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1. **REPORT DATE (DD-MM-YYYY)**
   1 Oct 04

2. **REPORT TYPE**
   Final Report

3. **DATES COVERED (From - To)**
   1 Oct 04 - 31 Mar 05

4. **TITLE AND SUBTITLE**
   COMPLETION OF THE PROJECT: 3-DIMENSIONAL PARTICLE TRACKING VELOCIMETRY IN SUPERSONIC FLOWS

5a. **CONTRACT NUMBER**

5b. **GRANT NUMBER**
   FA9550-05-1-0003

5c. **PROGRAM ELEMENT NUMBER**

6. **AUTHOR(S)**
   Hans Hornung

5d. **PROJECT NUMBER**

5e. **TASK NUMBER**

5f. **WORK UNIT NUMBER**

7. **PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)**
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8. **PERFORMING ORGANIZATION REPORT NUMBER**

9. **SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)**
   Air Force Office of Scientific Research
   875 North Randolph Street
   Arlington, VA 22203

10. **SPONSOR/MONITOR'S ACRONYM(S)**

11. **SPONSOR/MONITOR'S REPORT NUMBER(S)**
   AFRL:AFOSR-VA-TR-2016-D651

12. **DISTRIBUTION/AVAILABILITY STATEMENT**
   Distribution Statement A: Approved for public release. Distribution is unlimited

13. **SUPPLEMENTARY NOTES**

14. **ABSTRACT**
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15. **SUBJECT TERMS**

16. **SECURITY CLASSIFICATION OF:**
   - a. **REPORT** unclassified
   - b. **ABSTRACT** unclassified
   - c. **THIS PAGE** unclassified

17. **LIMITATION OF ABSTRACT**

18. **NUMBER OF PAGES**

19a. **NAME OF RESPONSIBLE PERSON**
   Hans Hornung

19b. **TELEPHONE NUMBER (Include area code)**

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39 18
Adobe Professional 7.0
Completion of the project: 3-Dimensional Particle Tracking Velocimetry in Supersonic Flows

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May 2004
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Abstract

The ability to map fluid velocity fields in three-dimensions would provide researchers with a much greater understanding of a wide variety of flows. To date, there is no such system that can do this reliably for a wide variety of flow parameters. For that purpose, some researchers have developed methods using a single camera [1] or several cameras (generally 3 or 4 using the epipolar technique) [2, 3, 4, 5]. Of particular interest is the work done by F. Pereira et al [3, 6] in developing a digital defocusing particle image velocimetry (DDPIV) system. Their system uses three individual CCDs placed at the vertices of an equilateral triangle, with all three cameras having overlapping fields of view. The test section can be placed anywhere in this overlap region. Every particle in the test section will be projected onto each CCD. If the CCD images are overlayed, the three images of a particle will be located at the three corners of a near-equilateral triangle. By measuring the size and position of the triangle, the three-dimensional position of a particle can be calculated. Sub-pixel accuracy of a particle image location is obtained by having the images unfocused, which drastically improves the overall resolution of the system. This design was patented by Gharib et al, and is assigned to the California Institute of Technology (U.S. Patent #6, 278, 947). Currently, this technology is being developed for commercial use by VioSense Corporation of Pasadena, California. F. Pereira et al also developed a sophisticated software toolkit which allows users to analyze the raw CCD images and transform them into a three-dimensional velocity field. The DDPIV system has been applied to bubbly flows.

In the last two years, the authors revisited some of the design choices made by F. Pereira et al, and improved upon some of them. Also, since the source-code of F. Pereira et al is proprietary, a completely new software package was developed. The current authors will investigate supersonic air flow, whereas F. Pereira et al investigated low-speed water flows. Of course, the software of both F. Pereira et al and the current authors share some of the same theoretical foundation; however, much of the new software is significantly different and substantially improved. These improvements provide increased accuracy of results. F. Pereira et al chose to use a statistical calculation for the velocity field; whereas, the current authors pursue direct particle tracking. The system developed by the current authors is referred to as three dimensional particle triangulation velocimetry (3DPTV) [7, 8].

The funding sought with this proposal is to enable the completion of the project.
2 Background

2.1 Principle of 3DPTV

Both the hardware setup of F. Pereira et al [3, 6] for DDPIV and the hardware setup of 3DPTV consist of a camera comprising three separate CCDs, which all view the same test volume from different angles. The front plate of the camera consists of a mask, three lenses, and three apertures, behind each of which there is a CCD. The need for more than two CCDs and suggestion that with three cameras an equilateral triangle is the optimum configuration is discussed by Maas et al [2]. In the previous setup, the center of a target plate is focused onto the center of each CCD. Figure 1 shows this for one of the CCDs. In this arrangement, the image of the test volume does not fill the frame of the CCD. In order to make full use of all the pixels of the CCD, 3DPTV uses offsets, in which the CCDs are moved away from the lens and away from the camera axis as shown in figure 1. Of course, full use of the CCDs could also be achieved by placing the target plate in the test volume, but this has some disadvantages, as is discussed later. The image of each point on the target plate will appear at the same place on each of the three CCDs, once the correction for the offsets has been made. It is important to note also that the lens is not parallel to the target plane or to the CCDs, the reason for that is to avoid lens aberration as much as possible.

In practice it is impossible to place the CCDs in their exact theoretical locations; therefore, careful alignment followed by calibration is required. In order to do this a well designed target is needed in addition to calibration software. For this purpose the authors designed a target with 100 equally spaced dots per square inch. With this target placed at the target plane, the CCDs should be placed such as to focus and match each of the three images as well as possible. The calibration software compares the measured and known dot locations, and then creates a mapping to correct for errors in CCD placement as well as lens aberrations and other effects.

Once the camera has been aligned and calibrated, the images of a particle that does not lie on the target plane will appear at the vertices of an equilateral triangle on the overlayed CCD images. The formation of triangles on the overlayed images is shown in figure 2. Clearly, it is only a matter of ray tracing to transform a triangle on the CCDs to a particle location.

Particle tracking is conceptually a simple task. Once all of the three dimensional positions are known, all that is required to determine the three components of the velocity
of the particles is to find corresponding particles in two successive frames separated by a known short time interval.

2.2 Accuracy of the system

Both theoretical and experimental error analysis were conducted to estimate the error one can expect when using the 3DPTV setup. Experimental data were obtained using a flat plate with an array of white dots on a black background. There were 100 dots per square inch. The plate was placed at \( z \)-positions of 469, 494, 519, 529, 539, 549, 559, 569, 594, and 619 mm from the camera. The plate was then moved in three equal increments of 2.54 mm in the \( y \)-direction for each of the \( z \)-locations, this resulted in a total of 6 displacement vectors for each \( z \)-position (3 displacements of 2.54 mm, 2 displacements of 5.08 mm and 1 displacement of 7.62 mm). Similarly the plate was moved in three equal increments of 2.54 mm in the \( z \)-direction for each \( z \)-position.

First, the mean error was evaluated. From theoretical considerations [8], this error is proportional to the displacement. Figure 3 shows the relative mean error obtained for 2.54 mm and 5.08 mm displacements, each data point is an average of 3 and 2 values, respectively. In this plot, the 7.62 mm results are not shown since they correspond to the average of the three 2.54 mm results.

Even though the mean errors are very small, all within 2%, they remain in general far above the results expected theoretically. One possible reason for the inaccuracies of the \( y \)-measurement in the \( z \)-displacement case, is that the whole camera was moved instead of the plate. Due to the weight of the camera and the lack of robustness of the micrometer stages, it might have changed its pitch angle very slightly when moved. An inaccuracy of
20 μm per mm in the y-measurement can be obtained by having a change in pitch angle of less than $4 \times 10^{-5}$ rad per mm (less than 0.0023° per mm). Furthermore, the resolution of the gauges that measured the displacements is 25.4 μm, it is therefore possible that part of the error comes from inaccuracies in the displacement of the plate.

The 90% deviation error of the displacements was also evaluated. The results are plotted in figure 4. Each symbol is the average of the 90% deviation errors from three and two data sets in the 2.54 mm and the 5.08 mm cases, respectively.

One can directly note that the results depend on the displacement distance of the plate. We can therefore use a formula whose form is suggested by theory,

$$\delta D = \sqrt{C_1^2 D^2 + C_2^2}. \quad (1)$$

Since, for most flow measurements, the displacement of the particles will be smaller than 2.54 mm, it is interesting to analyze the results for this particular displacement, as this represents the worst case scenario. Figure 5 therefore gives a higher bound for the 90% deviation error one can expect with the 3DPTV setup.

According to these results, the 90% deviation error in the test volume remains under 5.2 μm and under 66 μm in the x-, y-directions and in the z-direction, respectively.

To be able to compare with some of the results obtained theoretically, it is necessary to separate the constant part of the 90% deviation error, $C_2$, from its varying part, $C_1$. Using the results shown in figure 4, one can obtain these two terms.

The constant part of the 90% displacement error can be obtained theoretically. Both the theoretical values and the experimental data are shown in figure 6.
Figure 4: 90% deviation errors of plate displacement. White, grey, and black symbols refer to the errors in the $x$-, $y$-, and $z$-directions respectively. The 2.54 mm and the 5.08 mm cases are the average of 3 and 2 data points, respectively.

Figure 5: 90% deviation errors of plate displacements. White, grey, and black symbols refer to the errors in the $x$-, $y$-, and $z$-directions, respectively.
Figure 6: Fixed part of 90% deviation errors of plate displacements. White, grey, and black symbols refer to the errors in the $x$-, $y$-, and $z$-directions, respectively.

Another important result is the ratio of the constant parts of the error. According to theory, the ratio between the $x$ or $y$ constant part of the 90% deviation error, and the $z$ constant part of the 90% deviation error is given by a very simple parameter free formula [8]. The theoretical curve as well as the experimental values for this ratio are shown in figure 7.

The varying part of the 90% deviation error is presented in figure 8. Although these errors remain quite small (1 $\mu$m per mm in $x$ or $y$ and 10 $\mu$m per mm in $z$), we have to note that they remain higher than the predicted values. A possible reason for that is again the lack of robustness of the translation stages. In fact, if during the movement of the plate in the $y$-direction, its pitch angle changed by only $4.5 \times 10^{-4}$ rad per mm (0.026° per mm), the 90% deviation in $z$ would reach 10 $\mu$m. A small roll angle of $4.5 \times 10^{-5}$ rad per mm (0.0026° per mm) is sufficient to explain the inaccuracies in $x$ and $y$.

From all the results presented here, it is possible to obtain an upper bound for the velocity errors. The mean errors will be approximately

\begin{align*}
|\Delta U_x| &= \sqrt{24U_x^2 + 0.16U_y^2 + 205U_z^2} \times 10^{-3} \\
|\Delta U_y| &= \sqrt{0.16U_x^2 + 24U_y^2 + 205U_z^2} \times 10^{-3} \\
|\Delta U_z| &= \sqrt{54U_x^2 + 54U_y^2 + 25U_z^2} \times 10^{-3},
\end{align*}

(2) \hspace{1cm} (3) \hspace{1cm} (4)
Figure 7: Ratio of fixed part of 90% deviation errors of plate displacements. Empty and grey symbols refer to the ratio of the constant part of the 90% deviation error in the x- and y-directions, respectively, and the constant part of the 90% deviation error in the z-direction.

Figure 8: Variable part of 90% deviation errors of plate displacements. White, grey, and black symbols refer to the errors in the x-, y-, and z-directions, respectively.
whereas an estimation of 90% deviation errors of the velocities is

\[
\begin{align*}
\delta U_x &= \sqrt{95U_x^2 + 16U_y^2 + 9U_z^2 + \frac{11\times10^{-4}}{t_s^2}} \times 10^{-4} \quad (5) \\
\delta U_y &= \sqrt{16U_x^2 + 95U_y^2 + 9U_z^2 + \frac{11\times10^{-4}}{t_s^2}} \times 10^{-4} \quad (6) \\
\delta U_z &= \sqrt{85U_x^2 + 85U_y^2 + 1.6U_z^2 + \frac{18\times10^{-4}}{t_s^2}} \times 10^{-3} \quad (7)
\end{align*}
\]

where the velocities and the time between frame, \( t_s \), are in m/s and in s respectively.

These errors are very low and suggest that the biggest inaccuracy in supersonic flows will be due to the inertia of particles. In fact, due to their weight, particles have a tendency to soften high velocity gradients.
3 Statement of Work

1. Set up laser and optical system
2. Run jet experiments for 5 days (about 50 shots)
3. Process the data and write thesis

4 Statement of Objectives

The objective of the research is to validate the 3DPTV system. The camera and software are already tested and the accuracy that is obtained is more than satisfactory.

To validate the system, the full three dimensional velocity field of a sonic jet will be recorded.
5 Research Plan

5.1 3D Camera and software

The camera and software are already built and tested and the overall accuracy that is achieved is already estimated. The current research purpose is to validate the whole system with actual flow results. The camera is designed to investigate a $5 \times 5 \times 5$ cm$^3$ volume. For our current conditions, the accuracy of the camera was explained in the Background section.

5.2 Sonic Jet

The 10 m long, 5 cm diameter, Ludwieg tube that creates the sonic jet is in operation. Its mylar diaphragm is broken using a heated tungsten wire. The nozzle is designed to have a 7 mm sonic jet at the outlet. With the chosen pressure ratio, the jet is underexpanded and the flow becomes supersonic after the exit.

To track the flow, aerogel particles are used. These particles have a mean diameter of 8 $\mu$m. Due to their very low density (30 kg/m$^3$), these particles are particularly suitable to track supersonic flows.

The experiment will be conducted into ambient conditions. This allows near forward-scattering conditions (only 9° deflection), while avoiding any laser reflections on walls.

Atmospheric dust remains much brighter than the aerogel particles. To avoid their illumination on the images, a low-dust room was built inside the lab. Inside this room, air is constantly filtered using a standard HEPA filter. The particle density inside the room is reduced by more than 90% and gives a sufficiently low value to perform our experiments.

5.3 Optical setup

An optical setup was built to illuminate the volume. A CFR PIV-200 Big Sky laser (200 mJ) will be delivered by the end of May, on loan for 2 weeks. This laser has a flat top profile that is particularly suitable for the current experiments since it allows particles to be and remain illuminated the same way in the entire investigation volume.

5.4 Expected Results

Full 3D velocity fields will be taken. About 2000 vectors are expected which represent about 150 vectors per cm$^3$.

Good correlation of the data will validate the setup and prove the possibility to get instantaneous full three dimensional data of flows (not only in water flows but also in subsonic and supersonic air flows).
6 Key Personnel

1. Hans G. Hornung, PI, Kelly Johnson Professor of Aeronautics
2. Graduate Research Assistant, Nicolas Ponchaut

References


