TO:
Approved for public release; distribution is unlimited.

FROM:
Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; OCT 1958. Other requests shall be referred to Army Ballistic Research Laboratories, Aberdeen Proving Ground, MD.

AUTHORITY
BRL D/A ltr, 22 Apr 1981
ENERGY PARTITION IN THE EXPLODING WIRE PHENOMENON

F. D. Bennett

PROPERTY OF U.S. ARMY
STEEL BRANCH
FORT AND PULASKI, MD. 21010

Department of the Army Project No. 5B03-03-001
Ordnance Research & Development Project No. TB3-0108
BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND
ENERGY PARTITION IN THE EXPLODING WIRE PHENOMENON

F. D. Bennett

Requests for additional copies of this report will be made direct to ASTIA.

Department of the Army Project No. 5B03-03-001
Ordnance Research and Development Project No. TB3-0108

ABERDEEN PROVING GROUND, MARYLAND
ENERGY PARTITION IN THE EXPLODING WIRE PHENOMENON

ABSTRACT

Streak camera and oscillographic circuit damping data are presented for copper wires varying in diameter from 3 to 8 mils. A maximum of specific shock wave energy in the induced flow is found at a wire diameter different from that of a minimum in the total damping time of the circuit. This displacement is shown to be caused by the presence of residual circuit resistance. The argument is based on a critical analysis of optimum damping conditions in the exploding wire circuit. A maximum of apparent energy within the contact surface appears at about the same wire diameter as the minimum of total damping time. Discussion of the implications of the Taylor-Lin similarity theory indicates that lack of similarity of the flow is probably connected with the displacement of the maximum energies associated with shock wave and contact surface.
1. INTRODUCTION

We continue here the study of cylindrical shock waves initiated in an earlier paper\(^1\). There it was shown that modification of the rotating mirror technique makes the cylindrical shock wave visible over a considerable portion of its path. From a method of data plotting based on the Taylor-Lin similarity theory for cylindrical blasts one finds that a definite shock-energy can be associated with each trajectory.

E. David\(^2\) has recently given a detailed calculation of physical processes in the wire explosion, based partly on the experiments of Müller\(^3\) and partly on a hypothetical model constructed on the firmly established principles of thermodynamics and electromagnetic theory.

We are not concerned here with the finer details given by David, but rather with the cruder task to establish some facts relating to the partition of the stored condenser energy between 1) energy dissipated in the induced fluid motions and 2) energy consumed in apparent electrical resistance of the circuit. We shall show that a convincing correlation is possible between these two distinct methods of energy wastage, with the result that the validity of our method of assigning a shock energy is thereby rendered more probable.

\(^*\) Superscript numbers denote references listed at back of the report.
2. EXPERIMENTAL

Fig. 1 shows a sequence of streak camera pictures for 2 cm copper wires of 3, 4, 6.3 and 8 mils diameter. The traces were obtained by use of the backlighting technique already described in I.

Condenser energy at 28 kv is 118 joules and the ringing frequency of the circuit is about 0.5 mc. From the given capacitance, \( C = 0.3 \mu f \), and the damping curve for the shorted circuit alone (shown in I), it follows that the inductance of the circuit \( L_c \) is about \( 1/3 \mu h \) and the residual damping resistance \( R_c \) is approximately \( 1/4 \) ohm. Damping curves for each wire are shown opposite the corresponding flash photograph in Fig. 1.

The shock trajectory is most clearly seen in Fig. 1 (a) where separation from the contact surface occurs by about 0.8 \( \mu \) sec. For the 4 mil wire of 1 (c), the breadth of contact surface is increased and separation does not occur until \( t = 1.1 \mu \) sec. The 5 mil wire of I lies intermediate between this case and the 6.3 mil wire of 1 (e), where separation occurs at almost 2 \( \mu \) sec. Here the shock trajectory is less distinct than in the previous two cases and the contact surface appears to have gained strength relative to the shock. Finally, for the 8 mil wire the whole phenomenon is narrower, appears less energetic and shows a visible separation of the shock by about \( t = 0.6 \mu \) sec.

The corresponding oscillograms show interesting detailed behaviour, but from a gross point of view demonstrate mainly that total damping time passes through a minimum for wire diameter close to 6.3 mils. Wires of 9 mil diameter and higher failed to give a luminous effect.

Because of the interesting behaviour of the 6.3 mil wire, the complete flash is shown in Fig. 2. Here the wedge associated with the interior facing, second shock wave discussed in I is very prominent and occurs at \( t = 8 \mu \) sec well after the electrical oscillation is damped. Just ahead of the luminous tip of the wedge, faint traces of the incoming shock may be seen. These define the forward nappe of a cone whose outlines may be detected for perhaps 2 \( \mu \) sec prior to the reflection from the axis represented by the wedge tip.
3. ANALYSIS OF DATA

3.1 The Energy Plot

Measurements have been made of the shock and contact surface trajectories shown in Fig. 1. The ordinate in these flash pictures represents radial distance r while the abscissa, along the axis, represents elapsed time t. According to the method developed in I, a parabola-test plot, \((2R)^2\) vs \((t-t_0)\), of the shock data should yield a straight line if the experimental data conforms in an overall fashion to the Taylor-Lin theory for an ideal, cylindrical blast wave. It will be recalled that the Lin\(^{4}\) calculation is based on the assumptions of instantaneous energy release along a mathematical line into a perfect gas of constant specific heat ratio \(\gamma = 1.4\).

The actual wire explosion releases energy into a finite, massive cylinder and thence into an imperfect gas (viz. air) whose specific heat can vary; nevertheless, the physical situation of the experiment approximates to a reasonable degree the assumptions of the Taylor-Lin theory.

As the data of I demonstrate, the cylindrical shock wave formed by an exploding wire behaves over a large part of its trajectory like an ideal cylindrical blast wave with a virtual time of origin \(t_0\) and an apparent, axial energy release of \(E\) joules per cm easily obtainable from the slope of the data plot.

Rather than present parabola-test curves for each size of wire, a summary plot of the data is given in Fig. 3. Here the quantity \(Y = (2R)^2/(t-t_0)\) has been plotted for both the shock and the contact surface of each wire. As may be seen from the equation for the shock trajectory, viz.,

\[ R = S(\gamma) \left( \frac{E}{p_0} \right)^{1/4} (t - t_0)^{1/2}, \]

a plot of \(Y\) vs \(t\) should yield a horizontal straight line if the data closely approximates an ideal blast wave, and the value of \(Y\) will be given by \(m = 4S^2 \left( \frac{E}{p_0} \right)^{1/2}\) which is the slope one would obtain from the parabola-test curve. Assuming values of \(S(\gamma)\) and \(p_0\) appropriate for air, we can readily obtain values of \(E\) from the corresponding \(Y\) values. A scale of energy \(E\) appears at the right of Fig. 3.
Each wire size is represented by an especial symbol designated in the legend of the graph, and this symbol identifies two curves. In every case the higher curve represents the shock, while the lower represents the contact surface. Horizontal lines have been drawn to fit the data lying at or beyond $t = 1 \mu$ sec. On the left extremity of each line a vertical mark indicates the value of $t_o$ appropriate to the fitted data.

The points early in time tend to drop from higher values toward the final line. This transient behavior occurs because 1) the early data of a parabola-test plot define a knee-curve whose points approach the approximating, inclined straight line from above, and 2) the intercept of this line gives $t_o > 0$; so for $t = t_o$ the corresponding $Y$ value lies at positive infinity. Values of $Y$ for $t > t_o$ rapidly approach the final horizontal. Because this horizontal line defines an energy value for the blast wave, it is appropriate to call a $Y$ vs $t$ curve an "energy plot".

Several interesting facts may be seen by inspection of Fig. 3. In the first place, $Y$ values indicating highest shock wave energies of approximately 30-0.5 joules/cm occur for the 4 and 5 mil wires; but, in contrast, the corresponding contact surface curves fall well below that of the 6.3 mil wire.

The 4 and 5 mil contact surface points fall steadily further below the nominal horizontal line drawn at 14 joules/cm as time increases from $t = 1.5 \mu$ sec. The 3 mil contact surface data exhibits the same tendency but to a somewhat less pronounced degree beginning to deviate toward lower values at $t = 2 \mu$ sec.

In this connection notice that the initial times for the contact surfaces occur earlier than those for the shock waves, except for the 8 mil wire where the two coincide; thus the virtual starting times satisfy the inequality $t_o$ (contact surface) $\leq t_o$ (shock wave). The 6.3 and 8 mil wires are notable as cases where both shock and contact surface points fall well on horizontal lines over the entire range of data obtained.

3.2 The Damping Curves

Inspection of the oscillation damping curves in Fig. 1 and that of for the typical 5 mil wire given in I, shows a further distinction to be made between the 3, 4 and 5 mil wires when compared with the heavier 6.3 and 8 mil
wires. For the first three, the damping curve is smooth and quite continuous; for the latter two an abrupt interruption occurs after which the oscillations are almost negligible.

In Table 1 we collect for convenience the wire resistance at 20°C., the virtual times of origin of contact surface (cs) and shock (s), the total damping time, and the energy $E_s$ within the shock wave.

### TABLE 1

<table>
<thead>
<tr>
<th>Wire diam. (mils)</th>
<th>$R_w$ (ohms)</th>
<th>$t_o$ (cs) $\mu$ sec</th>
<th>$t_o$ (s) $\mu$ sec</th>
<th>Damping time $\Delta t$</th>
<th>$E_s$ (Joules/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>.075</td>
<td>.06</td>
<td>.31</td>
<td>8.2</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>.042</td>
<td>.28</td>
<td>.43</td>
<td>7.2</td>
<td>31</td>
</tr>
<tr>
<td>5</td>
<td>.027</td>
<td>.33</td>
<td>.46</td>
<td>6.3</td>
<td>30</td>
</tr>
<tr>
<td>6.3</td>
<td>.017</td>
<td>.40</td>
<td>.52</td>
<td>3.5</td>
<td>27</td>
</tr>
<tr>
<td>8</td>
<td>.011</td>
<td>.35</td>
<td>.35</td>
<td>3.5</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>.0084</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>.0068</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The damping times for 9 and 10 mil wire are included even though the 9 mil wire produced no visible flash and the 10 mil wire was not destroyed by the condenser discharge.

Inspection of the last two columns of Table 1 elicits two facts: 1) a maximum in the indicated energy $E_s$ occurs for a wire diameter near 4 mils, and 2) a minimum of total damping time occurs for a diameter between 6.3 and 8 mils. Furthermore, the damping-time trough is 50% broader than the shock energy peak on a scale of wire diameter. On a scale of diameter squared, which is proportional to wire mass, the trough is about twice the width of the peak.

### 3.3 Conclusions from the Data

The data just discussed indicate certain facts relating to the production of cylindrical shock waves by fine copper wires. As the type of wire material seems to play no essential role, providing it is a sufficiently
good conductor, these facts may be generalized to apply to other than copper wires. The result is a set of working hypotheses that extend our view of the exploding wire phenomenon and provide useful tools in the planning of future experiments. Elevated to this position, they may be stated as follows:

Under fixed conditions of condenser energy, wire material, wire length and type of ambient gas, fluid motions produced by an exploding wire have the following properties:

1) Both shock wave and contact surface follow closely parabolic paths over considerable intervals of time.

2) A maximum of \( E_s \) the specific shock wave energy, a maximum of the \( Y \) values associated with the contact surface, and a minimum in damping time of the exploding wire circuit occur with variation of wire diameter.

3) In general the stationary points of 2) do not necessarily coincide, although the maximum of the contact surface \( Y \) function appears close to the minimum of total damping time.
4. DISCUSSION

4.1 Maximum Shock Energy

A maximum in shock energy $E_s$ considered as a function of wire diameter is, as already noted, revealed in the data of Table 1. Its presence, although not surprising to intuition, is not easily explained; since the experimental concept of shock strength rests here on interpreting the latus rectum of the parabolic streak picture in terms of the constants, $S(\gamma) (E/\rho_o)^{1/4}$, given by the Lin calculation. The goodness of approximation of the actual physical situation to the hypotheses of the ideal fluid calculation given by Lin cannot be stated with precision at the present.

Accepting the interpretation of the data already given and reading from Table 1 the maximum energy of 31 joules/cm it is apparent that the 2 cm wire used in these experiments communicatés more than half $(2 \times 31/118)$ of the stored condenser energy into the fluid motion associated with the shock wave. End effects have been purposely neglected in arriving at this estimate.

4.2 Circuit Resistance and Minimum Damping Time

The presence of a minimum damping time at about 7 mils wire diameter is readily seen from Table 1. This fact implies that the resistance of the 7 mil wire, $R_w$, plus the residual resistance of the circuit $R_c$, forms a total that is in some sense, as yet undefined, well matched to the L-C parameters of the circuit.

As already mentioned in I, the residual resistance $R_c$ is about $1/4 \Omega$. Values of $R_c$ with the exploding wire shorted out, calculated from the logarithmic decrement of successive current peaks, show $R_c$ initially to start at about 0.25 $\Omega$ and to increase slightly up to 0.35 $\Omega$ at the tail of the damping curve.

A similar calculation made from the damping curves of the 3, 4 and 5 mil wires shows $R_c$ starting at about 0.30 $\Omega$ and increasing to at most 0.45 $\Omega$; which evidence may be interpreted to mean that by the time a clear oscillographic record is available for these wire sizes, that is after about 1 $\mu$ sec has elapsed, an electric discharge has been established in the expanding metal vapor and the subsequent damping is dominated by this discharge and the resistance residing in the remainder of the circuit.
David\textsuperscript{2} estimates that the conductivity of the copper wire should drop suddenly after the diameter has increased by a factor of two to four. When this condition is reached metallic conduction should no longer exist, but electron transfer could occur perhaps only by tunnel effect modified by local fields. After the expansion has proceeded sufficiently far, the pressure of the metal vapor will drop to a point where arc discharge can occur if enough energy remains in the circuit and the conductivity will increase again to a high value. In the event that little energy remains by virtue of its rapid expenditure in $i^2R$ heating, no arc can form after the wire diameter exceeds $2-4$ times its original value. The effect will be like that of opening a switch.

The oscillograms for the 6.3 and 8 mil wire indicate just such an effect after about 1 and 2 \(\mu\) sec respectively. The damping records show that these wires expand the condenser energy more rapidly than the 3,4, 5 mil series. We infer the cause to be better impedance match of the exploding wire to the circuit. The streak picture for 6.3 mils given in Fig. 2 tends to bear out this interpretation if the contact surface and second shock are regarded as being more energetic than in the other cases.

4.3 Choice of Optimum Circuit Resistance

The evidence just discussed supports the assumption that the 6.3 - 9 mil wires provide nearly optimum electrical resistance for maximum power and energy consumption in the circuit. The effective resistance is clearly far below the critical damping resistance; for, even though damped to a minimum total time, the circuit is still in the oscillatory regime.

In order to carry the discussion further, criteria bearing on the question of maximum power dissipation in an R-L-C circuit are needed. Investigation of the available literature shows that even for the case of fixed circuit resistance \(R\), the conditions for maximum power dissipation have not hitherto been precisely defined.

In connection with a study of exploding wires made in 1926, Anderson and Smith\textsuperscript{5} give the condition for maximum power dissipation in an R-L-C circuit with constant parameters to be approximately \(R = (L/C)^{1/2}\).
Because they provide no discussion of this result and because their subsequent statements about maximum current are open to objection, the question has been re-examined and some additional results obtained. The more precise and extended analysis is given below in Appendix I.

There it is found that for maximum power dissipation at the first current peak the total circuit resistance should be $R = 1.10 \frac{L}{C}^{1/2}$. If the heat losses are small enough so that the process can be considered adiabatic, then energy dissipation in a specified interval may be of primary importance. For example, if this interval extends past the first current peak to terminate when the sine factor in the current expression is unity, then $R = 1.24 \frac{L}{C}^{1/2}$ should hold. If a longer interval is important, the appropriate numerical factor will be larger than 1.24. As the interval becomes infinite, $R$ approaches the critical damping resistance given by $2 \frac{L}{C}^{1/2}$.

Now it is obvious that the resistance of an exploding wire is not constant; so that the criteria given above cannot strictly apply except in the case that power consumption at maximum current is the dominating term. Even here no energy can be dissipated save during a finite interval. Therefore we conclude that the optimum average resistance of the exploding wire circuit should satisfy the inequality $1.10 \leq R_{\text{opt}} \leq 1.30$ where the lower bound is set by maximum power dissipation at the first current peak and the upper bound is set slightly to exceed the value for maximum energy dissipation in the approximately quarter-cycle interval already discussed. The oscillogram for the 6.3 mil wire suggests an interval before current interruption somewhat larger than a quarter cycle in support of a choice of upper bound greater than 1.24.

With the L, C values for the present circuit this inequality works out to be $1.16 \leq R_{\text{opt}} \leq 1.36$ ohms. As already seen the residual circuit resistance is approximately $1/4 - 1/3 \Omega$; so by difference the exploding wire should furnish $0.9 - 1.2\Omega$ in order that the total resistance most rapidly dissipate the stored condenser energy.

Relying here on David's discussion of the resistance changes induced by heating of the copper wire, we estimate that wire resistance may increase by a factor of $10^2$ for a rise from room temperature up to $4000^\circ\text{K}$. Applying
this factor to the cold resistance given in Table 1 we find that the 6.3, 7 and 8 mil wires would have high temperature resistances in or near the range of values suggested as optimum in the previous paragraph. Wires of 5 mil diameter or smaller would appear to have high temperature resistances too high by a factor of two or more and larger than the critical damping resistance.

One would not expect the trough surrounding the minimum of damping time to be narrow since the maximum of power dissipation at the first current peak will merge smoothly with the relative maxima of energy dissipation as the effective time interval increases from zero. (See discussion in Section 2 of Appendix I.) The experimental damping times are in agreement with this qualitative prediction from theory; for damping times from 6.3 to 9 mils are roughly the same and the minimum is broad compared with the maximum of shock energy.

It appears highly probable that the nearly optimum wires reach temperatures of several thousand degrees Kelvin, perhaps even the critical point for copper under influence of magnetic pinch pressures, and that their high temperature resistance values are close to those demanded for maximum power or energy dissipation. The data appear to favor values higher than the lower bound.

4.4 Effect of Residual Circuit Resistance

We develop here an argument to show that the displacement of the shock energy peak from the minimum of damping time is entirely caused by the residual resistance in the circuit. Conceding temporarily the truth of this proposition then if this resistance is made to vanish, one would expect the shock with greatest axial energy release and the most rapid dissipation of condenser energy to occur at the same wire diameter, a criterion which will clearly be of importance where luminous effects (e.g. light sources) of minimum duration are desired.

As may be seen from Appendix I the necessary and sufficient conditions that the $i^2R$ power in the circuit be stationary, i.e. $dP = 0$, are that $\frac{\partial i}{\partial t} = 0$ and $\frac{\partial \ln \frac{1}{iR}}{\partial t} = -1/2R$. As before the resistance of the circuit is the sum $R = R_c + R_w$ where $R_c$ denote residual and exploding wire resistance.
respectively. Wire resistance $R_w$ is directly proportional to length $l$ and inversely proportional to radius $r$ squared; thus $R_w = \frac{kl}{r^2}$. The power expended per unit length of wire is given by $p_w = R_w \frac{i^2}{l} = k \left(\frac{i}{r}\right)^2$. Intuitively it is obvious that $p_w \Delta t$ must be related to the specific shock energy $E$ of the Lin theory because the energy expended in $i^2R_w$ heating partially reappears to supply the energy of the fluid motions induced by the explosion. Here $\Delta t$ represents a small time interval about the first current peak.

By logarithmic differentiation we find $d \ln p_w = 2 d \ln \left(\frac{i}{r}\right)$; but at the instant of power maximum, current is a function of $R$ alone and in particular $d \ln i = -(1/2) d \ln R$. Combining these two results leads us to $d \ln p_w = -d \ln (Rr^2)$. A stationary point for $p_w \neq 0$ can only exist when the right hand side of this equation vanishes. Otherwise, a non-zero value will obtain and an incremental change in wire diameter (resistance) can always be found which will increase $p_w$.

From the definitions, $Rr^2 = R_c r^2 + kl$ and we have finally, $\frac{dp_w}{p_w} = -(2R_c/R) \left(\frac{dr}{r}\right)$, from which it is obvious that $\frac{dp_w}{dr} \leq 0$ and vanishes if $R_c = 0$. We seen then, as stated at the beginning of this section, that non-vanishing residual circuit resistance implies a separation between the minimum of damping time and $E_{\text{max}}$ for the shock wave; furthermore, decreasing the wire diameter from the value necessary to satisfy $dP = 0$ causes $p_w$ to increase.

Assuming from the data of Table 1 that $E_{\text{max}}$ occurs at 4 mils and minimum damping time at 6.3 mils, with $R_c = 1/4$ Ω and $R_w = 1$ Ω, we find $\frac{dp_w}{p_w} = 2 \times (1/5) \times (2.3/6.3)$ which works out to a 15% increase in $p_w$ upon decreasing the wire diameter from 6.3 to 4 mils. Inspection of Fig. 3 shows a corresponding increase in the shock wave energy of about 12%; which in view of the uncertainties in the values used, may be considered as remarkable agreement.

It seems clear then that the 6.3 mil wire corresponds closely to a damping situation in which maximum energy is dissipated during an interval about the current peak. The fact that the neighboring wires that show efficient short damping times have lower cold resistances and presumably
lower high-temperature resistances suggests that the maximum integrated power dissipation of the upper bound may apply rather than the maximum power at peak current. Nevertheless the foregoing analysis based on the latter matching criterion brings out excellent agreement between the predicted specific power increase in the wire and the observed energy increase in the shock wave.

Because this agreement is found between quantities obtained by the two very different approaches taken through 1) the electrical theory of the R-L-C circuit and 2) fluid mechanical theory of the cylindrical blast wave, our confidence is strengthened in the use of the second approach to derive energy values for the experimental shock waves.

4.5 Contact Surface

We revert now to the question concerning the connection between an apparent energy within the contact surface and lack of similarity in the flow.

The similarity theory assumes that the cylindrical shock wave is a constant energy surface of the flow. Examination of the energy integral \[ \text{Lin's Eq. (23)} \] shows that for arbitrary radius \( r \) interior to the shock, the energy contained within \( r \) is a function only of \( \eta = r/R \). Thus parabolas given by \( \eta = \text{const.} \leq 1 \) are curves of constant energy in the \( r-t \) plane.

On the other hand, particle paths in the \( r-t \) plane define curves of constant mass, since by definition these paths may not intersect. Because each particle experiences a radial deceleration after passage of the shock, each inner particle must be doing work on the next outer particle of fluid. Therefore energy is flowing outward faster than the particles on any given cylinder of radius \( r \), and particle paths cut constant energy parabolas in the \( r-t \) plane in the direction of decreasing \( \eta \) values as time increases.

Now the contact surface experimentally determined from streak camera pictures may reasonably be assumed to approximate the outer, limiting particle path of the luminosity. If the flow is truly similar in the Taylor-Lin sense, then the contact surface should not coincide with a constant energy surface given by \( \eta = \text{const.} \), but should show a trend of \( \eta \) toward decreasing values. In contradiction to this conclusion the 6.3 and 8 mil data of Fig. 3
appear to lie well on curves $\eta = \text{const.}$. (The data for the smaller wires show a decreasing trend of $\eta$ with time.) Thus we may certainly say that the flows generated by the larger wires fail to exhibit similarity.

On the other hand, these flows have contact surfaces which plot like one of the constant energy surfaces of the ideal, similar flow corresponding to the experimental shock wave. Thus by comparison between the experimental flow and the ideal, similar flow of equal shock energy, the contact surface may be assigned an energy value which is the same as the energy inside the $\eta = \text{const.}$ surface of the ideal flow on which the experimental contact surface points fall. While this energy may not represent accurately the total energy residing within the experimental contact surface, it is nevertheless a well defined quantity derivable from the contact surface trajectory, and may be valuable for comparison of one contact surface with another. We adopt the foregoing definition of energy within the contact surface for all subsequent discussion.

To examine the contact surface data for the 3, 4 and 5 mil wires more closely, we need detailed information about the particle trajectories of the similarity flows. To this end recall that for the similar solutions found by Lin, the velocity of a particle at radius $r$ is given by $u = \phi(\eta) U$, where $U = dR/dt = \dot{R}$ gives the velocity of the shock and $\phi(\eta)$ is a numerically determined function.

The particle paths may be obtained by integrating the Lagrangian equation of motion $\dot{r} = u = \phi \dot{R}$. With the help of $\dot{r} = \eta \dot{R} + \dot{\eta} R$ we find the first order differential equation $d\eta = (\phi - \eta) d \ln R$. From Lin's Table I we find that $(\phi - \eta)$ is always negative. As a result we have $d\eta/dt = U(d\eta/dR) < 0$ and the decreasing trend of $\eta$ already noted is verified again.

The fractional change in $\eta$ is found from $d\eta/\eta = \eta^{-1}(\phi - \eta)(dR/R)$ from which it follows with the help of Lin's tabulated values that $|d\eta/\eta| \leq 0.275 (dR/R)$. The fractional change in $\eta$ divided by that of $R$ varies from 0.167 at $\eta = 1$ to 0.275 at $\eta = 0.6$ and appears to remain constant at this value as $\eta \to 0$. Thus we see that $\eta$ for a particle trajectory is a slowly varying function of $R$ and is to this extent crudely approximated by setting $\eta = \text{const.}$, as was suggested in I.
The particle paths can be obtained by a numerical integration of the equation 
\[
\int_{1}^{6} (\phi - x)^{-1} dx = \ln \left( \frac{R}{R_1} \right) = \frac{1}{2} \ln \left[ \frac{(t - t_0)}{(t_1 - t_0)} \right],
\]
where \(R_1\) and \(t_1\) are the shock radius and time respectively at which the shock passes over the particle in question, and \(t_0\) is the virtual time of origin of the shock as before. Values of the integral \(I = \ln \left( \frac{R}{R_1} \right)\) have been obtained by Simpson's rule from the numerical results in Lin's paper and are presented in Table 2. Using these values and the experimental values for \(t_1\) and \(t_0\), we have calculated theoretical and experimental values for the particle trajectory defined by the constant surface data for the 3,4 and 5 mil wires. These data are presented in the form of ratios \(\eta/\eta_{\text{calc.}}\) versus time in Table 3. Of the three wires the 4 mil comes nearest to similarity being only 2% low beyond 1 μ sec. The 5 mil wire is close to similarity in the latter part of the range while the 3 mil wire is never closer than 8% and as far as 16% below the calculated values. The experimental trajectories nearly always lie inside those calculated from theory. Note that the nearest approach to a similar flow occurs for the wire with the most energetic shock wave, viz., the 4 mil wire.

Turning now from discussion of the lack of similarity in the flows, we conclude by examining briefly the question of the relative amount of energy within the contact surface. The definition given above, which relates the experimental contact surface to an \(\eta = \text{const.}\) curve of the ideal, similarity flow of equal shock energy, provides a definite number for the contact surface but no assurance that this number represents the real energy within that surface.

The fact that the highest value occurs for the 6.3 mil wire whose streak picture, Fig. 2, shows what appears to be the most energetic phenomena within the contact surface, encourages the hypothesis that there is a proportionality, if not an identity, between the actual energy within the contact surface and the graphically determined value obtained from the energy plot.

Since the maxima of shock wave and contact surface Y values do not occur at the same wire diameter we are forced to suppose that the apparent energy within the contact surface is regulated not only by phenomena responsible for the shock wave, but also by other factors at present imperfectly known, which have to do with the electrical and mechanical phenomena of the wire explosion.
The experimental data of Figs. 1, 2 and 3 seem to support these hypotheses; for the most quickly damped condenser discharge is associated with the contact surface of highest indicated energy and with the most energetic appearing second shock wave. The most energetic shock wave, on the other hand, is associated with a contact surface definitely below the contact surface maximum on the energy plot. This suggests that while the maximum E value for a shock wave appears at a smaller wire diameter because of the distribution of electrical energy between internal and wire resistance, the condition of minimum damping time occurs at the same wire size as flow phenomena of maximum kinetic energy within the contact surface. That is to say, maximum dissipation of electrical power is accomplished partly through the simultaneous appearance of fluid dynamic effects different from those responsible for the generation of the shock wave. Not much more can be said at this point; for clearly, further experimental and theoretical information is needed.
### Table 2

Table 2  Numberical values of the integral $J = \int_{1}^{\eta} (\phi - x)^{-1} dx$ versus the parameter $\eta = r/R$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$1$</th>
<th>$\eta$</th>
<th>$1$</th>
<th>$\eta$</th>
<th>$1$</th>
<th>$\eta$</th>
<th>$1$</th>
<th>$\eta$</th>
<th>$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0</td>
<td>.94</td>
<td>.332</td>
<td>.82</td>
<td>.924</td>
<td>.55</td>
<td>2.420</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.995</td>
<td>.029</td>
<td>.93</td>
<td>.384</td>
<td>.80</td>
<td>1.021</td>
<td>.50</td>
<td>2.766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.990</td>
<td>.059</td>
<td>.92</td>
<td>.435</td>
<td>.78</td>
<td>1.119</td>
<td>.40</td>
<td>3.569</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.985</td>
<td>.088</td>
<td>.91</td>
<td>.485</td>
<td>.76</td>
<td>1.218</td>
<td>.30</td>
<td>4.627</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.980</td>
<td>.116</td>
<td>.90</td>
<td>.535</td>
<td>.74</td>
<td>1.319</td>
<td>.20</td>
<td>6.097</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.97</td>
<td>.172</td>
<td>.88</td>
<td>.633</td>
<td>.70</td>
<td>1.539</td>
<td>.10</td>
<td>8.673</td>
<td></td>
<td></td>
</tr>
<tr>
<td>.96</td>
<td>.226</td>
<td>.86</td>
<td>.731</td>
<td>.65</td>
<td>1.811</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.95</td>
<td>.280</td>
<td>.84</td>
<td>.827</td>
<td>.60</td>
<td>2.103</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3

Comparison of calculated and experimental trajectories for the contact surfaces of the 3, 4 and 5 mil wires.

<table>
<thead>
<tr>
<th>$t$ (\mu sec)</th>
<th>$\eta(\text{exp}) / \eta(\text{calc.})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>.88</td>
</tr>
<tr>
<td>1.5</td>
<td>.89</td>
</tr>
<tr>
<td>2.0</td>
<td>.92</td>
</tr>
<tr>
<td>2.5</td>
<td>.89</td>
</tr>
<tr>
<td>3.0</td>
<td>.86</td>
</tr>
<tr>
<td>3.5</td>
<td>.84</td>
</tr>
</tbody>
</table>

η(\text{exp}) / η(\text{calc.})
ACKNOWLEDGMENTS

The author wishes to thank J. Sternberg, R. Sedney, G. D. Kahl and D. B. Sleator for helpful discussions of the theoretical background of this paper.

The valuable suggestions and expert technical assistance of Mr. D. D. Shear in all of the experimental phases of this research are gratefully acknowledged.

F. D. Bennett
F. D. Bennett
REFERENCES

1. Bennett, F. D. Phys. of Fluids 1, 347, (1958). This paper will be referred to as I.


Fig. 1. Rotating-mirror pictures and damping curves for sequence of copper wires under constant ambient conditions and condenser energy. The 5 μ sec scale in (g) applies to all flashes. Each horizontal division of the oscilloscope scale corresponds to 2 μ sec.
Fig. 2. Rotating-mirror picture for exploded copper wire of 6.3 mil diameter to show details of the wedge luminosity representing the second shock wave.
Fig. 3. Energy plot of shock wave and contact surface data for the sequence of exploded copper wires shown in Fig. 1.
APPENDIX I
POWER DISSIPATION IN AN EXPLODING WIRE CIRCUIT

1. MAXIMUM AT PEAK CURRENT

Anderson and Smith discuss the typical R-L-C circuit used to produce the heavy current pulse under which a thin metal wire explodes. They state without proof that a maximum in the power dissipated by the circuit will occur when the circuit resistance R has a value near \((L/C)^{1/2}\). It may be inferred from their analysis that current \(i\) is to be maximized simultaneously with power although in the later step of obtaining the maximum squared current this assumption is ignored.

An explicit solution to this same problem is given here for the following reasons: 1) a complete discussion is not given by Anderson and Smith and cannot readily be found elsewhere in the literature, 2) the equation given by Anderson and Smith for the maximum current squared is not precise for reasons pointed out below; thus some doubt attaches to their earlier assertion about the best value of R, and 3) consideration of integrated power leads to some new "best values" of R which may sometimes be preferred.

Defining the quantities \(\alpha = R/2L\) and \(\omega = (1/LC - R^2/4L^2)^{1/2}\) one can write for the current in the circuit

\[
i = (V/\alpha L) e^{-\alpha t} \sin \omega t
\]

The stationary points of \(i\), or \(\ln i\) for convenience, are determined from

\[
\frac{\partial (\ln i)}{\partial t} = \omega \cos \omega t / \sin \omega t - \alpha = 0
\]

which gives

\[
cot \omega t = \alpha / \omega
\]

From (3) the sequence of times \(\{t_n\}\) may be determined to locate successive maxima and minima of current.

Instantaneous power dissipated in the circuit is \(i^2 R\) and the condition \(\partial (i^2 R)/\partial R = 0\) implies that

\[
\frac{\partial (\ln i)}{\partial R} = -1/2R
\]

26
With (1) and the definitions, Eq. (4) leads to the relation
\[ (t \cot \omega t - 1/\omega)(\partial \omega \partial R) - t(\partial \alpha \partial R) + 1/2R = 0 \tag{5} \]
or, with the partials explicitly evaluated, to
\[ 1/2\alpha - (\alpha/\omega)(t \cot \omega t - 1/\omega) - t = 0 \tag{6} \]
Eqns. (3) and (6) determine the conditions necessary to maximize power with respect to \( i \) and \( R \) simultaneously. Eqn. (3) is itself necessary to maximize \( i \) alone. We consider only the first maximum, since the wire is gone shortly thereafter.

If (3) be solved for \( t \) and used in (6) with definition \( p = \omega/\alpha \), there results the transcendental equation
\[ \tan^{-1} p = (p/2)(p^2 + 2)(p^2 + 1)^{-1} \tag{7} \]
Examination of (7) together with (3) shows that there is an infinity of increasing, positive roots, beginning with zero, which corresponds to the series of successive stationary current values. A graphical solution gives \( p_1 \approx 1.515 \) for the first non-zero root. From this it follows that
\[ R = 2(1 + p_1^2)^{-1/2} (L/C)^{1/2} \approx 1.10 (L/C)^{1/2} \]
If one defines \( 2(1 + p^2)^{-1/2} \) by \( f(p) \) in the general case, then \( 0 \leq f \leq 2 \) comprises all values for which oscillations may occur in the circuit. Values \( f = 0, 2 \) correspond to the undamped and critically damped cases for which \( p = \infty, 0 \) respectively, and \( f = 1.10 \) represents the resistance in which power is dissipated most rapidly at the first current maximum. It is now clear that Anderson and Smith's original assertion is equivalent to saying that for maximum power dissipation at the first maximum current, \( f \) must be close to one; furthermore the explicit solution shows that \( f \) is actually 10\% greater than one. For the successive current maxima \( f(p) \) decreases monotonically to zero.

The expression for current may be rewritten as
\[ i = (V_{V_{\text{max}}} V_{\text{max}}) e^{-\omega t/p} \sin \omega t \tag{8} \]
The stationary values occur when (3) is satisfied, i.e. when \( \omega t_m = \tan^{-1} p + mx \)
with \( m = 0, 1, 2 \ldots \) and \( \sin \omega t_m = (-1)^m p (1 + p^2)^{-1/2} \). With these relations, (8) becomes
\[ i_m = (-1)^m(V/\alpha L)(1 + p^2)^{-1/2} \exp \left[ -p^{-1}\tan^{-1}p - \alpha m \right] \] 

which furnishes the first maximum when \( m = 0 \). From the definitions one finds that \( \omega^2 = \alpha^2 p^2 = 1/\alpha L - \alpha^2 \) and \( \alpha L = (1 + p^2)^{-1/2}(L/C)^{1/2} \); thus

\[ i_m = (-1)^mV(\alpha L)^{1/2} \exp \left[ -p^{-1}\tan^{-1}p - \alpha m \right] \] 

The square of this expression corresponds to that given by Anderson and Smith except that their exponential term is incorrect and an incorrect factor of \( 4/3 \) enters because of their failure to evaluate at \( t_m \). Maximum power is obtained when \( m = 0 \), \( f = 1.10 \) and \( p = p_1 \). Thus

\[ P_{\text{max}} = 1.10 V^2 (\alpha L)^{1/2} e^{-1.30} \] 

For a typical case, \( L = 1/3 \) \( \mu \text{H} \), \( C = 0.3 \mu \text{F} \), \( V = 28 \text{ kV} \) and as before \( p_1 = 1.515 \). With these values \( (L/C)^{1/2} = 1.05 \) ohms, \( i_m = 15.3 \) kA and \( P_{\text{max}} = 2.24 \times 10^8 \) watts. If approximately this power is consumed during \( 10^{-7} \) sec., around the peak, then 22 joules are used during the interval.

With the exploding wire replaced by a copper strip, the circuit has a logarithmic decrement given approximately by \( \delta = \ln 2 = 2\pi/p \) from which computation gives \( R \approx 0.23 \) ohms. For maximum power at current peak, \( R = 1.10 (L/C)^{1/2} = 1.16 \). Thus for optimum conversion of the condenser energy in the explosion, the wire should furnish about 0.9 ohms.
Since available evidence indicates that the circuit containing an exploding wire of nearly optimum size is broken sometime during the latter part of the first half cycle, it is of interest to inquire whether power consumed during that interval can be maximized by selection of an appropriate circuit resistance. To this end the energy taken from the condenser at time $T$ is

$$ W = \int_0^T i^2 R \, dt $$

This equation can be integrated in compact form by letting $f/2 = g = \sin \phi$ and $\omega_o = (LC)^{-1/2}$; thus

$$ R = 2 \sin \phi \frac{(L/C)^{1/2}}{2} \quad , \quad (12a) $$

$$ \alpha = \omega_o \sin \phi \quad , \quad (12b) $$

$$ \omega = \omega_o \cos \phi \quad , \quad (12c) $$

and

$$ p = \omega/\alpha = \cot \phi \quad . \quad (12d) $$

With these relations and some elementary trigonometrical transformations (12) integrates to

$$ W = W_o \left\{ 1 - \sec^2 \phi \cdot e^{-2\omega T \tan \phi} \left[ 1 + \sin \phi \sin (2\omega T - \phi) \right] \right\} \quad , \quad (13) $$

where $W_o = (CV^2/2)$ represents the initial energy in the condenser. From the earlier requirement on $f$ note that $0 \leq \sin \phi \leq 1$; thus the negative term in (13) always remains negative. From the fact that the power $i^2 R$ is positive definite it follows that $W$ is a non-decreasing function of $T$.

To examine $W$ for stationary points with respect to $T$ and $R$, employ

$$ Q = \ln \left[ (W_o - W)/W_o \right] $$

for convenience. With this definition the first partials are $\partial W/\partial T = -W_o \cdot e^Q \cdot (\partial Q/\partial T)$ and $\partial W/\partial R = -W_o \cdot (4L/C)^{1/2} \sec \phi \cdot e^Q \cdot (\partial Q/\partial \phi)$. Since $e^Q \gg 0$, the stationary points will be given by setting $e^Q \partial Q/\partial T = \sec \phi \cdot e^Q \cdot (\partial Q/\partial \phi) = 0$. From the fact that $e^Q \to 0$ as $T \to \infty$ it is clear that as time increases the $W$ surface approaches the plane $W = W_o$ for any value of $R$. This is the absolute maximum of the function.

Performing the partial differentiation leads to two equations

$$ \sin \phi (\cos 2\omega T - 1) = 0 \quad (14) $$

and

$$ \sec \phi \left\{ \sin 2(\omega T - \phi) + 2\tan \phi \cdot [1 + \sin \phi \sin (2\omega T - \phi)] \right\} - 2\omega T \left[ 1 + \tan \phi \sin 2\omega T \right] = 0 \quad (15) $$

A number of cases arise from these.
In the first place (14) indicates that stationary points in time occur when \( \omega T = m\pi \), that is at points where the current is zero. It follows that the curves of \( W \) vs \( T \) have horizontal tangents at these points, beginning with \( T = 0 \) for \( n = 0 \), and rise smoothly in a series of steps to the maximum \( W_0 \). The size of the steps is not constant but diminishes monotonically as integer \( n \) increases.

When \( \sin \phi = 0 \) \((R = 0)\), Eq. (14) is satisfied and (15) leads to the condition that \( T = 0 \). Since \( W = 0 \) on planes \( T = 0 \) and \( \phi = 0 \) \((R = 0)\), the origin is clearly a true minimum.

If \( \sin \phi \neq 0 \), then as before \( \omega T = m\pi \); but (15) is only satisfied if \( n = 0 \). Thus the equation \( \omega T = \omega_0 \cos \phi \) \( T = 0 \) must hold. With \( \cos \phi \neq 0 \), the \( \phi \) axis emerges as a minimum line of the surface. This is to be expected since no power has been consumed at \( T = 0 \) for any value of \( \phi \) \((\text{resistance})\). The other choice rendering \( \cos \phi \) \( T = 0 \) is to set \( \phi = \pi/2 \) \( \text{i.e.} \) choose the critical damping resistance. But this value fails to satisfy (15) because \( \sec \phi \to \infty \) as \( \phi \to \pi/2 \). Investigation of the limit shows that the critical damping curve is not a stationary line of the \( W \) surface.

Our analysis shows that \( W \) has no true maxima or minima for interior values of the resistance; so that no choice can be made that will maximize the energy consumed by some finite time. On the other hand for some preferred value of \( T \), the \( W \) curve may have a relative maximum as a function of \( \phi \).

To investigate this possibility, we look for solutions to (15) other than the one already found.

Suppose, for example, we select those times for which \( \omega T = (2m + 1)\pi/2 \) with \( m = 0, 1, 2 \ldots \). If in addition \( \phi \leq \pi/2 \) is required, then the values of \( T \) so defined are finite and correspond, according to (1), to the points of tangency of the current curve with its exponentially decreasing envelopes. These points occur later in time than the corresponding current maxima or minima, \( \text{i.e.} \), later in time than the power maxima, and thus represent an integration to times well past the points of maximum power. The shortest interval \( T \), given by \( m = 0 \), will be the most interesting from standpoint of the exploding wire. It is worth noting that for \( \phi = \pi/2 \), the angular frequency vanishes, \( \text{i.e.} \), \( \omega = 0 \) and all \( T \) values for which \( \omega T = \text{const.} \) as \( \phi \to \pi/2 \) coincide with the point at infinity.

30
For the points of tangency, (15) vanishes if \(	an \phi = (2m + 1)\pi/4\). When \(m = 0\) we find \(\sin \phi = .618\) and \(R = 1.24(L/C)^{1/2}\). Since this value is about 10% larger than the value of \(R\) which gives maximum power consumption at peak current, we suspect a relative maximum of energy consumption, i.e., a maximum in the \(W\ vs\ R\) curve. Examination of (13) for values in the neighborhood of \(\phi = \tan^{-1}(\pi/4)\) while \(\omega T = \pi/2\) shows that this is indeed the case. Thus if integrated power, in other words total energy, released in the wire is the criterion of importance; then the optimum resistance \(R\) for the first point of tangency is slightly more than 10% larger than the value producing maximum power release at peak current. If the wire should last longer than the first half cycle, then even larger values of \(R\) would be appropriate corresponding to \(m \gg 1\); and in the limit of indefinitely many cycles, \(R\) approaches the critical damping value.

Should other values of interval \(T\) be chosen, then it is likely that different optimum values for \(R\) would be found, and it seems probable that these values will be larger or smaller than those just found depending on whether \(\omega T\) is taken larger or smaller than \((2m + 1)\pi/2\).
3. DISCUSSION

The conclusions which emerge from the preceding analysis of the under-damped R-L-C circuit are as follows:

(1) For maximum power dissipation at the first current peak a circuit resistance \( R = 1.10(L/C)^{1/2} \) should be used. This more precise value is 10% higher than that given by Anderson and Smith in 1926.

(2) For maximum energy dissipation in the interval terminated by the first contact of the current curve and its exponential envelope, the resistance should be \( R = 1.24(L/C)^{1/2} \). As the interval increases, the optimum value of resistance will increase toward the critical damping resistance.

It has not been our purpose here to demonstrate the relevance of these criteria for the experimental, exploding wire phenomenon. The complexity of this process defies exact analysis at present. This much may be said; the wire resistance is certainly not constant during the explosion, but must change rapidly and, in all probability, discontinuously with time. It may be conjectured that if the process is rapid enough to be essentially adiabatic then the larger value of (2) will apply; while if the explosion depends mainly on heat generated near the first current peak then the smaller value of (1) is appropriate. In intermediate cases the inequality \( 1.10 \leq R(L/C)^{-1/2} \leq 1.24 \) may be satisfied.
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 1            | Chief of Ordnance  
Department of the Army  
Washington 25, D.C.  
Attn: ORDTB - Bal Sec | 1            | Superintendent  
U.S. Naval Postgraduate School  
Monterey, California |
| 1            | Commanding Officer  
Diamond Ordnance Fuze Lab.  
Washington 25, D.C.  
Attn: ORDTFL - 012 | 1            | Director  
Air University  
Maxwell Air Force Base, Alabama  
Attn: Air University Library |
| 10           | Director  
Armed Services Technical Information Agency  
Arlington Hall Station  
Arlington 12, Virginia | 4            | Commander  
Wright Air Development Center  
Wright-Patterson Air Force Base, Ohio  
Attn: WCLGR-4 |
| 3            | British Joint Services Mission  
1800 K Street, N.W.  
Washington 6, D.C.  
Attn: Reports Officer | 3            | Director  
Advanced Research Lab.-GRD  
Air Force Cambridge Research Center  
L.G. Hanscom Field  
Bedford, Massachusetts  
Attn: W.G. Chace - CRZN  
M.A. Levine  
M. O'Day |
| 1            | Canadian Army Staff  
2450 Massachusetts Avenue  
Washington 8, D.C. | 1            | Director  
National Advisory Committee for Aeronautics  
Lewis Flight Propulsion Laboratory  
Cleveland Airport  
Cleveland, Ohio |
| 2            | Chief, Bureau of Ordnance  
Department of the Navy  
Washington 25, D.C.  
Attn: ReO | 1            | Director  
National Advisory Committee for Aeronautics  
Ames Laboratory  
Moffett Field, California  
Attn: Mr. V.J. Stevens  
Mr. Harvey Allen |
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 3 | Director  
National Advisory Committee for Aeronautics  
1512 H Street, N. W.  
Washington 25, D. C. |
| 2 | Director  
National Advisory Committee for Aeronautics  
Langley Memorial Aeronautical Laboratory  
Langley Field, Virginia |
| 1 | Technical Reports Library  
U. S. Atomic Energy Commission  
Washington 25, D. C.  
Attn: Director of Military Applications |
| 1 | Director, JPL Ord Corps Installation  
Department of the Army  
4800 Oak Grove Drive  
Pasadena, California  
Attn: Mr. Irl E. Newlan, Reports Group |
| 1 | Commanding General  
Frankford Arsenal  
Philadelphia 37, Pennsylvania  
Attn: Mr. Charles Lukens, Bldg. 150 |
| 1 | Commander  
Army Rocket and Guided Missile Agency  
Supporting Research Branch  
Redstone Arsenal, Alabama  
Attn: Capt. D. H. Steininger |
| 1 | Commanding General  
Army Ballistic Missile Agency  
Redstone Arsenal, Alabama  
Attn: Dr. T. A. Barr |
| 2 | Applied Physics Laboratory  
8621 Georgia Avenue  
Silver Spring, Maryland |
| 1 | California Institute of Technology  
Aeronautics Department  
Pasadena, California  
Attn: Prof. H. W. Liepmann |
| 1 | Cornell Aeronautical Lab.  
4455 Genesee Street  
Buffalo 5, New York  
Attn: Elma Evans, Librarian |
| 1 | Cornell University  
Graduate School of Aeronautical Engineering  
Ithaca, New York  
Attn: E. L. Resler |
| 1 | Carnegie Institute of Technology  
Department of Physics  
Pittsburgh 13, Pennsylvania  
Attn: Prof. E. M. Pugh |
| 1 | Combustion and Explosives Research, Inc.  
Alcoa Building  
Pittsburgh 19, Pennsylvania  
Attn: Prof. S. R. Brinkley |
| 1 | Case Institute of Technology  
Dept. of Mechanical Engineering  
University Circle  
Cleveland 6, Ohio  
Attn: Prof. G. Kuerti |
| 1 | Datamatic Corporation  
151 Needham Street  
Newton Highlands 61, Massachusetts  
Attn: Dr. R. F. Clippinger |
| 1 | Guggenheim Aeronautical Laboratory  
California Institute of Technology  
Pasadena, California  
Attn: Prof. L. Lees |
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Johns Hopkins University</td>
<td>1</td>
<td>Hartford Graduate Center</td>
</tr>
<tr>
<td></td>
<td>Dept. of Aeronautics</td>
<td></td>
<td>R. P. I.</td>
</tr>
<tr>
<td></td>
<td>Baltimore 18, Maryland</td>
<td></td>
<td>East Windsor Hill,</td>
</tr>
<tr>
<td></td>
<td>Attn: Prof. L. S. G. Kovaszny</td>
<td></td>
<td>Connecticut</td>
</tr>
<tr>
<td>1</td>
<td>James Forestal Research Center</td>
<td>1</td>
<td>Attn: Prof. R. Gordon</td>
</tr>
<tr>
<td></td>
<td>Princeton University</td>
<td></td>
<td>Campbell</td>
</tr>
<tr>
<td></td>
<td>Princeton, New Jersey</td>
<td></td>
<td>Stanford University</td>
</tr>
<tr>
<td></td>
<td>Of Interest to: Prof. S. Bogdonoff</td>
<td></td>
<td>Department of Mechanical</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Engineering</td>
</tr>
<tr>
<td>1</td>
<td>Lehigh University</td>
<td>1</td>
<td>Palo Alto, California</td>
</tr>
<tr>
<td></td>
<td>Dept. of Physics</td>
<td></td>
<td>Attn: Prof. D. Bershader</td>
</tr>
<tr>
<td></td>
<td>Bethlehem, Pennsylvania</td>
<td></td>
<td>United Aircraft Corporation</td>
</tr>
<tr>
<td></td>
<td>Attn: Prof. R. J. Emrich</td>
<td></td>
<td>Research Department</td>
</tr>
<tr>
<td>1</td>
<td>Massachusetts Institute of Technology</td>
<td>2</td>
<td>East Hartford 8, Connecticut</td>
</tr>
<tr>
<td></td>
<td>Dept. of Mechanical Engineering</td>
<td></td>
<td>University of Maryland</td>
</tr>
<tr>
<td></td>
<td>Cambridge 39, Massachusetts</td>
<td></td>
<td>Institute for Fluid</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Dynamics and Applied</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Mathematics</td>
</tr>
<tr>
<td>1</td>
<td>Massachusetts Institute of Technology</td>
<td>1</td>
<td>College Park, Maryland</td>
</tr>
<tr>
<td></td>
<td>Gas Turbine Laboratory</td>
<td></td>
<td>Attn: A. Weinstein</td>
</tr>
<tr>
<td></td>
<td>Cambridge 39, Massachusetts</td>
<td></td>
<td>S. I. Pai</td>
</tr>
<tr>
<td>1</td>
<td>Director Moore School of Electrical Engineering</td>
<td>1</td>
<td>University of Michigan</td>
</tr>
<tr>
<td></td>
<td>University of Pennsylvania</td>
<td></td>
<td>Department of Physics</td>
</tr>
<tr>
<td></td>
<td>Philadelphia 4, Pennsylvania</td>
<td></td>
<td>Ann Arbor, Michigan</td>
</tr>
<tr>
<td></td>
<td>Attn: Prof. S. Gorn</td>
<td></td>
<td>Attn: Prof. Otto Laporte</td>
</tr>
<tr>
<td>1</td>
<td>North American Aviation, Inc.</td>
<td>1</td>
<td>University of California</td>
</tr>
<tr>
<td></td>
<td>12214 Lakewood Boulevard</td>
<td></td>
<td>Low Pressures Research Project</td>
</tr>
<tr>
<td></td>
<td>Downey, California</td>
<td></td>
<td>Berkeley, California</td>
</tr>
<tr>
<td>1</td>
<td>Pennsylvania State University Physics Department</td>
<td>2</td>
<td>Attn: Prof. S. A. Schaaf</td>
</tr>
<tr>
<td></td>
<td>State College, Pennsylvania</td>
<td></td>
<td>University of Michigan</td>
</tr>
<tr>
<td></td>
<td>Attn: Prof. R. G. Stoner</td>
<td></td>
<td>Aeronautical Research Center</td>
</tr>
<tr>
<td>2</td>
<td>Purdue University</td>
<td>1</td>
<td>Willow Run Airport</td>
</tr>
<tr>
<td></td>
<td>Departmental of Mechanical Engineering</td>
<td></td>
<td>Ypsilanti, Michigan</td>
</tr>
<tr>
<td></td>
<td>Lafayette, Indiana</td>
<td></td>
<td>University of Oklahoma</td>
</tr>
<tr>
<td></td>
<td>Attn: Mr. Vollmer E. Bergdolt</td>
<td></td>
<td>Dept. of Physics</td>
</tr>
<tr>
<td></td>
<td>Prof. R. C. Binder</td>
<td></td>
<td>Norman, Oklahoma</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Attn: Prof. R. G. Fowler</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>University of Illinois</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Aeronautical Institute</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Urbana, Illinois</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Attn: Prof. B. L. Hicks</td>
</tr>
<tr>
<td>No. of Copies</td>
<td>Organization</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>--------------</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| 1             | Professor W. Bleakney  
                Palmer Physical Laboratory  
                Princeton University  
                Princeton, New Jersey |
| 1             | Professor F. H. Clauser, Jr.  
                Department of Aeronautics  
                Johns Hopkins University  
                Baltimore 18, Maryland |
| 1             | Professor H. W. Emmons  
                Harvard University  
                Cambridge 38, Massachusetts |
| 2             | Professor J. O. Hirschfelder  
                Department of Chemistry  
                University of Wisconsin  
                Madison 6, Wisconsin |
| 1             | Dr. A. E. Puckett  
                Hughes Aircraft Company  
                Culver City, California |
| 1             | Office of Technical Services  
                Department of Commerce  
                Washington 25, D. C. |
| 1             | Dr. Carl A. Rouse  
                Theoretical Division  
                UCRL  
                Livermore, California |
| 1             | Commander  
                Air Proving Ground Center  
                Eglin Air Force Base  
                Florida  
                Attn: PGEM |
Streak camera and oscillographic circuit damping data are presented for copper wires varying in diameter from 3 to 8 mils. A maximum of specific shock wave energy in the induced flow is found at a wire diameter different from that of a minimum in the total damping time of the circuit. This displacement is shown to be caused by the presence of residual circuit resistance. The argument is based on a critical analysis of optimum damping conditions in the exploding wire circuit. A maximum of apparent energy within the contact surface appears at about the same wire diameter as the minimum of total damping time. Discussion of the implications of the Taylor-Lin similarity theory indicates that lack of similarity of the flow is probably connected with the displacement of the maximum energies associated with shock wave and contact surface.

Bennett