THIS REPORT HAS BEEN DELIMITED AND CLEARED FOR PUBLIC RELEASE UNDER DOD DIRECTIVE 5200.20 AND NO RESTRICTIONS ARE IMPOSED UPON ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.
REPORT No. 990
JULY 1956

Comparison Of
Predicted And Observed Yaw
In Front Of The Muzzle
Of The 12-Inch Gun

R. H. KENT
H. P. HITCHCOCK

DEPARTMENT OF THE ARMY PROJECT No. 5803-03-001
ORDNANCE RESEARCH AND DEVELOPMENT PROJECT No. T83-01082
BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND
COMPARISON OF PREDICTED AND OBSERVED YAW IN FRONT OF THE MUZZLE OF THE 12-INCH GUN

R. H. Kent
H. P. Hitchcock
<table>
<thead>
<tr>
<th>Fig.</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yaw and orientation vs distance from gun</td>
</tr>
<tr>
<td></td>
<td>Loss of range due to yaw vs orientation</td>
</tr>
<tr>
<td>6</td>
<td>Plot of rounds 1 to 18 on 5-Ø plane, showing ranges</td>
</tr>
<tr>
<td>7</td>
<td>Plot of rounds 1 to 18 on 5-Ø plane, showing deflections</td>
</tr>
<tr>
<td>8</td>
<td>Plot of rounds 19 to 46 on 5-Ø plane, showing ranges</td>
</tr>
<tr>
<td>9</td>
<td>Plot of rounds 47 to 58 on 5-Ø plane</td>
</tr>
<tr>
<td>10</td>
<td>Effect of Range due to Yaw</td>
</tr>
</tbody>
</table>

* These plots, except figure 10, were inclosed with the firing records (Ref. 3). (The other figures are not needed.)
COMPARISON OF PREDICTED AND OBSERVED YAW IN FRONT OF THE MUZZLE OF THE 12-INCH GUN

ABSTRACT

The yaws of some 1070-pound projectiles, fired from a 12-inch Gun, in 1920, were determined by measuring the holes in beaver boards at a distance of 294 feet from the muzzle. Most of the rounds had yaws whose magnitude was less than the theoretical limit imposed by the maximum allowable clearance in the bore. The orientations were grouped about the theoretical value, calculated on the assumption that the front bourrelet followed the land that is at 6 o'clock at the origin. The range varies with both the magnitude and orientation of the yaw. Theoretically, the deflection also depends on the yaw, but this effect could not be detected.
INTRODUCTION

It came to Kent's notice some months ago that the British had been unaware of the fact that yaw is not completely random. This indicated that the British were unfamiliar with the work, described in his report, that Kent had done some twenty-five years ago.

THEORY

According to the small yaw theory of Fowler, Gallop, Lock and Richmond, the magnitude $\delta$ and orientation $\phi$ of the yaw of a spinning shell approximately satisfy the equations

$$\delta = (a \sin qt + b \cos qt)^{1/2}, \quad (1)$$

$$\phi = \phi_0 + \frac{AN}{2B} \frac{\alpha}{B} \tan^{-1} \left( \frac{\alpha}{B} \tan qt \right), \quad (2)$$

where

- $\alpha$ is the maximum yaw,
- $A$ the minimum yaw,
- $t$ the time from the first minimum yaw,
- $\phi_0$ the initial orientation,
- $A$ the axial moment of inertia,
- $B$ the transverse moment of inertia,
- $N$ the spin,

and

$$q = (AN/2B)(1 - 1/s)^{1/2} \quad (3)$$

if $s$ is the stability factor.

Damping is indicated by variations in $\alpha$ and $\beta$.

The spin is approximately

$$N = \pi \rho \omega / \mu a, \quad (4)$$
where

\( v_0 \) is the muzzle velocity,

\( 1/h \) the twist of rifling,

\( d \) the caliber.

For a short distance, the range is approximately

\[ x = v_0 t. \] (5)

Then, if

\[ \phi' = \pi A / B nd \] (6)

and

\[ \sigma = (1 - 1/8)^{1/2}, \] (7)

equations (1) and (2) may be expressed

\[ \delta = (\alpha^2 \sin^2 \phi' \sigma x + \beta^2 \cos^2 \phi' \sigma x)^{1/2}, \] (8)

\[ \phi = \phi_0 + \phi' x + \tan^{-1}(\frac{\sigma}{\beta} \tan \phi' \sigma x). \] (9)

The stability factor may be found from the period of yaw \( L \) by the relation

\[ (1 - 1/8)^{1/2} = B nd / A L. \] (10)

The curves on figure 1 are examples of the variation of \( \delta \) and \( \phi \) with \( x \).

The minimum yaw is usually very small: about the same as the yaw in the bore \( \epsilon \), permitted by the clearance between the bourrelets of the shell and the lands of the gun. If the front bourrelet follows one land and the rear bourrelet the opposite land, the maximum yaw may be calculated by the formula\(^2\)

\[ \alpha = (2B/A - 1) \epsilon / \sigma. \] (11)

The largest yaw to be expected at any distance may be found by using (8) and (11). Also, if it is assumed that the front bourrelet follows the land that is at \( 6 \) \( \phi \) clock at the origin, the orientation of the yaw at the muzzle is known, and hence the orientation at any distance can be found by (9).

6
The yaw due to initial disturbances damps out rapidly if the shell is well damped. While the yaw is appreciable, however, it has two effects on the trajectory: it increases the drag and changes the direction of motion. Both of these effects influence the range; the latter also influences the deflection. In the present study, it is assumed that the range, corrected to a standard muzzle velocity, and standard meteorological conditions, satisfies the formula

$$R = A + B\theta + C\cos(\theta + K),$$

(12)

where $R$ is the corrected range and $A$, $B$, $C$ and $K$ are constants. The magnitude and orientation of the yaw are measured at a given distance from the muzzle.

**EXPERIMENTAL DATA**

In December 1919 and January 1920, some 1070-pound Cast Iron Target Practice Projectiles (Dwg 75-10-15) were fired from a 12-inch Gun M1895 at an elevation of $18^\circ$. The velocity was measured by a Boulenge chronograph with screens at 124 and 294 feet. In order to determine the yaw, beaver boards were placed on the far side of the first screen and the near side of the second screen; however, the boards on the first screen were out by the wires so that the yaw could not be measured. On the second board, an increase in major axis from 12.1 to 13.1 inches was taken to represent a unit of yaw; this corresponds to an angle of $1.7^\circ$.

The observed ranges were corrected to a muzzle velocity of 2250 feet per second and standard meteorological conditions. The decrease in corrected range (meters) due to yaw was divided by the yaw (expressed in inches of major axis); the plot of this ratio as a function of orientation (Fig 5) shows that the range varies in accordance with equation (12).
Figures 6 to 9 are plots of orientation (degrees) versus magnitude of yaw (expressed as an increase in major axis, in inches) for three groups of rounds. Plots 6 and 7 pertain to the same rounds considered in figure 5: plot 6 gives the corrected range of each round and shows curves of equal range; plot 7 gives the observed deflections. The corrected ranges of another group are given on figure 8.

The yaw in the bore was computed from the following average dimensions:

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore Diameter</td>
<td>12.001 in.</td>
</tr>
<tr>
<td>Front bourrelet diam</td>
<td>11.982 in.</td>
</tr>
<tr>
<td>Rear bourrelet diam</td>
<td>11.930 in.</td>
</tr>
<tr>
<td>Bearing length</td>
<td>33.46 in.</td>
</tr>
</tbody>
</table>

The computed yaw in the bore is 0.077°.

It is estimated that the transverse moment of inertia is 10 times the axial moment of inertia, and that the stability factor is 1.5, since the observed nutational period is 430 feet (sec fig. 1). Hence the maximum yaw, computed by equation (11), is 2.5°. This is equivalent to an increase of 1.5 inches in major axis.

The increase in drag due to yaw near the muzzle is equivalent to a decrease in muzzle velocity. To determine this decrease for a maximum yaw of 2.5°, the following data are needed:

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005 deg⁻²</td>
<td>Yaw-drag coefficient K_D0 (estimated)</td>
</tr>
<tr>
<td>2250 fps</td>
<td>Muzzle velocity v_0 (observed)</td>
</tr>
<tr>
<td>0.1625 sec⁻¹</td>
<td>Drag function G_D (estimated)</td>
</tr>
<tr>
<td>8.10</td>
<td>Ballistic coefficient C_D (estimated)</td>
</tr>
<tr>
<td>1.10</td>
<td>Cross wind force coefficient K_L (estimated)</td>
</tr>
<tr>
<td>3.52</td>
<td>Yawing moment coefficient K_H (estimated)</td>
</tr>
<tr>
<td>0.07513 lb/ft³</td>
<td>Air density ρ (estimated)</td>
</tr>
<tr>
<td>1070 lb</td>
<td>Mass m (observed)</td>
</tr>
<tr>
<td>1370 lb-ft²</td>
<td>Transverse moment of inertia B (estimated)</td>
</tr>
</tbody>
</table>
The increase in drag due to yaw is a decrease of 12.7 meters.

The expression for the jump due to bore clearance and its orientation may be derived from an expression given by Sterne. It shows that if the initial value of $\phi$ is 270 degrees, the jump is in the vertical plane and downwards. Its magnitude is

$$\varepsilon = \frac{AN}{md} \frac{KL}{KM} \frac{1}{V_o} \left(\frac{B}{A} - 1\right)$$

Where $KM$ is the overturning moment coefficient

Hence, if the actual angle of orientation of the yaw at the muzzle of the gun is $\phi_0$ then the actual vertical component of the jump will be

$$-\varepsilon = \frac{AN}{md} \frac{KL}{KM} \frac{1}{V_o} \left(\frac{B}{A} - 1\right) \cos (\phi_0 - 270^\circ)$$

(13)

Hitchcock has discussed the displacement caused by initial yaw. This could be used in place of Sterne's formula.

The constants for this gun and ammunition (some repeated from page 8) appearing in equation (13) are as follows:

- $\varepsilon = 0.077^\circ = 1.37$ mils
- $A = 1.37$ lb ft$^2$
- $N = \frac{2\pi V_0}{md} = 565.5$ rad/sec
- $m = 1070$ lbs
- $d = 1.500$ ft
- $K_L = 1.10^*$
- $K_M = 1.91^*$
- $V_o = 2250$ ft/sec
- $B/A = 10^*$
- $\phi_0 = 226^\circ$

*Estimated

** The application of Hitchcock's technique produces results almost identical to those obtained by equation (13).
\[
\rho_o = 0.0751 \text{ lb/ft}^3 \\
\rho = 25 \text{ cal} \\
s = 1.91
\]

If the cofactor of \( \cos (\phi - 270^\circ) \) is represented by -J then the jump may be written

\[-J \cos (\phi - 270^\circ) \]

\[J = 0.25 \text{ mils} = 0.77'' \]

It follows that the vertical component of the jump will be \(-0.165 \text{ mils} = -0.56''\) and the horizontal component will be \(+0.160 \text{ mils} = 0.54''\).

At an elevation of \(10^\circ\), the firing table for the AP Projectile gives an increase in range of 9.8 meters for 1 minute increase in elevation. Therefore, the estimated increase in range for an increase of 0.75 minute in elevation is 7.35 meters.

The combined effect of drag and jump due to yaw is plotted on figure 10.

**DISCUSSION**

The theoretical increase in major axis at 294 feet is 1.5 inches. Most of the measured values were less than this, although some were more: the largest was 2.75 inches. It should be noted, however, that the yaw has passed its first maximum at this distance, as the period is 450 feet; therefore, a small increase in the distance to the maximum will cause an appreciable increase in the yaw at the second screen.

In other guns, when there have been enough cases to determine the actual maximum yaw, the observed value seldom exceeds the theoretical.

The theoretical mean orientation at 294 feet is \(35^\circ\). The observed orientations are grouped about this value although their dispersion is rather large: a few values are nearly \(120^\circ\) from the mean (two rounds are plotted with an orientation of \(295^\circ\) but this is only \(100^\circ\) from the mean in the negative direction). The assumption that the front bourrelet follows the land that is at 6 o'clock at the origin is further confirmed by the fact that the lands in this region wear more rapidly near the muzzle than the others in guns which fire separate-loading ammunition (this is not true for fixed ammunition).
Since the range varies with both magnitude and orientation of yaw, the dispersion in both of these quantities should be made as small as possible. For a given gun and ammunition, this can be done by ramming all rounds as uniformly as possible. The dispersion in deflection might also be reduced in this way, although the deflection has comparatively small dispersion anyway.

The evidence presented in this report confirming the hypothesis that in this gun and, presumably, in most guns firing separate loading ammunition, the yaw at the muzzle corresponds to an initial yaw at the breech at 6 o'clock is confirmed by observations of the muzzles of several guns that have shown erosion on the lands corresponding to 6 o'clock at the breech.

R. F. Kent

H. P. HITCHCOCK
REFERENCES


3. Test of Multisectional Charges for Seacoast Guns. APG: 2nd and Final Report on Ord Board Program 4062 (1920), including Supplementary Report on Jump Card Results Obtained in Connection with Firings Made on Dec 11, 16 and 31, 1919, and Jan 12, 1920.


6. Hitchcock, H. P. Integration of Equations of Motion of a Spinning Shell to Determine Displacement Due to Yaw. APG: ERL R-X95 (1922).

THEORETICAL CURVE

EQUATION: \[ \frac{A}{L} = \frac{\sin \beta}{\cos \alpha} \]

CURVE TO SHOW LOSS OF RANGE DUE TO YAW AS MEASURED AT THE SECOND SCREEN IN CONNECTION WITH TEST OF MULTI-SECTIONAL CHARGES FOR SEACOAST GUNS.

RESULTS OF ROUNDS 1 TO 9 ARE SHOWN WITH EXCEPTION OF ROUNDS 2, 3, AND 11. ROUNDS 2, 7, 8, AND 9 HAD BADLY FRAMED ROTATING BANDS.

THE UNIT YAW IS TAKEN TO BE THAT CORRESPONDING TO A LENGTH OF MAJOR AXIS OF 13.4" OF THE HOLE CUT IN THE SECOND SCREEN.

ORIENTATION \( \phi \) OF YAW AT SECOND SCREEN

FIGURE 2
PLOT OF ROUNDS 1 TO 18 ON S, \( \phi \) PLANE.
EXCEPT ROUNDS 2 AND 3.

\( \phi \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)

\( C \)
PLOT OF ROUNDS 1 TO 18 ON X, Y PLANE
EXCEPT ROUNDS 2, 3, AND 12.

NOTES: THE NUMBERS TO
THE POINTS ARE UNCORRECTED.
THE NUMBERS TO THE POINTS ARE THE NUMBERS OF THE ROUNDS.

FIGURE 7
THE NUMBERS ATTACHED TO THE POINTS ARE RANGES CORRECTED FOR VELOCITY AND WEATHER CONDITIONS.

THEORETICAL MAXIMUM YAW

PLOT OF ROUNDS 19 TO 48 ON THE B, Φ PLANE

FIGURE 8
PLOT OF RoundS 47 TO 58 ON THE \( \phi, \theta, \text{PLANE} \).

\[ \begin{array}{c}
\text{THEROETICAL MEAN ORIENTATION} \\
\text{THEROETICAL MAXIMUM YAW}
\end{array} \]

\[ \begin{array}{c}
\text{NOTE} \\
\text{THE NUMBERS TO THE RIGHT ARE THE NUMBERS OF THE ROUNDS.}
\end{array} \]

FIGURE 9
FIGURE 10  EFFECT ON RANGE DUE TO YAW FOR 1070-LC TP PROJECTILE M1911
Fired from 12-inch gun M1895. Mil. Yaw 2250. Eps. Elev. 18°
Normal range 17667 M. Maximum yaw 2.5°.
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 4             | Chief of Ordnance  
Department of the Army  
Washington 25, D. C.  
Attn: ORDTR - Bal Sec  
ORDX-AR - ORDTA |
| 10            | British Joint Services Mission  
1800 K Street, N. W.  
Washington 6, D. C.  
Attn: Mr. John Izzard, Reports Officer |
| 4             | Canadian Army Staff  
2450 Massachusetts Avenue, N. W.  
Washington 8, D. C. |
| 3             | Chief, Bureau of Ordnance  
Department of the Navy  
Washington 25, D. C.  
Attn: ReO |
| 2             | Commander  
Naval Ordnance Laboratory  
White Oak  
Silver Spring 19, Maryland |
| 1             | Commander  
Naval Ordnance Test Station  
China Lake, California  
Attn: Technical Library |
| 1             | Commander  
Naval Proving Ground  
Dahlgren, Virginia |
| 1             | Commandant  
U. S. Marine Corps  
Washington 25, D. C. |
| 1             | President  
Marine Corps Equipment Board  
Quantico, Virginia |

Director  
Armed Services Technical Information Agency  
Attn: Documents Service Center  
Knott Building  
Dayton 2, Ohio |

Assistant Chief of Staff for Development and Test  
Continental Army Command  
Fort Monroe, Virginia |

President  
COMARC Board No. 1  
Fort Sill, Oklahoma |

President  
COMARC Board No. 2  
Fort Knox, Kentucky |

President  
COMARC Board No. 3  
Fort Benning, Georgia |

Commandant  
T. Infantry School  
Fort Benning, Georgia |

Commandant  
The Artillery School  
Fort Sill, Oklahoma |

Commandant  
The Armored School  
Fort Knox, Kentucky |

Professor of Ordnance  
U. S. Military Academy  
West Point, New York |
<table>
<thead>
<tr>
<th>No. of Copies</th>
<th>Organization</th>
</tr>
</thead>
</table>
| 3            | Commanding General  
              | Picatinny Arsenal  
              | Dover, New Jersey  
              | Attn: Samuel Feltman Ammunition Laboratories |
| 1            | Commanding General  
              | Weapons Command  
              | Rock Island, Illinois |
| 1            | Commanding General  
              | Ordnance Ammunition Command  
              | Joliet, Illinois |

Best Available Copy