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MULTI-SCALE AND MULTI-PHYSICS SIMULATIONS USING THE MULTI-FLUID PLASMA MODEL

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UMass Dartmouth April 25th, 2017

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OUTLINE

1. Plasma
2. Multi-Fluid Plasma Model
   - Advantages
   - Limits of the model
3. Numerics: Blended Finite Element Method
   - Discontinuous Galerkin for ion/neutrals
   - Continuous Galerkin for electrons/fields
   - Initial tests
Plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior. 

“99% of matter in the universe is in the state of plasma"
TIME/Spatial scales.

Sean Miller, PhD dissertation, University of Washington (2016)
There are multiple plasma models.

- 3-Dimensions + 3-Velocities
- Evolve the particles position and velocity
- e.g. Particle-In-Cell models

- Ensemble average of particles distribution, \( f_s(x,v,t) \)
- Evolve the distribution function
- e.g. Vlasov-Maxwell models
The Boltzmann eqn:

\[ \frac{\partial f_s}{\partial t} + \mathbf{v} \cdot \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} = \frac{\partial f_s}{\partial t} \bigg|_c \]

Take the 0\textsuperscript{th}, 1\textsuperscript{st}, 2\textsuperscript{nd} moments of the Boltzmann Eqn.

\[ m_s \int \mathbf{v}^n \frac{\partial f_s}{\partial t} d\mathbf{v} + m_s \int \mathbf{v}^{n+1} \cdot \frac{\partial f_s}{\partial \mathbf{x}} d\mathbf{v} + q_s \int \mathbf{v}^n (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_s}{\partial \mathbf{v}} d\mathbf{v} = m_s \int \mathbf{v}^n \frac{\partial f_s}{\partial t} \bigg|_c d\mathbf{v} \]

Each moment of the Boltzmann eqn gives an equation for the moment variable, and introduces the next higher moment variable.

This process can go on indefinitely.
The Boltzmann equation evolves $f_s$.

\[
\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s u_s) = \left. \frac{\partial \rho_s}{\partial t} \right|_\Gamma
\]

\[
\frac{\partial \rho_s u_s}{\partial t} + \nabla \cdot (\rho_s u_s u_s + p_s \mathbf{I} + \Pi_s) = \frac{\rho_s q_s}{m_s} (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) - \sum_{s^*} R_{s,s^*} + \left. \frac{\partial \rho_s u_s}{\partial t} \right|_\Gamma
\]

\[
\frac{\partial \varepsilon_s}{\partial t} + \nabla \cdot ((\varepsilon_s + p_s) \mathbf{I} + \Pi_s) \cdot \mathbf{u}_s + \mathbf{h}_s = \frac{\rho_s q_s}{m_s} u_s \cdot \mathbf{E} + \sum_{s^*} Q_{s,s^*} + \left. \frac{\partial \varepsilon_s}{\partial t} \right|_\Gamma
\]

- System is truncated by relating higher moment variables to the lower ones.
- The fluids are coupled to each other and to the electromagnetic fields through Maxwell’s equations and interaction source terms.
PLASMA MODELS RANGE OF APPLICABILITY.

Sean Miller, PhD dissertation, University of Washington (2016)
**Advantages of the Model**

- Kinetic $\xrightarrow{\text{LTE, velocity moments}}$ MFPM $\xrightarrow{\epsilon_0 \to 0, \ m_e \to 0}$ MHD

**Ideal MHD Model is Valid When:**

- High collisionality, $\tau_{ii}/\tau \ll 1$
- Small Larmor radius, $r_{Li}/L \ll 1$
- Low Resistivity, $\left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{r_{Li}}{L}\right)^2 \frac{\tau}{\tau_{ii}} \ll 1$

**Multi-Fluid Plasma Model**

- Less computationally expensive than kinetic models
- Multi-fluid effects become relevant at small spacial and temporal scales
- Finite electron mass and speed-of-light effects are included
- There is charge separation is modeled
- Displacement current effects are resolved in the MFPM
The MFPM has dispersive sources.

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = S
\]

- The source Jacobian \( \frac{\partial S}{\partial Q} \) has imaginary eigenvalues.
- The equation system has dispersive sources.
- The dispersion is physical (may be difficult to distinguish from numerical dispersion).
- This dispersion is due to plasma waves that result from ion and electron plasma interactions with electromagnetic fields.
- An ideal numerical method for the MFPM should:
  - be high-order accurate
  - capture shocks
  - couple the flux and the sources
  - not impose strict time-step

Srinivasan et al, CCP 10 (2011)
BFEM simultaneously uses CG and DG.

Solution to the electron and EM fields is smooth and does not shock

Continuous Galerkin
Electron fluid and EM fields

\[ Q = \sum_i q_i v_i \]

Discontinuous Galerkin
Multiple ion and neutral fluids

\[ Q = \sum_i c_i v_i \]
For this implementation the balance law form is cast as
\[
\frac{\partial Q}{\partial t} + \frac{\partial \mathbf{F}}{\partial Q} \cdot \frac{\partial Q}{\partial x} = \mathbf{S} + \kappa \nabla^2 Q_d
\]

Lagrange polynomials are used for basis functions, \( v_r \)
\[
\int_\Omega v_r \frac{\partial Q}{\partial t} dV = \mathcal{L}_r(Q) = \int_\Omega v_r S dV - \int_\Omega v_r \frac{\partial \mathbf{F}}{\partial Q} \cdot \frac{\partial Q}{\partial x} dV + \kappa \int_\Omega v_r \nabla^2 Q_d dV
\]

\( \theta \)-method time integration
\[
\mathcal{R}(Q^n) = \mathbf{M} \frac{Q^{n+1} - Q^n}{dt} - \theta \mathcal{L}_r(Q^{n+1}) - (1 - \theta) \mathcal{L}_r(Q^n) = 0
\]

\( \theta = 0.5 \) is used for 2\(^{nd} \) order accuracy
\[
\mathbf{J}(Q^n) = \frac{\partial \mathcal{R}(Q^n)}{\partial Q^n}, \quad \mathbf{J}(Q^n) \Delta Q = -\mathcal{R}(Q^n), \quad Q^{n+1} = Q^n + \Delta Q
\]

Runge-Kutta Discontinuous Galerkin

\[
\frac{\partial Q}{\partial t} + \frac{\partial \vec{F}}{\partial x} = S
\]

- Legendre polynomials are used for basis functions, \( v_p \)
- The hyperbolic equation is multiplied by the basis function,

\[
\int_\Omega v_p \frac{\partial Q}{\partial t} dV = \mathcal{L}_p(Q) = \int_\Omega v_p S dV - \oint_{\partial \Omega} v_p \vec{F} \cdot dA + \int_\Omega \vec{F} \cdot \nabla v_p dV
\]

- Explicit Runge-Kutta time integration
- \( CFL = c \Delta t / \Delta x \leq 1/(2p - 1) \), \( p \) is the polynomial order

\[
Q^* = Q^n + \Delta t \mathcal{L}_p(Q^n),
\]

\[
Q^{n+1} = \frac{1}{2} Q^* + \frac{1}{2} Q^n + \frac{1}{2} \Delta t \mathcal{L}_p(Q^*).
\]

Loverich et al, CCP 9 (2006)
Convergence of the BFEM.

\[ \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} = 0, \quad Q(x, 0) = e^{-10(x-8)^2}, \quad ||\Delta Q||_2 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\hat{Q} - Q_i)^2} \]

Spatial Convergence

Simulations at fixed time-step

Temporal Convergence

Simulations at fixed CFL=1

\begin{align*}
\text{SOUSA (ERC/AFRL)} & \quad \text{DISTRIBUTION A: APPROVED FOR PUBLIC RELEASE; UNLIMITED DISTRIBUTION Clearance No. 17211}
\end{align*}
1D soliton is a two-fluid plasma problem

The solution is smooth, therefore artificial dissipation can be small

The simulation uses 512 second-order elements

\[ B_z = 1.0, \; T_e = T_i = 0.01, \; u_i = u_e = 0 \]

\[ n_e = n_i = 1.0 + e^{-10(x-6)^2} \]

Baboolal, Math. and Comp. Sim. 55 (2001)
\[
\frac{m_i}{m_e} = 1836, \quad \frac{c}{c_{si}} = 1000\sqrt{2}, \text{ FV 5000 cells}
\]

- DG Solution is very dispersive
- BFEM is less dissipative than the converged solution

Hakim et al, JCP 219 (2006)
BFEM COMPUTATIONAL COST SAVINGS

<table>
<thead>
<tr>
<th>case</th>
<th>$m_i/m_e$</th>
<th>$c/c_{si}$</th>
<th>DG time(s)</th>
<th>BFEM time (s)</th>
<th>BFEM cost over DG</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>10/√2</td>
<td>0.32</td>
<td>37.7</td>
<td>+11681%</td>
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<tr>
<td>2</td>
<td>100</td>
<td>10/√2</td>
<td>1.28</td>
<td>37.7</td>
<td>+2845%</td>
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<tr>
<td>3</td>
<td>500</td>
<td>10/√2</td>
<td>6.82</td>
<td>37.7</td>
<td>+452.8%</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>10/√2</td>
<td>12.4</td>
<td>38.2</td>
<td>+208.1%</td>
</tr>
<tr>
<td>5</td>
<td>1836</td>
<td>10/√2</td>
<td>23.5</td>
<td>40.4</td>
<td>+71.91%</td>
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<tr>
<td>6</td>
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<td>47.2</td>
<td>39.2</td>
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</tr>
<tr>
<td>7</td>
<td>3672</td>
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<td>520</td>
<td>265</td>
<td>-49.04%</td>
</tr>
<tr>
<td>8</td>
<td>3672</td>
<td>1000/√2</td>
<td>5274</td>
<td>2735</td>
<td>-48.14%</td>
</tr>
</tbody>
</table>

- Per time-step explicit DG is faster than BFEM, but it requires many more time-steps
- BFEM is more efficient only when time-step are considerably larger than explicit DG
BFEM Accuracy

$m_i/m_e = 1836$

- The BFEM seems to be less accurate than the DG implementation (∼ 50%)
- When the mass ratio is one, the two methods have the same level of accuracy
- The discrepancy is due to the fact that the semi-implicit BFEM does not resolve the plasma frequency in this problem
Electromagnetic Plasma Shock Problem

- Fast rarefaction wave (FR), a slow compound wave (SC), a contact discontinuity (CD), a slow shock (SS), and another fast rarefaction wave (FR)
- The problem exhibits limits of MHD and multi-fluid behavior by changing the Larmor radius, $r_L$
  - MHD: $r_L \rightarrow 0$
  - Multi-fluid: $r_L \sim L$

Brio and Wu, JCP 75 (1988)
Shock in Density but Smooth Fields.

- $t=0.05/\omega_{ci}$, $c/c_{si}=110$, $m_i/m_e=1836$

The main features of the problem are captured by all three methods. BFEM does not properly resolve the fast electromagnetic waves which require accurately resolving the electron dynamics.
\[ \Delta t_{\text{max}} = \min \left( \frac{\Delta x}{c_{se}}, \frac{\Delta x}{c_{si}}, \frac{\Delta x}{c}, \frac{0.1}{\omega_{ce}}, \frac{0.1}{\omega_{ci}}, \frac{0.1}{\omega_{pe}}, \frac{0.1}{\omega_{pi}} \right) \]

- \( \Delta t_{\text{max}} \) corresponds to the maximum value allowed for explicit methods based on the CFL condition.
- \( \Delta t = 42.9 \Delta t_{\text{max}} \) is the maximum time step allowed by the BFEM due to ion dynamics.
- Varying the artificial dissipation on the electron fluid, $\kappa_e$
- Wave-like behavior of the problem is better resolved
- Amplitude of the compound wave increases
- Right fast rarefaction wave is not visible
Varying the artificial dissipation on the EM-field, $\kappa_{EM}$

There is better agreement with the DG solution

This reinforces the point that the wave-like behavior arises from the interaction of the electron fluid with the electromagnetic fields
The blended finite element method (BFEM) is presented
- DG spatial discretization with explicit Runge-Kutta ($i^+$, $n$)
- CG spatial discretization with implicit Crank-Nicolson ($e^-$, fileds)
- DG captures shocks and discontinuities
- CG is efficient and robust for smooth solutions

Physics-based decomposition of the algorithm yields numerical solutions that resolve the desired timescales

DG method takes less computational time to advance the solution by one time-step, however $\Delta t$ is much smaller than that of the BFEM

Computational cost savings using the BFEM will only occur for relatively large implicit time-steps compared to explicit time-steps

Sousa and Shumlak, JCP 326 (2016) 56-75

Thank you.
Modeling each particle velocity and position is not practical. Instead an average is performed to give a statistical description. Calculate the number of particles per unit volume having approximately the velocity \( v \) near the position \( x \) and at time \( t \), distribution function \( f(v, x, t) \)

\[
\rho_s = m_s \int f_s(v)dv
\]

\[
\rho_s u_s = m_s \int vf_s(v)dv
\]

\[
P_s = p_s = m_s \int wwf_s(v)dv, \quad p_s = \frac{1}{3} m_s \int w^2 f_s(v)dv
\]

\[
H_s = m_s \int wwwwf_s(v)dv, \quad h_s = \frac{1}{2} m_s \int w^2wf_s(v)dv
\]

\[
w = v - u_s
\]
FIELD REVERSED CONFIGURATION THRUSTER

- Compact toroid, no central column
- Simple geometry and B-field configuration
- High power density (high plasma beta)
- Highly movable

A Rotating Magnetic Field (RMF) is used to form the FRC
- Strong magnetization of the electrons to the RMF produces a current
- To conserve the total flux the B-field at the center reverses

Advantages:
- Electrodeless and the plasmoid propellant is magnetically isolated from the walls
- Propellant is completely uncoupled from the driving and confining fields
- High plasma temperatures and densities significantly reduce ionization losses

1. Formation of high density FRC
2. Acceleration of the FRC by Lorentz force
3. FRC expands, converting thermal energy to directed energy
The formation of FRC using a rotating magnetic field (RMF)

- $\omega_{ci} < \omega < \omega_{ce}$
- $\nu_{ei} \ll \omega_{ce}$

**RMF BC**

$$I_{RMF} = I_t \cos(\omega t + \phi)$$
$$I_t = I_0 \left(1 - e^{-t/\tau}\right)$$

- $\phi_{A,B} = 0$, $\phi_{E,F} = \pi$
- $\phi_{C,D} = \pi/2$, $\phi_{G,H} = -\pi/2$

Bias Field is constant at $t=0s$
Initialization: $B_\omega = 90G$, $B_z = 50G$, $\omega = 5MHz$

Solve Maxwell’s eqns. with divergence constraints
Initialization: \( T_i = T_e = 30 \text{eV}, \ n_i = n_e = 10^{19} \text{m}^{-3}, \ m_i/m_e = 1836 \)

- Solve Multi-Fluid eqns.
- \( B_z \) is plotted

\[
\begin{align*}
 t &= 250 \text{ns} \\
 t &= 500 \text{ns} \\
 t &= 1500 \text{ns}
\end{align*}
\]
(NEAR) FUTURE WORK

Neutral-Plasma Model

- Interaction of multi-fluid plasma with gas dynamics neutral fluid
  - electron-impact ionization
    \[ e^- + n \rightarrow i^+ + 2e^- - \phi_{ion} \]
  - radiative recombination
    \[ e^- + i^+ \rightarrow n + h\nu \]
  - resonant charge exchange
    \[ i^+ + n \rightarrow n + i^+ \]

Meier and Shumlak, PoP 19 072508 (2012)

Collisional-Radiative Model

- Excitation/De-excitation rates

Le and Cambier, PoP 22 093512 (2015)