Building Program Verifiers from Compilers and Theorem Provers

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Automated Software Analysis

Program → Automated Analysis → Correct

Software Model Checking with Predicate Abstraction
  e.g., Microsoft’s SDV

Abstract Interpretation with Numeric Abstraction
  e.g., ASTREE, Polyspace

Incorrect
Turing, 1936: “undecidable”
How can one check a routine in the sense of making sure that it is right?

The programmer should make a number of definite assertions which can be checked individually, and from which the correctness of the whole programme easily follows.
SeaHorn

A fully automated verification framework for LLVM-based languages.

http://seahorn.github.io
SeaHorn Verification Framework

Distinguishing Features

- LLVM front-end(s)
- Constrained Horn Clauses to represent Verification Conditions
- Comparable to state-of-the-art tools at SV-COMP’15

Goals

- be a state-of-the-art Software Model Checker
- be a framework for experimenting and developing CHC-based verification
Related Tools

CPAChecker
- Custom front-end for C
- Abstract Interpretation-inspired verification engine
- Predicate abstraction, invariant generation, BMC, k-induction

SMACK / Corral
- LLVM-based front-end
- Reduces C verification to Boogie
- Corral / Q verification back-end based on Bounded Model Checking with SMT

UFO
- LLVM-based front-end (partially reused in SeaHorn)
- Combines Abstract Interpretation with Interpolation-Based Model Checking
- (no longer actively developed)
SeaHorn Philosophy

Build a state-of-the-art Software Model Checker

• useful to “average” users
  – user-friendly, efficient, trusted, certificate-producing, …
• useful to researchers in verification
  – modular design, clean separation between syntax, semantics, and logic, …

Stand on the shoulders of giants

• reuse techniques from compiler community to reduce verification effort
  – SSA, loop restructuring, induction variables, alias analysis, …
  – static analysis and abstract interpretation
• reduce verification to logic
  – verification condition generation
  – Constrained Horn Clauses

Build reusable logic-based verification technology

• “SMT-LIB” for program verification
SeaHorn Usage

> sea pf FILE.c
Outputs sat for unsafe (has counterexample); unsat for safe

Additional options

- `--cex=trace.xml` outputs a counter-example in SV-COMP'15 format
- `--track={reg, ptr, mem}` track registers, pointers, memory content
- `--step={large, small}` verification condition step-semantics
  - `small` == basic block, `large` == loop-free control flow block
- `--inline` inline all functions in the front-end passes

Additional commands

- `sea smt` -- generates CHC in extension of SMT-LIB2 format
- `sea clp` -- generates CHC in CLP format (under development)
- `sea lfe-smt` -- generates CHC in SMT-LIB2 format using legacy front-end
Verification Pipeline

Reference to figures or diagrams.
Constrained Horn Clauses

INTERMEDIATE REPRESENTATION
Constrained Horn Clauses (CHC)

A Constrained Horn Clause (CHC) is a FOL formula of the form

$$8 V . (Á \forall p_1[X_1] \forall \ldots \forall p_n[X_n] \rightarrow h[X]),$$

where

- $A$ is a background theory (e.g., Linear Arithmetic, Arrays, Bit-Vectors, or combinations of the above)
- $Á$ is a constrained in the background theory $A$
- $p_1, \ldots, p_n, h$ are $n$-ary predicates
- $p_i[X]$ is an application of a predicate to first-order terms
Example Horn Encoding

```plaintext
int x = 1;
int y = 0;
while (*) {
    x = x + y;
    y = y + 1;
}
assert(x ≥ y);

l0:
  x = 1
  y = 0

l1: b1 = nondet()

l2:
  x = x + y
  y = y + 1

l3:
  b2 = x ≥ y

l4:

lerr:

(1) p0.
(2) p1(x, y) ← p0, x = 1, y = 0.
(3) p2(x, y) ← p1(x, y).
(4) p3(x, y) ← p1(x, y).
(5) p1(x’, y’) ← p2(x, y),
    x’ = x + y,
    y’ = y + 1.
(6) p4 ← (x ≥ y), p3(x, y).
(7) perr ← (x < y), p3(x, y).
(8) p4 ← p4.
(9) ⊥ ← perr.
```
CHC Terminology

Rule

\[ h[X] \dashv p_1[X_1], \ldots, p_n[X_n], \dashv. \]

Query

\[ \text{false} \dashv p_1[X_1], \ldots, p_n[X_n], \dashv. \]

Fact

\[ h[X] \dashv \dashv. \]

Linear CHC

\[ h[X] \dashv p[X_1], \dashv. \]

Non-Linear CHC

\[ h[X] \dashv p_1[X_1], \ldots, p_n[X_n], \dashv. \]

for \( n > 1 \)
CHC Satisfiability

A **model** of a set of clauses $\models$ is an interpretation of each predicate $p_i$ that makes all clauses in $\models$ valid.

A set of clauses is **satisfiable** if it has a model, and is unsatisfiable otherwise.

A model is **A-definable**, if each $p_i$ is definable by a formula $\tilde{A}_i$ in A.
Relationship between CHC and Verification

A program satisfies a property iff corresponding CHCs are satisfiable
  • satisfiability-preserving transformations == safety preserving

Models for CHC correspond to verification certificates
  • inductive invariants and procedure summaries

Unsatisfiability (or derivation of FALSE) corresponds to counterexample
  • the resolution derivation (a path or a tree) is the counterexample

CAVEAT: In SeaHorn the terminology is reversed
  • SAT means there exists a counterexample – a BMC at some depth is SAT
  • UNSAT means the program is safe – BMC at all depths are UNSAT
FROM PROGRAMS TO CLAUSES
Hoare Triples

A Hoare triple $\{\text{Pre}\} \ P \ \{\text{Post}\}$ is valid iff every terminating execution of $P$ that starts in a state that satisfies $Pre$ ends in a state that satisfies $Post$.

**Inductive Loop Invariant**

\[
\begin{array}{c}
\text{Pre} \land \text{Inv} \quad \{\text{Inv}\land C}\quad \text{Body} \quad \{\text{Inv}\} \\
\hline
\text{Inv}\land C\quad \text{Body} \quad \{\text{Post}\}
\end{array}
\]

\[
\{\text{Pre}\} \ \text{while} \ C \ \text{do} \ \text{Body} \quad \{\text{Post}\}
\]

**Function Application**

\[
\begin{array}{c}
(\text{Pre}\land p=a) \land P \\
\hline
\text{P} \land \text{Body}_F \quad \{Q\} \\
\hline
\text{Q} \land r=a,b \land \text{Body} \quad \{\text{Post}\}
\end{array}
\]

\[
\{\text{Pre}\} \ b = F(a) \quad \{\text{Post}\}
\]

**Recursion**

\[
\begin{array}{c}
\{\text{Pre}\} \ b = F(a) \quad \{\text{Post}\} \ \land \ {\text{Pre}\} \ \text{Body}_F \quad \{\text{Post}\}
\end{array}
\]

\[
\{\text{Pre}\} \ b = F(a) \quad \{\text{Post}\}
\]
Weakest Liberal Pre-Condition

Validity of Hoare triples is reduced to FOL validity by applying a predicate transformer

Dijkstra’s weakest liberal pre-condition calculus [Dijkstra’75]

\[ \text{wlp} (P, \text{Post}) \]

weakest pre-condition ensuring that executing P ends in Post

\[ \{ \text{Pre} \} \; P \; \{ \text{Post} \} \text{ is valid } , \; \text{ Pre } ) \; \text{wlp} \; (P, \text{Post}) \]
Horn Clauses by Weakest Liberal Precondition

Prog = def Main(x) { body_M }, …, def P(x) { body_P }

wlp (x=E, Q) = let x=E in Q
wlp (assert (E), Q) = E \land Q
wlp (assume(E), Q) = E \Rightarrow Q
wlp (while E do S, Q) = I(w) \land 
    8w . ((I(w) \land E) \Rightarrow wlp (S, I(w))) \land ((I(w) \land :E) \Rightarrow Q))
wlp (y = P(E), Q) = p_{pre}(E) \land (8 r. p(E, r) \Rightarrow Q[r/y])

ToHorn (def P(x) {S}) = wlp (x0=x ; assume (p_{pre}(x)); S, p(x0, ret))
ToHorn (Prog) = wlp (Main(), true) \land 8\{P 2 Prog\} . ToHorn (P)
Example of a WLP Horn Encoding

{Pre: \( y \geq 0 \)}
\[
\begin{align*}
    x_o &= x; \\
    y_o &= y; \\
    \text{while } y > 0 \text{ do} & \\
    x &= x + 1; \\
    y &= y - 1; \\
{Post: x = x_o + y_o}
\end{align*}
\]

\( \text{ToHorn} \)

C1: \( I(x, y, x, y) \land y \geq 0 \).
C2: \( I(x+1, y-1, x_o, y_o) \land I(x, y, x_o, y_o), y > 0 \).
C3: \text{false} \land I(x, y, x_o, y_o), y \cdot 0, x \neq x_o + y_o

\{y \geq 0\} \ P \{x = x_{old} + y_{old}\} \text{ is true iff the query } C_3 \text{ is satisfiable}
Dual WLP

Dual weakest liberal pre-condition

\[ \text{dual-wlp} (P, \text{Post}) = \text{:wlp} (P, :\text{Post}) \]

\[ s^2 \text{dual-wlp} (P, \text{Post}) \text{ iff there exists an execution of } P \text{ that starts in } s \text{ and ends in Post} \]

\text{dual-wlp} (P, \text{Post}) \text{ is the weakest condition ensuring that an execution of } P \text{ can reach a state in Post} \]
Horn Clauses by Dual WLP

Assumptions

- each procedure is represented by a control flow graph
  - i.e., statements of the form \( l_i : S \); goto \( l_j \), where \( S \) is loop-free
- program is unsafe iff the last statement of Main() is reachable
  - i.e., no explicit assertions. All assertions are top-level.

For each procedure \( P(x) \), create predicates

- \( l(w) \) for each label, \( p_{\text{en}}(x_0,x,w) \) for entry, \( p_{\text{ex}}(x_0,r) \) for exit

The verification condition is a conjunction of clauses:

\[
\begin{align*}
p_{\text{en}}(x_0,x) & \land x_0 = x \\
l_i(x_0,w') & \land l_j(x_0,w) \land \forall \text{wlp} \ (S, :(w=w')) , \text{for each statement } l_i : S; \text{ goto } l_j \\
p(x_0,r) & \land p_{\text{ex}}(x_0,r) \\
\text{false} & \land \text{Main}_{\text{ex}}(x, \text{ret})
\end{align*}
\]
Example Horn Encoding

```plaintext
int x = 1;
int y = 0;
while (*) {
    x = x + y;
    y = y + 1;
}
assert(x ≥ y);

l₁ : b₁ = nondet()

l₂ : b₂ = x ≥ y
    x = x + y
    y = y + 1

l₃ : b₂ = x ≥ y
    x' = x + y,
    y' = y + 1

l₄ : l₅ :

l₆ : p₄ ← (x ≥ y), p₃(x, y).

l₇ : p₆ ← (x < y), p₃(x, y).

l₈ : p₄ ← p₄.

l₉ : p₉ ← p₆.
```

1. p₀.
2. p₁(x, y) ← p₀, x = 1, y = 0.
3. p₂(x, y) ← p₁(x, y).
4. p₃(x, y) ← p₁(x, y).
5. p₁(x', y') ← p₂(x, y),
   x' = x + y,
   y' = y + 1.
6. p₄ ← (x ≥ y), p₃(x, y).
7. p₆ ← (x < y), p₃(x, y).
8. p₄ ← p₄.
9. ⊥ ← p₆.
Single Static Assignment

SSA == every value has a unique assignment (a \textit{definition})
A procedure is in SSA form if every variable has exactly one definition

SSA form is used by many compilers
  \begin{itemize}
  \item explicit def-use chains
  \item simplifies optimizations and improves analyses
  \end{itemize}

PHI-function are necessary to maintain unique definitions in branching control flow

\begin{equation}
x = \text{PHI}(v_0:bb_0, \ldots, v_n:bb_n)
\end{equation}

\text{phi-assignment}

"x gets \(v_i\) if previously executed block was \(bb_i\)"
Large Step Encoding: Single Static Assignment

int x, y, n;

x = 0;
while (x < N) {
  if (y > 0)
    x = x + y;
  else
    x = x - y;
  y = -1 * y;
}

0: goto 1
1: x_0 = PHI(0:0, x_3:5);
y_0 = PHI(y:0, y_1:5);
if (x_0 < N) goto 2 else goto 6
2: if (y_0 > 0) goto 3 else goto 4
3: x_1 = x_0 + y_0; goto 5
4: x_2 = x_0 - y_0; goto 5
5: x_3 = PHI(x_1:3, x_2:4);
y_1 = -1 * y_0;
goto 1
6:
Example: Single Static Assignment

```
int x, y, n;
x = 0;
while (x < N) {
    if (y > 0)
        x = x + y;
    else
        x = x - y;
y = -1 * y;
}
```
Example: Large Step Encoding

0: goto 1
1: x_0 = PHI(0:0, x_3:5);
y_0 = PHI(y:0, y_1:5);
if (x_0 < N) goto 2 else goto 6

2: if (y_0 > 0) goto 3 else goto 4

3: x_1 = x_0 + y_0; goto 5

4: x_2 = x_0 - y_0; goto 5

5: x_3 = PHI(x_1:3, x_2:4);
y_1 = -1 * y_0;
goto 1

6:
Example: Large Step Encoding

\[ x_1 = x_0 + y_0 \]
\[ x_2 = x_0 - y_0 \]
\[ y_1 = -1 \times y_0 \]

1: \( x_0 = \text{PHI}(0:0, x_3:5); \)
   \( y_0 = \text{PHI}(y:0, y_1:5); \)
   if \((x_0 < N)\) goto 2 else goto 6

2: if \((y_0 > 0)\) goto 3 else goto 4

3: \( x_1 = x_0 + y_0 \) goto 5

4: \( x_2 = x_0 - y_0 \) goto 5

5: \( x_3 = \text{PHI}(x_1:3, x_2:4); \)
   \( y_1 = -1 \times y_0; \)
   goto 1
Example: Large Step Encoding

\[ x_1 = x_0 + y_0 \]
\[ x_2 = x_0 - y_0 \]
\[ y_1 = -1 \times y_0 \]

\[
\begin{align*}
B_2 & \rightarrow x_0 < N \\
B_3 & \rightarrow B_2 \land y_0 > 0 \\
B_4 & \rightarrow B_2 \land y_0 \leq 0 \\
B_5 & \rightarrow (B_3 \land x_3 = x_1) \lor (B_4 \land x_3 = x_2)
\end{align*}
\]

\[ B_5 \land x'_0 = x_3 \land y'_0 = y_1 \]

1: \( x_0 = \text{PHI}(0:0, x_3:5); \)
\[ y_0 = \text{PHI}(y:0, y_1:5); \]
if \( x_0 < N \) goto 2 else goto 6

2: if \( y_0 > 0 \) goto 3 else goto 4

3: \( x_1 = x_0 + y_0; \) goto 5

4: \( x_2 = x_0 - y_0; \) goto 5

5: \( x_3 = \text{PHI}(x_1:3, x_2:4); \)
\[ y_1 = -1 \times y_0; \]
\[ \text{goto 1} \]
Mixed Semantics

PROGRAM TRANSFORMATION
Deeply nested assertions
Deeply nested assertions

Counter-examples are long
Hard to determine (from main) what is relevant
Mixed Semantics

Stack-free program semantics combining:

- operational (or small-step) semantics
  - i.e., usual execution semantics
- natural (or big-step) semantics: function summary [Sharir-Pnueli 81]
  - \((\frac{3}{4}, \frac{3}{4}^-) \) \(2 \| f \|\) iff the execution of \(f\) on input state \(\frac{3}{4}\) terminates and results in state \(\frac{3}{4}'\)
- some execution steps are big, some are small

Non-deterministic executions of function calls

- update top activation record using function summary, or
- enter function body, forgetting history records (i.e., no return!)

Preserves reachability and non-termination

**Theorem:** Let \(K\) be the operational semantics, \(K^m\) the stack-free semantics, and \(L\) a program location. Then,

\[ K^2 \text{EF (pc=L)} , K^m^2 \text{EF (pc=L)} \quad \text{and} \quad K^2 \text{EG (pc\neq L)} , K^m^2 \text{EG (pc\neq L)} \]
```python
def main():
    x = nd();
    x = x+1;
    while(x>=0):
        x = f(x);
        if(x<0):
            Error;
    END;

def f(int y):
    ret y
    if(y¸10):
        y = y+1;
        y = f(y);
    else if(y>0)
        y = y+1;
    y = y-1
```

**Summary of f(y)**

\[(1 \cdot y \cdot 9 \ \land \ x'=x) \land (x \cdot 0 \ \land \ x'=x-1)\]
Mixed Semantics as Program Transformation

```plaintext
main ()
    p1 (); p1 ();
    assert (c1);
    p1 ()
    p2 ();
    assert (c2);
    p2 ()
    assert (c3);
```

mixed semantics

```plaintext
main_new ()
    if (*) goto p1_entry;
    else p1_new ();
    if (*) goto p1_entry;
    else p1_new ();
    if (¬c1) goto error;
    assume (false);

p1_entry :
    if (*) goto p2_entry;
    else p2_new ();
    p2_entry :
    if (¬c2) goto error;
    assume (false);
    error : assert (false);

p1_new ()
    p2_new ()
    assume (c2);

p2_new ()
    assume (c3);
```
Implementing Mixed Semantics in LLVM

Something about how this can be implemented as a simple transformation in LLVM

in the Lab, show how to do this transformation by hand by modifying the bitcode and using opt to execute the optimization
SOLVING CHC WITH SMT
A program $P = (V, \text{Init}, \frac{1}{2}, \text{Bad})$

- Notation: $F(X) = \exists \ u . (X \not\in \frac{1}{2}) \subset \text{Init}

$P$ is UNSAFE if and only if there exists a number $N$ s.t.

$$\text{Init}(v_0) \land \left( \bigwedge_{i=0}^{N-1} \rho(v_i,v_{i+1}) \right) \land \text{Bad}(v_N) \not\Rightarrow \bot$$

$P$ is SAFE if and only if there exists a safe inductive invariant $Inv$ s.t.

$$\text{Init}(u) \Rightarrow Inv(u)$$

$$Inv(u) \land \rho(u,v) \Rightarrow Inv(v)$$

$$Inv(u) \Rightarrow \neg \text{Bad}(u)$$
IC3/PDR Algorithm Overview

Input: Transition system $T = (Init, Tr, Bad)$

1. $F_0 \leftarrow Init$; $N \leftarrow 0$
2. repeat
   3. $G \leftarrow \text{PdrMkSafe}([F_0, \ldots, F_N], Bad)$
   4. if $G = []$ then return UNSAFE;
   5. $\forall 0 \leq i \leq N \cdot F_i \leftarrow G[i]$
   6. $F_0, \ldots, F_N \leftarrow \text{PdrPush}([F_0, \ldots, F_N])$
   // $F_0, \ldots, F_N$ is a safe $\delta$-trace
   7. if $\exists 0 \leq i \leq N \cdot F_i = \emptyset$ then return SAFE;
   8. $N \leftarrow N + 1$; $F_N \leftarrow \emptyset$
3. until $\infty$;
IC3/PDR in Pictures

Frame R₀  Frame R₁  lemma
IC3/PDR in Pictures

Inductive
IC3/PDR in Pictures

PDR Invariants

\[
R_i \rightarrow \text{: Bad} \quad \text{Init} \rightarrow R_i
\]

\[
R_i \rightarrow R_{i+1} \quad R_i \triangleleft \frac{1}{2} \rightarrow R_{i+1}
\]

Inductive
Spacer: Solving CHC in Z3

Spacer: solver for SMT-constrained Horn Clauses
• stand-alone implementation in a fork of Z3
• [http://bitbucket.org/spacer/code](http://bitbucket.org/spacer/code)

Support for Non-Linear CHC
• model procedure summaries in inter-procedural verification conditions
• model assume-guarantee reasoning
• uses MBP to under-approximate models for finite unfoldings of predicates
• uses MAX-SAT to decide on an unfolding strategy

Supported SMT-Theories
• Best-effort support for arbitrary SMT-theories
  – data-structures, bit-vectors, non-linear arithmetic
• Full support for Linear arithmetic (rational and integer)
• Quantifier-free theory of arrays
  – only quantifier free models with limited applications of array equality
RESULTS
SV-COMP 2015

4th Competition on Software Verification held (here!) at TACAS 2015

Goals

• Provide a snapshot of the state-of-the-art in software verification to the community.
• Increase the visibility and credits that tool developers receive.
• Establish a set of benchmarks for software verification in the community.

Participants:

• Over 22 participants, including most popular Software Model Checkers and Bounded Model Checkers

Benchmarks:

• C programs with error location (programs include pointers, structures, etc.)
• Over 6,000 files, each 2K – 100K LOC
• Linux Device Drivers, Product Lines, Regressions/Tricky examples
• http://sv-comp.sosy-lab.org/2015/benchmarks.php
Results for DeviceDriver category

Time in s

BLAST
CBMC
CPAchecker
ESBMC
SeaHorn
SMACKCorral
UAutomizer
UKojak

Accumulated score

0 500 1000 1500 2000 2500
Conclusion

SeaHorn ([http://seahorn.github.io](http://seahorn.github.io))
- a state-of-the-art Software Model Checker
- LLVM-based front-end
- CHC-based verification engine
- a framework for research in logic-based verification

The future
- making SeaHorn useful to users of verification technology
  - counterexamples, build integration, property specification, proofs, etc.
- targeting many existing CHC engines
  - specialize encoding and transformations to specific engines
  - communicate results between engines
- richer properties
  - termination, liveness, synthesis
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