Chiral superfluidity with p-wave symmetry from an interacting s-wave atomic Fermi gas

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ABSTRACT
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Chiral superfluidity with p-wave symmetry from $\ldots$

Block 13: Supplementary Note
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Chiral superfluidity with $p$-wave symmetry from an interacting $s$-wave atomic Fermi gas

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Chiral $p$-wave superfluids are fascinating topological quantum states of matter that have been found in the liquid $^3$He-A phase and arguably in the electronic Sr$_2$RuO$_4$ superconductor. They are fundamentally related to the fractional 5/2 quantum Hall state, which supports fractional exotic excitations. Past studies show that they require spin-triplet pairing of fermions by $p$-wave interaction. Here we report that a $p$-wave chiral superfluid state can arise from spin-singlet pairing for an $s$-wave interacting atomic Fermi gas in an optical lattice. This $p$-wave state is conceptually distinct from all previous conventional $p$-wave states as it is for the centre-of-mass motion, instead of the relative motion. It leads to spontaneous generation of angular momentum, finite Chern numbers and topologically protected chiral fermionic zero modes bounded to domain walls, all occurring at a higher critical temperature in relative scales. Signature quantities are predicted for the cold atom experimental condition.
opological superconductors, like the type of $p_x + ip_y$-wave pairing studied in the liquid $^3$He (ref. 1) and strontium ruthenate$^2$, are among the most desirable unconventional many-body states in condensed matter physics$^3$. In two dimensions (2Ds), their topological properties are fundamentally linked to a class of fractional quantum Hall states of non-Abelian statistics$^4$. Studies of vortices in such materials point to fascinating braiding statistics and applications in topological quantum computing. The fate of topological superconductivity in 2D electronic matter remains, however, debatable. In the field of ultracold atoms, this phase was predicted to appear near the $p$-wave Feshbach resonance in Fermi gases$^5$. However, the life time of such systems is severely limited by the three-body heating or ultracold chemical reactions. Another approach which future experimental breakthroughs are desired to suppress is the type of Fulde–Ferrell–Larkin–Ovchinnikov $^6$-wave superconductivity in 2Ds has stood largely open for many-body states in condensed matter physics$^3$. In two orbital bands, respective. Recently, the research of higher orbital bands (the expression for $H_0$ describes tunnelling pictorially represented in Fig. 1b,c (the expression for $H_0$ is standard and is given in Supplementary Note 1) and $H_{\text{int}}$ is the Hubbard interaction,

$$H = H_0 + H_{\text{int}},$$  (1)

where $H_0$ describes tunnelling pictorially represented in Fig. 1b,c (the expression for $H_0$ is standard and is given in Supplementary Note 1) and $H_{\text{int}}$ is the Hubbard interaction,

$$H_{\text{int}} = -U \sum_{\mathbf{R}} \left\{ C^{\dagger}_{p_x} (\mathbf{R}) C^*_{p_x} (\mathbf{R}) - \frac{1}{2} (C^{\dagger}_{p_y} (\mathbf{R}) C^*_{p_y} (\mathbf{R}) + C^{\dagger}_{p_y} (\mathbf{R}) C^*_{p_y} (\mathbf{R}) ) \right\} - 1.$$  (2)

Here $C^{\dagger}_{p_x} (\mathbf{R})$ is a fermionic annihilation operator for the localized $s$, $p_x$, or $p_y$ orbital on $A$ sites and the corresponding annihilation operator on the $B$ sites is denoted as $C^{\dagger}_{p_y} (\mathbf{R})$ with $\mathbf{R}$ labels the lattice site. The interactions between $s$ and $p$ orbitals originate from interactions between two hyperfine states, which are tunable by the $s$-wave Feshbach resonance in ultracold atomic gases. We focus on the case with attractive interaction where superconducting pairing is energetically favourable. Although the annihilation operators $C^{\dagger}_{p_x} (\mathbf{R})$ do not appear in the interaction $H_{\text{int}}$, they play important roles in the tunnelling Hamiltonian $H_0$ by determining the orientation of Fermi surfaces. With spin $\downarrow$ fermions residing on both $A$ and $B$ sites, and spin $\uparrow$ ones on $A$ sites only, Fermi surfaces of the $s$-band for spin $\uparrow$ and the $p$-band for spin $\downarrow$ are approximately two squares right on top of each other. Such a momentum-space geometry leads to nearly perfect Fermi surface nesting, which is the key to the $p$-wave superconducting pairing between $s$ and $p$ orbitals at weak interaction.

The system, as described by the Hamiltonian in equation (1), exhibits lattice rotation $C_4$ and reflection symmetries. For the reflection in the horizontal and vertical direction (see Fig. 1a), the fermionic operators transform as $R_{xy} \equiv \{ C^{\dagger}_{p_x} (\mathbf{R}) \rightarrow -C^{\dagger}_{p_y} (\mathbf{R}), \; C^{\dagger}_{p_y} (\mathbf{R}) \rightarrow C^{\dagger}_{p_x} (\mathbf{R}) \}$ and $R_{xy} \equiv \{ C^{\dagger}_{p_x} (\mathbf{R}) \rightarrow C^{\dagger}_{p_x} (\mathbf{R}), \; C^{\dagger}_{p_y} (\mathbf{R}) \rightarrow C^{\dagger}_{p_y} (\mathbf{R}) \}$, respectively. Under the lattice rotation, $C^{\dagger}_{p_x} (\mathbf{R}) \rightarrow C^{\dagger}_{p_y} (\mathbf{R} - R_y R_x), \; C^{\dagger}_{p_y} (\mathbf{R}) \rightarrow C^{\dagger}_{p_x} (\mathbf{R} - R_y R_x)$. These symmetries, reflection symmetries in particular, play an essential role in the following theory.

Ginzburg–Landau theory. From the analysis of Cooper’s problem (see Supplementary Note 2 and Supplementary Fig. 1), we conclude that condensation of Cooper pairs at $Q = (\pi/a, \pi/a)$ is most energetically favourable in the ground state, where $a$ is the lattice constant. This peculiar momentum selection for Cooper pairs is related to the Fermi surface nesting at half-filling. Then, it is convenient to introduce two slowly varying bosonic fields $\Delta_s (\mathbf{x})$ and $\Delta_p (\mathbf{x})$, which represent Cooper pairs $(-1)^N \int d^2 \mathbf{x} [U^{\dagger}_{p_x} (\mathbf{R}) C^{\dagger}_{p_x} (\mathbf{R})] \; (\text{and} \; -1)^N \int d^2 \mathbf{x} [U^{\dagger}_{p_y} (\mathbf{R}) C^{\dagger}_{p_y} (\mathbf{R})]$ in the low-energy limit, respectively, with $\mathbf{x}$ a coarse grained coordinate for the lattice labelling $\mathbf{R}$. The pairing fields $\Delta_s (\mathbf{x})$ and $\Delta_p (\mathbf{x})$ are parity odd and transform as $\Delta_s (\mathbf{x}, y) \rightarrow -\Delta_s (\mathbf{x}, -y), \; \Delta_p (\mathbf{x}, y) \rightarrow \Delta_p (\mathbf{x}, -y)$ and $\Delta_s (\mathbf{x}, y) \rightarrow \Delta_s (\mathbf{x}, -y), \; \Delta_p (\mathbf{x}, y) \rightarrow -\Delta_p (\mathbf{x}, -y)$ under reflection in the horizontal and vertical directions (see Fig. 1a), respectively. The symmetry of the pairing fields plays a crucial role in determining the form of the free energy, to be demonstrated in the following. A two-flavour Ginzburg–Landau free energy respecting all the symmetries is given as follows:

$$F[\Delta_s, \Delta_p] = \int d^2 \mathbf{x} [f_{\text{mean}} (\mathbf{x}) + f_{\text{Gaussian}} (\mathbf{x})],$$  (3)
with \( f_{\text{Max}} = r(\Delta_1^4 + |\Delta_2|^4) + g_1(\Delta_1^4 + |\Delta_2|^4) + g_2|\Delta_1|^2 |\Delta_2|^2 + g_3(\Delta_1^4 + |\Delta_2|^4) + h.c. \) and \( f_{\text{Gaussian}} = K(\Delta_1^2 + |\Delta_2|^2 + |\Delta_1|^2 + |\Delta_2|^2)^2 \). Here \( r, g_1, g_2, g_3 \) and \( K \) are phenomenological coefficients to be related to the Hamiltonian (equation (1)) by microscopic calculations.

This free energy generalizes the theory of two-gap superconductivity as proposed in the context of transition metals\(^{31}\). We have neglected temporal fluctuations of Cooper pair fields and such a treatment is valid at non-zero temperature away from quantum critical regime. In this theory, we want to emphasize two key points owing to the reflection symmetries: first, \( \Delta_p \) and \( \Delta_s \) are decoupled at quadratic level; second, linear derivatives makes modulations in \( \Delta_p \) suppressed, and condensation of Cooper pairs at \((\pi/a, \pi/a)\) is expected to be stable against weak perturbations such as having small \( t_3 \) (Fig. 1c) or slightly doping away from half-filling. For finite \( t_2 \) and \( t_3 \) (Fig. 1c), the stability (that is, \( K > 0 \) in equation (3)) is confirmed in our numerics (see Supplementary Note 3 and Supplementary Figure 2). The rotation symmetry plays another key role. For example, it protects the degeneracy of the two components \( \Delta_p \) and \( \Delta_s \) that is, the splitting terms such as \( |\Delta_1|^2 - |\Delta_2|^2 \) are not allowed in the free energy. In short, the point group \( D_4 \) symmetry protects the form of the free energy in equation (3). Here we point out that the preserved space lattice rotation symmetry makes our model different from the spin-orbit-coupled systems, where such symmetry is absent.

With the phenomenological coefficients \( r \) and \( g_1 \) obtained from integrating out fermions (see Supplementary Fig. 2), we find a phase diagram shown in Fig. 1d. With moderate attraction \( U < 7t_0 \), a first-order phase transition from the \( p_x \pm i p_y \) to \( p_x \pm i p_y \) phase occurs when the diagonal hopping \( t_3 \) is above some critical value. Surprisingly, when the attraction is strong enough \( U > 7t_0 \), we find that even infinitesimal \( t_3 \) makes the \( p_x \pm i p_y \) favourable, opening a wide window for this non-trivial state. When \( t_3 = 0 \), the system has \( U(1) \times U(1) \) symmetry, which means no phase coherence between the two components \( \Delta_p \) and \( \Delta_s \). We also find that the superconducting gaps \( \Delta_p \) and \( \Delta_s \) have anisotropy in momentum space (see Supplementary Note 5 and Supplementary Fig. 4). Here we emphasize the crucial role of \( t_3 \) in determining the relative phase between Cooper pairs \( \Delta_p \) and \( \Delta_s \). As it mixes the \( p_x \) and \( p_y \) orbitals, the particle numbers of these two orbitals are no longer separately conserved, and will consequently lock the relative phase between these two pairing fields. For large \( t_3 \), the \( p_x \pm i p_y \) state mostly minimizes the free energy and becomes energetically favourable. Surprisingly, this \( p_x \pm i p_y \) phase is found to occupy a large region in the phase diagram (Fig. 1d). In the strong interaction regime, Cooper pairs behave like tightly bound repulsive molecules and the energetic selection of the \( p_x \pm i p_y \) phase can be understood from an analogue of Hund’s rule for p-orbital bosons\(^{32}\), where maximizing the angular momentum generically saves the interaction energy cost.

The p-wave superfluidity proposed here refers to a centre-of-mass p-wave pairing state. The distinction of such pairing from the conventional spin-triplet pairing \( p_x \pm i p_y \) states is illustrated in Fig. 1e,f. To further distinguish this state from the conventional relative p-wave pairing that usually involves two different sites (for example, pairs between spinless fermions) owing to the fermionic statistics, we consider a general pairing form \( \Phi_{\text{COM}}(x, x') \), which represents the Cooper pair between parity even and odd orbitals (that is, \( s \) and \( p \)) orbitals from any two sites. Under parity transformation, the pairing transforms as \( \Phi_{\text{COM}}(x, x') \rightarrow -\Phi_{\text{COM}}(x', -x) \), rather than \( \Phi_{\text{COM}}(x, x') \rightarrow -\Phi_{\text{COM}}(x', x) \), which distinguishes the centre-of-mass from the conventional relative p-wave pairing. A remarkable feature immediately born out of this pairing mechanism is the centre-of-mass p-wave superfluidity arises directly from a purely s-wave two-body interaction, requiring neither engineered p-wave interactions nor induced effective ones. This leads to a significantly improved transition temperature, which is confirmed by our study of the finite temperature phase transitions for the model Hamiltonian (equation (1); see Supplementary Note 4 and Supplementary Figure 3). For example, taking a typical density of the \( ^{40}K \) gas\(^{33,34}\) and the potential depths of the optical lattice to be \( V_J/E_R = 3 \) and \( V_p/2E_R = 5 \) for \( s \) and \( p \) orbitals (see the lattice potentials in Methods), respectively, we estimate the Kosterlitz–Thouless transition temperature can reach around 100 nK or higher, being within the experimental temperature scope\(^{35}\).
Gapless chiral fermions. We now show that the $p_x \pm ip_y$ superfluid state possesses important measurable signatures because of the broken time reversal $Z_2$ symmetry, which belongs to the Ising universality class. Following the standard procedure, our calculation finds that the state is topologically nontrivial by a non-zero Chern number, which is 1 and $-1$ for the $p_x + ip_y$ and $p_x - ip_y$ state, respectively. The topological properties are manifested in the existence of gapless chiral fermions, emergent on a domain wall connecting topologically distinct regions. In experiments, Ising domains of $p_x + ip_y$ and $p_x - ip_y$ are expected to spontaneously form as have been observed in the recent cold atom experiment studying ferromagnetic transitions\textsuperscript{36}. In the following, we show that a domain wall defect carrying gapless fermions as bounded surface states is experimentally accessible.

Considering a lattice geometry in the presence of a domain wall decorated superconducting background as in Fig. 2a, the mean-field Hamiltonian is given by

$$
H_M = H_0 - U \sum_R \{ C_{\downarrow \uparrow}^A(R) C_{\uparrow \downarrow}^{A*}(R) - C_{\uparrow \downarrow}^A(R) C_{\downarrow \uparrow}^{A*}(R) \}
+ \frac{1}{2} \sum_R \{ C_{\downarrow \downarrow}^A(R) C_{\uparrow \uparrow}^{A*}(R) + C_{\uparrow \uparrow}^A(R) C_{\downarrow \downarrow}^{A*}(R) \}
+ \frac{1}{2} \sum_R \{ C_{\downarrow \downarrow}^A(R) C_{\downarrow \downarrow}^{A*}(R) + C_{\uparrow \uparrow}^A(R) C_{\uparrow \uparrow}^{A*}(R) \}
+ \frac{1}{2} \sum_R \{ C_{\uparrow \downarrow}^A(R) C_{\downarrow \uparrow}^{A*}(R) + C_{\downarrow \uparrow}^A(R) C_{\uparrow \downarrow}^{A*}(R) \}.
$$

(4)

Here we emphasize that the centre-of-mass $p$-wave superfluidity, as described in equation (4), exhibits an unbroken spin $U(1)$ symmetry under the transformation $C_{\downarrow \uparrow}^A \rightarrow e^{i\theta} C_{\downarrow \uparrow}^A$ and $C_{\downarrow \downarrow}^A \rightarrow e^{-i\theta} C_{\downarrow \downarrow}^A$. Such an unbroken continuous symmetry is absent in the conventional relative $p$-wave pairing state for either spinless or spin-orbit-coupled fermion systems. The energy spectrum of fermionic excitations is obtained by diagonalizing equation (4). With the periodical boundary condition chosen in the $x$ direction (Fig. 2a), the momentum $k_x$ is a good quantum number and the energy spectra in Fig. 2b is thus labelled by $k_x$.

For the same reason as in quantum Hall insulators, the number of gapless chiral modes moving along the interface is topologically determined by the difference of the Chern numbers in regions on either side of the interface\textsuperscript{37}; in this case, $|\Delta C| = 2$. This conclusion is confirmed in our numerics. As shown in Fig. 2b, we find four gapless chiral modes, with two localized on the domain wall (purple colour) and the other two on the outer edges of the lattice (red colour). From their spectra $\epsilon_n(k_x)$, the two chiral modes on the domain wall have positive group velocities, which lead to anomalous mass flow along the domain wall\textsuperscript{38}.

Experimental signatures. Several different methods can be used to observe our proposed chiral $p$-wave superfluidity experimentally. For instance, one of the direct experimental evidence for this topological superfluids, as in quantum Hall insulators, is the existence of the gapless chiral edge states. The domain wall supporting gapless chiral fermions between two Ising domains $p_x + ip_y$ and $p_x - ip_y$ can spontaneously form in atomic gases in a trap. These gapless fermionic states lead to signatures in local density of states (LDOS), which can be measured by radio-frequency (RF) spectroscopy. The LDOS is calculated as $\rho(E) = \sum_{n=1}^{\infty} \int dE_n |v_n^+|^2 \delta(E - \epsilon_n) + |v_n^-|^2 \delta(E + \epsilon_n)$, where $(v_n^+, v_n^-)^T$ is the eigenvector corresponding to the eigenenergy $\epsilon_n$ of Hamiltonian equation (4) and $v$ runs over all the Wannier orbitals $(s, p_x, p_y)$ on $A$ and $B$ sites. We find that these gapless modes manifest themselves by a peak in LDOS located at the position of the domain wall, as shown in Fig. 2c–e. The reason for that peak of LDOS is the existence of localized gapless surface.

Figure 2 | Topological domain wall defect. (a) Schematic picture of a lattice system in the presence of a domain wall. (b) Energy spectrum of the system with a domain wall defect, when $t_y/t_0 = 8$, $t_x/t_0 = 2$, $t_y/t_0 = 0.1$ and $U/t_0 = 9$. The purple and red branches correspond to the modes at the domain wall and the edge of the lattice, respectively. (c–e) The local density of states (LDOS) defined in the main text. The peak of LDOS located at domain wall is shown by red colour in c and further shown with $E/t_0 = 0$ and $y/a = 49$ in d and e, respectively. The LDOS is in units of $1/t_0$. © 2014 Macmillan Publishers Limited. All rights reserved.
states, reminiscent of the quantum Hall edge states. This spatially localized zero-energy peak in LDOS can be detected using spatially resolved RF spectroscopy technique\textsuperscript{39}. Taking a laser wavelength of $\lambda = 1.024$ nm typical for the current optical lattices, the width of the LDOS peak is estimated about 2 $\mu$m. This is greater than the reported spatial resolution (about 1.4 $\mu$m) in the current experimental RF measurement\textsuperscript{39}, which makes the detection of this signal experimentally accessible. Besides detecting edge properties, the bulk Chern number can also be used to demonstrate the topological nature of our proposed chiral $p$-wave superfluidity. For the detection of Chern numbers, one may adopt the existing proposals either from time-of-flight measurement\textsuperscript{40–43} or from Bloch oscillation technique\textsuperscript{34,45}.

**Discussion**

We now discuss the sharp distinction between our proposal to realize chiral $p$-wave superfluidity and that in previous studies\textsuperscript{10–18}. Our proposed chiral $p$-wave superfluid state is a centre-of-mass $p$-wave superconducting pairing state. That is fundamentally different from other conventional $p$-wave superfluidity in previous studies, where the pairing order parameter has $p$-wave symmetry in the relative motion. The concept difference leads to two major distinct features in centre-of-mass $p$-wave state. One is that our proposal requires neither spin-orbit coupling nor an induced second-order effective $p$-wave interaction, automatically avoiding the challenge of their experimental realization in atomic Fermi gases. The centre-of-mass $p$-wave superfluidity proposed here arises directly from a purely $s$-wave two-body interaction. Hence, a short-ranged contact interaction as widely implemented in cold gases should satisfy well. This also significantly improves the transition temperature. The other feature is that the centre-of-mass $p$-wave superfluidity predicted here exhibits some unbroken symmetries such as spin $U(1)$ symmetry and space lattice rotation symmetry. Such symmetries are present in the conventional relative $p$-wave pairing state (for example, in spin-orbit-coupled fermion systems).

We would like to stress that although the spin dependence of the studied lattice potentials is important to support our $p$-wave superconductivity in the weak interaction region, such lattice potentially causing experimental challenges could be avoided by considering strong interactions such as the resonance (see in Methods). The key ingredient for topological properties to emerge is rather from mixing Wannier orbitals of opposite parities. The connection between topological superconductivity and insulating or semimetallic phases\textsuperscript{46,47} involving the idea of mixing parities remains an intriguing question for future research.

In summary, we find a topological $p$-wave superfluid state in a spin imbalanced atomic Fermi gas with an $s$-wave interaction. Its pairing symmetry and topological origin differ from the previously known superconducting or superfluid phases. The $p$-wave symmetry refers to the centre-of-mass motion, not to the relative motion as in the well-known $^3$He superfluid. The appearance of chiral fermionic zero modes bounded to domain walls is predicted as a concrete experimental signature for this novel state.

After the submission of this manuscript, we became aware of a related paper\textsuperscript{48}, which puts forward an idea of generating $p$-wave interaction on the optical lattice.

**Methods**

**Spin-dependent optical lattice.** Experimentally, one may consider the existing proposals for realizing spin-dependent optical lattices\textsuperscript{24,25}. Alternatively, the recent progress in group-II (alkaline-earth-metal) atoms points to the possibility of achieving even greater tunability by using the ground $S_0$ and long-lived metastable $3P$ atomic levels, taking advantage of the strong orbit dependence of AC Stark effect in such systems. This should in principle be able to make the lattices for different components being completely independent (so maximally spin-dependent lattice) by selection of the appropriate wavelengths\textsuperscript{25}. To be more specific, for the model presented here, we consider the lattice configurations for spin up ($s$ orbital) and down ($p$ orbital) fermions are $V_s(x,y) = -V_s\cos(k\cdot(x+y)) + V_s\cos(k\cdot(x-y))$ and $V_p(x,y) = -V_p\cos(k\cdot(x) + \cos(k\cdot(y))$, respectively. Here $k = \frac{2\pi}{\lambda}$, $V_s$ and $V_p$ are the lattice strength, and $\lambda$ is the wavelength of laser. The phase-stable lattices should in principle be realized through the phase control technique developed in recent experiments\textsuperscript{49,50}. It is worthwhile to note that this special lattice configuration is to make the Cooper pair favourable even when interaction is weak. However, with strong interactions as in the resonance regime, such special lattice is expected to be unnecessary. The reason is that when the strong pairing-induced energy gap is a significant fraction of or is even comparable to the band width, Fermi surface nesting is no longer important for pairing.

**References**


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Author contributions
W.V.L. proposed the study of cross-band pairing during visits and discussions with B.W. X.L. and B.W. made breakthrough in theoretical ideas with inputs from B.W. and W.V.L. B.L. performed all calculations with X.L., under the supervision of W.V. L. and B.W. All authors worked on theoretical analysis and contributed in completing the paper.

Additional information
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Supplementary Figures

Supplementary Figure 1: Energy spectra of Cooper’s bound state. Bound state energy $E_x(Q)/t_0$ varies as a function of center-of-mass momentum Q, when $t_1/t_0 = 8$, $t_2/t_0 = 2$, $t_3/t_0 = 0$ and $U/t_0 = 10$.

Supplementary Figure 2: Coefficients in Ginzburg-Landau theory. a, The coefficient $r$ vs. interaction strength $U/t_0$; b and d, The coefficient $g_3$ as a function of $t_3/t_0$; c, The coefficient $K$ vs. $t_3/t_0$. 
Supplementary Figure 3: **Finite-temperature phase diagram.** The solid line illustrates the KT transition temperature. The mean-field transition temperature is shown by the dot line. The regions for $p_x \pm ip_y$ and $p_x \pm p_y$ superfluid state are separated by the dash line. Here, the lattice strengths are $V_s/E_R=5$ and $V_p/2E_R=5$ with recoil energy $E_R = \frac{\hbar^2}{2m(a^2)}$ and $a_s$ is the $s$-wave scattering length. The inset plot shows that increasing of lattice strength will decrease superfluid transition temperature.

Supplementary Figure 4: **Structure function $S_y(\theta_{k_F})$ of superconducting gap near Fermi surfaces.** a, $U/t_0=7$, $t_1/t_0=8$, $t_2/t_0=0$ and $t_3/t_0=0$; b, $U/t_0=7$, $t_1/t_0=8$, $t_2/t_0=2$ and $t_3/t_0=0.1$; c, $\theta_{k_F}$ on the Fermi surface of $s$-band. Due to $C_4$ symmetry, the structure of $S_y$ is readily given by a $\pi/2$ rotation.
Supplementary Note 1. Hopping Term

The hopping term $H_0$ can be written as

$$H_0 = \sum_{R} \left[ C^\dagger(R) T_0 C(R) + C^\dagger(R) T_{1x} C(R + e_x) ight] + C^\dagger(R) T_{1y} C(R - e_x) + C^\dagger(R) T_{1y} C(R + e_y) + C^\dagger(R) T_{1y} C(R - e_y) + C^\dagger(R) T_{2} C(R + e_x - e_y)$$

where the matrices $T$ and $T'$ are given as

$$T_0 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -t_2 & 0 & 0 & 0 \\ 0 & 0 & t_1 & 0 & 0 \end{pmatrix}, \quad T_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -t_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -t_2 & 0 & 0 & 0 \\ 0 & 0 & t_1 & 0 & 0 \end{pmatrix}, \quad T_{1x} = \begin{pmatrix} -t_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & t_3 & 0 & 0 & -t_2 \\ 0 & 0 & 0 & t_3 & 0 \end{pmatrix},$$

$$T_{1y} = \begin{pmatrix} -t_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -t_3 \\ 0 & t_1 & 0 & 0 & -t_3 \\ 0 & 0 & -t_2 & -t_3 & 0 \end{pmatrix}, \quad T'_{1y} = \begin{pmatrix} -t_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -t_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -t_3 \\ 0 & t_1 & 0 & 0 & -t_3 \\ 0 & 0 & -t_2 & -t_3 & 0 \end{pmatrix}.$$ (2)

Here $t_0$ is the hopping amplitude between $s$ orbital fermions; $t_1$ and $t_2$ are the longitudinal $\sigma$-bond and transverse $\pi$-bond hopping amplitudes for $p$ orbitals, respectively; $t_3$ is the hopping amplitude between $p_x$ and $p_y$ orbitals and $C(R) = \begin{pmatrix} C_A^s(R) \\ C_A^p(R) \\ C_B^s(R) \\ C_B^p(R) \\ C_P^s(R) \end{pmatrix}$ is the fermion annihilation operator located at lattice site $R$.

Supplementary Note 2. Cooper’s Problem

From hopping term $H_0$, we find that there are five Bloch bands. The corresponding operators $\alpha_n(k)$ and $\alpha_{pn}(k)$ for $s$ and $p$ bands are introduced, respectively. Because the width of $p$-band is much larger than that of $s$-band, intuitively we know that the condensation of Cooper pairs between these two bands at center-of-mass momentum $Q = (\pi/a, \pi/a)$, which is the energy minimal of $p$-band, will be energetically favorable. Besides this intuitive picture, systematically, this conclusion is borne out by solving energy spectra of Cooper’s bound states [1], which are defined as $|\Phi\rangle = \sum_{k, k', n} \phi_n(k, k')^\dagger \alpha_{pn}(k) \alpha_n^\dagger(k') |\Omega\rangle$, where $|\Omega\rangle$ is the vacuum state and $\phi_n(k, k')$ is the two-particle wavefunction. The summation $\sum_{k'}$ here is over modes above the Fermi level [1]. Due to translational symmetry, the center-of-mass momentum $Q = k + k'$ is a good quantum number, which is used to label the energy spectra obtained from the eigenvalue problem, $H |\Phi(Q)\rangle = E(Q) |\Phi(Q)\rangle$. Resulting from lattice rotation symmetry $C_4$, there are two branches of Cooper’s bound states, which are related to each other by rotation. These two branches are most clear in the limit of $t_3 \to 0$, i.e., without coupling between $p_x$ and $p_y$ orbitals. In this case, particle numbers of $p_x$ and $p_y$ orbitals are separately conserved. One type of bound state is formed by $p_x$ and $s$ orbital fermions leading to an energy dispersion $E_x(Q)$; while the other formed by $p_y$ and $s$ orbitals leads to a dispersion $E_y(Q)$.

As shown in Supplementary Figure 1, we find that the bound state energy $E_x(Q)$ varies as a function of center-of-mass momentum $Q$ and the energy minimal is located at $Q = (\pi/a, \pi/a)$. Due to $C_4$ symmetry, the energy minimal of $E_y(Q)$ also locates at $(\pi/a, \pi/a)$. Condensation of Cooper pairs at $Q = (\pi/a, \pi/a)$ is energetically favorable. The effect of finite coupling $t_3$ between $p_x$ and $p_y$ orbitals has been discussed in the framework of effective field theory [2].
**Supplementary Note 3. Path Integral Approach**

To calculate free energy from the path integral method [2], we introduce the Grassman fields $\Psi(R, \tau)$ and $\bar{\Psi}(R, \tau)$ and express the grand partition function of the system as

$$Z = \int D\bar{\Psi} D\Psi \exp(-S[\bar{\Psi}, \Psi])$$

(3)

with $\Psi(R, \tau) = \begin{pmatrix} \Psi^A(R, \tau) \\ \Psi^B(R, \tau) \end{pmatrix}$. The quartic term in the interaction term of action $S$ can be decoupled with the Hubbard-Stranovich transformations,

$$\tilde{\Delta}_x(R, \tau) = U\Psi^A_p(R, \tau)\Psi^A_s(R, \tau),$$

$$\tilde{\Delta}_y(R, \tau) = U\Psi^B_p(R, \tau)\Psi^B_s(R, \tau).$$

(4)

Then the partition function can be written as

$$Z = \int D\tilde{\Delta}_x D\tilde{\Delta}_s D\tilde{\Delta}_y D\tilde{\Delta}_p D\Psi D\bar{\Psi} \exp(-S[\bar{\Psi}, \tilde{\Delta}_x, \tilde{\Delta}_s, \tilde{\Delta}_y, \tilde{\Delta}_p]).$$

(5)

The action in Eq. (5) is

$$S[\bar{\Psi}, \Psi, \tilde{\Delta}_x, \tilde{\Delta}_s, \tilde{\Delta}_y, \tilde{\Delta}_p] = \int d\tau dR \left\{ \frac{|\tilde{\Delta}_x(R, \tau)|^2}{U} + \frac{|\tilde{\Delta}_y(R, \tau)|^2}{U} \right\} - \int d\tau d\bar{R} \bar{\Psi}(R, \tau)G^{-1}(R, \tau; \bar{R}, \tau')\Psi(\bar{R}, \tau'),$$

(6)

where $\int dR = \sum_R$. After doing an unitary transformation of fermionic fields, we replace $\tilde{\Delta}_x$ and $\tilde{\Delta}_y$ in Eq. (6) by two slowly varying and time-independent bosonic fields $\Delta_x(x)$ and $\Delta_y(x)$, respectively. Integrating the fermionic fields, we get an effective action

$$S_{\text{eff}}[\Delta_x, \Delta_y] = \int d\tau d^2x \left\{ \frac{|\Delta_x|^2}{U} + \frac{|\Delta_y|^2}{U} - \ln |\det \hat{G}^{-1}[\Delta_x, \Delta_s, \Delta_y, \Delta_p]| \right\}$$

(7)

where $\hat{G}^{-1}$ is the inverse Green’s function. By calculating free energy from Eq. (7), we obtain coefficients $r, g_1, g_2$ and $g_3$. As shown in Supplementary Figure 2a, since the low temperature limit is much smaller than the Fermi energy, the coefficient $r$ changes sign from positive to negative with increasing $U/L_0$, which implies a second order phase transition from normal to a superfluid state with $\Delta_{x,y} \neq 0$ at mean field level (Fig. 1d). Our numerics also find that $0 < g_1 < g_2/2$ and $|g_3| < g_1$. Minimizing the free energy gives a field configuration with $|\Delta_x| = |\Delta_y|$. The relative phase between $\Delta_x$ and $\Delta_y$ is fixed by $g_3$ as shown in Supplementary Figure 2b and d. The coupling $g_3 > 0$ makes the relative phase locked at $\pm \pi$ and leads to a $p_x \pm ip_y$ superfluid state where the ‘$\pm$’ sign is spontaneously chosen; while $g_3 < 0$ favors a $p_x \pm ip_y$ state (Fig. 1d).

**Supplementary Note 4. Finite Temperature Phase Transition**

It is well known that in 2D the transition from the normal to superfluid state is of the Kosterlitz-Thouless type. To obtain the KT transition temperature, we should rewrite the complex order parameters $\Delta_x(x) = \Delta_0 e^{i\theta_x(x)}$ and $\Delta_y(x) = \Delta_0 e^{i\theta_y(x)}$ with the phase fluctuations $\theta_x$ and $\theta_y$ [3]. Introducing new variables $\theta = \frac{1}{2}(\theta_x + \theta_y)$ and $\Delta \theta = \theta_x - \theta_y$, from the Gaussian fluctuation part of free energy in Eq. (3), we derive the well-known XY model [4, 5] in terms of $\theta$ as $\Delta F[\Delta_x, \Delta_y] = \int d^2x \hat{K}(T)[(\partial_x \theta)^2 + (\partial_y \theta)^2]$. Here, the relative phase $\Delta \theta$ is determined by the sign of $g_3$ at finite temperature. Specifically, $\Delta \theta$ is locked at $\pm \frac{\pi}{2}$ for $(p_x \pm ip_y)$ or $(p_x \pm ip_y)$. The KT transition temperature is determined by the formula $k_BT_{KT} = \frac{\pi}{\theta^2} \hat{K}(T = T_{KT})$. Solving this equation self-consistently, we get the KT transition temperature and plot it in Supplementary Figure 3. We find that in the weak-coupling regime $T_{KT}$ approaches the mean-field transition temperature $T_{\text{Mean}}$ as determined by $r = 0$ at finite temperature. With stronger interaction, there is a large derivation of the two as expected [6], for the reason that mean field analysis underestimates fluctuation effects. Our numerics also find that $k_BT_{KT}$ can reach $3.21L_0$ accompanying with increasing of interaction strength when the lattice strengths are $V_s/E_R = 5$ and $V_p/2E_R = 5$ for $s$ and $p$ orbitals, respectively. As shown in the inset plot of Supplementary Figure 3, we also find that decreasing of lattice strength will increase $T_{KT}$. 
Supplementary Note 5. Anisotropic Superconducting Gap

In this section, we discuss the superconducting gap for fermions resulting from this unconventional pairing. The superconducting gap is calculated by solving the Mean field Hamiltonian without a domain wall defect. The anisotropy of the gap which is a remarkable property being absent in the conventional $s$-wave superconductors is characterized by the structure functions

$$S_x(k) = \frac{U}{N} < C_{p}^A (-k + Q) C_{s}^A (k) >$$

and

$$S_y(k) = \frac{U}{N} < C_{p}^A (-k + Q) C_{s}^A (k) >$$

where $N$ is the total site.

We find $S_{x/y}(k)$ near the Fermi surface is highly anisotropic, that is it strongly depends on the polar angle of $k$, $\theta_{k_F}$, as shown in Supplementary Figure 4. In Supplementary Figure 4a, when $t_2 = 0$ and $t_3 = 0$, the Fermi surfaces of $p$ and $s$ orbital fermions are matched very well when $0 \leq \theta_{k_F} \leq \pi/4$, so the gap is almost at the same maximum value when $\theta_{k_F}$ in that region. However, when $\pi/4 < \theta_{k_F} < \pi/2$, the gap decreases by increasing $\theta_{k_F}$ due to the mismatch of Fermi surfaces. The situation is different for $t_2 \neq 0$ and $t_3 \neq 0$, where the Fermi surfaces are mismatched. The gap is non-monotonic when $\theta_{k_F}$ varies from 0 to $\pi/2$, and it is maximal at $\theta = \pi/4$ (Supplementary Figure 4b). This peculiar non-monotonic behavior is related to van-Hove singularities [7] which lead to large density of states nearby.

Supplementary References