**ABSTRACT**

Hypersonic flight introduces extreme head loads into the leading-edges of a vehicle. Determining these loads is challenging. It requires accounting for the aerothermodynamic features of both the flow and thermal state of the surface. These features can be influenced by pressure and viscous effects, real-gas and low density effects, ionization, radiative heating, surface radiation cooling, and surface catalytic behavior.
ABSTRACT

Hypersonic flight introduces extreme heat loads into the leading edges of a vehicle. Determining these loads is challenging. It requires accounting for the aerothermodynamic features of both the flow and thermal state of the surface. These features can be influenced by pressure and viscous effects, real-gas and low density effects, ionization, radiative heating, surface-radiation cooling, and surface catalytic behavior. It also requires accounting for the complex layout of the structure and materials concept which includes high-temperature materials and coatings and internal thermal insulation, for gap and cove heating, and for active cooling. Hence, the accurate thermal analysis of hot structures requires not only a state-of-the-art nonlinear heat transfer tool for modeling temperature-dependent material properties, contact resistance between parts and across welds, and radiation within cavities, but also a tight coupling between aeroheating, thermal, and structural models. Incomplete forms of such an integration have been attempted in the past using loosely-coupled solution procedures that were either computationally inefficient or numerically unstable. The main objective of this proposal is to develop an alternative, higher-fidelity, multidisciplinary computational approach to thermal analysis of hot structures that is numerically stable, efficient, and compatible with the aerothermoelastic simulation environment AERO deployed at the Edwards Air Force Base to enable the accurate assessment of the effects of heat loads on structural integrity and aeroelastic stability. The proposed approach centers around a four-field formulation of aerothermoelastic problems, a conservative discretization of appropriate transmission conditions on non-matching interfaces, and advanced steady and unsteady conjugate heat transfer algorithms for accelerating vehicles. The anticipated long-term outcome of this research is the enabling of a state-of-the-art analysis tool for predicting steady and unsteady structural temperatures and their gradients, heat loads, and structural deformations and stresses associated with hypersonic systems in order to increase the safety and efficiency of their testing.

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1 OBJECTIVES

The objectives of this three-year research proposal are to: (a) extend the three-field formulation of nonlinear aeroelasticity that is the foundation of the AERO software deployed at the Edwards Air Force Base to a four-field arbitrary Lagrangian/Eulerian framework for coupling convective heat transfer over and conduction heat transfer within a hypersonic vehicle, its structural dynamics, and the motion of the Computational Fluid Dynamic (CFD) mesh, (b) implement in AERO’s thermal analyzer AERO-H finite element based computational models for temperature-dependent material properties, contact resistance between structural parts, across welds, and at structural joints, and for surface-radiation and radiation within cavities, (c) develop fast computational algorithms for AERO-H and its coupling with the comprehensive flow solver AERO-F in order to support robust and computationally efficient steady and unsteady Conjugate Heat Transfer (CHT) analyzers, (d) verify the resulting computational tool with the solution of aerothermodynamics problems associated with the Rankine-Hugoniot-Prandtl-Meyer- (RHPM-) flyer, and (2) demonstrate its potential with the multidisciplinary thermal analysis of a Cruise and Acceleration Vehicle (CAV) with airbreathing propulsion such as the hypothetical hypersonic air transportation vehicle Orient Express.

To this effect, the following research goals and corresponding statement of work are formulated.

1.1 RESEARCH GOALS

1) Four-Field Computational Framework for Aerothermoelastic Analysis. The three-field formulation of nonlinear computational aeroelasticity introduced a decade ago by the Principal Investigator (PI) models a fluid/structure interaction problem by three coupled partial differential equations: those governing the fluid subsystem written in an Arbitrary Lagrangian/Eulerian (ALE) coordinate system, those governing the motion of the fluid mesh, and those governing the dynamic equilibrium of the structural subsystem. The corresponding computational framework is adopted today by a large segment of the computational fluid/structure interaction community. It can address many subsonic, transonic, and supersonic aeroelastic problems including flutter and limit cycle oscillations, the prediction of steady and unsteady loads and control surface effects in level flight and maneuvering, aerelastic tailoring, and performance analysis. However, it cannot treat hypersonic problems because it: (a) simplifies the treatment of the equilibrium of the thermal surface to an isothermal/adiabatic and radiation-free wall-boundary condition, and (b) does not account for thermal loads in the formulation of the equilibrium of the structure and does not account for structural deflections and therefore shape changes in the aeroheating analysis. Here, the research goals are to: (a) extend this computational framework to a four-field formulation where heat conduction and surface-radiation are modeled and accounted for, and suitable transmission conditions with the surrounding hypersonic viscous flow are introduced, and (b) develop a conservative method for discretizing these transmission conditions on non-matching discrete interfaces in order to properly exchange aerothermodynamic, elastodynamic, and thermal (or thermostructure) data between the fluid, structure,
and thermal analyzers.

2) Enhancement of AERO's thermal analyzer AERO-H. AERO-H is the basic linear finite element heat transfer analysis module of the AERO simulation platform. Relying on it for the thermal analysis of hot aerospace structures requires first enhancing it as follows: (a) developing computational models for contact resistance between structural parts, across welds and at structural joints, and for active cooling, (b) incorporating the treatment of surface-radiation and radiation within cavities, (c) incorporating a bulk fluid model for computing average temperatures in cavities to compute the corresponding convective flux boundary conditions, and (d) developing a Newton-based nonlinear computational infrastructure for addressing the nonlinearities arising from the treatment of radiation and enclosure radiation surface fluxes.

3) Fast and Reliable Coupled Solvers for Steady and Unsteady CHT analyzers. The simultaneous solution of the equations governing convective heat transfer over a hypersonic vehicle and those governing conduction heat transfer within the vehicle can be unpractical. For this and other reasons usually related to computational efficiency, a loosely-coupled procedure has been typically employed for solving these coupled nonlinear equations in the context of Conjugate Heat Transfer (CHT) analysis. In such a procedure, the heat transfer equation in the fluid and that in the structure are time-integrated by different schemes tailored to their different mathematical models, and the resulting discrete equations are typically solved by one Gauss-Seidel iteration. Such a strategy simplifies explicit/implicit treatment, subcycling, load balancing, software modularity, and replacements as better mathematical models and methods emerge in the fluid and structure disciplines. Unfortunately, such a strategy is in general at least one order less time-accurate than its underlying aerothermodynamics and thermal time-integrators, and is often either numerically unstable or performance-limited by severe time-step restrictions. Here, the research goal is to develop better coupling solution methods for steady and unsteady CHT, equip them with fast algebraic equation solvers for implicit schemes, analyze them in terms of accuracy and numerical stability, implement them in the AERO code and assess their performance in terms of computational efficiency.

4) Verification and Demonstration. Here, the first objective is to assess the outcome of the research performed under the three research tasks described above and verify its integration in the AERO simulation platform. For this purpose, a highly simplified flight vehicle configuration known as the Rankine-Hugoniot-Prandtl-Meyer- (RHPM-) flyer will be considered. This is an infinitely thin flat plate at angle of attack with a radiation-adiabatic surface. The flow past this plate can be determined by means of simple shock-expansion theory. Semi-analytical and computational aerothermodynamic results are available for this simplified model [1] and can be used for verifying the results produced by the expanded AERO code. The second objective is to demonstrate the potential of the numerical tools developed under this research project with the multidisciplinary thermal analysis of a Cruise and Acceleration Vehicle (CAV) with airbreathing propulsion accelerating from Mach 7 to Mach 12 at small angle of attack, for which the flow field and thermal surface are viscosity effect dominated and the
rarefaction and thermo-chemical effects are weak.

1.2 STATEMENT OF WORK

A four-field formulation of aerothermoelastic problems associated with hypersonic flows will be derived. This formulation will include: a modified version of the Navier-Stokes equations written in an Arbitrary Lagrangian Eulerian (ALE) coordinate system that is suitable for the prediction on moving grids of high-temperature hypersonic flows characterized by $K_n < 0.01$, where $K_n$ denotes here the Knudsen number; a non-linear pseudo-structural system for modeling the motion of the fluid mesh; a nonlinear heat flow equation for modeling heat conduction in the structure, surface-radiation, and radiation in the cavities of the structure; and a nonlinear form of the structural equations of dynamic equilibrium that accounts for thermal loading and geometrical nonlinearities. These four computational models will be coupled by the appropriate kinematic, temperature, stress, and temperature flux transmission conditions at the fluid/structure interface.

A computational framework associated with the above four-field formulation will also be developed and analyzed. This framework will include: a method for the discretization of the transmission conditions coupling the aerothermodynamic and thermal subproblems that is variationally consistent with the discretizations of these two subproblems; a fast, strongly-coupled, Newton-Krylov scheme for the iterative solution of the coupled system of equations arising from the steady-state Conjugate Heat Transfer (CHT) analysis of hot aerospace structures; a state-of-the-art, computationally efficient, and loosely-coupled implicit staggered procedure for the solution of the coupled system of equations arising from the unsteady CHT analysis of hot aerospace structures that is formally second-order time-accurate and characterized by good numerical stability properties; and a version of the domain decomposition based iterative solver FETI-DP tailored to linearized systems of equations arising from the discretization of thermal problems.

AERO-H, the basic thermal module of the Computational Fluid Dynamic- (CFD-) based aeroelastic code deployed at the Edwards Air Force Base, will be enhanced and equipped with: computational models for contact resistance between structural parts, across welds and at structural joints, and for active cooling; a finite element treatment of surface-radiation and radiation within cavities; a bulk fluid model for predicting average temperatures in cavities to compute the corresponding convective flux boundary conditions; and a Newton-based nonlinear computational infrastructure for addressing the nonlinearities arising from the treatment of radiation and enclosure radiation surface fluxes.

All computational methods outlined above will be integrated into the AERO code and verified with the solution of multidisciplinary aerothermodynamics problems defined for a Rankine-Hugoniot-Prandtl-Meyer- (RHPM-) flyer. The potential of the expanded AERO will then be assessed with the multidisciplinary thermal analysis of a Cruise and Acceleration Vehicle (CAV) accelerating from Mach 7 to Mach 12 at small angle of attack.
2 TECHNICAL PROPOSAL

2.1 RESEARCH EFFORT

2.1.1 Introduction

The motivations for building flight vehicles that will travel in the atmosphere at hypersonic speeds have grown during the last two decades [2,3]. In particular, the Air Force is currently interested in hypersonic systems with a strike capability. Flight testing and clearing these systems is particularly challenging, because of the speeds at which they operate. When anything unexpected occurs, the time to react is so short that the tested system must be destroyed. Furthermore, ground-based experimental facilities such as shock tunnels and arc-jets are typically unable to reproduce the flight conditions that these vehicles experience during hypersonic travel. For all these reasons, accurate computational models are required for designing these vehicles, determining their stability and structural integrity throughout their flight phases, and assisting in their flight testing.

Hypersonic vehicles are hot aerospace structures that are expected to withstand intense heat loads. Predicting these loads calls for multidisciplinary computational models that account at least for the aerodynamics of the vehicle’s shape, the thermomechanics of its structure, and the thermodynamics of its flight environment. However, even accounting for all these effects may still lead to an incomplete overall computational model. Indeed, the surface temperature of a flight vehicle can affect the external flow by changing the amount of energy absorbed by the structure. Furthermore, the temperature gradients in the structure can induce structural deformations that can alter the flow field, surface pressures, and heating rates. For these reasons, a significant coupling can occur between the hypersonic flow field, heat transfer in the structure, and structural response. For example, tests conducted in the Mach 7 8-ft High-Temperature Tunnel at the NASA LaRC [4] showed that panels bowed-up into the flow to produce heating rates that are up to 1.5 times greater than flat-plate predictions [5]. This and other examples highlight the important role of fluid/structure/thermal coupling even when the immediate objective is only the thermal analysis of a hypersonic vehicle, and certainly when the objective is the certification of hot structures that are expected to experience severe aerodynamic heating. Therefore, advances in computational methods are needed not only for modeling hypersonic flow fields and heat transfer processes, but also for modeling and simulating the coupled aerothermomechanics of hypersonic flight. This was recognized, among others, by the NASA LaRC which initiated more than a decade ago the development of LIFTS, an integrated fluid/structure/thermal analyzer using finite element methods.

The state-of-the-art of thermal analysis has advanced during the last decade, particularly in the area of convective heat transfer over and conduction heat transfer within a solid body. In many numerical simulations of heat transfer applications where the external and internal temperature fields are coupled, Conjugate Heat Transfer (CHT) analysis [6–9] is now often performed instead of imposing a constant wall temperature or a heat flux boundary condition. CHT couples a Navier-Stokes equation solver — with or without turbulence modeling — and a heat conduction analyzer. Most if not all CHT analyses reported in the literature have relied on the most primitive form of loose coupling of the fluid solver and heat con-
duction analyzer. Unfortunately, basic loosely-coupled solution algorithms are known to be computationally inefficient. For steady-state applications, they typically require more iterations than otherwise possible to achieve convergence. For unsteady problems, their numerical stability is often a significant concern. For example, the analysis of the basic loosely-coupled CHT algorithm performed in [10] reveals that its accuracy is not a major problem but that for its numerical stability, it is very important that the flow computation sets the heat flux boundary condition for the heat conduction analysis and the heat conduction analysis sets the surface temperature for the flow computation. Large numerical instabilities have also been reported in [8] for the solution of coupled fluid/thermal problems associated with the predictions of ablating hypersonic vehicles using a primitive loosely-coupled solution procedure.

On the other hand, the current state-of-the-art of coupling fluid, thermal, and structural analyzers for hypersonic vehicles is not significantly different from that of fifteen years ago [5,11,12], except perhaps for specific advances in subtopics such as fluid/thermal approaches for ablation (for example, see [13]). Recent efforts appear to have focused on “software integration” more than on “coupled field analysis” — that is, on ensuring that the output of one analyzer can be used as input for another analyzer (for example, see the recent works published in [14,15]), instead of ensuring that the appropriate transmission conditions are correctly enforced and by the best numerical solution algorithms. Such ad-hoc approaches are not only low-fidelity, but also computationally inefficient. Changing this paradigm can produce significant payoff. For example in the field of aeroelasticity, the attention paid to rigorous coupling at both the continuous and discrete levels is the reason why today, the AERO code [16,17] is an order of magnitude faster than many counterparts, independently from the speed of the computing platform (for example, see [18]). Operating at such computational efficiency is essential for flight test centers.

Therefore, the main objective of this research effort is to address the issues raised above in order to advance the state-of-the-art of computational methods for the multidisciplinary thermal analysis of hot, hypersonic, aerospace structures in view of assisting their future flight testing.

2.1.2 Research Plan

2.1.2.1 Scope

Different hypersonic vehicles raise different aerothermodynamic design and test problems. Aerothermodynamic phenomena and heat loads can have different importance for different classes of hot vehicles. For this reason, four major classes of hypersonic vehicles were introduced in [1]:

- Winged re-entry vehicles (RV), like the Space Shuttle and the X-38. These flight vehicles are launched typically by means of rocket boosters. Their aerothermodynamic features and multidisciplinary design challenges are pressure-effects and low-density effects dominated. For this class of hot vehicles, real-gas effects and surface-radiation cooling also play a major role.

- Cruise and acceleration vehicles (CAV) with airbreathing propulsion such as the hypothetical hypersonic air transportation vehicle Orient Express. For this class of flight
vehicles, the Flight Mach number would lie in the range \(7 \leq M_\infty \leq 14\). Viscosity effects such as the laminar-turbulent transition and turbulence at altitudes below 40 to 60 km, and surface-radiation cooling play a major role in the aerothermodynamic behavior of these hot vehicles. The real-gas effects are however weak in this case.

- Ascent and re-entry vehicles with airbreathing (and rocket) propulsion like the NASP/X30. These are only partly viscous-effects dominated vehicles whose aerothermodynamic behavior is strongly influenced by low-density and real-gas effects.

- Aeroassisted orbital transfer vehicles (AOTV) for which ionisation, radiation, real-gas and low-density effects play a major role.

Because of time and budget constraints, the proposed research effort will focus on the case of CAVs. These have slender bodies, fly at small angles of attack and are surface-radiation cooled. Their flow fields are viscosity-effect dominated; rarefaction and thermochemical effects are weak. For the analysis of such vehicles, it will be assumed that the continuum approach is valid.

Relatively high Mach number flow simulations over slender bodies have been performed using the Euler equations and were found to give good agreement with experimental data [19]. In [20], the effects of air chemistry on waverider aerodynamics were studied and found to be small for the examples considered therein. Viscous simulations using the Navier-Stokes equations with a turbulence model and perfect gas assumptions [21, 22] have also been used for many high Mach number calculations and were shown to accurately reproduce experimentally measured surface pressure, heating rate, and skin friction. For these reasons, the Navier-Stokes equations equipped with a turbulence model and the perfect gas assumption will be used during the initial phase of this research project. In a second phase, a more accurate model for high-temperature hypersonic flows will be constructed by modifying the initial one as follows:

- The conservation of mass equation will be replaced by a species conservation equation for each species in the flow. The latter equation has a form that is similar to that of the continuity equation but contains a source term that predicts the production and/or destruction of each of the species.

- The total momentum equations will be kept unchanged from the perfect gas case except that the molecular viscosity will be that of the mixture.

- The standard energy equation will be augmented with heat conduction terms from the vibrational states of the flow, and an additional energy equation for the vibrational modes will be introduced.

The above modifications lead to a two-temperature model for the fluid that has been shown to work well for many hypersonic flows [23].

Thermal Protection Systems (TPS) are an important constituent of any overall hypersonic reusable vehicle. Accurate models to predict the heat transfer from the high-enthalpy flow to the TPS are necessary to prevent failure of this mission critical system, and also to realize the performance objectives of the vehicle along the flight path. TPS can be ablative or non-ablative, depending on the mission requirement. Non-ablative systems are relatively easy to
model using well-known finite element approximations to the heat conduction equation \[24, 25\]; only the simulation of the flow between the TPS tiles (gap flow) is a delicate task that will be naturally addressed by the outcome of this research proposal. Ablative systems provide another level of complexity. The interface between the fluid and the structure moves at the speed of regression. The additional chemical reactions caused by the burning of the ablative material incur the possibility of energy exchange between the products of ablation and the gases in the boundary layer of the fluid flow. The usage of moving grids for tracking the ablation surface was examined in \[26\] using an iterative technique and a model to predict the chemical process of ablation was developed in \[27\]. Again, because of time and budget constraints, the proposed research project will focus only on non-ablative systems.

2.1.2.2 Four-Field Computational Framework for Aerothermoelastic Analysis

**Research Issues.** The three-field formulation of nonlinear computational aeroelasticity introduced a decade ago by the PI models a fluid/structure interaction problem by three coupled partial differential equations: those governing the fluid subsystem written in an Arbitrary Lagrangian/Eulerian (ALE) coordinate system, those governing the motion of the fluid mesh, and those governing the dynamic equilibrium of the structural subsystem. The corresponding computational framework is adopted today by a large segment of the computational fluid/structure interaction community. It can address many subsonic, transonic, and supersonic aeroelastic problems including flutter and limit cycle oscillations, the prediction of steady and unsteady loads and control surface effects in level flight and maneuvering, aeroelastic tailoring, and performance analysis. However, it cannot treat hypersonic problems because it: (a) simplifies the treatment of the equilibrium of the thermal surface to an isothermal/adiabatic and radiation-free wall-boundary condition, and (b) does not account for thermal loads in the formulation of the equilibrium of the structure and does not account for structural deflections and therefore shape changes in the aeroheating analysis.

**Approach.** Consider the cross section of the wing of a hypersonic vehicle shown in Fig. 1. The coupled aerothermoelastic behavior of this vehicle will be formulated as a four-field coupled problem governed by four equations of the form:

\[
\frac{\partial (Jw)}{\partial t} \bigg|_\xi + J \nabla \cdot (F(w) - \frac{\partial x}{\partial t} w) = J \nabla \cdot R(w) \tag{1a}
\]

\[
\rho_s \frac{\partial^2 u_s}{\partial t^2} - \text{div}\left(\sigma_S(\varepsilon_S(u_s), \frac{\partial \varepsilon_S}{\partial t}(u_s) , \theta_S)\right) = b \tag{1b}
\]

\[
\rho \frac{\partial^2 x}{\partial t^2} - \text{div}(\bar{\sigma}(\bar{\varepsilon}(x))) = 0 \tag{1c}
\]

\[
\rho_s c_s \frac{\partial \theta_s}{\partial t} - \text{div}(\kappa_s \nabla \theta_s) - q_v = 0 \tag{1d}
\]

Eq. (1a) is the ALE conservative form of the Navier-Stokes equations (equipped with a turbulence model and later with a two-temperature model as outlined earlier). Here, \( t \) denotes the time, \( x(t) \) denotes the time-dependent position or displacement of a fluid grid point (depending on the context of the sentence and the equation), \( \xi \) its position in a reference configuration, \( J = \text{det}(dx/d\xi) \), \( w \) is the fluid state vector using the conservative variables,
Figure 1: Schematic description in the continuum regime of the thermal surface of a hypersonic vehicle (cross section of a wing is shown): tangential fluxes and non-convex radiation cooling effects are neglected [1]. $q_F$: heat flux in the fluid at the wall. $q$: heat flux into the wall. $q_{rad}$: surface radiation heat flux.
and $F$ and $R$ denote respectively the convective and diffusive ALE fluxes. Eq. (1b) is the thermoelastodynamic equation where $u_S$ denotes the displacement field of the structure, $\rho_S$ its density, $\sigma_S$ and $\epsilon_S$ denote respectively the stress and strain tensors, $\theta_S$ denotes the temperature field in the structure, and $b$ represents the body forces acting on the structure. Eq. (1c) governs the dynamics of the fluid grid. It is similar to an elastodynamic equation because the dynamic mesh is viewed here as a pseudo-structural system. A tilde notation is used to designate the fictitious mechanical quantities [28, 29]. Eq. (1d) is the heat transfer equation that governs the thermal response of the structure, where $\rho_S$, $c_S$, and $\kappa_S$ denote respectively the density, specific heat, and heat conduction coefficient of the structure, and $q_v$ denotes the volumetric heating. For the sake of notational simplicity, the various Dirichlet and Neumann boundary conditions intrinsic to each of the fluid, structure, and heat transfer problems are omitted except for the surface radiation boundary condition

$$q_n = \eta \epsilon f (\theta_S - \theta_r^4)$$

and enclosure radiation boundary condition

$$q_n = \eta \epsilon \theta_r^4 - \alpha G$$

which play an important role in this research project. In Eq. (2) and Eq. (3) above, $q_n$ denotes the flux in the direction normal to the surface, $\eta$ denotes the Stefan-Boltzmann constant, $\epsilon$ is the emissivity of the surface, $f$ is the form factor from the surface to the reference surface, $\theta_r$ is the temperature of the reference surface, $\alpha$ denotes the absorptivity of the surface and $G$ represents the surface irradiation.

Eq. (1a) and Eq. (1c) are directly coupled. If $u_F$ denotes the ALE displacement field of the fluid and $p$ its pressure field, $\sigma_F$ the fluid viscous stress tensor, $\Gamma$ the fluid/structure interface boundary (wet boundary of the structure), and $n$ the normal at a point to $\Gamma$, the fluid and structure equations are coupled by the transmission — or interface boundary — conditions

$$\sigma_{S,n} = -pn + \sigma_F.n$$

$$\frac{\partial u_S}{\partial t} = \frac{\partial u_F}{\partial t}$$

The first of these two transmission conditions states that the tractions on the wet surface of the structure are in equilibrium with those on the fluid side of $\Gamma$. The second of Eqs. (4) expresses the compatibility between the velocity fields of the structure and the fluid at the fluid/structure interface.

The equations governing the structure and dynamic mesh motions are coupled by the continuity conditions

$$x = u_S$$

$$\frac{\partial x}{\partial t} = \frac{\partial u_S}{\partial t}$$

The first of these two transmission conditions states that the tractions on the wet surface of the structure are in equilibrium with those on the fluid side of $\Gamma$. The second of Eqs. (4) expresses the compatibility between the velocity fields of the structure and the fluid at the fluid/structure interface.
Neglecting tangential heat fluxes, a possible temperature jump and other possible external heat radiation sources such as the fluid (gas) itself, Eq. (1a) and Eq. (1d) are then coupled by the following additional transmission conditions

\[
\begin{align*}
\kappa_S \nabla \theta_S \cdot n - \kappa_F \nabla \theta_F \cdot n + \eta \epsilon \theta_S^2 - \theta_F^2 &= 0 \quad \text{on } \Gamma \\
\theta_S &= \theta_F \quad \text{on } \Gamma
\end{align*}
\]

(6a)  (6b)

The first of the two above equations describes the general balance of the surface heat fluxes. The second expresses the continuity of the temperature field at \( \Gamma \).

The discretization on non-matching meshes of the transmission conditions (4) by a conservative method is described in [30]. Furthermore, this discretization is already implemented in the AERO code. A similar approach that is variationally consistent with the discretizations of the fluid and thermal subproblems will be adopted for discretizing the transmission conditions (6).

2.1.2.3 Enhancement of AERO's thermal analyzer AERO-H

Research and Development Issues. AERO-H is the linear finite element heat transfer analysis module of the AERO simulation platform. Currently, it lacks the following modeling capabilities, all of which are essential for the thermal analysis of hot aerospace structures: contact resistance between structural parts, across welds and at structural joints, active cooling, surface-radiation and radiation within cavities (see Fig. 1.), and a bulk fluid model. It is also a linear module, whereas the radiation boundary conditions (2) and (3) introduce nonlinearities in the thermal problem.

Approach. Finite element based computational models will be developed and incorporated in AERO-H to address all its deficiencies outlined above.

Contact resistance between structural parts, across welds, and at structural joints will be modeled as an imperfect contact between two solid surfaces, which can take account of surface roughness. Through such an interface, heat transfer follows different paths: effective conduction through solid-to-solid contact, poor conduction through gas-filled interstices, and inefficient thermal radiation across gaps. This contact will be treated in AERO-H by setting the gap flux across the interface proportional to the temperature drop

\[
q_{n_1} = -q_{n_2} = h_c \left( \theta_{S_1} - \theta_{S_2} \right)
\]

(7)

where the subscripts 1 and 2 designate the two sides of the interface, \( h_c \) is the contact conductance which is similar to a heat transfer coefficient and whose value depends on the temperatures of the two materials at the contact surface, the materials in contact, the surface finish and cleanliness, the pressure at which the surfaces are forced together, and the substance or lack of it in the interstitial spaces [31].

Surface radiation and radiation within cavities will be modeled in AERO-H by the finite element discretization of the boundary conditions (2) and (3), respectively.

At the surface of fluid-filled cavities such as those illustrated in Fig. 1, the convective flux boundary condition can be written as

\[
q_n = h \left( \theta_S - \bar{\theta}_F \right)
\]

(8)
where $\bar{\theta}_F$ represents the mean value of the temperature of the still fluid in the cavity and $\theta_S$ is the local temperature of the surface of the cavity. AERO-H will be equipped with a bulk fluid model for computing $\bar{\theta}_F$. The bulk fluid temperature is an average temperature through the cavity (or reservoir) that is obtained by solving the integral conservation equation

$$\frac{d}{dt}(V \rho c \bar{\theta}_F) = \int_{\partial V} q_n dA$$

(9)

where $\rho$ and $c$ denote the density and the specific heat of the bulk fluid, respectively, and $V$ designates the bulk fluid volume.

Finally, to handle the nonlinearities arising from the treatment of the radiation (2) and enclosure radiation (3) surface fluxes, AERO-H will also be equipped with a Newton-based nonlinear computational infrastructure and a version of the scalable, Gordon Bell Award winner, iterative algorithm FETI-DP [32] for solving the linearized systems of equations arising at each Newton iteration.

2.1.2.4 Fast and Reliable Coupled Solvers for the Steady and Unsteady CHT analyzers

Research Issues. Here, the research issues center around the computationally efficient solution of the four coupled systems of ordinary differential equations arising from the semi-discretization of the four-field formulation of aerothermoelastic problems outlined in Eqs. (1a-1d). Taking into account the current capabilities of the AERO code, the research effort will focus on the computationally efficient solution of the coupled fluid (1a) and thermal (1d) equations for both steady and unsteady problems. This will lead to the design of state-of-the-art steady and unsteady CHT analyzers.

Approach. Partitioned procedures and corresponding staggered algorithms [33–35] are often used [36–38] to solve coupled systems of semi-discrete equations such as those arising from the four-field formulation of aerothermoelastic problems outlined above.

In a partitioned procedure for aerothermoelastic computations, the fluid, structure, and thermal subsystems are time-integrated by different schemes that are tailored to their different mathematical models and solved by a staggered numerical algorithm which is not to be confused with a loosely-coupled solution algorithm. An elementary but popular partitioned procedure for solving aerothermoelastic problems is the Conventional Serial Staggered (CSS) procedure whose generic cycle can be described as follows (see Fig. 2): (1) time-advance the fluid solver, (2) transfer the aerodynamic forces to the structure and the aeroheat fluxes to the thermal subsystem associated with the structure, (3) update the structural temperature under the new aeroheat flux supply, (4) send the new temperature field to the structure, (5) compute the structural displacement under the new fluid and thermal loads, (6) update the fluid mesh. The staggered solution algorithm supporting this partitioned procedure can also be described as a loosely-coupled solution algorithm. However, when equipped with carefully designed inner- or sub-iterations that are performed between each pair of consecutive time-stations [39–41], this staggered algorithm is also often referred to as a strongly-coupled solution algorithm, even though it remains a partitioned solution method.
Figure 2: Generic cycle of the CSS procedure. Here, $T_S = \theta_S$ and $\theta = \theta_S - \theta_{reference}$

For any coupled problem, the advantages of partitioning and staggering are numerous. Indeed, this approach reduces the computational complexity per time-step, simplifies explicit/implicit treatment, facilitates subcycling, eases load balancing, achieves software modularity, enables the exploitation of off-the-shelf software components, and makes replacements relatively painless when better mathematical models and methods emerge in the fluid, structure, and thermal subdisciplines. Yet for nonlinear aeroelastic applications, partitioned procedures in general, and loosely-coupled solution algorithms in particular, are often heavily criticized in the literature for their lack of sufficient time-accuracy and sufficient numerical stability. For this and other reasons, loosely-coupled solution methods are discouraged by both the proponents of monolithic schemes and the advocates of strongly-coupled solution algorithms.

In a monolithic scheme (or what is sometimes referred to in the literature as a fully implicit scheme) for fluid/structure interaction problems, the structure equations of motion are typically assumed to be linear and re-cast in first-order form, then combined with the fluid equations of motion into a single system of first-order semi-discrete equations. Then, this system is solved by a single preferred time-integrator (for example, see [42]). When feasible, such a strategy is usually simpler to analyze mathematically than a partitioned procedure with either a loosely- or strongly-coupled staggered solution algorithm and delivers in principle the time-accuracy of the chosen time-integrator. For these reasons, it is an appealing solution strategy. This approach — which is the ultimate form of strong coupling — can be extended to fluid/thermal and fluid/structure/thermal problems. However, it does not acknowledge the differences between the mathematical properties of the fluid, structure, and thermal semi-discrete subsystems. Furthermore, it tends to ignore the issues of software modularity, availability, and integration, even though each of these issues can be in practice a major obstacle. Most importantly, the monolithic approach is memory greedy and can be computationally inefficient. Perhaps for all these reasons, monolithic schemes for nonlinear aeroelastic applications have been demonstrated so far mostly for simple problems.

Whether they are related to accuracy or numerical stability, the observed deficiencies of a loosely-coupled solution algorithm are usually blamed on the “loose” aspect of its coupling mechanism, rather than on one or several of its key components such as the chosen fluid or
thermal time-integrator, or the algorithm adopted for updating the position of the dynamic fluid-mesh. For this reason, it is often attempted to correct these deficiencies by performing inner- or sub-iterations between each pair of consecutive time-stations. As stated earlier, when equipped with these inner-iterations, the staggered solution algorithm is often referred to as a strongly-coupled solution method. However, inner-iterations increase the complexity of the computer implementation of a coupled fluid/thermal or fluid/structure/thermal analysis as well as the computational cost of each of its time-steps. Furthermore, it is not clear that a better computational efficiency cannot be obtained simply by reducing the time-step and performing the simulation with a state-of-the-art loosely-coupled version of the chosen staggered solution method. In other words, the computational efficiency of strongly-coupled solution algorithms is debatable except when no loosely-coupled solution algorithm can perform the target fluid/thermal or fluid/structure/thermal simulation using a reasonable time-step.

It is well-known that the time-accuracy of the CSS procedure is in general at least one order lower than that of its underlying single discipline time-integrators. However, it was shown in [33] for simple linear problems, and in [43] for complex nonlinear ones, that carefully constructed predictors can be introduced to fix this issue. Hence, in this research task, provably second-order time-accurate, loosely-coupled, and therefore computationally efficient staggered solution procedures will be designed for solving the coupled semi-discrete aerothermoelastic equations using simple mathematical constructs. To this effect, the sources of degradation of time-accuracy for the simplest loosely-coupled solution algorithms will be identified and remedies for them will be designed. These sources go well beyond the loose aspect of the coupling between the chosen single discipline time-integrators. To this effect, the computational framework developed in [43] for analyzing formally the time-accuracy of loosely-coupled fluid/structure time-integrators where the fluid subsystem is solved in moving grids will be extended to address the fluid/thermal and fluid/structure/thermal systems.

It is also well-known that the numerical stability limit of the CSS procedure can be much more restrictive than that of the single discipline solvers. For this reason, several ad-hoc strategies have been published in the literature for improving the stability properties of the CSS procedure. Most of them consist essentially in inserting some type of predictor/corrector iterations within each cycle of this procedure, in order to compensate for the time-lag between the fluid and structure solvers [39, 40].

In [34], a formal numerical stability analysis of partitioned procedures for the solution of fluid/structure interaction problems was attempted to improve the understanding of their behavior and design better alternatives to the CSS method. However, because the dependence of the structure equations of equilibrium on the motion of the fluid dynamic mesh is implicit rather than explicit and the fluid equations of motion can be strongly nonlinear, this analysis was confined to the mathematical investigation of a one-dimensional aerelastic model problem. This model problem was obtained by linearizing the governing equations around a position of aerelastic equilibrium. Furthermore, the fluid-mesh motion equation was replaced by transpiration fluxes at the fluid/structure interface and therefore the model problem was formulated as a two-field and two-way coupled fluid/structure interaction problem. Then, it was proved that an unconditionally stable partitioned procedure, that furthermore retains the order of time-accuracy of its underlying flow and structure time-integrators, can be constructed by superposing a subiteration-free but carefully constructed
corrector scheme to the basic CSS method. Based on this mathematical analysis, guidelines were established for exchanging aerodynamic and elastodynamic data in the presence of sub-cycling, in a manner that preserves the unconditional stability and order of time-accuracy of a given partitioned procedure. Unfortunately, it was not possible to extend all of these ideas to complex three-dimensional fluid/structure interaction problems where the fluid is discretized on moving grids.

In [44], an alternative approach for improving the maximum allowable time-step of the CSS procedure that does not increase its computational cost per cycle was described. This approach is based on introducing two computationally economical factors for compensating the time-lag between the fluid and the structure subsystems: (1) a non-trivial prediction of the displacement field, and (2) a non-necessarily trivial transfer of the aerodynamic forces to the structure. More specifically, it was shown in [44] that given two time-integration schemes for the fluid and structure equations of motion, the displacement predictor and transferred force can be designed to achieve a $p$-order "energy-transfer-accurate" CSS procedure. The higher $p$ is, the closer is the CSS procedure to conserving the transfer of energy through the fluid/structure interface. Using this approach, third-order energy-transfer-accurate loosely-coupled procedures were constructed and shown to sustain as large time-steps as those afforded by strongly-coupled or monolithic schemes, without having to pay the usual penalties (see above) associated with these approaches.

Therefore, the stability-oriented design framework presented in [44] will be extended to the case of fluid/thermal and fluid/structure/thermal problems and combined with the analysis framework of [43] to develop state-of-the-art loosely-coupled staggered procedures for the solution of CHT and aerothermoelectric analysis problems that feature both second-order time-accuracy and excellent numerical stability properties.

For steady fluid/structure applications where there is no concept of real time-lag — and therefore no real opportunity for introducing compensators such as predictors and correctors — Schur-Newton-Krylov solvers [45] have recently been shown to be effective coupled solvers. This approach will also be explored for steady CHT analysis.

### 2.1.2.5 Verification and Demonstration

To verify the computational models to be developed and integrated in AERO as described in the previous sections, a highly simplified flight vehicle configuration known as the Rankine-Hugoniot-Prandtl-Meyer- (RHPM-) flyer will be considered [1]. This is an infinitely thin flat plate at angle of attack with a radiation-adiabatic surface. The flow past this plate can be determined by means of simple shock-expansion theory. Semi-analytical and computational aerothermodynamic results are available for this simplified model [1] and can be used for verifying the results produced by the expanded AERO code.

To demonstrate the potential of the numerical tools developed under this research project, the multidisciplinary thermal analysis of a Cruise and Acceleration Vehicle (CAV) with airbreathing propulsion accelerating from Mach 7 to Mach 12 at small angle of attack will be considered. For these vehicles, the flow field and thermal surface are usually viscosity-effect dominated and the rarefraction and thermo-chemical effects are weak and therefore the adopted assumptions are justified.
2.1.3 Project Schedule, Milestones and Deliverables

The development of the mathematical aspects of the four-field formulation of aerothermoe­lastic problems and the enhancements of the AERO-H thermal solver will be completed during the first year of funding. The development and integration in the AERO simulation platform of the conservative method for the discretization of the fluid/thermal transmission conditions and the fast coupled solvers for the steady and unsteady CHT analyzers will be completed during the middle of the third year of funding. The verification and demonstration work will be performed during the second half of the third year of funding. Updates to the AERO code will be delivered to the Flight Test Center at the Edwards Air Force Base at the end of each quarter of each of the three years of funding.

References


### 2.2 PRINCIPAL INVESTIGATOR TIME

#### 2.2.1 Time Committment to this Research Project

The PI of this proposed research project is Professor Charbel Farhat. He will dedicate at least 2% of his academic time and 16% of his summer time to the proposed research effort. Professor Farhat will also supervise one full-time graduate student who will contribute to the proposed research project. This graduate student will be immersed in the AERO research group. Therefore, he/she will benefit from the expertise of the critical mass that is available for integrating the research findings into the AERO code deployed at the Edwards Air Force Base.

#### 2.2.2 Current and Pending Support

Professor Farhat is currently the PI of the following research grants which extend beyond March 1st, 2007:

• Grant: A Dynamic Data-Driven System for Structural Health Monitoring and Critical Event Prediction. Agency: National Science Foundation. Commitment: 5% AY and 0.5 month summer.

• Grant: High-Resolution Methods for the Solution of Direct and Inverse Acoustic Scattering Problems. Agency: Office of Naval Research. Commitment: 10% AY and 0.5 month summer.

• Grant: Unsteady CFD Analysis of a Formula One Car. Agency: Toyota Motor Corporation. Commitment: 5% AY and 0.5 month summer.

• Grant: Scalable Substructuring Methods for Linear and Nonlinear Dynamics Problems. Agency: Sandia National Laboratories. Commitment: 5% AY.


2.3 FACILITIES
The PI operates at Stanford University a High-Performance Computing and Visualization Laboratory that can serve as a development and application platform for the proposed research. The laboratory is equipped with a Linux Cluster system with 200 Intel Xeon 3.056 GHz processors and 400 GB of memory. This parallel processor is connected to a Panasas Storage Cluster with 5 terabytes of disk space and direct node-to-disk access, and to several front-end and visualization systems.

2.4 KEY PERSONNEL
The key personnel for this proposed research project includes Professor Charbel Farhat and a graduate student.

2.4.1 Charbel Farhat

Biographical Sketch
Charbel Farhat is Professor of Mechanical Engineering, Professor, by courtesy, of Aeronautics and Astronautics, and Professor in the Institute for Computational and Mathematical Engineering, all at Stanford University. Previously, he held the positions of Professor and Chair of Aerospace Engineering Sciences and Director of the Center for Aerospace Structures at the University of Colorado at Boulder. He holds a Ph.D. in Civil Engineering from
the University of California at Berkeley (1987). He is the recipient of several prestigious awards including the Institute of Electrical and Electronics Engineers (IEEE) Computer Society Gordon Bell Award (2002), the International Association of Computational Mechanics (IACM) Computational Mechanics Award (2002), the Department of Defense Modeling and Simulation Award (2001), the US Association of Computational Mechanics (USACM) Medal of Computational and Applied Sciences (2001), the IACM Award in Computational Mechanics for Young Investigators (1998), the USACM R. H. Gallagher Special Achievement Award for Young Investigators (1997), the IEEE Computer Society Sidney Fernbach Award (1997), the American Society of Mechanical Engineers (ASME) Aerospace Structures and Materials Best Paper Award (1994), and the United States Presidential Young Investigator Award (1989).

Professor Farhat is Associate Editor of the International Journal for Numerical Methods in Engineering. He also serves on the editorial board of eleven other international scientific journals, and on the technical assessment board of several national research councils and foundations. He is a Fellow of the American Society of Mechanical Engineers (2003), Fellow of the International Association of Computational Mechanics (2002), Fellow of the World Innovation Foundation (2001), Fellow of the US Association of Computational Mechanics (2001), and Fellow of the American Institute of Aeronautics and Astronautics (1999). He has been an AGARD lecturer on aeroelasticity and computational mechanics at several distinguished European institutions, and a keynote speaker at numerous international scientific meetings. He is the author of over 200 refereed publications on aeroelasticity, acoustics, fluid/structure interaction, computational fluid dynamics on moving grids, computational structural mechanics, numerical analysis, applied mathematics, and parallel processing. His research program has been and is currently funded by several government and private agencies including the National Science Foundation, the Air Force Office of Scientific Research, the NASA Langley Research Center, the NASA Ames Research Center, the NASA Lewis Research Center, the Naval Research Laboratory, the Office of Naval Research, the Department of Energy, the Department of Defense's High Performance Computing Modernization Program, the Defense Advanced Research Projects Agency, the Sandia National Laboratories, TRW, the FMC Corporation, the Lockheed-Martin Corporation, High Performance Technologies, and the Toyota Motor Corporation.

Selected Publications


2.5 COST PROPOSAL

The budget includes yearly support for: a full-time graduate student trained in coupled field problems; 2% of the academic time and 16% of the summer time of the PI to supervise and contribute to this research project; and travel to attend the AFOSR Test and Evaluation Portfolio Review and a technical conference pertaining to the proposed research effort.