Missing links between the solar photospheric magnetic field and the upper atmosphere

Kiyoto Shibasaki
SOLAR PHYSICS RESEARCH INC

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Final Report

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# Abstract

To uncover missing links between the photosphere and the upper atmosphere, two approaches were taken: observational and theoretical. As the observational approach, full disk chromospheric magnetic field maps were synthesized using the 17 GHz image data taken by the Nobeyama Radioheliograph. It was found that extended enhanced magnetic field regions in the chromosphere are associated with radio brightness enhancement even outside active regions. Generally, association between magnetic enhancements at the photosphere and at the chromosphere (associated with enhanced radio brightness) is good but not always. To understand these relations, we need to understand mechanisms for radio brightness enhancement and magnetic field strength. As the theoretical approach, the magneto-hydrodynamic theory (MHD) is revisited. The solar chromosphere is in the transition from the low to high plasma beta condition, or the gas pressure is not negligible relative to the magnetic pressure. However, the existing MHD theory is based on the assumption of low-beta. It is necessary to modify the existing MHD to be applicable to the high-beta plasma. For this purpose, magnetic moment of thermal plasma was calculated and added to the MHD equation. It has been believed that the magnetic moment is negligible in the thermal plasma due to the frequent collisions among particles and the random velocity distribution. However, it is shown that even under the highly collisional conditions and the random velocity distribution, the magnetic moment does exist and plays important roles in the solar atmosphere. By adding the magnetic moment to MHD, we can understand the lifting of hot plasma into the upper atmosphere against the gravity. Generally the magnetic field is weaker upwards, hence the magnetic Kelvin force acting on the magnetic moment is directed upward.
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“Missing Links between the Solar Photospheric Magnetic Field and the Upper Atmosphere”

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**Name of Principal Investigators (PI and Co-PIs):** Kiyoto Shibasaki  
- e-mail address: shibasaki.kiyoto@md.ccnw.ne.jp (shibasaki.kiyoto@nao.ac.jp)  
- Institution: Solar Physics Research Inc.  
- Mailing Address: Matsushin chou 2-24, Kasugai city, Aichi prefecture, 486-0931 Japan  
- Phone: +81-568-34-5433  
- Fax: +81-568-34-5433

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**Abstract:** To uncover missing links between the photosphere and the upper atmosphere, two approaches were taken, observational and theoretical. As the observational approach, full disk chromospheric magnetic field maps were synthesized using the 17 GHz image data taken by the Nobeyama Radioheliograph. Radio waves around 17 GHz are emitted mainly from the chromosphere, hence we can measure magnetic field strength (line-of-sight component) there. The emission from the plasma in the magnetic field with temperature gradient is circularly polarized proportional to magnetic field strength even if the emission is optically thick. High quality circular polarization images are essential to measure the magnetic field strength due to small signal. Various techniques were tested and could reach the level of 0.1 percent circular polarization degree near the disk center corresponding to 30 gauss. It is found that extended enhanced magnetic field regions in the chromosphere are associated with radio brightness enhancement even outside active regions. Generally, association between magnetic enhancements at the photosphere and at the chromosphere (associated with enhanced radio brightness) is good but not always. To understand these relations, we need to understand mechanisms for radio brightness enhancement and magnetic field strength.

As the theoretical approach, the magneto hydrodynamic theory (MHD) is revisited. The solar chromosphere is in the transition from the low to high plasma beta condition, or the gas pressure is not negligible relative to the magnetic pressure. However, the existing MHD theory is based on the assumption of low-beta. It is necessary to modify the existing MHD to be applicable to the high-beta plasma. For this purpose, magnetic moment of thermal plasma was calculated and added to the MHD equation. It has been believed that the magnetic moment is negligible in the thermal plasma due to the frequent collisions among particles and the random velocity distribution. However, it is shown that even under the highly collisional conditions and the random velocity distribution, the magnetic moment does exist and plays important roles in the solar atmosphere. By adding the magnetic moment to MHD, we can understand the lifting of hot plasma into the upper atmosphere against the gravity. Generally the magnetic field is weaker upwards, hence the magnetic Kelvin force acting on the magnetic moment is directed upward. This mechanism can explain the relation between the radio brightness and the magnetic field strength in the chromosphere. In this theoretical study, it is shown that the Bohr-van Leeuwen theory (uniform thermal plasma does not have the magnetic moment) is not valid. This means that the MHD theory loses one of the important foundations and the theory should be rewritten to include the magnetic moment and to revise the many MHD closures.

**Introduction:** Activities in the interplanetary space are controlled by the solar upper atmosphere, the corona. The solar corona has million degree temperature even though the lower atmospheres have lower temperature. The photosphere and the chromosphere are roughly 6,000 K and 10,000 K respectively. This temperature inversion is called the coronal heating problem. Not only the corona, but also the chromosphere has the same problem. We need non-thermal mechanism(s) to
heat the corona and the chromosphere.

The solar chromosphere is an important layer which connects the surface of the Sun, the photosphere, and the upper atmosphere. However, due to its complexity of the structure and transitional nature of plasma properties, it is hard to clarify how the photosphere is linked with the upper atmosphere. Important links are magnetic field and plasma flow. At the photosphere, the magnetic field is controlled by the gas dynamics due to the high gas pressure or the high plasma beta (= gas pressure / magnetic pressure). In the corona, on the other hand, the gas dynamics is controlled by the magnetic field due to the low gas pressure or the low plasma beta. The chromosphere is in between. Under the high-beta condition, the magnetic field is not potential nor force free. Extrapolation of the measured photospheric magnetic field into the corona assuming potential or (non-) linear force free is not enough, even though it is frequently used. To estimate the coronal magnetic field, we need to know the magnetic field distribution at the upper chromosphere where the plasma beta becomes small. Magnetic field measurements so far done are mainly at the photosphere or at the lower chromosphere. We need measurements at the upper chromosphere.

Microwave is emitted mainly from the upper chromosphere where the temperature is about 10,000 K. The emission is optically thick thermal free-free by the collisions of thermal electrons with ions. The opacity of the circularly polarized emission depends on the magnetic field strength. The circularly polarized emission with the same sign with the gyration direction of the electrons has larger opacity than that of the other. When combined with the temperature gradient of the solar atmosphere, the larger opacity mode becomes optically thick at the higher altitude where the temperature is higher in the chromosphere. This is the mechanism how the circularly polarized emission is generated in the chromosphere outside sunspot umbrae. The observed circular polarization degree (= polarized intensity / total intensity) is proportional to the magnetic field strength. To get the full disk chromospheric magnetic field strength, the full disk circular polarization degree maps need to be synthesized. I used data from the Nobeyama Radioheliograph at 17 GHz (1.76 cm wavelength) to get the full disk circular polarization degree map. Due to the weak circular polarization degree, except sunspots, synthesis of high dynamic range images are required and the major efforts are how to get the high dynamic range circular polarization maps.

To interpret the observational results, we need theoretical tools. The most important tool is the Magneto Hydrodynamic (MHD) theory. However, the existing MHD theory is applicable to the low (negligibly small) beta plasma in a restricted sense. This is because the MHD is originally constructed to study the interaction between the magnetic field and the electrically conducting fluid, such as the liquid metal. Hence, the magnetic moment of plasma is not included in the MHD. This is mentioned in the text book by the founder of the MHD (Alfven and Falthammar [1963], “Cosmical Electrodynamics” 2nd ed. Chapters 4 and 5). Most of the MHD text books mention that the magnetic permeability of plasma is close to that of the vacuum, which means the magnetic moment is negligible. Assumption of the small magnetic moment of plasma is equivalent to the low-beta plasma. To apply the MHD to the plasma atmosphere in the photosphere and the chromosphere, we need to revise the MHD theory to be applicable to the high-beta plasma.

In this program, 1) I developed a chromospheric magnetic field measurement method using the radio technique and applied to the data taken by the Nobeyama Radioheliograph. Also, 2) I proposed a new mechanism to supply the hot plasma from below by revisiting the fundamental plasma physics, the MHD.

Experiment:

1. Chromospheric magnetic field measurements

The Nobeyama Radioheliograph (NoRH) is a solar dedicated radio interferometer operating at 17 GHz since 1992 and 17/34 GHz since 1995. We use the intensity (brightness) data at 17 and 34
GHz and the circular polarization data at 17 GHz. Radio interferometers measure visibilities (spatial Fourier components) of radio brightness and circular polarization distribution on the sky plane. To synthesize images, measured visibility data are calibrated, inverse Fourier transformed and CLEANed to remove artifacts caused by the array configuration. We took four processes to get high dynamic range images: 1. selection of good datasets, 2. good calibration, 3. deep CLEANing, and 4. image averaging.

The array configuration of NoRH has many redundant antenna pairs (same baseline length in the same direction) for calibration. The visibilities from the redundant combination of the antennas are used to estimate the antenna based errors. The calibration data sets are taken simultaneously with the data sets for imaging. As the quality of the synthesized images depends strongly on the accuracy of the estimated errors, the calibration data sets are averaged to minimize the random errors caused by the system noise. We averaged 600 datasets (10 min.) to estimate the phase and the amplitude of the antenna based errors. The visibility data sets for imaging are also averaged during the same period as for the error estimation and then calibrated using the estimated errors. After applying the inverse Fourier transformation to the averaged and error corrected visibilities, the image restoration called CLEAN is applied to minimize the influence of side lobes caused by array configuration. We set a very low CLEAN criterion level to suppress the influences of the bright radio sources on the Sun (active regions). Thus, we can get the high quality images every 10 min. both intensity (I) and circular polarization (V). These images are averaged for many hours during the one day observation. We averaged 25 images (local noon + - 2 hrs) rather than 48 (8 hrs) to avoid degradation due to the low elevation of the Sun in the sky and image smearing due to the differential rotation of the Sun. The final circular polarization degree map is calculated by dividing the V-map by the I-map. In these processes, we assume that the Sun is quiet during 4 hours of imaging. We selected the days of very quiet Sun and also the good observing condition (the weather and the instrument). An example is shown in the attached material (Ref. 1, page 15). Comparison with the photospheric magnetic field image taken by MDI / SOHO around the same time on the same day (page 16) shows general agreement between the chromospheric magnetic field measured by NoRH. Due to low spatial resolution of NoRH (around 20 arc sec.) and smearing caused by the solar rotation, the chromospheric magnetic field images are somewhat blurred. Around the disk center, the noise level of the circular polarization degree is in the order of 0.1 percent. However, we can see systematic errors (~0.5 percent) near the limb. To go further, it is necessary to identify the causes of these errors and remove them. One possible cause is a small shift between both polarized (right-handed circular and left-handed circular) images due to the built-in parameters in the image synthesis program package. We are not yet successful in improving measuring circular polarization near the limb. This work was done mainly during the first year.

2. Revision of MHD theory

To understand the chromosphere, where plasma pressure is not negligible relative to the magnetic pressure (finite or high plasma beta), it is necessary to revise the existing MHD theory. The MHD theory does not include the magnetic moment of the thermal plasma due to the low plasma beta assumption and other reasons. Most part of the study was presented at the workshop on “Solar Physics with Radio Observation” September 9-10, 2016, Nagoya U. (Ref. 2) and some part is published as a RHESSI science nugget (Ref. 4).

http://sprg.ssl.berkeley.edu/~tohban/wiki/index.php/The_Kelvin_Force_and_Loop-Top_Centroratio

Also the work is summarized as a full paper (draft, Ref. 3) which will be submitted to Physical Review Letter or other appropriate journal.

For the revision of the MHD theory to be applicable to the chromosphere, we introduce the magnetic moment of the thermal plasma. Not only the magnetic moment but also drift currents caused by the magnetic field intensity gradient and the curvature are needed to be a self-consistent theory. To understand the magnetic properties of the plasma as the continuous matter, we have to know how the constituents of the plasma behave in the presence of the magnetic field. Our plan is to revise the fluid MHD equations applicable also to the high beta plasma which is equivalent to include...
the magnetic moment. It has been believed that the magnetic moment disappears in the thermal plasma under frequent collisions (collision rate > gyro frequency). However, as is shown in the attached Ref. 2 (page 12) and Ref. 3 (page 4), interruption of the gyration motion by the collisions do not influence the magnetic moment because the magnetic moment is defined at each moment of time. Another reason why it has been believed that the magnetic moment does not exist in the thermal plasma is due to the Bohr-van Leeuwen theory. They showed that the statistically averaged magnetic moment disappears in the uniform thermal plasma. However, as is explained in Ref. 3 (page 5), if we take into account the curved motion of each particle due to the Lorentz force, the statistically averaged magnetic moment does not disappear. The magnetic Kelvin force is the force acting on the magnetic moment in the presence of the field intensity gradient along the field. As the plasma is diamagnetic, the plasma is pushed where the magnetic field is weak along the field. This force is not included in the existing MHD. By introducing this new term in the MHD equation of motion, we can discuss the plasma motion along the field (Ref. 2 page 18 and Ref. 3 page 6). Generally, the magnetic field weakens upwards from the photosphere, the plasma are pushed upwards spontaneously. In the closed magnetic field, plasma can be trapped around the loop top (Ref. 4 and Ref. 3 page 8). In the open magnetic field, the temperature dependent plasma up-flow is explained including the solar wind (Ref. 3 pages 8-10). Density scale length will be greatly deviated from the hydrostatic scale length by the Kelvin force depending on the temperature and the intensity gradient of the magnetic field strength between the photosphere and the chromosphere. By combining the measured chromospheric magnetic field shown above and the available photospheric magnetic field measured using optical methods (from satellites and ground), we can estimate the vertical gradient of the field intensity between the two layers. Two approaches (observational and theoretical) will be merged to understand the links between the photosphere and the upper atmosphere. The second year was devoted to the theoretical work of revising the existing MHD equation.

Results and Discussion: We successfully got the full disk chromospheric magnetic field maps using the NoRH data with the accuracy of 0.1 percent which corresponds to 30 Gauss around the disk center (Ref. 1). Near the limb, the systematic errors still remain. To get high dynamic range images for both intensity and circular polarization, we tried various combinations of data averaging for calibration and for imaging. CLEANIN level was adjusted as low as possible. Comparison with the photospheric magnetic field image taken by MDI / SOHO around the same time on the same day (Ref. 1 page 16) shows general agreement between the chromospheric magnetic field measured by NoRH. To get the magnetic field intensity gradient between the chromosphere and the photosphere, further studies are needed. Comparison between the measured magnetic field based on different principles (radio and optical) requires very careful treatments.

It is shown that the magnetic moment does exist even in the highly collisional and randomly moving charged particles of the thermal plasma in the uniform magnetic field (Refs. 2-4). Also, the validity of the Bohr-van Leeuwen theory is denied (Ref. 3). By introducing the magnetic moment to the existing MHD theory, the Kelvin force is added to the equation of motion. Due to the added Kelvin force, we can discuss plasma motion along the magnetic field, the excess density scale length, or even the reversal of the density scale length. With the new MHD equations, the observed features on the sun in radio can be understood. In areas where magnetic field is concentrated and the vertical gradient is strong, we expect that upward Kelvin force is comparable to the downward gravity force, hence the density scale length is longer than the other area. The radio brightness will enhance due to the thick layer of density enhancement. Not only the radio features, but also fundamental questions of solar atmosphere such as the mass circulation in the atmosphere and coronal / chromospheric heating mechanisms can be discussed using the new MHD (Ref. 3).

The introduction of the magnetic moment to MHD causes very strong influences to the framework of the MHD theory, not only the Kelvin force. As is known that the causality relation between the electric current and the magnetic field is reversed in MHD (the magnetic field is the source of the current) relative to the classical electromagnetism (the electric current is the source of the magnetic field). This reversed causality in MHD can be corrected by introducing the magnetic moment and by
revising several elements called the MHD closures. These are the most important results of this program. I continue to reformulate the existing MHD theory to be compatible with the classical electromagnetism. To really understand the physics of solar activity and its influence to the interplanetary space, we need solid theoretical foundations.

**List of Publications and Significant Collaborations that resulted from your AOARD supported project:**

d) conference presentations without papers,

Ref. 1: Kiyoto Shibasaki, “Full disk chromospheric magnetic field imaging” presented at “Solar workshop on the Sun's Chromosphere” March 15-18, 2016 | NSO/LASP, Boulder, CO, USA


e’) draft manuscript not yet published,

Ref. 3: Kiyoto Shibasaki, “Magnetic moment of thermal plasma and the Kelvin force: - Driving force of plasma up-flow and loop-top concentration -”, Draft paper to be submitted to Phys. Rev. Letter

c) papers published in non-peer-reviewed journals and conference proceedings,


f) provide a list any interactions with industry or with Air Force Research Laboratory scientists or significant collaborations that resulted from this work.

I have been collaborating with Dr. White, S. M. (DR-03 USAF AFMC AFRL/RVBXS) on chromospheric magnetic field measurement using NoRH data. He uses a different method to synthesize radio images called AIPS for high dynamic range imaging.

**Attachments:**

Refs. 1 - 4 are attached.
Full disk chromospheric magnetic field imaging

Kiyoto Shibasaki (Solar Physics Research Inc.)
shibasaki@md.ccnw.ne.jp
http://www.md.ccnw.ne.jp/sprinc/

Outline

1. Motivation
2. Circular Polarization Measurements by the Nobeyama Radioheliograph
3. Coronal magnetic field (post flare loop)
4. Active region chromospheric magnetic field
5. Full disk chromospheric magnetic field measurement
6. Summary
Motivation

• To understand solar activity influence to the interplanetary space and to the Earth upper atmosphere, coronal magnetic field is one of the key information.

• As the coronal magnetic field is controlled by the magnetic field at the chromosphere, we need to measure global chromospheric magnetic field. We measure chromospheric magnetic field independently from optical methods.

• Starting from daily full disk chromospheric magnetograms, we plan to synthesize synoptic charts and eventually a butterfly diagram.

Introduction to radio polarimetry

• Interaction of moving charged particles with B due to Lorenz force ($ \mathbf{F} = q \mathbf{v} \times \mathbf{B}$)
  • Gyration motion around $\mathbf{B}$
  • Electron cyclotron frequency
    $f_H$ (MHz) = $2.8 \times B$ (Gauss) ($10 \, G \sim 30\, MHz$)

• Circularly polarized EM wave interacts with gyrating electrons
  • Classical treatment (no Quantum Mechanics !)
  • simple inversion (pol. deg $\rightarrow B_||$)

• Continuum emission (no lines)
  • No Doppler effect
  • magnetic fields in Hot, Turbulent and Moving plasma can be measured

• $B_||$ only, low spatial resolution
Thermal free-free emission

\[ T_B = \int T \exp(-\tau) d\tau \quad \text{TB: Observed brightness temperature} \]
\[ \tau = \int \kappa \, dl, \quad \kappa = \text{const} \times n_e^2 T^{-3/2} \]
\[ T_B \sim T \tau \sim EM / \sqrt{T} \quad (\tau \ll 1, \text{optically thin uniform cloud}) \]
\[ EM = \int n_e^2 dl \]

• In magnetic field:
\[ \kappa_{o,x} = \kappa(1 \pm f_H |\cos(\alpha)| / f) \]
(for ordinary and extraordinary mode)
\[ T_{b,o,x} = T_b (f \pm f_H |\cos(\alpha)|) \]
\[ p = (T_{b,x} - T_{b,o}) / (T_{b,x} + T_{b,o}) \sim n(f_H / f)|\cos(\alpha)| \]
where \( n = -\partial(\ln(T_b)/\partial(\ln(f))) \), Bogod and Gelfreikh, 1980

Inversion at 17 GHz

• Polarization degree and magnetic field
\[ p = n \times (f_H / f)|\cos(\alpha)|, \quad T_b \propto f^{-n} \]
\[ B_\parallel (\text{Gauss}) = 60 / n \times p(\%) \quad \text{at 17 GHz} \]
• Optically thin case: \[ n=2 \]
\[ B_\parallel (\text{Gauss}) = 30 \times p(\%) \]
\[ (1\% \sim 30 \text{ Gauss}) \]
• Optically thick case:
  - uniform temperature \[ n = 0, \quad p = 0 \]
  - Optically thick case: \[ n=0.15 \quad (\text{QS at 17 GHz and 34 GHz}) \]
  - temperature gradient \[ B_\parallel (\text{Gauss}) = 400 \times p(\%) \]
\[ (40 \text{ Gauss} \sim 0.1\%) \]
Instrument: Nobeyama radioheliograph

- Frequency: 17 GHz (I and V), 34 GHz (I)
- Field of view: Full disk
- Spatial resolution: 10 arcsec (17GHz), 5 arcsec (34GHz)

\[ B_I [G] = 10700 \frac{1}{n\lambda_{[cm]}} \frac{V}{I} \]

2-D Radio magnetic filed

Coronal magnetic field (Post flare arcade)
Event on Oct. 22, 2000

Upper Left: 17GHz intensity (I)

Loop like structure corresponds to bright arcade top. Each flare loop is in the plane of line of sight.

Upper Right: 17GHz circular polarization (V)

Lower Left: Circular polarization degree (V/I) ~ 0.3%
Mag. field distribution (~10 Gauss)

Active Region Chromospheric Magnetic field (optically thick case with temperature gradient)

Iwai and Shibasaki (PASJ, 2013)
Observation: circular polarization

Radio Magnetic filed

\[ B_l[G] = 10700 \frac{1}{n\lambda_{[cm]}} \frac{V}{I} \]

<table>
<thead>
<tr>
<th></th>
<th>HMI (G)</th>
<th>Radio (G)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>FP+</td>
<td>568</td>
<td>116</td>
<td>0.20</td>
</tr>
<tr>
<td>FP-</td>
<td>-456</td>
<td>-217</td>
<td>0.47</td>
</tr>
</tbody>
</table>
Full disk chromospheric magnetic field measurement

- Optically thick case with temperature gradient
  
  \( n = 0.15 \) (QS between 17 GHz and 34 GHz)
  
  \( B_{\parallel}(\text{Gauss}) = 400 \times p(\%) \)
  
  (40 Gauss \( \sim 0.1\% \))

- Due to low degree of circular polarization:
  - Major concerns are: how to suppress sidelobes, noise, systematic instrumental circular polarization, etc.
    1. Average visibilities and images
    2. Minimize sidelobes by deep CLEANing
    3. Avoid radio burst influence and bad data (manually or automatically)
    4. Use image rotation during one-day observation

Upper Left: 17GHz intensity (I)
Upper Right: 17GHz circular polarization (V)
Lower Left: 34GHz intensity (I)
Radio polarization (by Iwai)

Fig. Radio intensity at 17 GHz.
(20 min averaging)

Fig. Radio circular polarization at 17 GHz.

Polarization is up to 3%

- 10 sec integration (visibility) map
- I-map: Max=29,415 K
- V-map:
  - Sigma=63 K
  - Max,min=319/-415 K

2016/03/15 ChromoAID@Boulder

DISTRIBUTION A. Approved for public release: distribution unlimited.
Left: I-map (10min/img * 1 img)
Right: V-map (same as above)
Display range: 
-200 K ~ + 200 K

Left: V-map (gray) & I-map (contour, [8, 12, 16, 20]*10^3 K)
Right: V-map (gray) & E-W cross cut through disk center, horizontal lines interval: 50K

Left: I-map (10min/img * 25 img)
Right: V-map (same as above)
Display range: 
-200 K ~ + 200 K

Left: V-map (gray) & I-map (contour, [8, 12, 16, 20]*10^3 K)
Right: V-map (gray) & E-W cross cut through disk center, horizontal lines interval: 50 K
6. Summary

• Magnetic field measurements using radio method is explained and examples of actual measurements using Nobeyama Radioheliograph are presented
  • Coronal magnetic field in a post flare arcade of loops (optically thin, uniform temperature)
  • Chromospheric magnetic field in an active region (optically thick with temperature gradient)
• Ideas on how to synthesize full disk chromospheric magnetic field map are discussed:
  • Minimization of influences from sidelobes, noise, radio bursts, etc. (0.1 % or better circular polarization degree is required)
Abstract

Global solar cycle activity is strongly correlated with magnetic field in the solar atmosphere. To understand physical mechanisms of the strong correlation between them, information of magnetic field strength in the solar upper atmosphere, the transition region and the corona, is inevitable. However, magnetic field measurements in these regions are difficult due to tenuous atmosphere. Measurement of magnetic field in the chromosphere is possible with some limitations. So far, chromospheric magnetic field is measured by optical methods. In this work, we use radio technique to measure chromospheric magnetic field independently from optical technique.

Nobeyama Radioheliogrpah (NoRH) has been observing the full disk images of the sun at 17 and 34 GHz. At 17 GHz, both circularly polarized emissions are obtained. The emission mechanism is thermal free-free in the quiet sun and most parts of active regions except above sunspot umbrae. Even though thermal emission from the chromosphere is optically thick, we can detect circularly polarized component due to the combination of temperature gradient and opacity difference between the two circular polarizations. We can invert the circular polarization degree map into the line-of-sight component of the magnetic field in the chromosphere. The inversion is rather simple, but how to get images with low noise and low instrumental polarization is another thing. By lowering noise and instrumental effects, we tried to measure full disk images of the chromospheric magnetic field including weak and diffuse field in the quiet sun.
Solar Radio Physics and MHD

Kiyoto SHIBASAKI
Solar Physics Research Inc.
shibasaki.kiyoto@md.ccnw.ne.jp
http://www.md.ccnw.ne.jp/sprinc/

Abstract

• Circularly polarized radio waves interact with charged particles in magnetic field even under highly collisional chromosphere. This nature is used to measure magnetic field. However, the standard MHD theory assume that charged particles are not magnetized due to frequent interruption of gyration. Hence, magnetic moment of plasma is not included in MHD. In this talk, it is shown that magnetic moment does not disappear even under highly collisional condition and that a new MHD is needed to understand behavior of plasma in the solar atmosphere.
Outline

1. Necessity to revise the classical MHD
2. On MHD (limitations and causality reversal)
3. Magnetic moment of thermal plasma
4. Electric currents in the magnetized plasma and self-consistent field
5. Magnetic moment and Kelvin Force: the force to push plasma upwards along the magnetic field in the solar atmosphere
6. Summary and conclusions

Necessity to revise the MHD to understand solar (radio) physics

• Interaction between circularly polarized radio wave with electron gyration around magnetic field
  • Measurement of magnetic field in the dense solar atmosphere (chromosphere)

• Up-flow of plasma in unipolar magnetic field region
  • Good correlation between enhanced magnetic field and microwave radio brightness in unipolar region (coronal hole)

• Good correlation between unsigned total magnetic flux and integrated microwave flux density
On MHD

• MHD was introduced to study the interaction of electrically conducting media, such as liquid metal, with magnetic field. To apply MHD to thermal plasma, there are some limitations. (diamagnetic moment due to thermal random motion is not included)

• “Cosmical Electrodynamics” (2nd ed.) by Alfvèn and Fallback [1963]
  • When the electrically conducting material consists of ionized gas (plasma), the thermal motion of the individual particles produces several important phenomena (diamagnetism, ambipolar diffusion, etc.). Such phenomena fall outside the scope of magneto-hydrodynamics taken in a restricted sense and are discussed in the chapters on plasma physics (Chapters 4 and 5)

Magnetic moment in MHD textbooks

1. Parker[2007] p.54 last paragraph
   • The essential point for our conversation of electric and magnetic fields in the cosmos is that the hot gases that are everywhere in the cosmos have very little electric polarizability and very little magnetic susceptibility, so $D=E$ and $H=B$ to good approximation.

2. Cowling[1976] p.3 line 4
   • The material is assumed nonmagnetic: thus its permeability $\mu$ has the value $4\pi \times 10^{-7}$ henry/m characteristic of free space.

3. Priest[1982] p.73 after Eq(2.4)
   • where constitutive relations $H = B/\mu$, $D = \epsilon E$ have been used to eliminate the magnetic field (H) and electric displacement (D). (For solar plasma, $\mu$ and $\epsilon$ are invariably approximated by their vacuum values, $\mu_0$ and $\epsilon_0$, respectively)

   • The magnetic permeability of the media considered in magnetohydrodynamics differs only slightly from unity, and the difference is unimportant as regards the phenomena under discussion. We shall therefore take $\mu = 1$ through the present chapter.

5. Chen[1984] p. 55 line 1, p.56 line 8
   • In a plasma, the ions and electrons comprising the plasma are the equivalent of the “bound” charges and currents. Since these charges move in a complicated way, it is impractical to try to lump their effects into two constants $\epsilon$ and $\mu$. Consequently, in plasma physics, one generally works with the vacuum equations [3-1] – [3-4], in which $\sigma$ and$\mu$ include all the charges and currents, both external and internal.
   • In a plasma with a magnetic field, each particle has a magnetic moment $\mu$, and the quantity $M$ is the sum of all these $\mu$ in $1\text{m}^3$. But we now have
     $$\mu_a = \frac{m_1 v_1^2 \sigma_a}{B} \propto \frac{1}{B} \quad M \propto \frac{1}{B}$$

The relation between $M$ and $H$ (or $B$) is no longer linear, and we cannot write $B = \mu_0 H$ with $\mu_0$ constant. It is therefore not useful to consider a plasma as a magnetic medium.
Causality reversal in MHD

According to Dungey “Cosmic Electrodynamics” 1958

§ 1.4 Causal relationships
- Laboratory electrodynamics: \( J \) causes \( H \)
- Cosmic electrodynamics: \( H \) causes \( J \)

§ 1.5 Orbits of particles
- The calculation of the orbits of individual particles is not as important as might be thought at first sight, ....
- ..., when the field depends on the orbits, a very difficult self-consistent field problem ensures.
- It may here be noted that any effects due to the actual dipole moments of the various particles in the gas are negligible.

§ 2.1 Need for a statistical treatment

Orbit approach is abandoned and causality is reversed

(* In magnetic reconnection, current disruption causes magnetic reconfiguration \((J\) controls \(H\)). Causality is reversed again)

Toward normal causality (Classical Electromagnetism)

1. Calculate magnetic moment and magnetization current of thermal plasma
2. Calculate drift current of thermal plasma
3. Determine self-consistent magnetic field using the relation of currents: total current = magnetization current + drift current
4. Solve inconsistencies in MHD caused by reversed causality
5. Revise MHD equations for plasma (pMHD) with normal causality (consistent with classical electromagnetism)
6. Apply pMHD to the solar atmosphere and answer unsolved problems
Magnetic moment of the media

- Magnetic moment in a small volume (definition)

\[ \Delta M = \sum \frac{1}{2} \int Nf(v)(r \times j)dv \Delta V \]

Where,
- \( \Sigma \): sum of ions and electrons
- \( N \): number density
- \( f(v) \): velocity distribution function
- \( r \): position of charged particle from its guiding center
- \( j \) (current) = \( qv \), \( q \): charge, \( v \): velocity
- integral: in the velocity space perpendicular to \( B \)
- \( \Delta V \): small volume (contains enough number of particles)

- Assuming that \( f(v) \) is the isotropic Maxwellian distribution for ions and electrons with the same temperature

\[ M = \frac{\Delta M}{\Delta V} = \sum \frac{q}{2} \int Nf(v)(r \times v)dv \]

\[ = - \sum \int Nf(v) \frac{mv^2}{2B} dv \delta B = - \frac{2Nk_B T}{B} b = - \frac{P}{B} b \]

- Equation of motion under Lorentz force and collisional impact force:

\[ m \frac{dv}{dt} = q(v \times B) + f_c, \quad \delta(v - r \times \omega) = \frac{f_c}{m} \delta t \]

\[ \omega = \frac{qB}{m}, \quad r \times v = - \frac{v^2}{\omega} b = - \frac{mv^2}{\omega qB} \]

- By the collisional impact, \( v \) will be modified hence \( r \) and the guiding center, but the Maxwellian distribution of \( v \) does not change.

- This result is consistent with the result calculated from Larmor motions of individual particles (magnetic moment = circular current \( x \) area)
Magnetic moment of thermal plasma

\[ M = -\frac{2Nk_B T}{B} \mathbf{b} = -\frac{P}{B} \mathbf{b} \]

Where,
- \( M \): magnetic moment per unit volume
- \( B \): magnetic flux density (magnetic field)
- \( N \): number density of electrons (= ions, assuming protons)
- \( k_B \): Boltzmann constant
- \( T \): temperature (assuming same for ions and electrons)
- \( P \): gas pressure
- \( \mathbf{b} \): unit vector along \( B \)

- Magnetic moment is independent of mass and charge, dependent only on temperature
- Plasma is the non-linear diamagnetic media

Influence of collisions between plasma particles to the magnetic moment

- Magnetic moment is defined at each moment (average is in velocity space, not in time), hence the complete gyration motion is not required. Frequent interruption of gyration by collisions do not influence the magnetic moment.
- The condition for plasma magnetization, “cyclotron frequency >> collision frequency” is not needed.
Magnetic moment and plasma beta

• The ratio between magnetic moment and magnetic field is:

\[
\frac{\mu_0 M}{B} = -\frac{1}{2} \frac{P}{B^2/2\mu_0} = -\frac{\beta}{2}
\]

• Low-beta assumption is equivalent to ignoring magnetic moment

Magnetic field in the plasma

• Magnetic field in the magnetized media is as follows
  B: magnetic flux density (magnetic field)
  \(\mu_0\): magnetic permeability of the vacuum
  H: magnetic intensity
  M: magnetic moment

\[
B = \mu_0 \left( H + M \right)
\]

• In case of thermal plasma
  • B (left) is the function of B (right), i.e. B is a self-consistent field.

\[
B = \mu_0 \left( H - \frac{P}{B} b \right)
\]

• Using the magnetic permeability of the vacuum for plasma is equivalent to ignoring magnetic moment of plasma.
Self-consistent field equation

• Total current = magnetization current + drift current

\[ \nabla \times \left( \frac{\mathbf{B}}{\mu_0} \right) = \text{magnetization current + drift current} \]

Both magnetization current and drift current are functions of B, this equation can be the equation to determine B self consistently.

Currents and self-consistent field

• Magnetization current

\[ \mathbf{J}_m = \nabla \times \mathbf{M} = \nabla \times \left( -\frac{P}{B} \mathbf{b} \right) = \left( -\frac{\nabla P}{B} + \frac{P}{B^2} \nabla B + \frac{P}{B} \kappa \right) \times \mathbf{b} - \frac{P}{B} \left( \mathbf{b} \cdot \left( \nabla \times \mathbf{b} \right) \right) \mathbf{b} \]

• Drift current = grad B drift current + curvature drift current

\[ \mathbf{J}_d = \mathbf{J}_{\nabla B} + \mathbf{J}_c = \left( \frac{P}{B^2} \nabla B + \frac{P}{B} \kappa \right) \times \mathbf{b} \]

• Total current

\[ \mu_0 \mathbf{J}_{\text{total}} = \nabla \times \mathbf{B} = \nabla \times \left( B \mathbf{b} \right) = \nabla B \times \mathbf{b} + B \nabla \times \mathbf{b} \]

\[ = \left( \nabla B - B \kappa \right) \times \mathbf{b} + B \left( \mathbf{b} \cdot \left( \nabla \times \mathbf{b} \right) \right) \mathbf{b} \]

• Relations derived from the current equation (Total = magnetization + drift)
  • Currents originated from field intensity gradient and curvature are cancelled.
  • Total (gas + magnetic) pressure balance is established (self-consistent field)

\[ \frac{\nabla B}{\mu_0} + \frac{\nabla P}{B} = B \frac{1}{\mu_0} \frac{B_0}{R} \frac{B_0}{\mu_0} \frac{1}{R} \left( \frac{B^2}{2\mu_0} + P \right) \]

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Forces acting on magnetized plasma

1. Ampere force
\[ F_A = (J_m + J_d) \times B = \left( -\frac{\nabla P}{B} \times b \right) \times B = (\nabla P)_\perp \]
F_A balances with perpendicular gas pressure gradient force

2. Kelvin force
\[ F_K = (M \cdot \nabla) B = -\frac{P}{B} \left( \nabla || B \right) b \]
Original Kelvin force uses \( \mu_0 H \) rather than B. However, recent studies show that B should be used.

• The Kelvin force does not appear in the standard MHD equations because magnetic moment is not included in the MHD.

Motion of plasma along vertically oriented field with intensity gradient

• MHD equation of motion (modified)
\[ \rho \left\{ \frac{\partial v}{\partial t} + (v \cdot \nabla) v + v (\nabla \cdot v) \right\} = -\nabla P + J \times B + (M \cdot \nabla) B + \rho g - \alpha N v \]

In case of steady flow along vertically oriented magnetic field:
\[ \rho v (\nabla \cdot v) = -\frac{\partial P}{\partial z} - \frac{P}{B} \frac{\partial B}{\partial z} - \rho g - \alpha N v \]

In case of no flow:
\[ -\frac{\partial P}{\partial z} - \frac{P}{B} \frac{\partial B}{\partial z} = \rho g \]

where, \( \rho \), \( g \), \( N \), \( \alpha \) are the mass density, the gravity acceleration, the number density and the friction coefficient per particle.
Static relation

- Static relation of isothermal plasma filled in vertically oriented magnetic field (a leg of a coronal loop)

\[ \frac{1}{N} \frac{\partial N}{\partial z} - \frac{1}{B} \frac{\partial B}{\partial z} = \frac{mg}{2k_BT}, \quad \frac{1}{\ell_N} + \frac{1}{\ell_B} = \frac{1}{H} \]

\( P = 2Nk_BT, \rho = mN, \) where \( N \) is the number density, \( T \) is the temperature, \( m \) is the proton mass, and \( k_B \) is the Boltzmann constant.

- The static relation is reduced to the relation between density \( (\ell_N) \), magnetic field intensity \( (\ell_B) \), and hydrostatic \( (H) \) scale lengths.

- Generally, \( N \) and \( B \) decrease upwards hence all terms are positive.

The condition for loop top plasma concentration

- Leg part of a coronal loop filled with the thermal plasma is discussed:

\[ \frac{1}{\ell_N} + \frac{1}{\ell_B} = \frac{1}{H}, \quad \frac{1}{\ell_N} = -\frac{1}{N} \frac{\partial N}{\partial z}, \quad \frac{1}{\ell_B} = -\frac{1}{B} \frac{\partial B}{\partial z}, \quad \frac{1}{H} = \frac{mg}{2k_BT} \]

- Without the Kelvin force, the density scale height is equal to the hydrostatic scale height (classical MHD).

- The magnetic scale length is roughly the size of the active region or the loop size, and is independent of plasma properties (temperature, density).

- When the temperature exceeds a certain value (large \( H \)), the density scale length need to be negative, or the density has to increase upwards.

- This is the condition for loop top plasma concentration.

- Even if the density scale length is positive, it is larger than the hydrostatic scale length due to the Kelvin force.
The temperature condition for loop top concentration

• The condition for negative density gradient, or loop top concentration, is:

\[ H > \ell_B , \quad T > \frac{mg}{2k_B} \ell_B = \frac{1.67 \times 10^{-27} \times 2.74 \times 10^2}{2 \times 1.38 \times 10^{-23}} \ell_B = 1.66 \times 10^{-2} \ell_B \]

• In the loop with the size normalized by 100Mm (\( \ell_B \)), the condition of temperature (\( T_\alpha \), measured in the unit of million K) for loop top concentration is:

\[ T_\alpha > 2 \times \ell_8 \]

• In case of very high temperature plasma (such as solar flares), highly concentrated plasma is expected

\[ \ell_N \sim -\ell_B \]

Steady flow in open magnetic loop

• We study the steady flow of plasma along vertically oriented open magnetic field. We assume that the plasma is incompressible and the density gradient is negligible.

\[ \frac{P}{B} \frac{\partial B}{\partial z} - \rho g - \alpha N v = 0 \]

\[ v = \frac{P}{\alpha N \ell_B} - \frac{\rho g}{\alpha N} = \frac{2k_B}{\alpha \ell_B} T - \frac{mg}{\alpha} \]

\[ v_3 = \frac{2 \times 1.38 \times 10^{-23} \times 10^6}{\alpha \ell_8} \times T_\alpha - \frac{1.67 \times 10^{-27} \times 2.74 \times 10^2}{\alpha \times 10^1} \]

\[ = \frac{2.76 \times 10^{-28}}{\alpha \ell_8} \times T_\alpha - \frac{4.58 \times 10^{-28}}{\alpha} = a \times T_\alpha - b \]

\[ a = \frac{2.76 \times 10^{-28}}{\alpha \ell_8} , \quad b = \frac{4.58 \times 10^{-28}}{\alpha} \]
Kelvin force

- Plasma with magnetic moment get Kelvin force from magnetic field with intensity gradient.
- Generally, magnetic field weakens upwards, hence the force push the plasma upwards.
- This is the origin of ubiquitous hot plasma upflows in the solar atmosphere.

Summary and conclusions

- In cMHD:
  - There are some limitations (ignore magnetic moment)
  - Causality between current and magnetic field is reversed (reversed again in magnetic reconnection)
- To return to normal causality, magnetization current and drift current are calculated and the self-consistent field equation is formulated.
  - Using this equation, it is shown that currents originated from field intensity gradient and curvature are cancelled and that total pressure balance is established.
- Kelvin force acting on plasma can explain upward flow of hot plasma in the solar atmosphere
- Further studies
  - Set of equations for pMHD should be revised and inconsistent concepts should be corrected.
  - Apply these equations to study high-beta plasma behavior (such as instabilities) in the solar atmosphere should be studied.
Abstract
Magnetic property of thermal plasma in the solar atmosphere is studied. It is shown that even in highly collisional thermal plasma, magnetic moment does exist and plays important roles in solar physics. The magnetic Kelvin force, which works on magnetic moment in inhomogeneous magnetic field, drives flow of plasma along magnetic field lines. Due to diamagnetism, plasma is pushed toward weak field region, generally upwards. If the Kelvin force exceeds the gravity force due to high temperature, upward plasma flow is expected. Also, hot plasma concentration around the loop top is expected in the closed loop. Using observational results, physical parameters of the solar atmosphere are derived.

1. Introduction
For studies of high energy phenomena on the Sun, it is necessary to understand behaviors of accelerated charged particles by solar flares. In the presence of magnetic field, Lorentz force is the major force acting on the particles and influence of collisions with the surrounding plasma particles could be small. Under this condition, orbit theory of charged particles is a powerful tool and various adiabatic invariants can be found. Circular currents due to circular motions of charged particles around the magnetic field are equivalent to the magnetic moment and this moment is one of the most important invariants. Due to invariance, charged particles are trapped around the magnetic loop top by the mirror force. Bright loop top microwave emission (e.g. Melnikov, Shibasaki, and Reznikova, 2002) can be explained by this mechanism.
Behaviors of thermal plasma which we can see in soft X-ray and EUV are treated by the Magneto hydrodynamic (MHD) theory (Priest, 2014). In MHD, magnetic moment is not included (Cowling, 1976). Due to frequent collisions among thermal charged particles, circular motions are interrupted and it is believed that the magnetic moment goes out. The condition for charged particles to have magnetic moment (or to be magnetized) is that the mean-free-path is long compared to the gyro-radius or the collision frequency is small compared to the gyro-frequency. However, as is shown in the following section, the magnetic moment does not go out even in highly collisional thermal plasma. Thermal plasma should be treated as a magnetic media and the Kelvin force, which acts on magnetized media, should be included in MHD equations. With the Kelvin force in MHD, we can interpret various phenomena in the solar atmosphere such as: loop top plasma concentration (Acton et. al., 1992; Feldman et. al., 1994), temperature dependent plasma up flows (Peter and Judge, 1999; Milligan and Dennis, 2009), solar wind acceleration, etc.

In the following sections, magnetic moment of thermal plasma is derived and the MHD equation of motion is revised by adding the Kelvin force acting on the magnetic moment. Various observed dynamical phenomena of thermal plasma along magnetic field lines are discussed using the revised MHD equation of motion.

2. Magnetic moment of thermal charged particles and the Kelvin force

2.1 The equation of motion of a charged particle

The equation of motion of a charged particle in a constant or slowly varying (in space and time) magnetic field \( \mathbf{B} \) is (1). Forces acting on the particle such as the gravity force and the electric force are put into \( \mathbf{F} \) and are assumed to be constant or vary slowly in space and time. The mass, the electric charge and the velocity of the particle are \( m, q, \) and \( \mathbf{V} \) respectively. The SI system of units is used.

\[
m \frac{d\mathbf{V}}{dt} = q \mathbf{V} \times \mathbf{B} + \mathbf{F}. \tag{1}
\]

The velocity can be divided into the parallel (along \( \mathbf{B} \)) component and the perpendicular component \( (\mathbf{V} = \mathbf{V}_\parallel + \mathbf{V}_\perp) \). The perpendicular component can be divided into the constant (or slowly varying) component \( \mathbf{v}_0 \) and the time dependent component \( \mathbf{v} \) : \( \mathbf{V}_\perp = \mathbf{v}_0 + \mathbf{v} \). The parallel component of (1) is not influenced by \( \mathbf{B} \) and the following
two equations are derived from the perpendicular component.

\[ \mathbf{v}_0 = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}, \text{ and} \]

\[ m \frac{d\mathbf{v}}{dt} = q\mathbf{v} \times \mathbf{B}. \]

According to Equation (2), the particle drifts with the constant (or slowly varying) velocity perpendicular to both \( \mathbf{F} \) and \( \mathbf{B} \). It is interesting to note that this drift motion does not depend on the result of the time varying part (3), hence the drift motion exists at each moment of time. The time varying \( \mathbf{v} \) can be expressed as the product of the magnitude \( v \) and the unit vector \( \mathbf{v}_1 \) directed to the velocity: \( \mathbf{v} = v \mathbf{v}_1 \). The time derivative of \( \mathbf{v} \) is,

\[ \frac{d\mathbf{v}}{dt} = \frac{dv}{dt} \mathbf{v}_1 + \frac{v^2}{\rho} \mathbf{n}_1, \]

where, \( \rho \) and \( \mathbf{n}_1 \) are the curvature radius and the unit vector directed to the normal direction respectively. By combining Equations (3) and (4), the following two relations corresponding to each direction are derived.

\[ m \frac{dv}{dt} = 0, \text{ and} \]

\[ m \frac{v^2}{\rho} = qv\mathbf{B}. \]

Equation (5) tells us that the kinetic energy is constant in the plane perpendicular to \( \mathbf{B} \). From Equation (6), we can get the curvature radius of the motion.

\[ \rho = \frac{mv}{qB}. \]

The curvature radius can be defined at each moment of time. The concept of the guiding center is not needed to derive both the drift motion and the curvature radius.

2.2 Magnetic moment

According to the classical theory of electromagnetism (e.g. Panofsky and Phillips, 1962), the magnetic moment of a particular volume containing currents \( \mathbf{j}_m \) is defined as,

\[ \Delta \mathbf{M} = \frac{1}{2} \int (\xi \times \mathbf{j}_m) dV, \]

where, \( \xi \) is the molecular coordinate and the integration is carried out in the volume. In the case of the charged particle, we can replace \( \xi \) by \(-\rho \mathbf{n}_1\) and \( \mathbf{j}_m \) by \( q\mathbf{v} \), and integrate over the velocity space of thermal plasma particles. The resultant magnetic
moment per unit volume is,

\[ \mathbf{M} = -\sum \int \frac{m_\alpha v^2}{2B} \mathbf{b}_\alpha f_\alpha (v) \, dv, \]  

(9)

where, \( \sum \) is the sum of various species. In case of thermal plasma, the velocity distribution function \( f_\alpha (v) \) is the two-dimensional Maxwellian distribution. Even though the particle motions are random, the magnetic moments are aligned to \(-\mathbf{b}_1\) direction. For simplicity, we assume that the plasma consists of electrons and protons and they have the same uniform temperature. In this case, the magnetic moment becomes very simple.

\[ \mathbf{M} = -\frac{P}{B} \mathbf{b}_1, \]  

(10)

where, \( P = 2Nk_B T \) is the gas pressure, \( k_B \) is the Boltzmann constant, \( N \) and \( T \) are the number density of protons (also electrons) and the temperature respectively. The magnetic moment does not depend on particle charge nor mass if they are in the thermal equilibrium state. In the above derivation of the magnetic moment, time averaging or integration is not applied. The magnetic moment is defined at each moment, hence the complete gyration is not required. The concept of the guiding center is not needed for magnetic moment. Collisions among plasma particles do not suppress the magnetic moment. If we add the friction term proportional to the velocity \((-\alpha v)\) to Equation (1), the motion slows down and the magnetic moment will decrease. However, thermal random motion do not decay that means thermal particles do not cool down. Dissipated energy by the friction cannot be used to heat up other particles. This is the limitation by the second law of thermodynamics. By collisions, thermal particles get or loose energy and the resultant velocity distribution is the same Maxwellian. The magnetic moment is no longer constant.

According to Equation(10), the thermal plasma is diamagnetic and nonlinear. Magnetic field in a media is,

\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}), \]  

(11)

where, \( \mu_0 \) is the magnetic permeability of the vacuum. In the linear media, magnetic moment is proportional to \( \mathbf{H} \), hence \( \mathbf{B} = \mu \mathbf{H} \). The coefficient \( \mu \) is called the magnetic permeability of the media. However, as the plasma is not a linear media, we
cannot define the magnetic permeability of the thermal plasma. We have to discuss the magnetic moment separately. The ratio between $M$ and $B$ is,

$$\frac{\mu_0 M}{B} = -\frac{\mu_0 P}{B^2} = -\frac{1}{2} \beta,$$

(12)

where $\beta$ is the plasma beta. Because $B$ is a function of $B$ itself, $B$ in the thermal plasma is a self-consistent field.

According to the Bohr-van Leeuwen theorem, magnetic moment should not appear in a thermal plasma (Van Leeuwen, 1921; Van Vleck, 1932; Nielsen, 1972). In their calculations, the position vectors of free electrons and velocity vectors are treated independently. Hence, the averaged velocity or the current at a certain position is zero and the magnetic moment is zero also. However, the position vector is determined by the velocity as is shown in Equation (7). The origin of the position vector should be the curvature center of the motion and is not common to all the particles. The position vectors and the velocity vectors should not be treated separately. In case the plasma surrounded by a conductor, the current at the boundary due to reflecting particles cancels the magnetic moment inside. In the solar atmosphere, reflection current does not flow at the boundary, hence the magnetic moment does exist.

2.3 The Kelvin force and the MHD equation of motion

The force acting on the magnetic moment in non-uniform magnetic field is called the Kelvin force. Original form of the force is $F_k = (\mathbf{M} \cdot \nabla) \mu_0 \mathbf{H}$ (Landau, Lifshitz, and Pitaevskii, 1984). However, the recent study by Odenbach and Liu (2001) shows that $\mathbf{B}$ should be used rather than $\mu_0 \mathbf{H}$. We use this expression and call it as the Kelvin force. The Kelvin force acting on the magnetized plasma is,

$$\mathbf{F}_k = (\mathbf{M} \cdot \nabla) \mathbf{B} = -\frac{P}{B} (\nabla \parallel B) \mathbf{b}_1.$$

(13)

This force should be added to the MHD equation of motion. The revised MHD equation of motion is as follows.

$$\rho_m \left\{ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right\} = -\nabla P + \mathbf{J} \times \mathbf{B} + (\mathbf{M} \cdot \nabla) \mathbf{B} + \rho_m g - \alpha \mathbf{N} \mathbf{u} + \mathbf{F},$$

(14)

where $\rho_m$, $\mathbf{u}$, $\mathbf{J}$, $g$, $\alpha$, $\mathbf{F}$ are, the mass density, the velocity of the fluid element, the current density, the gravity acceleration, the friction coefficient, and other forces respectively. The Kelvin force is the extension of the mirror force to the fluid.
3. Dynamics of the thermal plasma along the magnetic field line

By adding the Kelvin force to the MHD equation of motion, plasma dynamics along the magnetic field are influenced. The MHD equation of motion along the magnetic field is as follows. It is assumed that no other forces are acting on the plasma.

\[
\rho_m \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial s} \right) = -\frac{\partial P}{\partial s} - \frac{P}{B} \frac{\partial B}{\partial s} - \rho_m g \cdot \cos(\theta) - \alpha N u, \tag{15}
\]

where \( s \) and \( \theta \) are the distance along the magnetic field and the angle between the magnetic field and the vertical directions respectively.

3.1 Hot plasma confinement around the loop top

In this subsection, we discuss plasma density distribution along a closed magnetic loop without flow. By putting \( u = 0 \) in Equation (15), we can study the force balance along a closed magnetic loop.

\[
\frac{\partial P}{\partial s} = -\frac{P}{B} \frac{\partial B}{\partial s} - \rho g \cos(\theta). \tag{16}
\]

This gives the density distribution along the loop by assuming the uniform temperature distribution.

\[
\frac{\partial}{\partial s} \ln(N) = \frac{1}{\ell_B} - \frac{\cos(\theta)}{H_g}, \tag{17}
\]

where, \( \frac{1}{\ell_B} \equiv -\frac{1}{B} \frac{\partial B}{\partial s}, \frac{1}{H_g} \equiv \frac{m_p g}{2k_B T} \) and \( m_p \) is the proton mass. The density profile is determined by the balance between the magnetic scale length \( (\ell_B) \) and the hydrostatic scale height \( (H_g) \). It does not depend on the magnetic field strength. With the Kelvin force, the sign of the density gradient can change. The condition for the positive density gradient is,

\[
H_g \geq \ell_B \cos(\theta). \tag{18}
\]

The critical temperature which divide the sign of the density gradient is,

\[
T_c = \frac{m_p g}{2k_B} \ell_B \cos(\theta). \tag{19}
\]
Higher than the critical temperature, the plasma density increases upwards. The density scale length around the critical temperature is extended. The density profile along the loop can be derived by integrating Equation(17).

Figure 1 shows a model magnetic field generated by a magnet placed horizontally at (0, -1) and a selected magnetic field line by the thick line. The coordinate unit is relative and the hydrostatic scale height is set to 10. It is assumed that the magnetic field is not influenced by the plasma. The density profile along the loop is shown in Figure 2. This is very similar to the X-ray intensity profiles along various loops observed by XRT/Hinode (Kano and Tsuneta, 1996).

Fig. 1  A model magnetic field generated by a magnet placed horizontally at (0, -1). The thick line is the selected magnetic field line.
Fig. 2  The density profile along the selected field line shown in Fig. 1, assuming that the hydrostatic scale length is 10. The plotted density is relative to the density at the foot point. The horizontal coordinate “s” is the distance along the field line.

Here, it is assumed that the magnetic moment does not deform the existing magnetic field and this assumption is equivalent to the low beta plasma assumption. If the pressure increase around the loop top is not negligible compared to the magnetic pressure (finite plasma beta), the magnetic field strength decreases due to diamagnetism and the field lines expand to keep the total pressure balance (gas + magnetic pressure = constant). Bulge type plasma concentrations can be formed, which are often observed in the post flare loops (Doschek, Strong, and Tsuneta, 1995).

3.2 Plasma motion along an open magnetic field line

In this subsection, we discuss a constant velocity plasma flow along an open magnetic field line. We assume that the motion is not bounded by the top and the bottom boundaries, hence the pressure gradient force can be ignored. The resultant equation of motion is,

$$\alpha Nu = -\frac{P}{B} \frac{\partial B}{\partial s} - \rho_m g \cdot \cos(\theta).$$  \hspace{1cm} (20)

The velocity is regulated by the friction force. The velocity is proportional to the temperature.

$$u = -\frac{2k_B}{\alpha \ell_B} T - \frac{m_p g}{\alpha} \cos(\theta).$$  \hspace{1cm} (21)

The dividing temperature between up and down flows corresponds to the critical
temperature (19). There are many observational reports of temperature dependent plasma upflows in active regions and the quiet sun (e.g. Peter and Judge, 1999; Teriaca, Banerjee, and Doyle, 1999; Dadashi, Teriaca, and Solanki, 2011). The reported dividing temperatures are between 1 MK and 0.5 MK, which correspond to the magnetic scale length $\ell_B \cos(\theta)$ of 86 and 43 arc sec. respectively. These values are in the order of active region sizes.

Milligan and Dennis (2009) studied temperature dependence of the plasma flow velocity generated by an impulsive solar flare. They found the following relations for upflows (2 - 16 MK) and downflows (0.5 - 1.5 MK).

\[ -u_{up} (\text{km/s}) = 8 - 18T(MK), \quad \text{and} \]
\[ -u_{down} (\text{km/s}) = 60 - 17T(MK). \]  

The critical temperature is roughly 2 MK and the temperature coefficient is 18 (or 17). Using these observed values, we can estimate the magnetic scale length and the friction coefficient from Equations (19) and (21). Estimated values are, $\ell_B = 172 / \cos(\theta)(")$ and $\alpha = 1.4 \times 10^{-29} \cos(\theta)(\text{kg/m})$. Using these values, we can calculate the constant term as 35 (km/s). However, the observed values are 8 km/s for upflows and 60 km/s for downflows and a velocity jump exists at the critical temperature. The discontinuity at the critical temperature cannot be explained by the Kelvin force. We need another mechanism to explain the discontinuity. If we assume that there are mean upflow of 27 km/s and downflow of -25 km/s beside the temperature dependent flows, we can explain the observed constant terms ($35 \cdot 27 = 8, 35 + 25 = 60$), because the friction force acts on the velocity difference. However, the reason why the average flow velocities are different for up- and down-flows is not clear.

3.3 Solar wind acceleration

In the previous subsection, we assume that the velocity is regulated by the friction force. If the friction force is negligible, the velocity changes. Under the steady state condition, the equation of motion along the field is,
\[ \rho_m \frac{\partial u}{\partial s} = -\frac{P}{B} \frac{\partial B}{\partial s} - \rho_m g \cdot \cos(\theta). \]  

(24)

For the radial magnetic field, Equation (24) is,

\[ \frac{\partial}{\partial r} \left( \frac{1}{2} m_p u^2 \right) = \frac{4k_B T}{r} - m_p g_0 \left( \frac{\nu_0}{r} \right)^2, \]

(25)

where, \( r, \nu_0, \) and \( g_0 \) are the radial distance, the solar radius, and the surface gravity acceleration respectively. The right hand side of Equation (25) is the force acting on plasma particles and the left hand side is the kinetic energy increase rate with height. The original solar wind equation derived by Parker (1958) has the same forces but the velocity acceleration term is different. The difference is due to the fact that he used pressure gradient force to drive the solar wind, but we used the Kelvin force. With the Kelvin force, the solar wind equation can be applied to asymmetric cases. Here, frequent collisions and constant temperature are assumed. If collisions are negligible, we should treat the magnetic moment as constant and the equation of motion is,

\[ \frac{\partial}{\partial r} \left( \frac{1}{2} m_p u^2 + \mu_m B - m_p g_0 r^2 \frac{1}{r} \right) = 0. \]

(26)

This is the energy conservation equation (kinetic + magnetic + gravity potential = constant) along the distance. To explain flow speeds exceeding the thermal speed, non-thermal energy supply mechanisms such as waves and others are needed on top of the Kelvin force.

4. Discussion

In the previous sections, we discussed about behavior of thermal plasma in the magnetic field, but not the supply mechanism. How to supply heated plasma from below is called the coronal heating problem which is one of the major unsolved problem in solar physics (recent reviews are: Klimchuk, 2006; Reale, 2014). We can put a limitation to the proposed mechanisms and suggest a new mechanism. When the plasma is heated in the higher atmosphere, heated plasma will be pushed directly upwards as the solar wind due to the Kelvin force. Evaporation or ablation of lower atmosphere by heat conduction will be inefficient. Heating mechanisms, if they exist, need to work close to the temperature minimum layer. Direct supply of hot plasma from below the photosphere through the magnetic flux tube is another possible mechanism to supply hot plasma into the chromosphere and above. Generally, the magnetic field strength
decreases upwards even below the photosphere, hence the Kelvin force push the hot plasma upwards from below the photosphere into the upper atmosphere along the magnetic field line.

Magnetic moment is one of the most important properties of plasma. It influences plasma dynamics not only along the magnetic field by the Kelvin force but also perpendicular to the magnetic field by the Lorentz force. Plasma currents flowing perpendicular to the magnetic field are the magnetization current ($\nabla \times \mathbf{M}$) and the drift current. These currents are not related to electric field hence the Ohm’s law cannot be applied. High-beta plasmas confined in curved magnetic fields are unstable due to these currents. Plasma disruptions can occur, called high-beta disruption (localized interchange instability or ballooning instability), which is very similar to solar flares (Shibasaki, 2001). For such studies, the magnetic moment has to be included in MHD.

5. Conclusions

In this paper, it is shown that the magnetic moment does exist even in highly collisional thermal plasma and the magnetized plasma is subjected to the Kelvin force. By adding the Kelvin force to the MHD equation of motion, hot plasma concentration around the loop top and temperature dependent plasma up and down flows are explained even though the model is very simple and calculation is rather crude. Without the Kelvin force, we need special mechanisms to explain such phenomena separately. It is also shown that the revised MHD equation of motion can be used to discuss the solar wind acceleration. Many other phenomena can be discussed with the help of the Kelvin force, such as the coronal heating problem. Magnetic moment influences plasma dynamics not only along the field but also perpendicular to the field including the disruption. Inclusion of the magnetic moment in MHD is inevitable for studies of solar physics and plasma physics.

References


The Kelvin Force and Loop-Top Concentration

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Introduction

The Lorentz force (https://en.wikipedia.org/wiki/Lorentz_force) causes charged particles in a magnetized plasma to gyrate in a plane perpendicular to the direction of the field. This consequence is easy to understand if the Larmor frequency (https://en.wikipedia.org/wiki/Larmor_precession#Larmor_frequency) is greater than the collision frequency; the particle will move in a circle in 2D, or in a helical path in 3D. In the absence of strong gradients this gyrational motion is the first of the three adiabatic invariants (https://en.wikipedia.org/wiki/Adiabatic_invariant#Plasma_physics) of charged particles in a plasma, an extremely convenient concept for understanding the Van Allen Belts (https://en.wikipedia.org/wiki/Van_Allen_radiation_belt), for example.

But what happens when the collision frequency increases and the plasma becomes a fluid, in the MHD (http://www.scholarpedia.org/article/Magnetohydrodynamics) approximation to plasma kinetic physics? Does the Lorentz force cease to be important? This question underlies the important Bohr-Van Leeuwen theorem (https://en.wikipedia.org/wiki/Bohr–van_Leeuwen_theorem), first reported in the independent PhD theses of Niels Bohr (http://www.google.com/doodles/niels-bohrs-127th-birthday) and Hendrika Van Leeuwen (https://www.revolvy.com/topic/Hendrika%20Johanna%20van%20Leeuwen).

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According to this theorem, the bulk magnetism of a solid cannot be explained by classical physics and must have a quantum-mechanical explanation.

The theorem, based on statistical mechanics and classical physics, asserts that the net magnetism of a body in thermal equilibrium simply vanishes. Thus, quite surprisingly, any magnetic effect on an equilibrium solid body (diamagnetism, paramagnetism, ferromagnetism) requires an explanation based on quantum physics. How does this result apply to an inhomogenous plasma? It would be safe to say that these concepts do not often get addressed in practical research affairs in solar physics. But what if they were decisively important? In a magnetic fluid the a fluid approximation (MHD), this is called the Kelvin force. In a plasma this represents the bulk force resulting from the magnetic moment (http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/magmom.html) associated with the gyrations of the individual particles. In this Nugget we describe a possible application of the Kelvin force to solve a perplexing problem that arose largely from the soft X-ray observations of the Yohkoh SXT (http://ylstone.physics.montana.edu/ystorage/nuggets/index.html) soft X-ray telescope: what supports loop-top concentrations of plasma in solar flares?

--- The Observations ---

The loop-top concentrations in solar flares can be seen in a number of ways: soft X-rays (commonly), microwaves (Ref. [1]), and white light. Figure 1 provides an illustration that is little known or appreciated: a "white light prominence" (Ref. [2]). This rare kind of event is hard to observe, but probably also very common; typically one sees loop-top concentrations better at wavelengths with high image contrasts. White-light views such as this one are hampered by the glare of the photosphere, but the emission mechanism is probably the same as for a coronagraphic white-light view: Thomson scattering (http://en.wikipedia.org/wiki/Thomson_scattering), and therefore a direct measure of column density. What we see seems like a large mass elevated mysteriously to coronal heights by the flare. Such a concentration is inconsistent with hydrostatic equilibrium.

The Mathematics

We do not present a detailed derivation here, but note that the Larmor motion of the constituent particles produces a magnetic moment $\mathbf{M}$ and results in a net magnetic field $\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$, where $\mathbf{B}$ is the magnetic induction and $\mathbf{H}$ the magnetic field strength as spelled out in Maxwell's equations. The $\mathbf{M}$
term is not present in classical MHD theory and, in the real world, must be included. This leads to the Kelvin force

\[
F_K = (\mathbf{M} \cdot \nabla) \mathbf{B} = -\frac{P}{B} (\nabla_p B) \mathbf{b},
\]

and then to a revised formula for the MHD equation of motion:

\[
\rho \frac{d\mathbf{u}}{dt} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla P + \mathbf{J} \times \mathbf{B} + (\mathbf{M} \cdot \nabla) \mathbf{B} + \rho g - \alpha \mathbf{u} + \mathbf{F},
\]

where the terms have their usual meanings; \(\alpha\) is the frictional force and \(\mathbf{F}\) represents any other forces in the system. To put this in perspective we can compare it with the gravitational force very simply: if the vertical gradient of the magnetic field exceeds the gradient of gravity, it is upward and tends to levitate the plasma. Because of this it provides a simple explanation for the observed loop-top concentrations, noted recently by Ref. [4] with the comment "The origin of these brightenings and their dynamics is not well understood to date." The sketch in Figure 2 illustrates the situation.

Based on this, we have simulated a simple dipole flux tube, as shown in Figure 2, and find that with the inclusion of the Kelvin force the excess mass concentration at loop top results naturally. Note that we assume a low [plasma beta] so that the additional force does not alter the shape of the dipole flux tube. The result is consistent with the many observations of loop-top structures in soft X-ray images, for example the \textit{Yohkoh} results in Ref. [3].

There are of course many other possible applications of this revision to the MHD equation of motion.

\[\text{Figure 2: Toy model of a dipole magnetic field (left), with one flux tube identified; on the right, the plasma concentration resulting from adding the Kelvin force. Without this the flux tube would be in hydrostatic equilibrium. In this simulation the hydrostatic scale length is set to 10.}\]

**Conclusions**

The Kelvin force represents the mirror action of a gradient in the magnetic field of plasma volume. It has long been neglected for subtle reasons, including the influential but frequently misunderstood Bohr-van Leeuwen theorem. The magnetic moment is one of the most important properties of a plasma. It
influences plasma dynamics not only along the magnetic field by the Kelvin force, but also perpendicular to the magnetic field by the Lorentz force. A magnetic moment does exist even in a highly collisional and thermally relaxed plasma. We have described one application, but there are others.

The Kelvin force dominates the gravitational force wherever the vertical gradient of B exceeds the gradient of gravity; this is almost everywhere in the corona, since gravity originates at Sun center and the coronal magnetic field mainly via a multipole structure originating near the photosphere.

References


[2] "White-light flare observed at the solar limb" (http://adsabs.harvard.edu/abs/1992PASJ...44...55H)


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