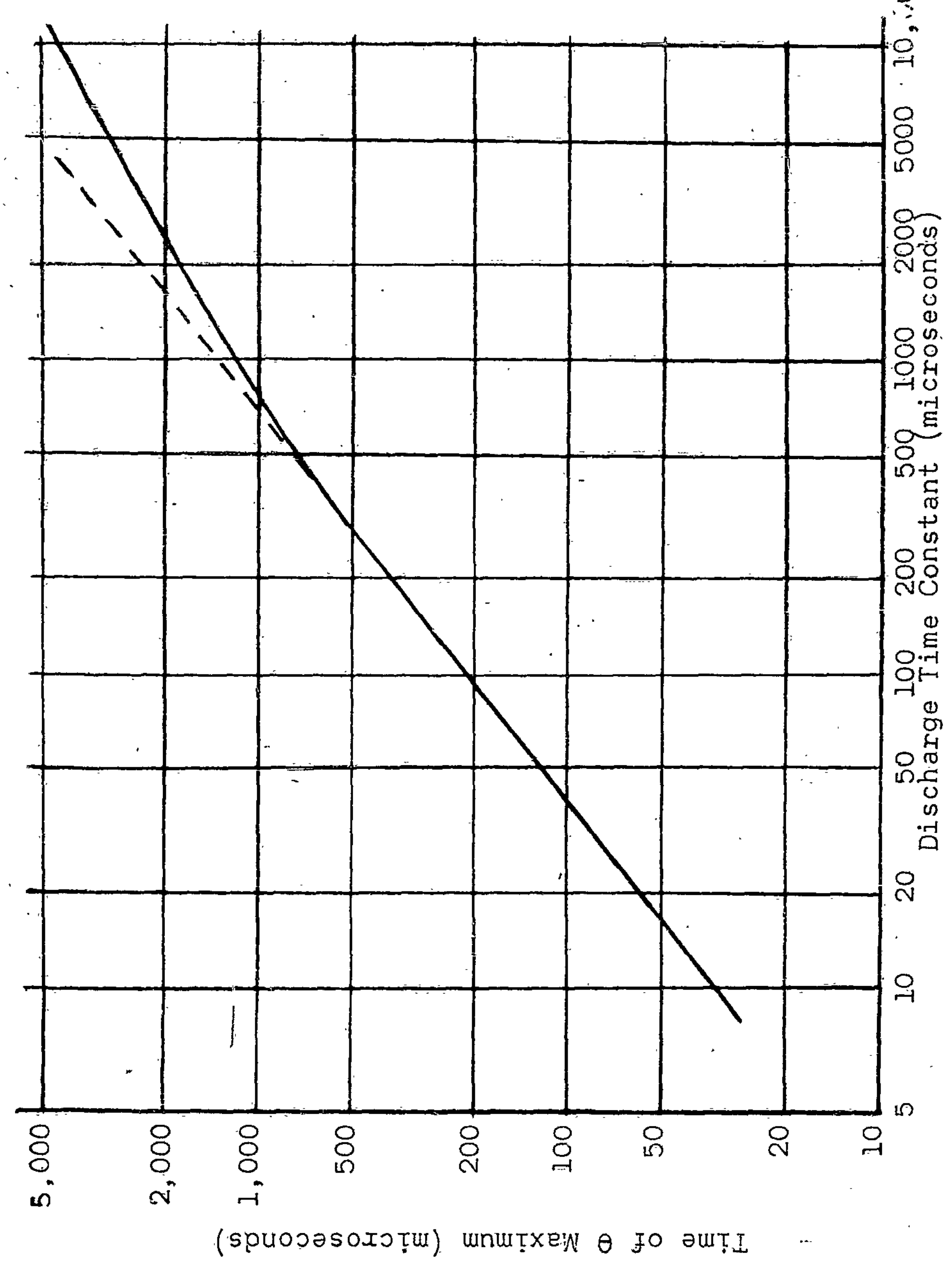


Figure 11. Time of Maximum Temperature after Start of Discharge as a Function of Discharge Time Constant for the Squib Mk 1

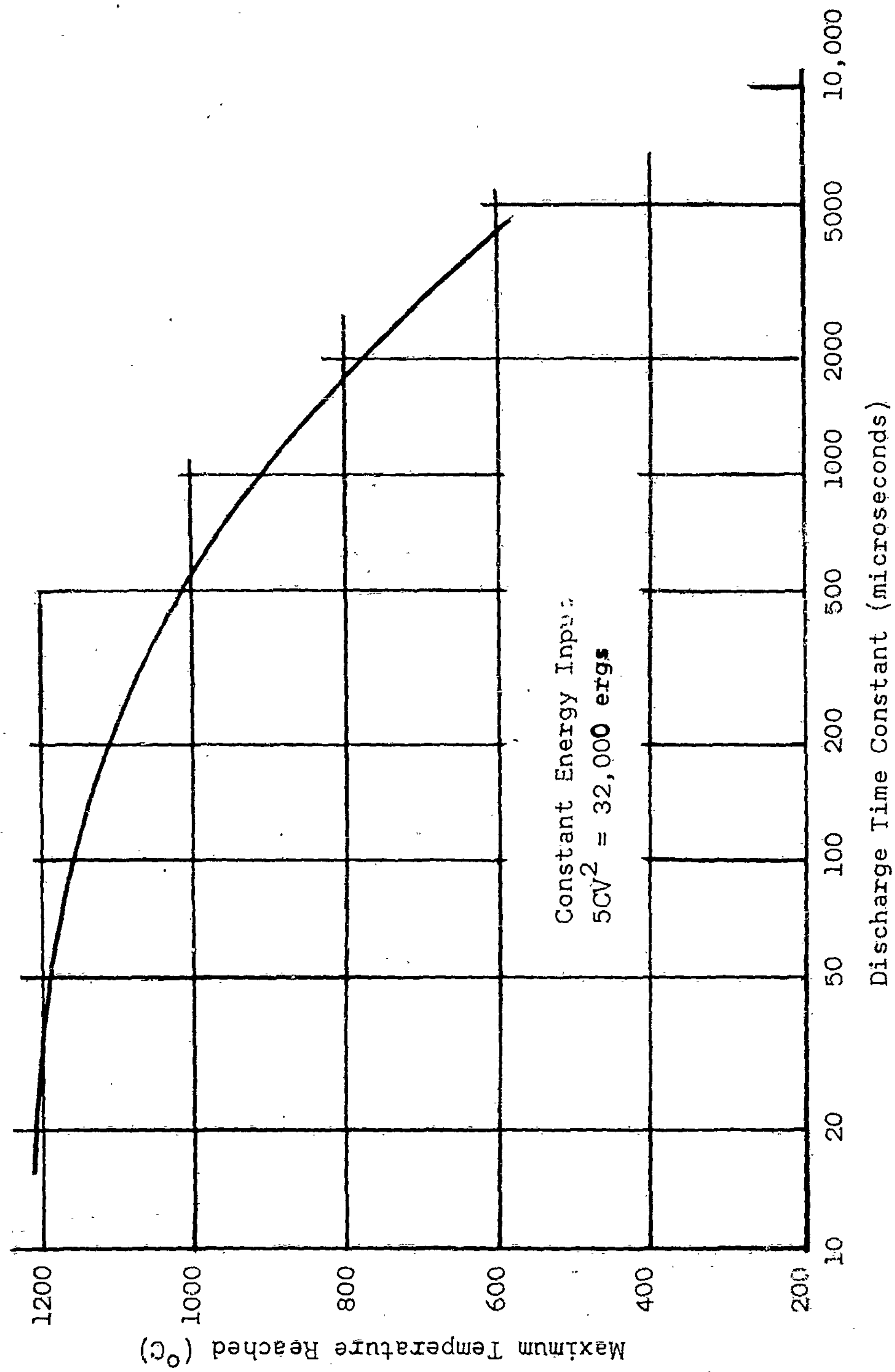


Figure 12. Maximum Temperature Achieved vs. Discharge Time Constant at Constant Energy Input for a Squib Mk 1.

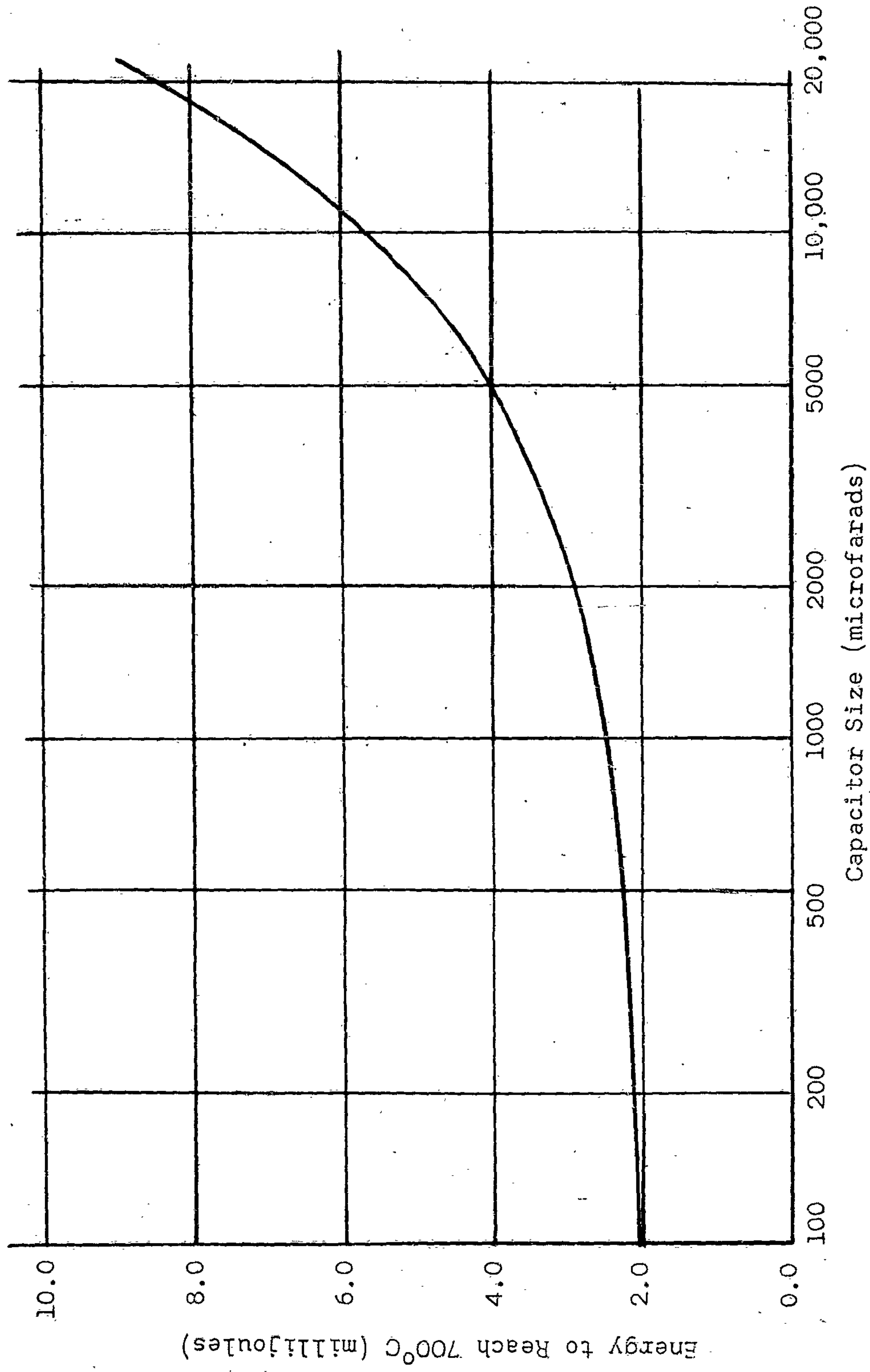
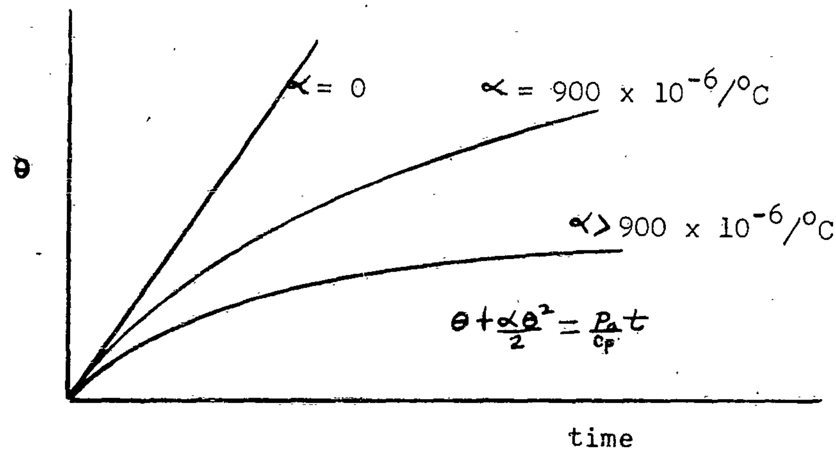
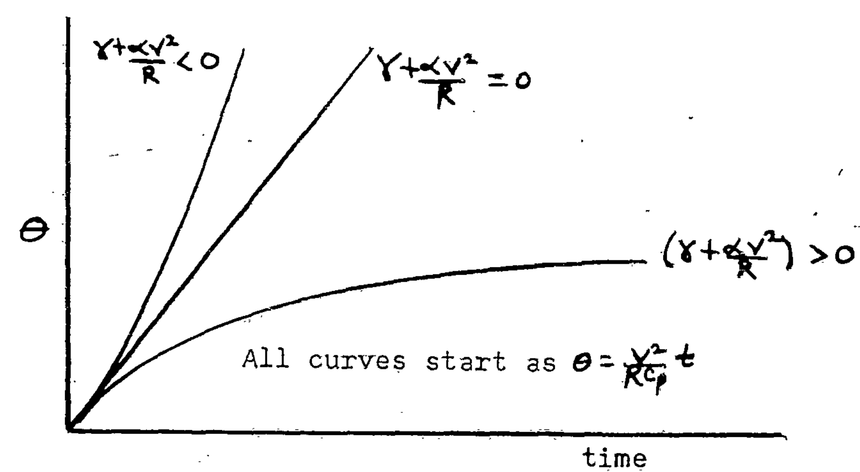


Figure 13. Theoretical Energy Required to Reach 700°C in Squib Mk 1 vs. Size of Capacitor in Capacitor Discharge of Energy



(a) Adiabatic temperature increase with constant voltage firing.



(b) Temperature increase with temperature dependent power absorbtion term included in the heat loss factor.

Figure 14. Constant Voltage Firing Temperature vs. Time Traces

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