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COMPRESSIVE BUCKLING CURVES FOR SANDWICH PANELS WITH ISOTROPIC FACINGS AND ISOTROPIC OR ORTHOTROPIC CORES

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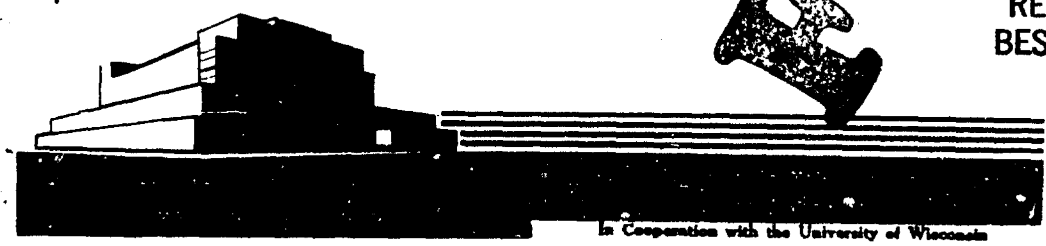

In Cooperation with the University of Wisconsin

COMPRESSIVE BUCKLING CURVES FOR FLAT SANDWICH PANELS WITH ISOTROPIC FACINGS AND ISOTROPIC OR ORTHOTROPIC CORES[1]

By

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Abstract

~~This report presents~~ curves and formulas for use in computing the buckling of flat panels of sandwich construction under edgewise compressive loads, are presented. Included are curves for constructions with isotropic facings and either isotropic or orthotropic cores.

Introduction

The derivation of formulas for the buckling loads of rectangular sandwich panels subjected to edgewise compression is given in Forest Products Laboratory Report No. 1583-B.[3] These formulas apply to panels with orthotropic facings and cores. They are given also in section 4.2.1.1 of ANC

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[1] This progress report is one of a series (ANC-23, Item 57-3) prepared and distributed by the Forest Products Laboratory under U. S. Navy Bureau of Aeronautics Nos. NAer 01628 and 01684 and U. S. Air Force No. AF 18(600)-102, Amendment A6(55-286). Results here reported are preliminary and may be revised as additional data become available.

[2] Maintained at Madison, Wis., in cooperation with the University of Wisconsin.

[3] Ericksen, W. S. and March, H. W. Effects of Shear Deformation in the Core of a Flat Rectangular Sandwich Panel. Compressive Buckling of Sandwich Panels Having Facings of Unequal Thickness. Forest Products Laboratory Report No. 1583-B, 1950.

Bulletin 23, Part II.[4] These formulas, reduced to apply to sandwich panel with isotropic facings and cores, are given with design curves in section 3.2.1.1 of the ANC bulletin. In the computation of these curves, the stiffness of the individual facings was neglected, so that the values of the parameters for each edge condition could be represented by a single family of curves.

In the determination of design values of the mechanical properties of honeycomb cores of cells of hexagonal shape, it was found that the modulus of rigidity associated with the directions perpendicular to the foil ribbons of which the honeycomb is made and the length of the cells is roughly 40 percent of the modulus associated with the directions parallel to these foil ribbons and the length of the cells.[5] This fact makes it possible to represent parameters for the design of sandwich panels with isotropic facings and honeycomb cores by a reasonable number of families of curves. Also, these curves, along with those for isotropic facings and cores, provide means of estimating critical loads for sandwich panels with some other orthotropic core materials.

A method is included for determining from these curves the buckling stress of sandwich panels for which the stiffness of the individual facings is not neglected.

Formulas Used

The notation used in this report is illustrated in the sketch of figure 1. The lengths of the edges of the panels are denoted by $\underline{a}$ and $\underline{b}$. The load is applied to the edges of length $\underline{a}$ and acts in the direction parallel to the edges of length $\underline{b}$. The natural axes of the orthotropic core are parallel to the edges $\underline{a}$ and $\underline{b}$. The modulus of rigidity of the core $\underline{G_{C\beta z}}$ is associated with the axes in the direction parallel to edges $\underline{b}$ and in the direction perpendicular to the panel. The modulus of rigidity $\underline{G_{C\alpha z}}$ is associated with the axes in the direction parallel to edges $\underline{a}$ and in the direction perpendicular to the panel.

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[4] U. S. Forest Products Laboratory. Sandwich Construction for Aircraft. ANC Bulletin 23, Part II. Second Edition, 1955.

[5] Kuenzi, E. W. Mechanical Properties of Aluminum Honeycomb Cores. Forest Products Laboratory Report 1849. 1955.

The value of the stress in the facings at which panel buckling occurs is given in the nomenclature of ANC Bulletin 23,[4] by:

$$f_{Fcr} = E_F \frac{\pi^2 D}{a^2 H} K \tag{1}$$

Assuming that the modulus of elasticity of the core is so small that its stiffness can be neglected, then $\underline{H}$ and $\underline{D}$ are defined by

$$H = E_F (t - t_C) \tag{2}$$

$$D = \frac{E_F t_{F1} t_{F2} (t + t_C)^2}{4\lambda_F (t - t_C)} \tag{3}$$

where

$$\lambda_F = 1 - \mu_F{}^2$$

and $\underline{t}$, $\underline{t_C}$, $\underline{t_{F1}}$, and $\underline{t_{F2}}$ are thicknesses of the sandwich, the core, one facing, and the other facing, respectively, and $\underline{E_F}$ and $\underline{\mu_F}$ are the modulus of elasticity and Poisson's ratio for the facings. The value of $\underline{K}$ is given by

$$K = K_F + K_M \tag{4}$$

in which

$$K_F = \frac{E_F}{\lambda_F} \frac{t_{F1}{}^3 + t_{F2}{}^3}{12D} (C_1 + 2C_2 + C_3) \tag{5}$$

and

$$K_M = \frac{C_1 + 2C_2 + C_3 + VA\left(\frac{r}{C_4} + 1\right)}{1 + V\frac{r}{C_4}\left(C_1 + \frac{C_2}{3}\right) + V\left(C_3 + \frac{C_2}{3}\right) + V^2 \frac{rA}{C_4}} \tag{6}$$

Substitution of equation (3) into equation (5) results, after simplification, in:

$$K_F = B(C_1 + 2C_2 + C_3) \tag{7}$$

In formulas (5), (6), and (7),

$$A = C_1C_3 - C_2^{\,2} + \frac{C_2}{3}(C_1 + 2C_2 + C_3) \tag{8}$$

$$B = \frac{1}{3}\left(\frac{t_{F1}}{t_{F2}} + \frac{t_{F2}}{t_{F1}} - 1\right)\left(\frac{t - t_c}{t + t_c}\right)^2 \tag{9}$$

$$V = \frac{\pi^2\, t_c t_{F1} t_{F2} E_F}{\lambda_F\, a^2\, (t - t_c)\, G_{C\beta z}} \tag{10}$$

$$r = \frac{G_{C\beta z}}{G_{C\alpha z}} \tag{11}$$

The values of $\underline{C_1}$, $\underline{C_2}$, $\underline{C_3}$, and $\underline{C_4}$ depend upon the ratio $\underline{b/a}$, the number of half-waves ($\underline{n}$) into which the panel buckles, and the panel edge conditions.

For a panel with all edges simply supported:

$$C_1 = C_4 = \frac{b^2}{n^2a^2}, \qquad C_2 = 1, \qquad C_3 = \frac{n^2a^2}{b^2}$$

For a panel with loaded edges simply supported and other edges clamped:

$$C_1 = \frac{16}{3}\,\frac{b^2}{n^2a^2}, \qquad C_2 = \frac{4}{3}, \qquad C_3 = \frac{n^2a^2}{b^2}, \qquad C_4 = \frac{4}{3}\,\frac{b^2}{n^2a^2}$$

For a panel with loaded edges clamped and other edges simply supported:

$$C_1 = C_4 = \frac{3}{4}\frac{b^2}{a^2} \quad \text{for } n = 1$$

$$C_1 = C_4 = \frac{1}{n^2 + 1}\,\frac{b^2}{a^2} \quad \text{for } n > 1$$

$$C_2 = 1, \quad C_3 = \frac{n^4 + 6n^2 + 1}{n^2 + 1} \frac{a^2}{b^2}$$

For a panel with all edges clamped:

$$C_1 = 4C_4 = 4\frac{h^2}{a^2} \quad \text{for } n = 1$$

$$C_1 = 4C_4 = \frac{16}{3(n^2 + 1)} \frac{b^2}{a^2} \quad \text{for } n > 1$$

$$C_2 = \frac{4}{3}, \quad C_3 = \frac{n^4 + 6n^2 + 1}{n^2 + 1} \frac{a^2}{b^2}$$

The critical buckling stress, f_{Fcr}, given by equation (1) is that for which an integral value of n ($n = 1, 2, 3 \ldots$) gives a minimum value of K in equation (4). Conservative but reasonably accurate values will be obtained if K_F and K_M are minimized separately with respect to n and these minimum values added. This procedure reduces greatly the number of curves required for design purposes.

Curves giving minimum values of K_M for various values of the parameters involved are given in figures 2 to 13. For most values of b/a, K_F is small and K_M is an excellent approximation of K. For small values of b/a, K_F can contribute and then K_F can be computed by the formula

$$K_F = BK_{MO} \tag{12}$$

where K_{MO} is determined from equation (6) or the curves of figures 2 to 13 for $V = 0$. Then K is determined by equation (4).

Values of K_{MO} for small values of b/a are not included in figures 2 to 13 because they are too great to be conveniently plotted. Figure 13 supplies these values in a logarithimic plot.

Discussion of Design Curves

Each family of curves in figures 1 to 12 consists of a plot of K_M against b/a for various values of V. The families differ because they apply to different edge conditions and different values of r. Each individual curve is really a subfamily, since there is a complete curve of K_M for b/a for each value of n; but, because the critical value of K_M is given by the particular curve for the n that gives the least value of K_M, only the portions of the curves that show least values are plotted. The parameter b/a is used in the left half of the curve sheets, and the parameter a/b in the right half. Thus values of K_M for values of b/a from zero to infinity may be read.

When b/a is zero, K_M is equal to $1/V$ and n is unity. As the value of V increases, the value of K_M decreases. There is a value of V that first causes K_M to equal $1/V$ for all values of b/a and causes n to become infinite. This particular value is indicated on each curve sheet by the plotted straight horizontal line. For greater values of V, K_M remains equal to $1/V$ and n remains infinite. The value of K_M equal to $1/V$ is known as the shear instability value.

Figures 2, 3, and 4 exhibit families of curves for panels simply supported at their edges. In figure 2, r has the value of 0.4. In figure 3, r is unity, and in figure 4, r has the value 2.5. Values of K_M for other values of r may be estimated by reading from the curves the K_M values for these three values of r and plotting them against r. The estimate may be made by reading the value of K_M at the required value of r from a smooth curve sketched through the points.

Similarly, figures 5, 6, and 7 apply to panels with the loaded edges simply supported and the other edges clamped; figures 8, 9, and 10 apply to panels with the loaded edges clamped and the other edges simply supported; and figures 11 1[illegible] nd 13 apply to panels with all edges clamped.

The curves for V equal to zero are, of course, independent of r, and only one curve is obtained for each set of edge conditions. For convenience, these curves are duplicated in the figures applying to the same edge conditions.

Figure 14 is a logarithimic plot of these four curves for small values of b/a and supplements figures 2 to 13 in this range. These curves are particularly useful in supplying values of K_{MO} for the determination of K_F by means of equation (12). The dashed lines in figure 14 are useful in determining values of K_{MO} at still smaller values of a/b. The abscissas are divided by multiples of 10, the ordinates are multiplied by these same multiples of 10, and the relations indicated by these lines are used.

Figure 15 shows a comparison of K_M with K calculated by this method for a panel having simply supported edges, r equal to unity and a B of 0.01. It shows that, though K_F is very small for most values of b/a, it should not be ignored when these values are small.

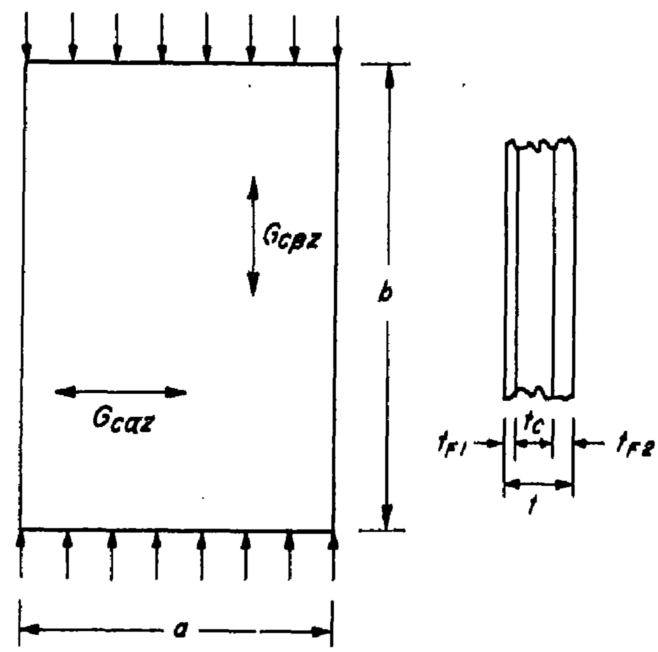

Figure 1. --Notation for sandwich panel dimensions and core moduli of rigidity.

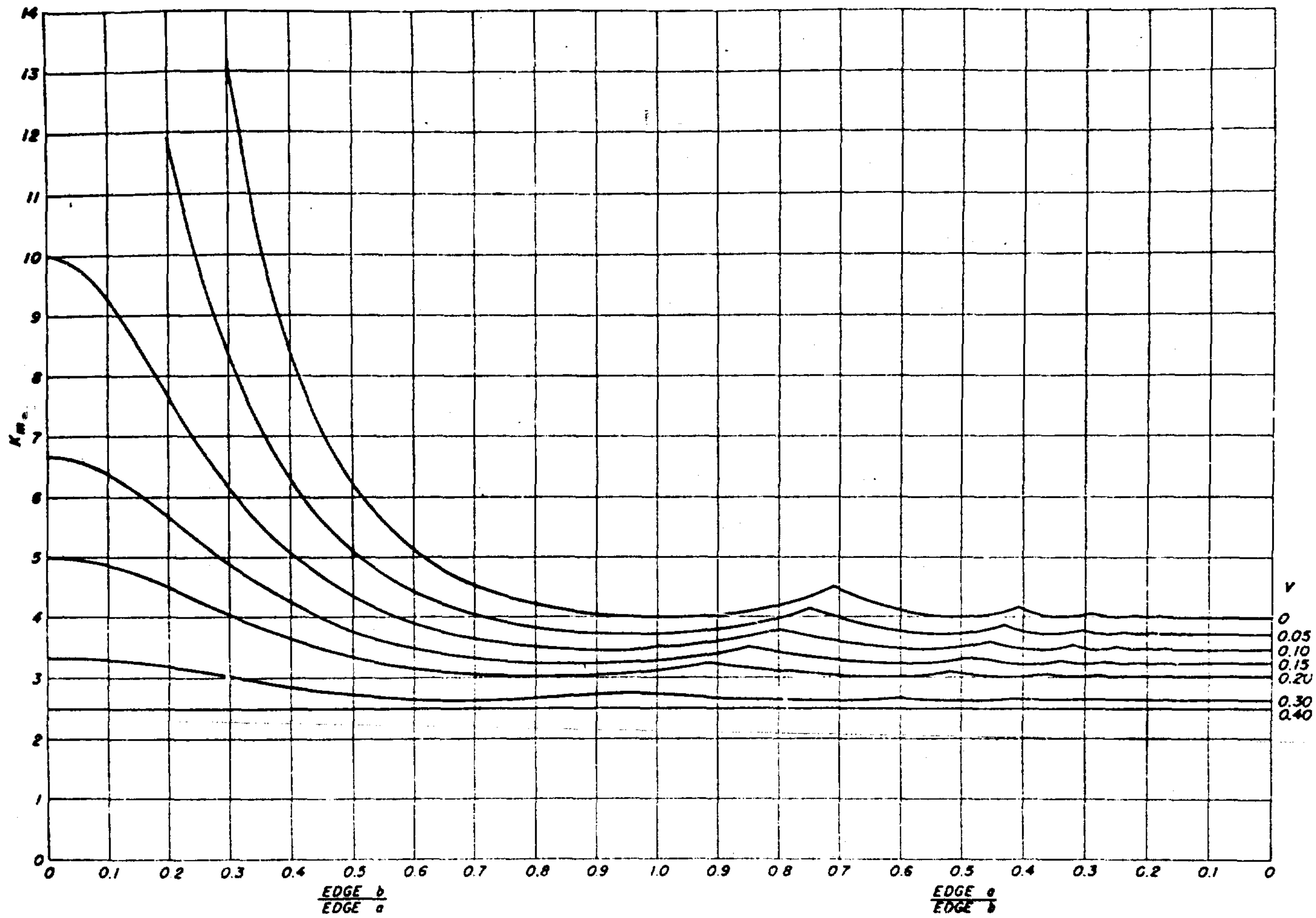

Figure 2. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges simply supported; $r = 0.4$.

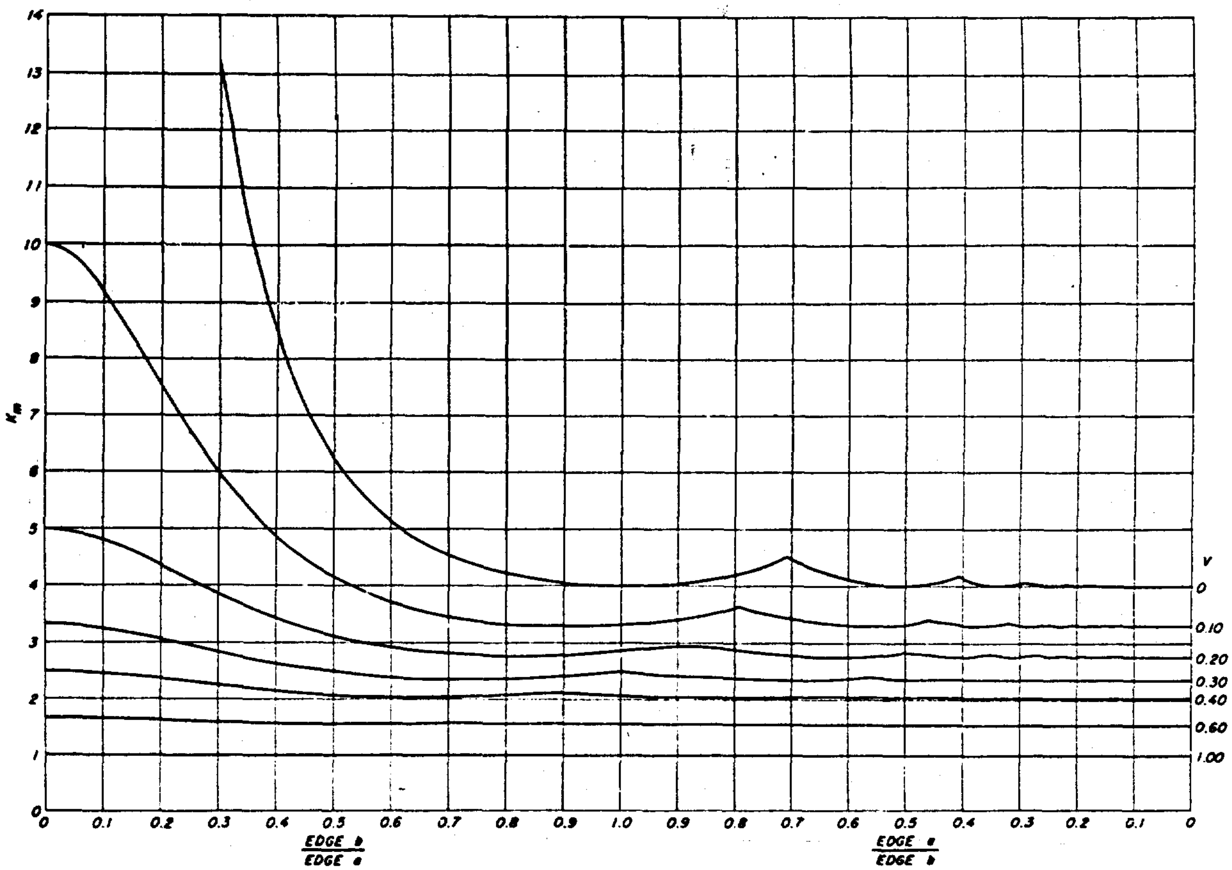

Figure 3. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges simply supported; $\overline{r} = 1.0$.

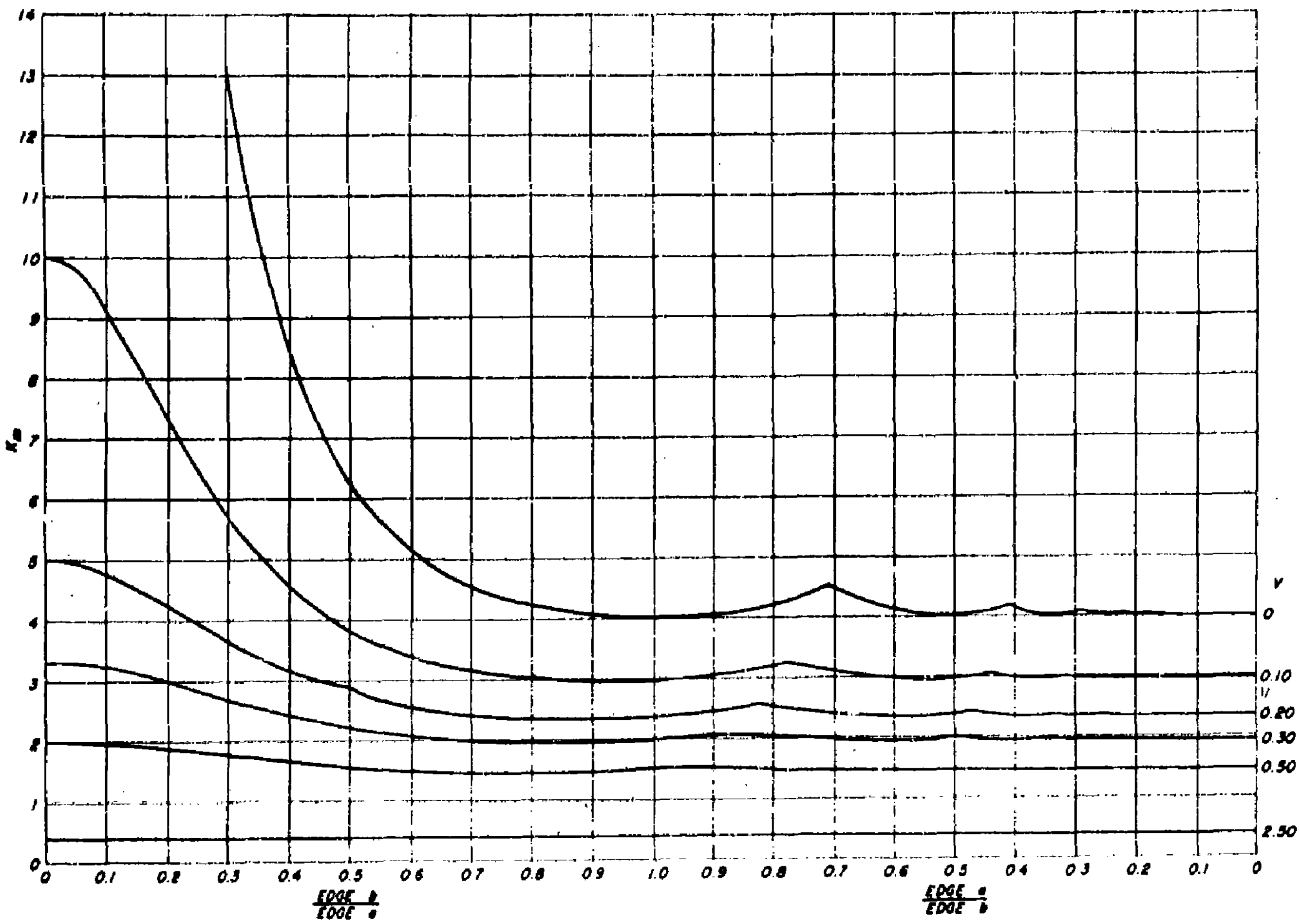

Figure 4. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges simply supported; $\bar{r}$ = 2.5.

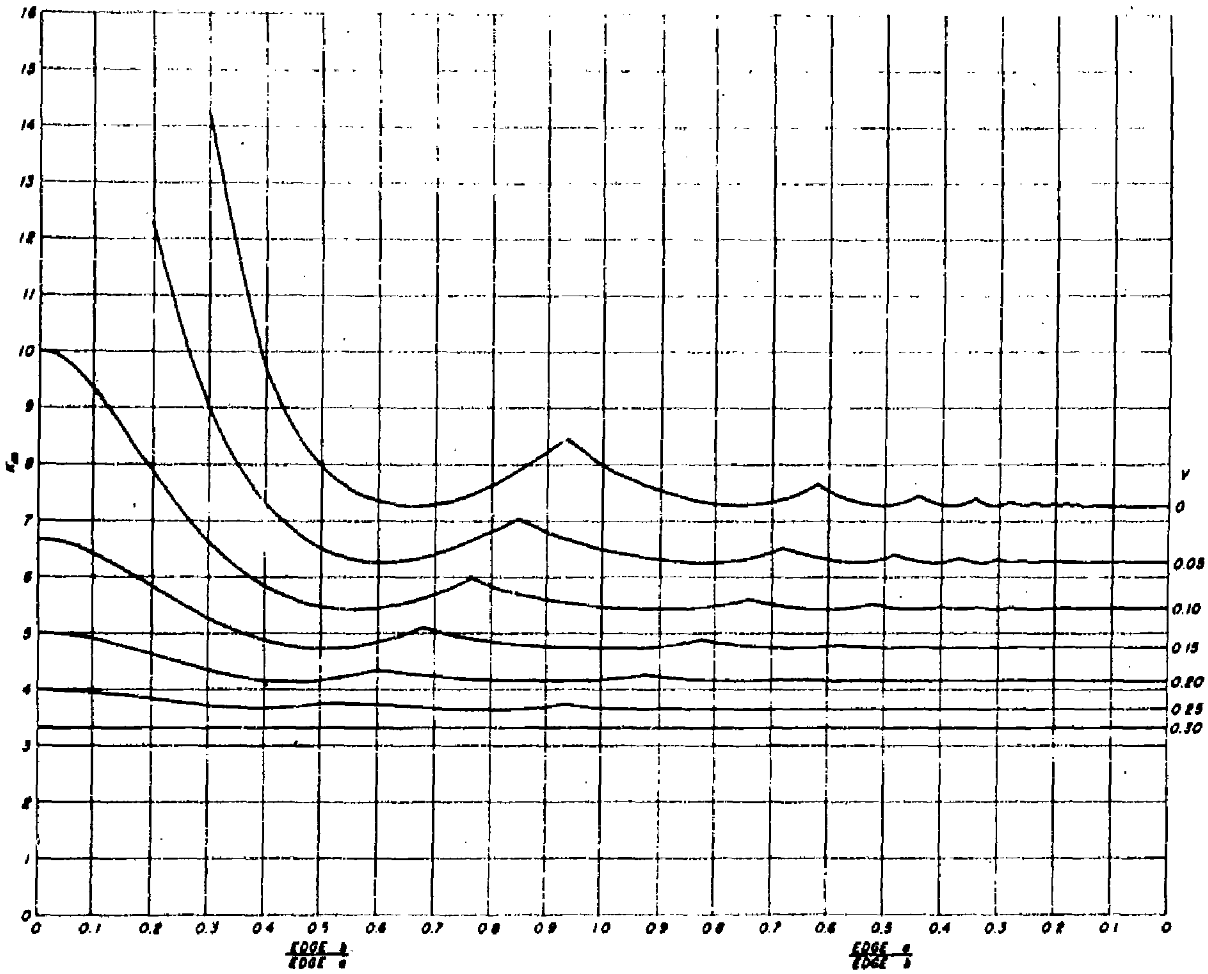

Figure 5. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. Loaded edges simply supported, other edges clamped; r = 0.4.

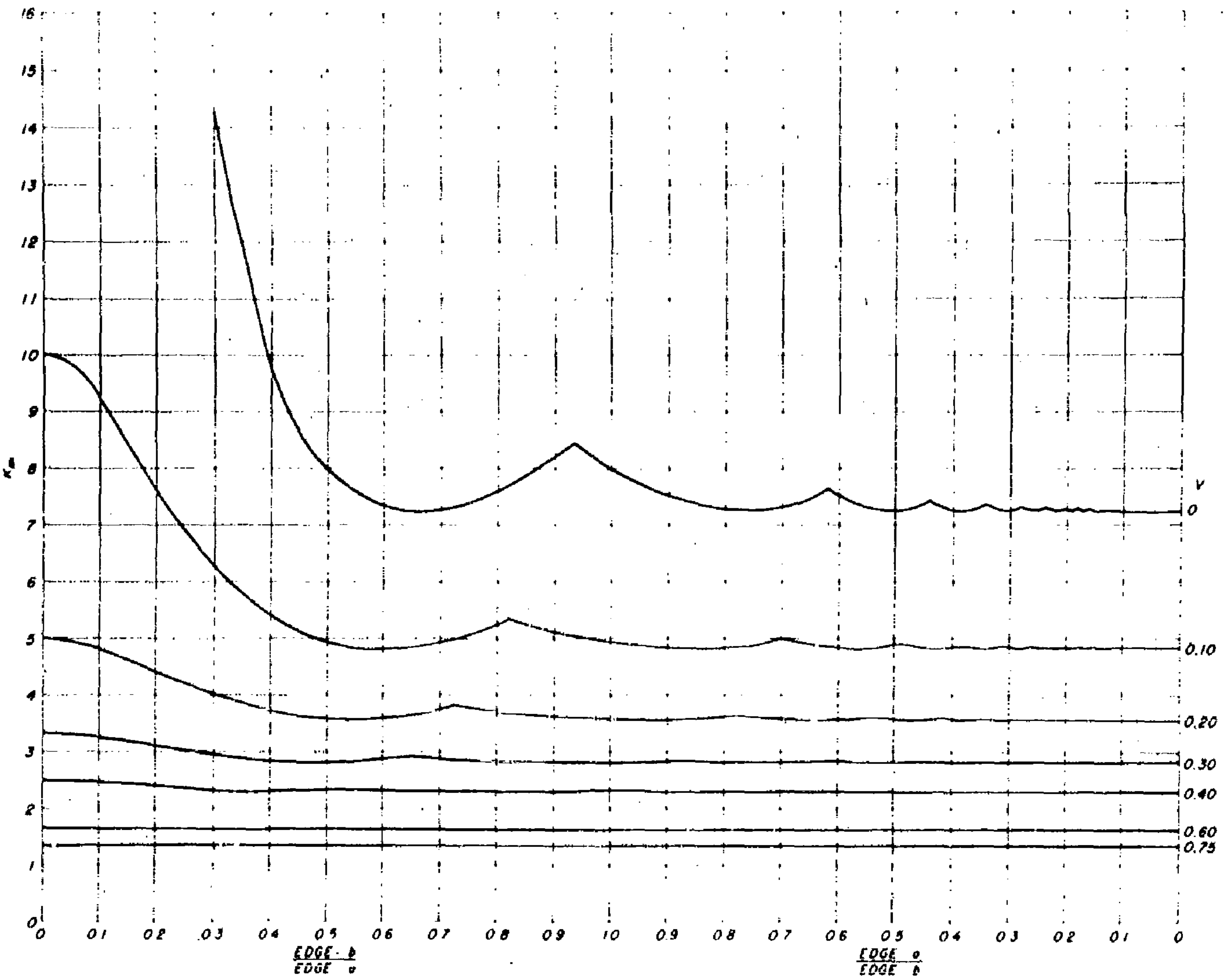

Figure 6. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. Loaded edges simply supported, other edges clamped; r = 1.0.

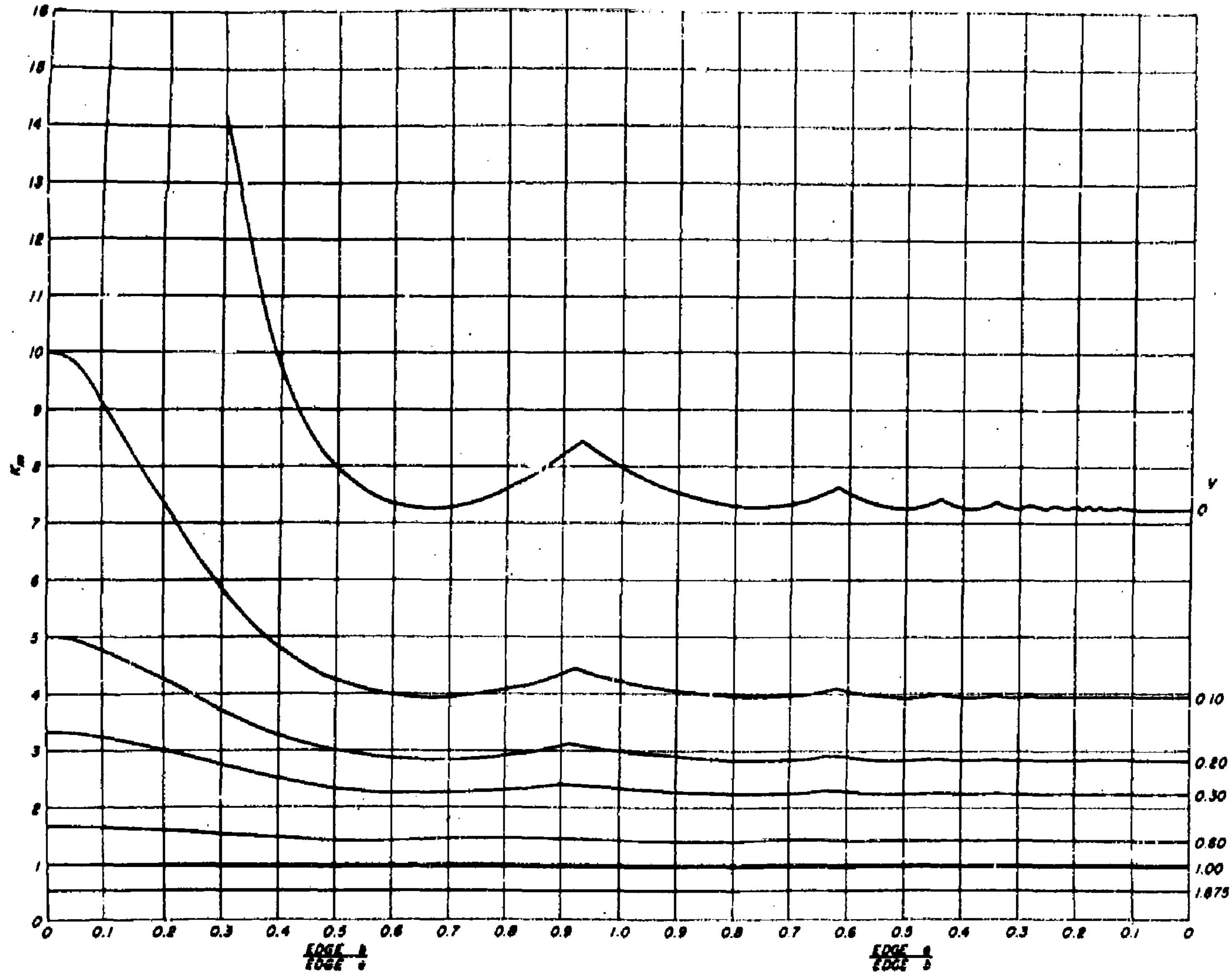

Figure 7. --Values of the parameter $\overline{K_M}$ from equation (6) plotted against the aspect ratio of the panel. Loaded edges simply supported, other edges clamped; r = 2.5.

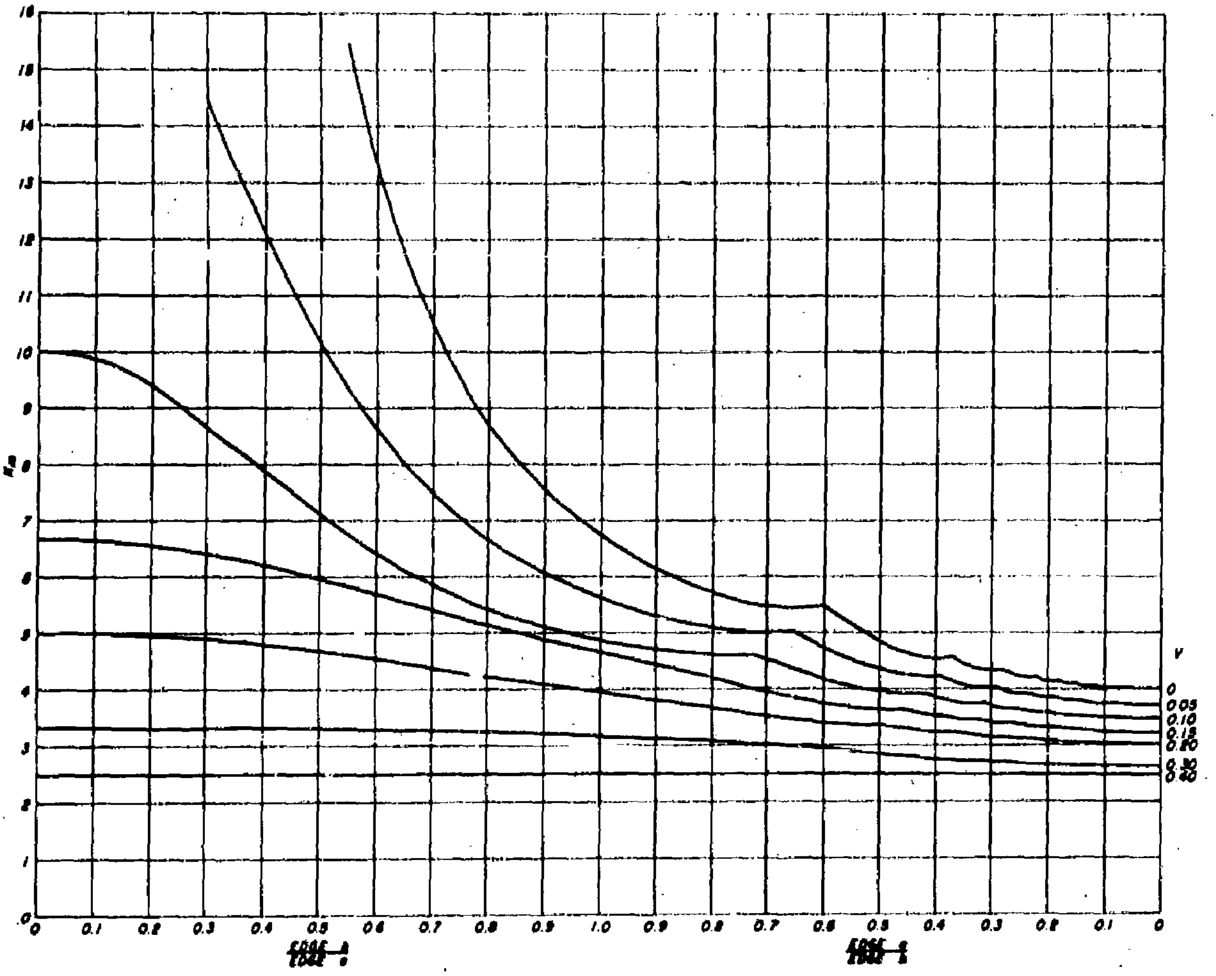

Figure 8. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. Loaded edges clamped, other edges simply supported; r = 0.4.

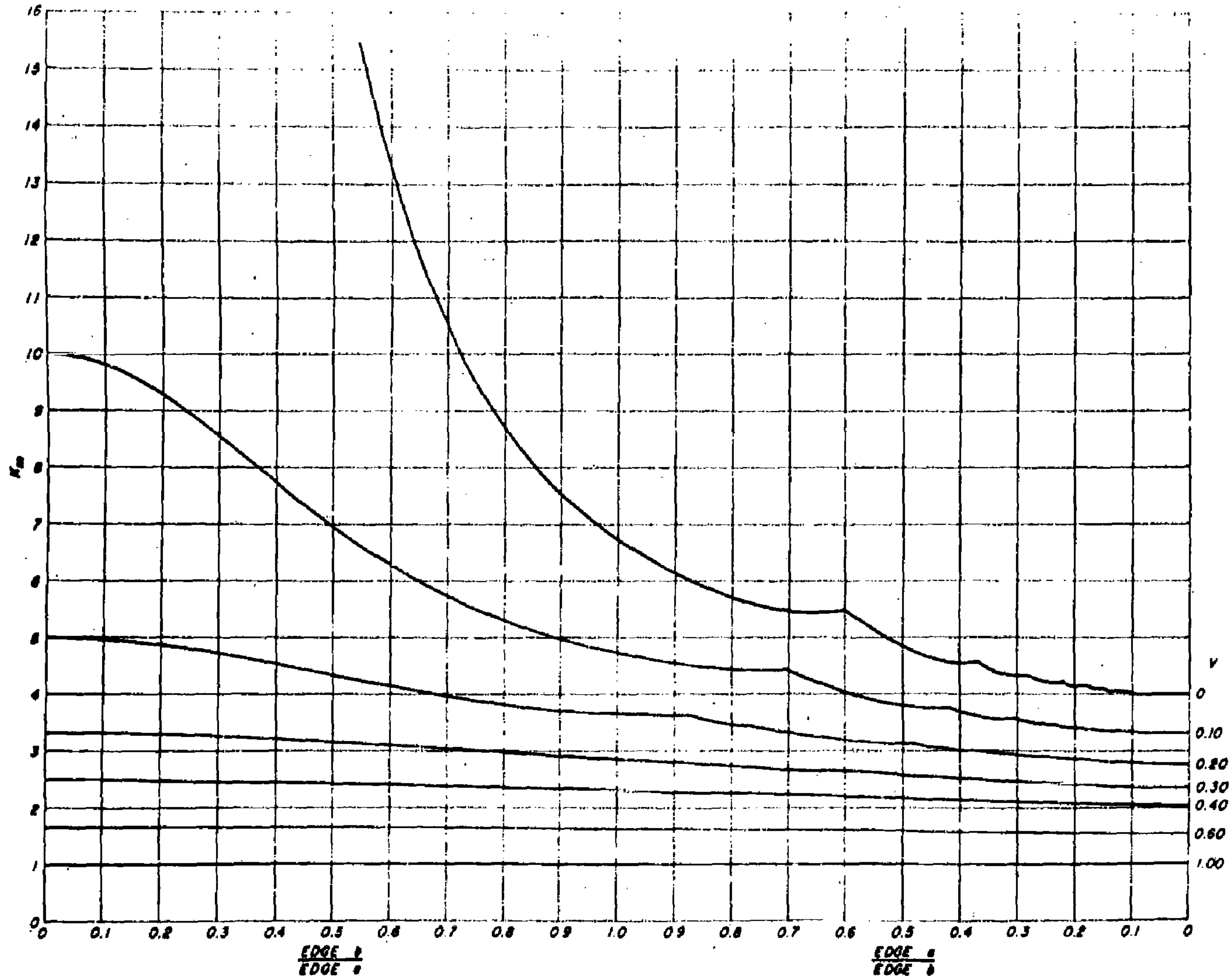

Figure 9. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. Loaded edges clamped, other edges simply supported; $r = 1.0$.

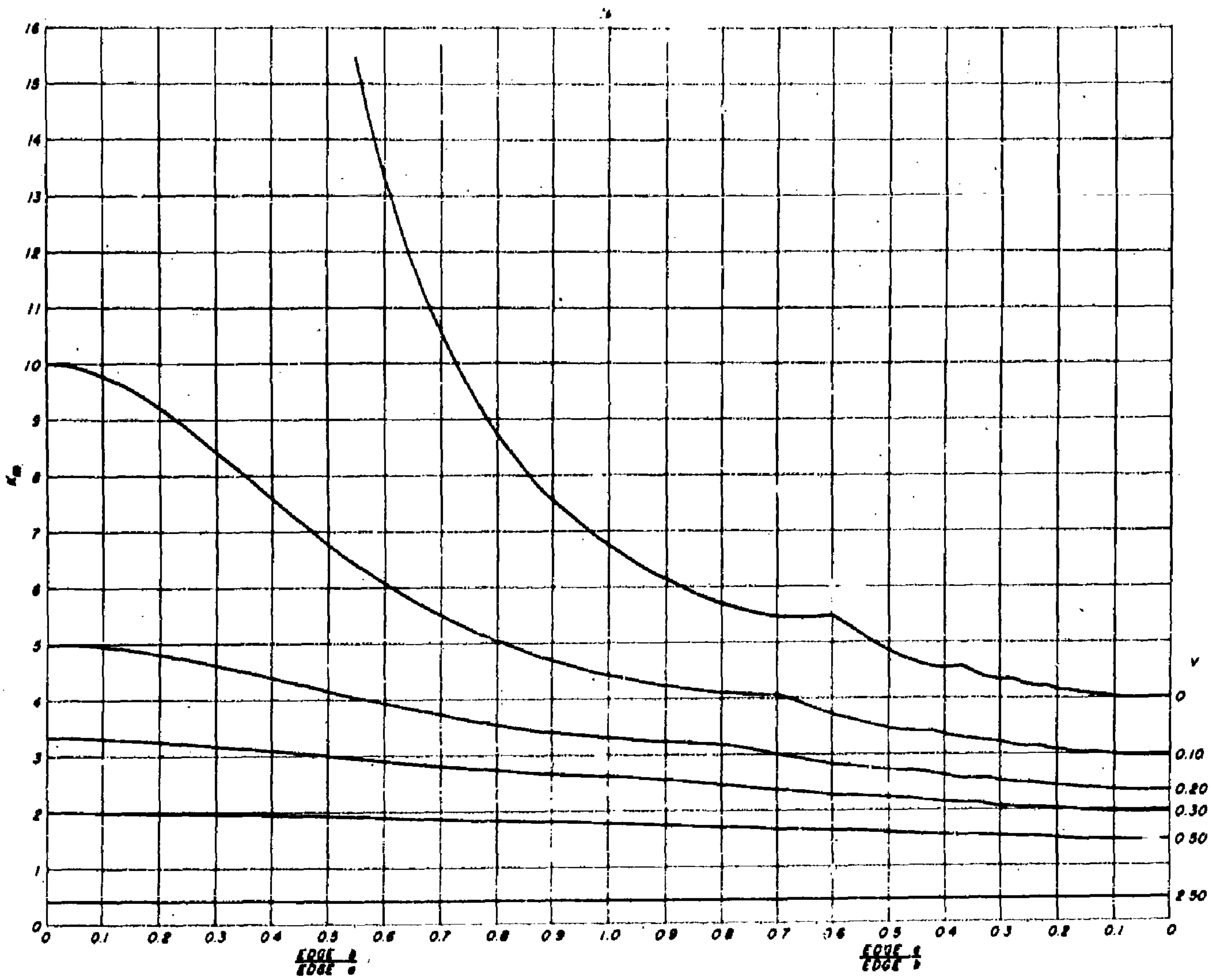

Figure 10. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. Loaded edges clamped, other edges simply supported; $r = 2.5$.

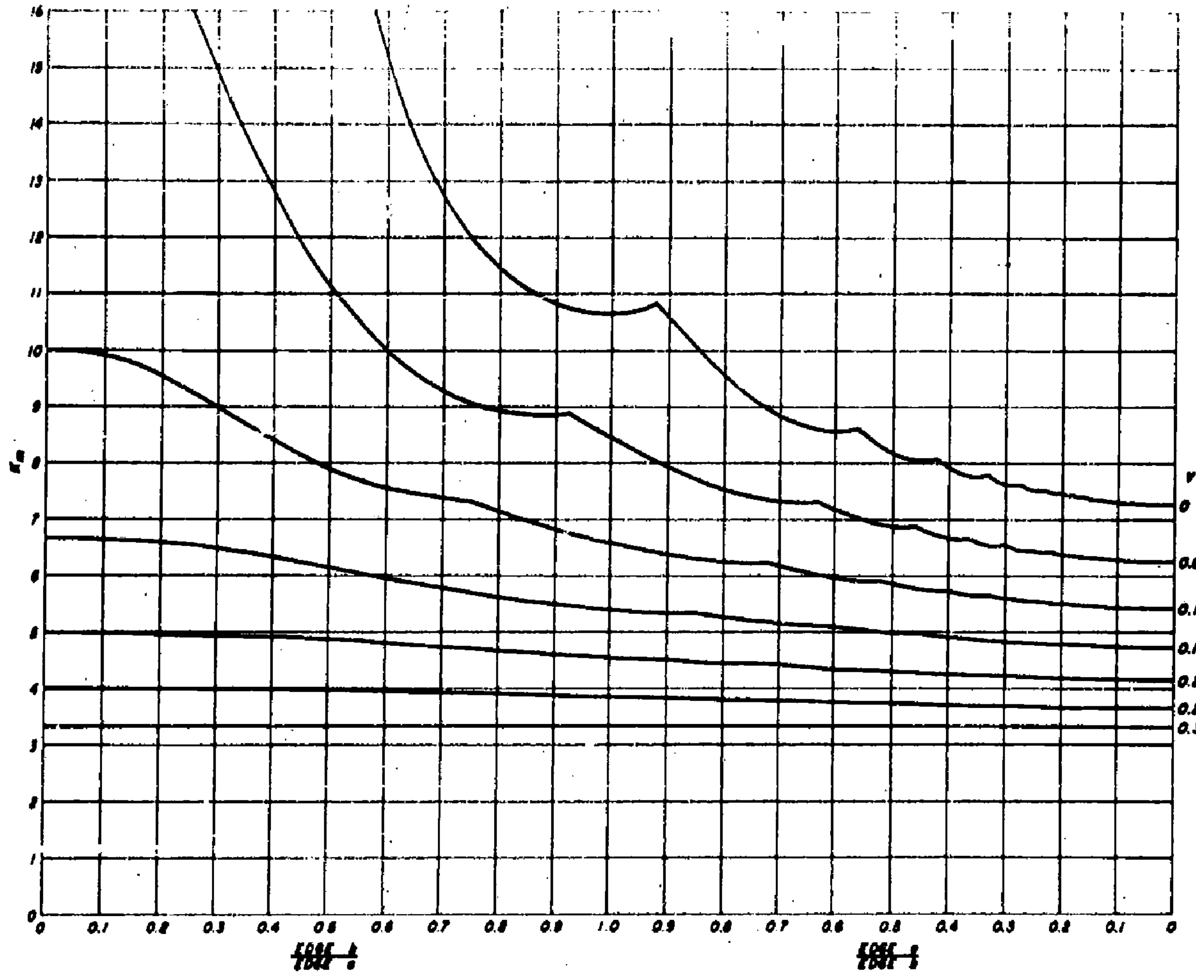

Figure 11. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges clamped; r = 0.4.

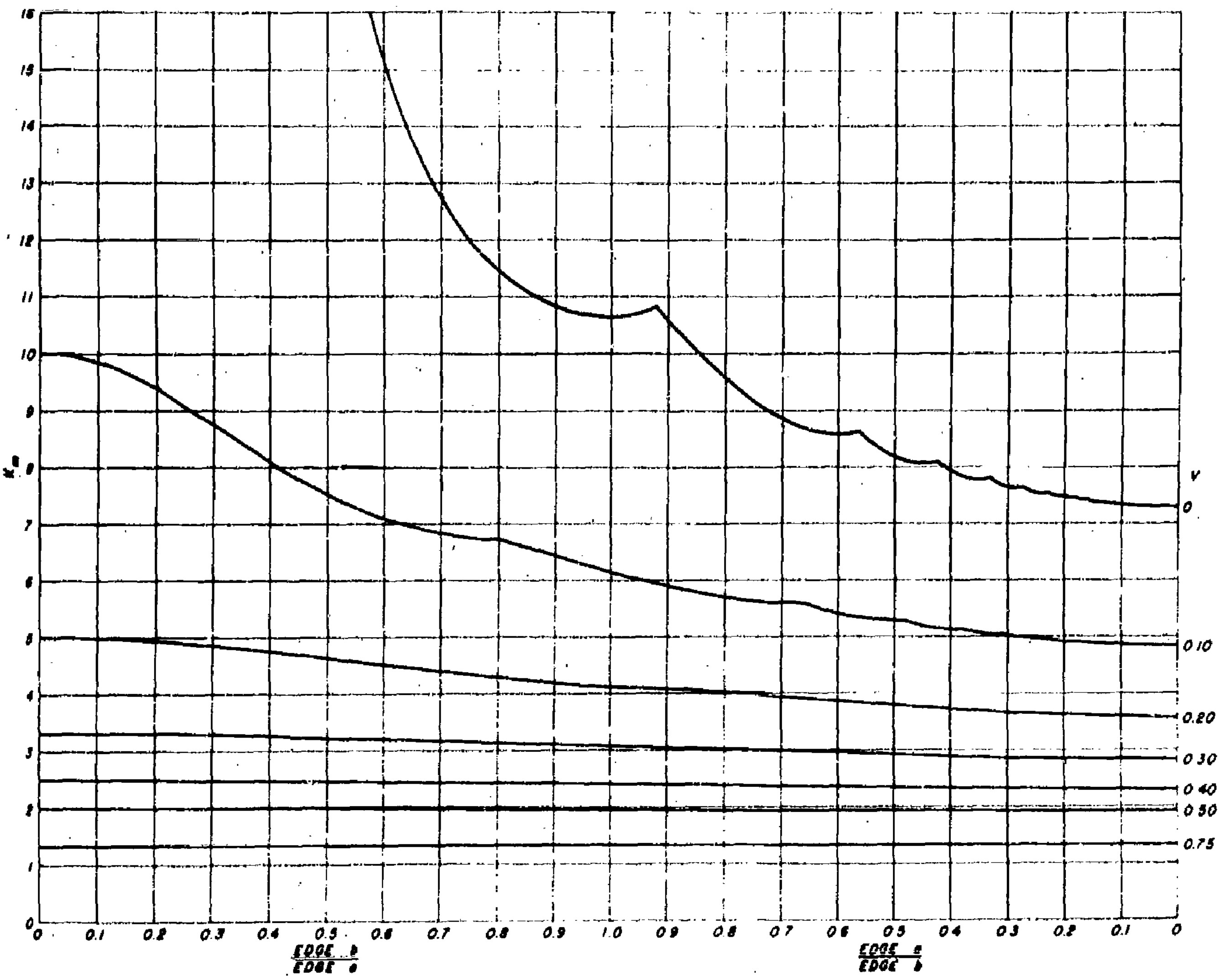

Figure 12.--Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges clamped; $r = \overline{1.0}$.

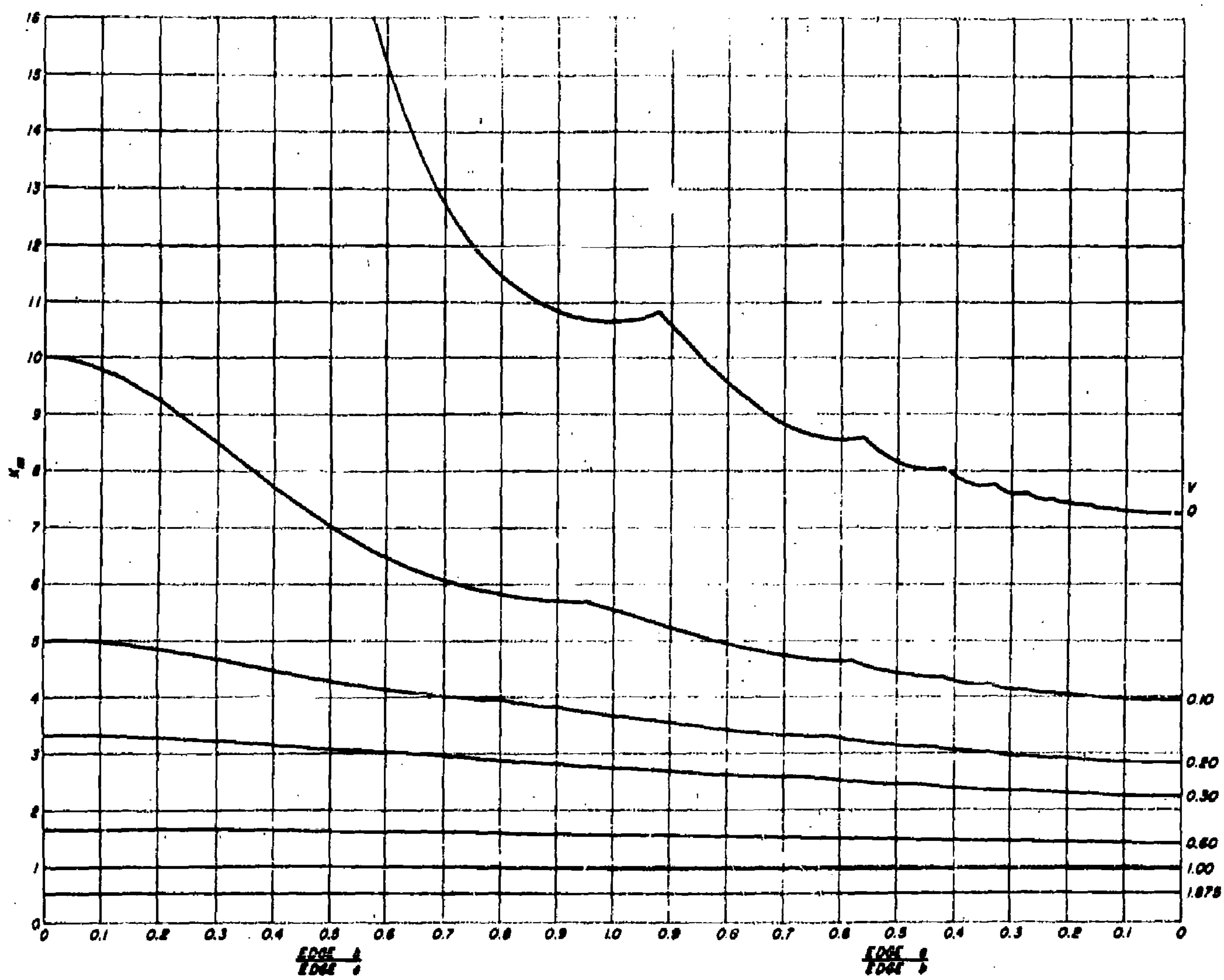

Figure 13. --Values of the parameter K_M from equation (6) plotted against the aspect ratio of the panel. All edges clamped; r = 2.5.

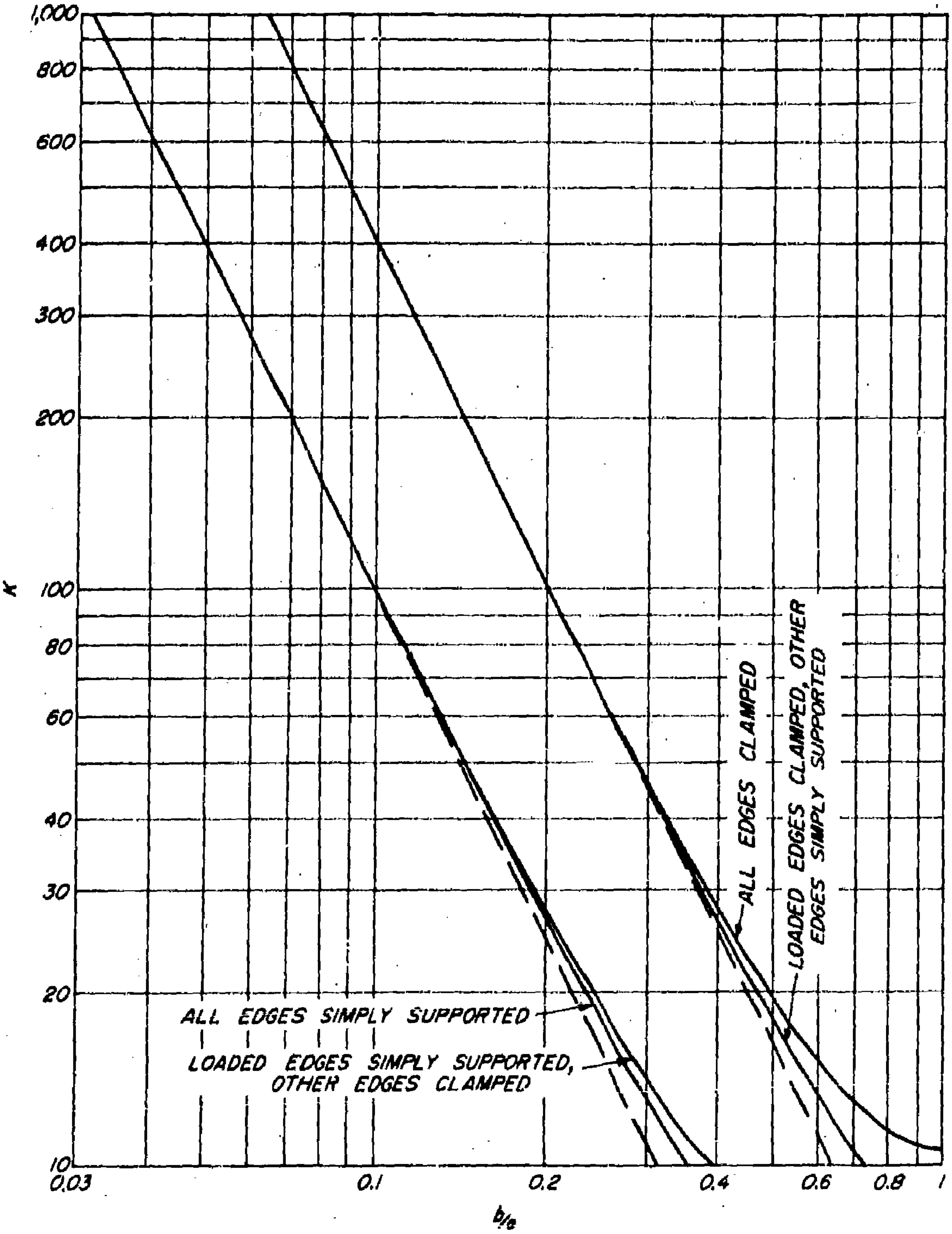

Figure 14. --Values of $\underline{K_M}$ for $\underline{V}$ equal to zero ($\underline{K_{MO}}$) for the four sets of edge conditions

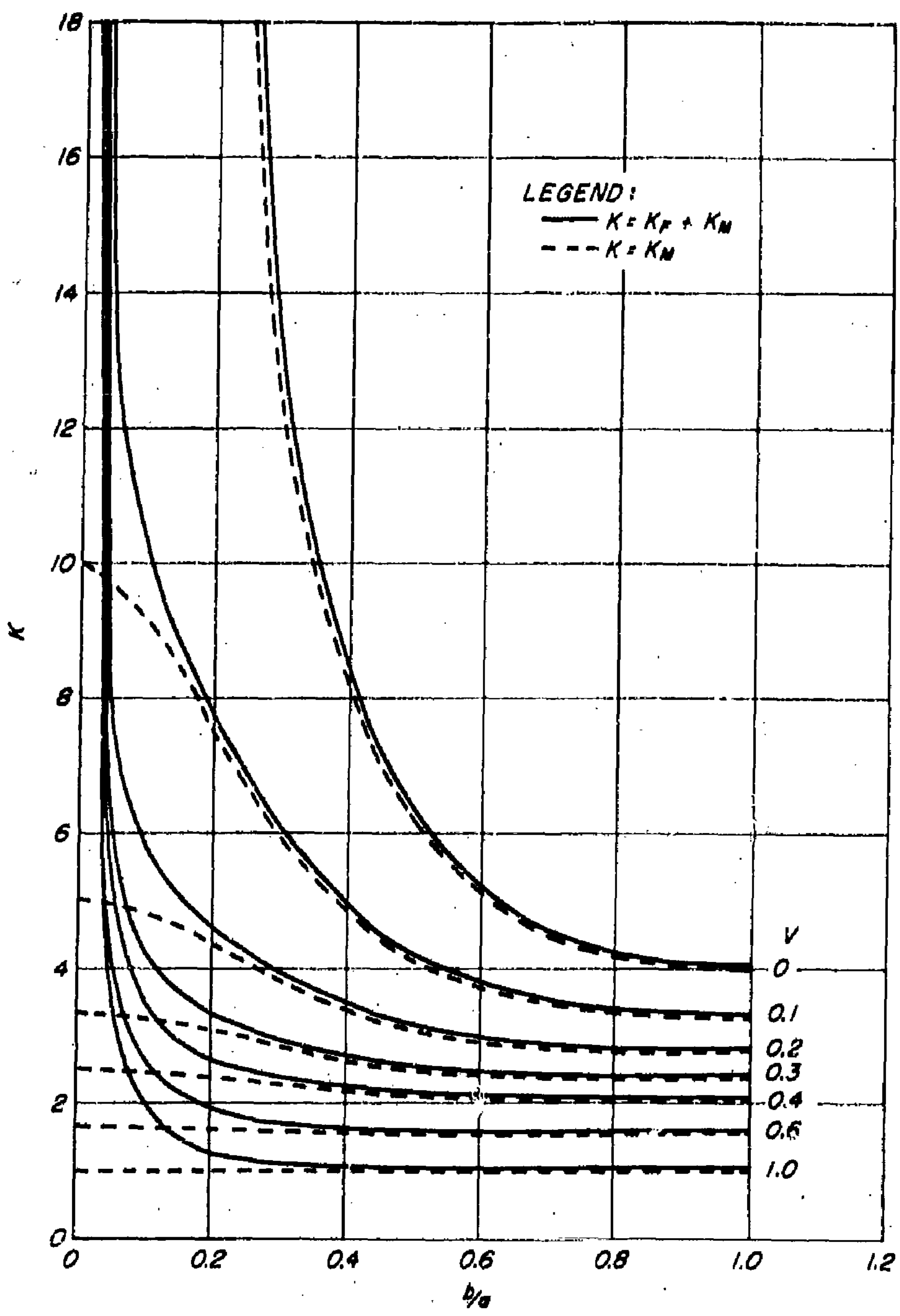

Figure 15. --Comparison of $\underline{K}$ with K_M for flat sandwich panel in edgewise compression. All edges simply supported. B = 0.01. r = 1.0.