RADAR SIGNAL/IMAGE PROCESSING ENHANCEMENTS USING ALPHASTABLE TECHNIQUES

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ABSTRACT

In conventional radar signal and image processing, the background clutter and noise are assumed to follow the Gaussian model. Recent research has found that many nonhomogeneous types of clutter and noise, such as sea clutter, do not fit the Gaussian model well because of impulsive outliers or the so called “sea spike. These types of clutter and noise lend themselves to a heavy tail in amplitude distribution; consequently, the conventional matched filter does not perform well. Most recent research has shown that the $\alpha$-stable model is a better model, and most radar clutter is modeled well by the $\alpha$-stable statistics. A robust family of $\alpha$-stable matched filters is a natural extension of the conventional matched filter with the capability of suppression the clutter to reveal targets. An optimal $\alpha$-stable matched filter extracted from this family of filters is being developed in a simple closed form. This optimal $\alpha$-stable matched filter significantly improves target detection in both simulated data and real clutter data. Moreover, the $\alpha$-stable matched filter is computationally efficient. It can be applied in wide varieties of radar signal and image processing.

Introduction

In conventional radar signal/image processing, the background clutter/noise is assumed to follow the Gaussian model. Indeed Gaussian is a good model for homogeneous clutter/noise such as in the desert. Under this assumption, it has been shown that the conventional matched filter is optimal in target detection (Ref.[1]). However, recent research has found that many types of clutter/noise, such as sea clutter, do not fit the Gaussian model well because of impulsive outliers or the so called “sea spike” (Ref.[2]). These types of clutter/noise lend themselves to a heavy tail in amplitude distribution. Consequently, the conventional matched filter does not perform well. Radar engineers have been exploring other models such as the K-distribution or the Weibull distribution to fit these types of clutter/noise (Refs [3], [4]). Most recent research has shown that the $\alpha$-stable model is a better model and the associated $\alpha$-stable matched filter enhances target detection in radar signal and image processing applications (Refs [5], [9]). Extended from the conventional matched filter, the $\alpha$-stable matched filter is actually a family of matched filters lending itself to the robustness in filter optimization. This paper develops a simple close form in determining the optimal matched filter from the family of filters.

1 Approved for public release: distribution is unlimited
Background: $\alpha$-stable Model and $\alpha$-stable Matched Filter

The symmetric $\alpha$-stable model has three parameters (Ref. [6]); namely, the location parameter $\delta$ to specify the point of symmetry, the dispersion parameter $\gamma$ to specify the spread of data around $\delta$, and the characteristic exponent parameter $\alpha$ ($0 < \alpha \leq 2$) to specify the heaviness of the tail. It is to realize that, as a special case, when $\alpha = 2$ the $\alpha$-stable model is a Gaussian model. Properties of the Gaussian model such as the bell shape, symmetry, and Central Limit Theorem carry naturally to the symmetric $\alpha$-stable model. Thus, the $\alpha$-stable model is a natural extension of the Gaussian model. It stands out from the Gaussian model by providing a unique parameter $\alpha$ that characterizes the heaviness of the tail of the clutter. It is shown in Ref. [5] that the real sea clutter called “HPC” (with radar look down angle of 8 degrees and sea state of 3) obtained from NSWC fits on the $\alpha$-stable model better than the Gaussian model, the K-distribution, and the Weibull distribution. The $\alpha$-stable model is also shown in Ref. [5] to fit four other types of real radar clutter data well.

Let $u(t)$ be the radar transmit waveform and $x(t)$ be the received signal. Then the conventional matched filter is expressed as:

$$u^*(-t) \otimes x(t)$$

(1)

and the $\alpha$-stable matched filter (Ref [9]) is expressed as:

$$u^*(-t) \otimes \frac{x(t)}{|x(t)|^{2-p}}$$

(2)

where $\otimes$ is the convolution operation, * is the complex conjugate, and $0 < p \leq \alpha$.

It should be noted that the $\alpha$-stable matched filter is actually a family of filters with parameter $p$, lending itself to the robustness in filter optimization. The $\alpha$-stable matched filter distinguishes itself from the conventional one by multiplying a suppression factor $1/|x(t)|^{2-p}$ to the received signal for the purpose of suppressing the “spiky” clutter. For a Gaussian clutter ($\alpha = 2$), the optimal matched filter is the one with $p = 2$ in Eq.(2). For a skikier clutter ($\alpha < 2$) the $p$ corresponding to the optimal $\alpha$-stable matched filter in Eq.(2) should be reduced accordingly to achieve the goal of suppressing the spikier clutter. Thus, the $\alpha$-stable matched filter is a natural extension of the conventional matched filter with a parameter $p$ as an extra dimension for detection optimization.

Performance of $\alpha$-Stable Matched Filter

The simulated and real data from the popular linear chirp waveform radar are used to evaluate the performance of the $\alpha$-stable matched filter (Ref [9]). Specifically, the NP-3 SAR
waveform with linear chirp rate of -30 MHz/sec, pulse duration of 4 micro-seconds, and sampling rate of 125 MHz is used in the simulation. The following simulation steps are designed:

1. Select, for each pulse, 512 range bins of simulated or real clutter data \( c(t) \);
2. Inject a target at 256-th range bin;
3. Form the received signal \( x(t) = s(t) + c(t) \), where \( s(t) \) is the simulated received target return from the transmitted waveform with amplitude adjusted to a desired SNR (signal to clutter/noise ratio);
4. Perform signal processing using the \( \alpha \)-stable matched filter: \( y(t) = \alpha(x(t)) \) as shown in Eq. (2);
5. Declare target detection only if \( |y(256)| \) is larger than a threshold;
6. Perform Monte Carlo for \( N \) pulses.

With a given SNR in step 3, the threshold needed in step 5 can be computed in accordance with selected PFAs (Probability of False Alarm). The Monte Carlo simulation is then performed to result in the ROC (Receiver Operating Characteristics) curves in terms of PFA vs. probability of detection. Figure 1 shows the ROC curves resulted from four \( \alpha \)-stable matched filters using 1024 pulses of simulated clutter data with \( \alpha = 1.74, \gamma = 0.97, \delta = 0.0 \) (same \( \alpha, \gamma \) and \( \delta \) as the HPC sea clutter) and SNR = -20 dB. It is shown in Figure 1 that the probabilities of detection at PFA = 0.01 are 0.37, 0.80, 0.83, and 0.02 for \( p = 0.5, 1.0, 1.5, \) and 2, respectively. This simulation result shows that by using the \( \alpha \)-stable matched filter with the parameter \( p \) in between 1 and 1.5 the probability of detection increases dramatically over the conventional matched filter \((p = 2)\).

Note that Figures 1 only shows the performance of \( \alpha \)-stable matched filters for four \( p \) values. Naturally, it is very desirable to obtain the \( p \) corresponding to the optimal matched filter. This paper develops a close form in determining optimal \( p \).

**Optimal \( \alpha \)-Stable Matched Filter**

As discussed earlier, the family of \( \alpha \)-stable matched filters is defined in Eq. (2) with parameter \( p \). It is unlikely that an analytic method can be obtained in determining the optimal matched filter from the family of filters. In this paper, Monte Carlo simulation approach is used to estimate the optimal \( p \) with a large number of trials, say \( N = 5000 \). Recall that if a clutter is well modeled by the \( \alpha \)-stable statistics, it will be characterized by the three parameters \( \alpha, \gamma, \) and \( \delta \), and hence the optimal matched filter \( p \) is a function of these three parameters. It is shown that actually the optimal \( \alpha \)-stable matched filter depends only on the parameter \( \alpha \). Figure 2, in a simulation run with \( \alpha = 1.5, 1.6, 1.7, 1.8, 1.9, 1.92, 1.94, 1.96, 1.98, \) and 2.0, shows the performance curves of the family of \( \alpha \)-stable matched filters in terms of probability of detection for different values of \( p \). Through extended analysis of the performance curves, the following properties are observed:
1. the performance curves are smooth,
2. the $\alpha$-stable matched filter can significantly outperforms the conventional matched filter ($p=2$); this is especially true for spikier clutter,
3. there is a plateau of optimal/near optimal region for each performance curve; i.e. there is a wide region of $p$ for which the match filters are optimal or near optimal,
4. the optimal $\alpha$-stable matched filter is independent of PFA,
5. the optimal $\alpha$-stable matched filter is independent of SNR.

The above phenomena and analysis reveal that if radar clutter fits the $\alpha$-stable model, the optimal $\alpha$-stable matched filter is solely a function of the parameter $\alpha$, i.e. $Po = f(\alpha)$, where $Po$ is the $p$ in Eq.(2) corresponding to the optimal matched filter. Naturally, it is very desirable to find a close form for the function $f$. For $\alpha = 1.5, 1.6, 1.7, 1.8, 1.9, 1.92, 1.94, 1.96, 1.98, and 2.0$, the simulated optimal $Po$ are indicated by “*” on the performance curves in Figure 2. It is also observed that when $\alpha = 1.5$, $Po \sim 3\alpha/4 = 1.125$, and when $\alpha = 2$, $Po = 2$. One simple family (with parameter $q > 1$) of functions that pass through these two points and fits simulated optimal $(\alpha, p)$ data, indicated by “*” on the performance curves in Figure 2, is:

$$Po = f(\alpha) = 1.125 + 0.875 \times (1 - (1 - 2 \times (\alpha - 1.5))^{\frac{1}{q}})$$

(3)

Through vast simulation runs, it is found that, with $q = 3.5$, Eq.(3) provides an excellent close form in estimating the optimal $Po$. These optimal $Po$ via Eq.(3) for various $\alpha$ values are shown in Table 1. They are indicated by “o” on the performance curves in Figure 2. It is to note that even though the closed form optimal and the simulated optimal may not coincide with each other for all $\alpha$-stable clutter they are both on the plateau of optimal/near optimal region. In fact, from Figure 2, the differences in probability of detection between them for all performance curves are all within 0.004. Thus the closed form optimal $Po$ derived from Eq.(3) provides a simple and quick way to extract an optimal matched filter out of the entire family of $\alpha$-stable matched filters. Using this optimal matched filter for the target detection processing via Eq.(2), the optimal $\alpha$-stable matched filter outperforms the conventional matched filter significantly. Taking from the results shown in Figure 2, Table 1 shows the gain in probability of detection of the optimal $\alpha$-stable matched filter over the conventional matched filter for different $\alpha$-stable clutter. As expected, the spikier the clutter (smaller $\alpha$ value) the more gain the optimal matched filter produces.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>1.92</th>
<th>1.94</th>
<th>1.96</th>
<th>1.98</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Po (optimal)</td>
<td>1.125</td>
<td>1.179</td>
<td>1.244</td>
<td>1.327</td>
<td>1.448</td>
<td>1.482</td>
<td>1.523</td>
<td>1.575</td>
<td>1.651</td>
<td>2.0</td>
</tr>
<tr>
<td>SNR</td>
<td>-25</td>
<td>-23.5</td>
<td>-22</td>
<td>-20.5</td>
<td>-19</td>
<td>-19</td>
<td>-18.5</td>
<td>-18</td>
<td>-17.5</td>
<td>-17</td>
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<tr>
<td>Prob</td>
<td>0.763</td>
<td>0.706</td>
<td>0.637</td>
<td>0.581</td>
<td>0.553</td>
<td>0.480</td>
<td>0.527</td>
<td>0.558</td>
<td>0.589</td>
<td>0.640</td>
</tr>
<tr>
<td>Prob Gain</td>
<td>0.761</td>
<td>0.704</td>
<td>0.633</td>
<td>0.572</td>
<td>0.535</td>
<td>0.452</td>
<td>0.411</td>
<td>0.375</td>
<td>0.204</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 1 Detection probability by the optimal matched filter vs. the conventional filter
Performance of Optimal $\alpha$-Stable Matched Filter on Real Data

NP3-SAR (Synthetic Aperture Radar) data available in NAWCAD Radar Laboratory was used to evaluate the performance of the optimal $\alpha$-stable matched filter on real data. A subset of L-band SAR sea clutter data known as the “Puerto Rico L9p5Ihh” of size 512 range bins by 2048 pulses was observed. Fitting this sea clutter data by the $\alpha$-stable model pulse by pulse, except for a few anomaly pulses, the parameter estimation of $\alpha$ is fairly consistent, with an average value of 1.759 and standard deviation of 0.1. With this $\alpha$ the corresponding optimal $\alpha$-stable matched filter via Eq(3) is the one with $Po = 1.2897$. By using the Puerto Rico real data for Monte Carlo simulation, the optimal $\alpha$-stable matched filter outperforms the conventional matched filter by a gain of 0.60 in probability of detection, which is comparable to the result shown in Table 1.

$\alpha$-Stable Matched Filter for Image Formation

SAR provides 2-dimensional imagery, of which the axes are commonly referred to as the range and the azimuth. To form a SAR image two basic processing steps are needed; namely, the range compression and the azimuth compression. Each compression is processed using an appropriate matched filter. If the clutter of the image is spiky and the clutter fits a particular $\alpha$-stable model well, then instead of using the conventional matched filter, one can expect that the use of appropriate $\alpha$-stable matched filter(s) for either or both range and azimuth compression would result in improvement in target detection. To test the efficacy of the $\alpha$-stable matched filter in image processing, a 512x512 simulated SAR raw data is created. The data contain simulated $\alpha$-stable clutter pulse by pulse with $\alpha = 1.5$, and a simulated weak target at the center of the image with SNR = -35. The conventional image formation process is then performed on the data to form the image. The resulting image shows no target, just the noisy clutter. An $\alpha$-stable image formation matched filter consisting of the range compression filter with optimal $Po$ derived from Eq.(3) and the conventional azimuth compression matched filter, is then applied to the same data. The resulting image shows a recognizable target with the clutter being successfully suppressed.

The above result is appealing but more work needs to be done in quantifying the performance of $\alpha$-stable method versus the conventional method in terms of standard measurements such as location registration, phase accuracy, resolutions, mainlobe to sidelobe ratio, integrated mainlobe to sidelobe power ratio etc. In addition, real data should be tested to conclude the effectiveness of $\alpha$-stable image formation matched filters.
Conclusion

In general most radar clutter are modeled by the $\alpha$-stable statistics well. Robust family of $\alpha$-stable matched filters is a natural extension of the conventional matched filter. An optimal $\alpha$-stable matched filter is developed in a simple closed form in this paper. This optimal $\alpha$-stable matched filter significantly improves target detection in probability of detection for simulated data as well as real clutter data. Moreover, the $\alpha$-stable matched filter is computationally efficient. This technology can be used in wide varieties of radar signal and image processing. The process of implementing the $\alpha$-stable technology in a platform is very simple; it can be outlined by the following three steps:

(a) periodically model the received signal to update the $\alpha$ parameter;
(b) compute a new optimal $\alpha$-stable matched filter using Eq.(3);
(c) employ the new optimal $\alpha$-stable matched filter (i.e. new optimal $Po$) in Eq.(2) for target detection.

References

Figure 1 Performance of $\alpha$-Stable Matched Filter on Simulated Clutter (PFA vs Pd)
Figure 2. Performance of the family of $\alpha$-Stable Matched Filters for simulated clutter with $\alpha = 1.5, 1.6, 1.7, 1.8, 1.9, 1.92, 1.94, 1.96, 1.98,$ and 2