Unified Theory and Algorithm for Solving Challenging Problems in Mathematical Physics and Complex Systems with Applications

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Final Report

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Unified Theory and Algorithm for Solving Challenging Problems in Mathematical Physics and Complex Systems with Applications

Supported by this AOARD grant, the PI and his post-doctor and co-workers have successfully developed/improved a breakthrough canonical duality theory and its associated algorithms for solving a large class of challenging problems in mathematical physics and complex systems. Within one year, he has published 1 book by Springer, 1 journal special issue (Springer), and about 29 papers (11 are journal papers). The most significant achievement is the solution to the well-known knapsack problem, which is listed as one of 21 NP-complete problems in computer science. By using the canonical duality theory, this 0-1 integer programming problem can be equivalently converted to a non-smooth concave maximization problem with only one dual variable, which can be solved very easily, therefore, this so-called NP-complete problem can be obtained analytically via this canonical dual solution. Application to computational physics leads to a powerful deterministic algorithm for solving the most challenging bi-level mixed integer programming problem in structural topology optimization.
Final Report for AFOSR Grant (AOARD) FA2386-16-1-4082  
July 1, 2016 – December 31, 2017

PI: David Y Gao, Alex Rubinov Professor of Mathematics, Federation University Australia

Title: A Breakthrough Theory and Algorithms for Solving Chaotic Dynamics and NP-Hard Problems in Mathematical Physics and Complex Systems

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2. Professor Z.C. Cai, Purdue University

Other Research Collaborators:
1. Dr. Eldar Hajilarov, Federation University
2. Dr. Vittorio Latorre, University of Rome.

Accomplishments/New Findings:
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Another important achievement of this project is a revolutionarily new global optimal method for solving chaotic dynamical systems. Instead of traditional linear iteration methods, a nonlinear dynamical system is first formed as a global optimization problem via the least squares method. Then using the canonical duality theory, this nonconvex minimization problem can converted as a concave maximization problem in dual space, which can be solved easily via the well-developed convex optimization techniques. If this canonical dual has a unique solution, the nonlinear system is not chaotic. Otherwise, the primal global optimization problem is NP-hard, and in this case, the nonlinear system has chaotic solutions. Therefore, the connection between chaos in complex systems and the NP-hardness in computer science is revealed for the first time.

Remaining significant achievements include a unified model for multi-scale complex systems, a unified methodology for solving a class of fully nonlinear partial differential equations, and a unified algorithm for solving a class of so-called NP-Hard problems in decision science and global optimization.

The projects proposed in the proposal are fully completed.
Fig. 1. Design domain for a long cantilever beam with external load

Fig. 2. Topology optimization results obtained by SIMP (a), BESO (b), and the canonical duality theory CDT (c).
Co-sponsored by Federation University, this AFOSR grant has been used to support one senior research fellow, one PhD student, several international visitors, and international research activities.

Keynote, Plenary, and Invited Lectures at International Conferences.
1. Invited Lecture, Symposia on Intelligent Technologies for Advancing and Safeguarding Australia, 15 August 2017, Deakin University, Geelong, Australia.
2. Invited Lecture, 12th International Symposium on Health Informatics and Management, 2-3 June, 2017, National Chiao Tung University, Taiwan.

Contributions within Discipline:
The canonical duality methodology can be used for modeling multi-scale complex systems. The canonical duality theory produces analytical solutions to a class of fully nonlinear partial differential equations in large deformation solid mechanics, post-buckling of thin-walled structures and phase transitions problems. The algorithm can be used for solving a large class of challenging problems in mathematical physics and complex systems.

Contributions to Other Disciplines:
Canonical duality theory has been used successfully in neural networks optimization, sensor communication systems, filter design, signal processing, machine learning, chaotic dynamical systems, decision making, supply chain, scheduling problems, and computational mechanics, light-weight structural design, etc.

Publications:
Books/Special Issue Published:

Papers published in international journals:

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**Papers to appear:**


5. Ruan, N. and Gao, DY. On Modelling and Complete Solutions to General Fixpoint Problems in Multi-Scale Systems with Applications