Abstract—Non-cooperative moving targets appear defocussed in SAR images and the blurring effect due to the unknown motion leads to low detection capabilities. In this work, clutter suppression and ISAR processing are combined to obtain well focused images of extended non-cooperative moving targets embedded in strong clutter, by exploiting Multichannel SAR (M-SAR) systems. Clutter mitigation and radar motion compensation are performed by means of the proposed Space Doppler Adaptive Processing. Then ISAR processing is used to compensate the unknown target motion. Two principal issues will be addressed. First, a technique to apply ISAR processing after clutter mitigation is presented, and then a suboptimal approach for clutter mitigation is proposed to overcome computational and statistical issues associated with estimation of clutter cross-power spectral matrix. Results of the processing applied to simulated data are provided in order to show the effectiveness of the proposed techniques.

I. INTRODUCTION

The perfect knowledge of the relative motion between the radar platform and target scene allows the synthesis of a long synthetic array that is exploited to obtain high cross-range resolution images of the observed scene. The synthesis of a long aperture is obtained by the coherent processing of the echoes received during the overall observation interval, which are linearly combined after the compensation of the phase shift induced by the relative motion. This is the basic principle of Synthetic Aperture Radar systems (SAR). In that scenario, a moving target within the observed area might not be easily detectable and, even if detected, the formation of the corresponding high resolution image is not straightforward.

Another way to look at the problem of forming a long synthetic array to achieve high cross-range resolution in non-cooperative target images is Inverse Synthetic Aperture Radar (ISAR). ISAR systems do not base their functioning on the knowledge of target motion whilst they exploit this motion to form the synthetic aperture. ISAR processing can be used also to obtain well focused images of moving targets within SAR images as described in [6], [7]. It is worth pointing out that in order to apply ISAR techniques to SAR data, a non-cooperative target must be detected first, and the detection can be a critical step, especially in the case of strong ground clutter.

A considerable gain over conventional detection processing can be obtained by coherent processing of the echoes received from different antenna elements at different time instants. This is known as Space Time Adaptive Processing (STAP) and can significantly improve detection performances of low velocity and small targets in strong clutter environment or in presence of strong interferers. This technique is widely treated for a general airborne radar case [4], but in recent years, with the development of imaging systems with multichannel capabilities, many researchers have studied the applicability of STAP processing in a multichannel radar imaging scenario [3], [8], [11].

In this paper, a different approach is presented. Space Doppler Adaptive Processing (SDAP) is derived using the Range-Doppler technique. This algorithm performs static clutter suppression and platform motion compensation leading to a defocussed image of the target. Then, the use of ISAR autofocusing on the output of the SDAP algorithm is proposed in order to compensate for the unknown part of the motion due to the target and obtain high resolution images of non-cooperative moving targets. The second aim of this work is to achieve sub-optimal SDAP that allows less data to be used when estimating the cross-power spectral density matrix. A multichannel signal model is provided in Section II. Optimal and sub-optimal processors are described in Section III while results on simulated data are shown in Section IV.

II. MULTICHANNEL SIGNAL MODEL

Let us consider the acquisition geometry shown in Figure 1 where the radar system observes a moving target within a static background.

For a typical $P$-channel SAR system, usually one antenna transmits pulses and all $p$ antennas receive the echoes reflected from the illuminated scene. For such a multistatic scenario, an equivalent monostatic scenario where each antenna transmits and receives can be adopted to simplify the notation [9].

The Discrete Fourier Transform (DFT) of the received signal after range compression for the $p$-th channel, $p = 1, 2, \ldots, P$, can be expressed as follows:

$$S_p(n, m) = S_{t,p}(n, m) + S_{c,p}(n, m) + N_p(n, m) \quad (1)$$

where $S_{t,p}(n, m)$ is the contribution due to the moving target, $S_{c,p}(n, m)$ is the contribution due to the static scene (clutter), and $N_p(n, m)$ is the noise contribution.
\( N_p(n,m) \) is additive noise and \( n \) and \( m \) denote the frequency index and the pulse index respectively.

For an array size much smaller than the radar-target distance, the LOS (line of sight) is the same for each element so moving target and static scene components can be written as follows:

\[
S_{t,p}(n,m) = e^{-j \frac{2 \pi}{R_{0t,p}}(n,m) \sum_{i=1}^{N_s} \sigma_i e^{-j \frac{2 \pi}{R} |x_i + \Omega_{\text{eff},t} y_i| n T_R} \}
\]

\( S_{c,p}(n,m) = e^{-j \frac{2 \pi}{R_{0c,p}}(n,m)} \cdot \iint_{(x,y)} \sigma(x,y) e^{-j \frac{2 \pi}{R} |x + \Omega_{\text{eff},c} y T_R|} dxdy \) \tag{3}

where \( x_i \) and \( y_i \) are the coordinates of the \( i \) – \( th \) scatterer in the target’s reference system, \( R_{0t,p} \) is the slant range distance between the \( p – th \) antenna and the target reference point and \( \Omega_{\text{eff},t} \) is total rotation vector modulus due to the relative motion of the target and the static scene respectively projected on a plane orthogonal to the LOS. Finally, \( \sigma_i \) is the amplitude of the \( i – th \) target scatterer, \( \sigma(x,y) \) is the ground reflectivity and \( T_R = \frac{1}{PRF} \) is the pulse repetition interval. Also \( R_{0c,p} \) is the distance between the \( p – th \) antenna and the reference point on the ground and \( \Omega_{\text{eff},c} \) is the rotation vector modulus due to the platform motion [2].

A statistical description of the static clutter is needed in order to define the adaptive clutter mitigation process. Clutter is modeled as a random process with assigned features, depending on the observation geometry and the characteristics of the observed surface. The observation of the earth’s surface by a linear array distributed along the flight direction involves a particular correlation between samples of the signal received from different array elements at different time instants.

A model for the clutter space-time covariance matrix \( R_{st} \) is available in [1] where:

\[
E \{ S_{c,i}(n,m) S_{c,k}^*(n,l) \} = P_c p_s [(m-l) v_p T_R + (i-k) d] \rho_d [(m-l) T_R] \tag{4}
\]

The indexes \( m, l \) and \( i, k \) denote the slow time instants (i.e., pulse numbers) and the element numbers respectively while \( P_c \) is the clutter power, \( v_p \) is the platform velocity, \( d \) is the distance between adjacent receivers and \( \rho_d(\Delta t) = e^{-\frac{\Delta t^2}{2\sigma_d^2}} \) and \( \rho_v(\Delta t) = e^{-\frac{\Delta t^2}{2\tau}} \) are the space and time correlation coefficients respectively.

Let us define the signal and target Doppler spectrum at the generic \( p – th \) channel as follows:

\[
\tilde{S}_{t,p}(n,l) = DFT_n \{ S_{t,p}(n,m) \}
\]

\[
\tilde{S}_{c,p}(n,l) = DFT_n \{ S_{c,p}(n,m) \}
\]

The generic element of the clutter cross-power spectral matrix \( R_{st,D} \) is defined as follows, where \( l, m \) and \( i, k \) denote Doppler frequency bins and element numbers respectively:

\[
E \{ \tilde{S}_{c,i}(n,l) \tilde{S}_{c,k}^*(n,m) \} \tag{7}
\]

\( R_{st,D} \) is related to \( R_{st} \) by means of a unitary transformation matrix \( T \) that is defined as the Kronecker product of the time-frequency Fourier transformation matrix \( T_t \) and the identity matrix of dimension \( P, IP \).

\[
R_{st,D} = TR_{st}T^H \tag{8}
\]

\[
T = T_t \otimes I_P \in C^{MP \times MP} \tag{9}
\]

The generic element of \( T_t \in C^{M \times M} \) is

\[
\tilde{t}_t(l,m) = e^{-j2\pi l \Delta f d T_R}
\]

with \( m, l = [-\frac{M}{2}, \frac{M}{2} - 1] \), \( \Delta f_d = \frac{1}{MT_R} \) and \( M \) is the number of Doppler frequency bins.

III. Clutter Suppression and Imaging

In Figure 2 the functional block of the clutter suppression and imaging processing is shown. A stacking step is needed to define both the signal vector and the reference vector in the Doppler domain [11]:

\[
\mathbf{\tilde{S}}(n,l) = \frac{1}{P} \left[ \tilde{S}_{t}(n,l), \ldots, \tilde{S}_{P}(n,l) \right]^T \in C^{P \times 1} \tag{11}
\]

\[
\mathbf{\tilde{S}}_{\text{ref}}(n,l) = \frac{1}{P} \left[ \tilde{S}_{t_{\text{ref}}}(n,l), \ldots, \tilde{S}_{P_{\text{ref}}}(n,l) \right]^T \in C^{P \times 1} \tag{12}
\]
where:

\[ \hat{S}(n) = \begin{bmatrix} \tilde{S}^T(n,0), \cdots, \tilde{S}^T(n,M-1) \end{bmatrix}^T \in C^{MP \times 1} \] (13)

\[ \tilde{G}(n,l) = \begin{bmatrix} \tilde{S}^{T}_{\text{ref}}(n,l), \cdots, \tilde{S}^{T}_{\text{ref}}(n,l-M+1) \end{bmatrix}^T \in C^{MP \times 1} \] (14)

where

\[ \tilde{S}_{\text{p,ref}}(n,l) = FT_m \{ S_{p,\text{ref}}(n,m) \} = \] 

\[ = FT_m \{ e^{-j \frac{2\pi}{T} R_{\text{ref}}(m)} \} \] (15)

The SMI (Sample Matrix Inversion) implementation of the optimum space Doppler adaptive processing is given by the incorporation of the clutter suppression operation in the classical range-Doppler focusing algorithm [2]:

\[ F(n,l) = \tilde{W}^H(n,l)\tilde{S}(n) \] (16)

where:

\[ \tilde{W}(n,l) = \delta \tilde{R}_{sD}^{-1} \tilde{G}(n,l) \in C^{MP \times 1} \] (17)

\[ \tilde{R}_{sD} \] is an estimate of the exact clutter cross-power spectral matrix \( R_{sD} \) and is defined as:

\[ \tilde{R}_{sD} = \frac{1}{N_r} \sum_{n_r=1}^{N_r} Z(n_r)\tilde{Z}^H(n_r) \in C^{MP \times MP} \] (18)

and

\[ \delta = \frac{1}{\sqrt{\tilde{G}^H(n,l)\tilde{R}_{sD}^{-1}\tilde{G}(n,l)}} \] (19)

It is worth highlighting that the proposed algorithm is different from the post Doppler STAP described in [4] because of the definition of the reference vector in (14).

The vector \( Z(n_r) \) in (18) denotes target-free data in the \( n_r \)-th range cell. It is worth pointing out that processing in (16) performs both clutter suppression and image focusing. In order to obtain well focused images of the moving target in the scene, the reference vector \( \tilde{G}(l,n) \) must take into account both platform and target own motion. It is quite obvious that in a real scenario the motion due to a non-cooperative moving target is unknown. In this case a first motion compensation is achieved by applying the ISAR focusing algorithm based on the contrast maximization [5] to the data provided at the output of the Space Doppler processor.

Issue to be addressed concerns the estimation of the interference cross-power spectral matrix \( \tilde{R}_{sD} \). In order to obtain an average performance loss of roughly 3 dB, \( N_r = 2PM \) target-free range cells are needed [10]. With typical values of \( PRF = 1kHz \), \( T_{\text{obs}} = 1s \) and \( P = 3 \), \( N_r = 6000 \) range cells are needed. Assuming a range resolution of \( 0.5m \), this results in an area of homogeneous clutter of \( 3Km \) in the range dimension being required which may not always be achievable in practice.

### A. Sub-optimum approach

A possible sub-optimum approach consists of splitting the filtering operation in (16) as shown in Figure 3. This operation consists of applying a windowing operation to the space-time data and applying the Space Doppler processing on each window. In this case the dimension of the cross-power spectral matrix to be estimated is limited by the length of the window. It is worth pointing out that this windowing operation does not lead to any losses if only the image formation processing is taken into account (\( \tilde{W}(n,l) = \tilde{G}(n,l) \)) while in the clutter suppression processing this consists of using only the block diagonal sub-matrices of the overall cross-power spectral matrix as illustrated in figure Figure 4.

This operation can be written as follows where \( L \) and \( i = 0, 1, \cdots, \lfloor \frac{L}{2} \rfloor - 1 \) denote the window length and the window index respectively:

\[ F_w(n,l) = \sum_i \tilde{W}^H_i(n,l)\tilde{S}_i(n) \] (20)

where:

\[ \tilde{W}_i(n,m) = \delta_i \tilde{R}_{sD,i}^{-1} \tilde{G}_i(n,l) \in C^{LP \times 1} \] (21)

and

\[ \tilde{G}_i(n,l) = \begin{bmatrix} \tilde{S}_{\text{ref}}(n,l-iL) \\ \tilde{S}_{\text{ref}}(n,l-(iL+1)) \\ \vdots \\ \tilde{S}_{\text{ref}}(n,l-((i+1)L-1)) \end{bmatrix} \in C^{LP \times 1} \] (22)

\[ \tilde{S}_i(n) = \begin{bmatrix} \tilde{S}(n,iL) \\ \tilde{S}(n,iL+1) \\ \vdots \\ \tilde{S}(n,(i+1)L-1) \end{bmatrix} \in C^{LP \times 1} \] (23)
\[
\delta_i = \frac{1}{\sqrt{G_i^{-1}(n,l) R_{sD,i}(n,l)^\dagger R_{sD,W}(n,l) - 1 sD,i}}
\]

In this case only \(2LP\) target free range cells are needed, instead of \(2LM\), to obtain a good estimation of \(\hat{R}_{sD,i}\). The window length \(L\) is limited by the error in the estimation of the whole cross-power spectral matrix due to the blocking operation illustrated in Figure 4 where \(\hat{R}_{sD,W}\) is the approximation of the cross-power spectral matrix taken into account when the sub-optimum approach is applied.

IV. SIMULATION RESULTS

In this section the processing schemes described above are applied to simulated data. Since real data are not available a multichannel SAR simulator is set up. The useful signal is the response from an extended non-cooperative moving target. The target consists of 35 point scatterers and the unknown motion includes a translation part and a rotation part (yaw, pitch, roll). The system and target parameters are listed in Table I and Table II respectively.

| \(v_p\) | platform velocity | 200m/s |
| \(h\) | platform height | 4000m |
| \(\theta_{inc}\) | incidence angle | 45° |
| \(f_0\) | carrier frequency | 10GHz |
| \(B\) | signal bandwidth | 300MHz |
| \(T_{obs}\) | observation time | 1s |
| \(PRF\) | pulse repetition frequency | 1KHz |
| SCR | signal to clutter ratio | 30dB |
| \(P\) | Number of channels | 3 |

Table 1

SYSTEM PARAMETERS

| \(v_t\) | target velocity | 18m/s |
| \(\theta_t\) | target orientation | -30° |
| \(A_{pitch}\) | Pitch Amplitude | 2° |
| \(A_{yaw}\) | Yaw Amplitude | 1.5° |
| \(T_{roll}\) | Roll Period | 8s |
| \(T_{pitch}\) | Pitch Period | 11s |

Table II

TARGET PARAMETERS

As shown in 6 (a) the effect of clutter is to completely mask the ISAR image of the target whilst after the proposed SDAP the ISAR image of 6 (b) is almost equal to that of the clutter-free image shown in 6 (c).

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The output of optimum SDAP using the full matrix formed using all 1000 pulses as described in Section III is shown in Figure 7 where in contrast to the results shown in Figure 6 the target motion is unknown. It is quite obvious that SDAP...
considering only the platform motion leads to a defocused image of the target. The improvement in detection capability after clutter suppression is evident. In order to obtain the high resolution image ISAR processing has to be applied. It is worth pointing out that such results are obtained by using $N_r = 2PM$ target-free range cells to estimate the cross-power spectral matrix $\hat{R}_{sD}$.

As can be seen when using a relatively large number of pulses, i.e. $L = 100$, results shown in Figure 8 (a) closely approximates results shown in Figure 7 (c) which was obtained using 1000 pulses. As indicated by results in Figure 8 (a) and (b) decreasing the window length $L$, the distortion due to the approximation on the covariance matrix become more evident. It is worth pointing out that such results are obtained by using $N_r = 2PL$ target-free range cells to estimate $\hat{R}_{sD,1}$.

V. CONCLUSION

ISAR processing can be applied to obtain well focused images of non-cooperative moving targets detected in SAR images. In strong clutter environments detection can be a critical step because of the spreading of the useful energy due to the lack of knowledge of the target motion. An approach to reducing clutter based on SDAP of the outputs of a multichannel SAR has been proposed and demonstrated to significantly improve detection in presence of clutter. When target motion is unknown combined SDAP/ISAR processing results in well formed images even in presence of high levels of clutter. A sub-optimal approach has been developed in order to overcome computational and statistical problems in the estimation of the space time-cross-spectral matrix. The effectiveness of the proposed algorithms has been proven on simulated data.

REFERENCES


