From: Vagle, Diane P CTR USAF USAFA USAFA/DFER  
Sent: Tuesday, December 24, 2013 8:47 AM  
To: Solti, James P Civ USAF USAFA USAFA/DFER  
Subject: FW: eSSS: PA approval for AIAA paper (Mehdi Ghoreyshi)  

This public release needs approval.

v/r

Diane

From: Cummings, Russ M Dr USAF USAFA DF/DFAN  
Sent: Monday, December 23, 2013 8:56 AM  
To: Vagle, Diane P CTR USAF USAFA USAFA/DFER  
Cc: Lofthouse, Andrew J LtCol USAF USAFA USAFA/DFAN; Reed, Shad A LtCol USAF USAFA USAFA/DFAN; Cummings, Russ M Dr USAF USAFA DF/DFAN  
Subject: FW: eSSS: PA approval for AIAA paper (Mehdi Ghoreyshi)

From: Lofthouse, Andrew J LtCol USAF USAFA USAFA/DFAN  
Sent: Monday, December 23, 2013 8:53 AM  
To: Cummings, Russ M Dr USAF USAFA DF/DFAN; Reed, Shad A LtCol USAF USAFA USAFA/DFAN  
Subject: eSSS: PA approval for AIAA paper (Mehdi Ghoreyshi)  

Russ and Shad,
This is the PA approval for Mehdi’s second paper. Please forward on to DFER (Diane Vagle, Col Kraus, Dr. Solti) when ready (and CC me).

Thanks,

Andrew

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ACTION OFFICER: Lt Col Lofthouse, USAFA/DFAN
Phone: DSN 333-9526
Due: 20131227
SUBJECT: Clearance for Material for Public Release
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1. PURPOSE. To provide security and policy review on the document at Tab 1 prior to release to the public.

2. BACKGROUND.

Authors: Mehdi Ghoreyshi, Andrew J. Lofthouse, Russ Cummings
Title: Sampling Strategies for Reduced-Order Modeling of Nonlinear and Unsteady Aerodynamics

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ANDREW J. LOFTHOUSE, Lt Col, USAF
Director, Modeling & Simulation Research Center

1. AIAA Paper
Sampling Strategies for Reduced-Order Modeling of Nonlinear and Unsteady Aerodynamics

Mehdi Ghoreyshi,* Andrew J. Lofthouse† and Russell M. Cummings‡

Modeling and Simulation Research Center, U.S. Air Force Academy
USAF Academy, Colorado 80840

The sampling strategies for creation of reduced order aerodynamic models is the focus of this paper. The methods of factorial design, Latin hypercube sampling, and designs based on optimality criteria are used to define samples in the angle of attack and free-stream Mach number space. The reduced-order models are based on Duhamel’s superposition integral using response (indicial) functions. The indicial functions are calculated using a prescribed grid motion and unsteady Reynolds-Averaged Navier-Stokes simulations starting from an initial steady-state condition that corresponds to each sample. A time-dependent surrogate model is then developed to fit the relationship between flight conditions (Mach number and angle of attack) and simulated step functions. To illustrate the method, the pitch moment of X-31 aircraft undergoing pitching motions is considered. The flight speeds in this paper are in the range of subsonic and transonic. The shock waves and vortical flows over the X-31 aircraft makes modeling pitch moment a very challenging task. The results show that a sample design with more data points at transonic speeds predicted better accuracy. Also, designs based on optimality criteria showed better accuracy than traditional factorial designs.

I. Introduction

The transient aerodynamic response due to a step change in a forcing parameter, such as angle of attack or pitch rate is a so-called “indicial function”. Assuming that the indicial functions are known, the aerodynamic forces and moments induced in any maneuver can be estimated by using the well-known Duhamel superposition integral.1 Tobak et al.2,3 and Reisenthal et al.4,5 detailed the superposition process for the modeling of unsteady lift and pitch moment from angle of attack and pitch rate indicial functions. Ghoreyshi and Cummings6 extended this model to include lateral forces and moments as well. However, this model is subject to the limitations of the identification methods of indicial functions. These functions can be derived from analytical, CFD, or experimental methods.7 Limited analytical expressions of indicial functions exist for two-dimensional airfoils.8 For incompressible flows, Wagner9 was the first who detailed the analytical unsteady lift of a thin airfoil undergoing a plunging motion using a single indicial function (the so-called Wagner’s function) with its exact values defined in terms of Bessel functions. For unsteady, compressible flow past two-dimensional airfoils, Bisplinghoff et al.10 also described an exponential approximation to the exact solutions of the indicial functions at different Mach numbers. However, these analytical expressions are not valid for aircraft configurations due to the three-dimensional tip vortices.

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Direct and indirect methods are used to estimate indicial functions. Leishman\textsuperscript{11} has presented an indirect technique for identifying indicial functions from aerodynamic responses due to harmonic motions. However, the derived indicial functions using indirect methods depend largely on the quality of motion, e.g. amplitude and frequency. Experimental tests are practically nonexistent for direct indicial function measurements due to wind tunnel constraints. An alternative is to use CFD, but special considerations are required to simulate step responses in CFD. Singh and Baeder\textsuperscript{12} used a surface transpiration approach to directly calculate the angle of attack indicial response using CFD. Ghoreyshi et al.\textsuperscript{13} also described an approach based on a grid motion technique for CFD-type calculation of linear and nonlinear indicial functions with respect to angle of attack and pitch rate. Ghoreyshi and Cummings\textsuperscript{6} later used this approach to generate longitudinal and lateral indicial functions for a generic unmanned combat air vehicle and used these functions for predicting the aerodynamic responses to aircraft six degree of freedom maneuvers. In this paper, the transonic indicial functions of the X-31 aircraft are calculated using CFD and the grid motion approach.

For motions at low angles of attack, and assuming incompressible flow, only a single indicial function with respect to each forcing parameter needs to be generated. For flows at transonic speed, however many indicial functions need to be generated for each combination of angle of attack and free-stream Mach number. The generation of all these functions using CFD is expensive and makes the creation of a ROM very time consuming. Note that these models are still cheaper than full-order simulations because the ROMs based on indicial functions eliminate the need to repeat calculations for each frequency. Ghoreyshi et al.\textsuperscript{14} proposed a surrogate modeling based on the Kriging technique\textsuperscript{15} to model indicial functions as a function of angle of attack and free-stream Mach number. They applied this method to predict X-31 pitch moment values at transonic speeds. However, the samples were generated using simple full factorial design. The objective of this work is to investigate the effects of sample designs on the reduced order model predictions.

II. Formulation

A. CFD Solver

The flow solver used for this study is the Cobalt code\textsuperscript{16} that solves the unsteady, three-dimensional and compressible Navier-Stokes equations in an inertial reference frame. These equations in integral form are\textsuperscript{17}

\[
\frac{\partial}{\partial t} \int \int \int \mathbf{Q} d\mathbf{V} + \int \int (\mathbf{f} + \mathbf{g} + \mathbf{h}) \cdot n d\mathbf{S} = \int \int (\mathbf{t} + \mathbf{g} + \mathbf{t}) \cdot n d\mathbf{S}
\]

(1)

where \( V \) is the fluid element volume; \( S \) is the fluid element surface area; \( n \) is the unit normal to \( S; \); \( i, j, \) \( \) and \( k \) are the Cartesian unit vectors; \( \mathbf{Q} = (\rho, \rho u, \rho v, \rho w, \rho e) \) is the vector of conserved variables, where \( \rho \) represents air density, \( u, v, w \) are velocity components and \( e \) is the specific energy per unit volume. The vectors of \( \mathbf{f}, \mathbf{g}, \) and \( \mathbf{h} \) represent the inviscid components and are detailed below

\[
\begin{align*}
\mathbf{f} &= \begin{pmatrix} \rho u \rho u^2 + p, \rho v w, \rho w w, \rho (e + p) \end{pmatrix}^T \\
\mathbf{g} &= \begin{pmatrix} \rho v u, \rho v^2 + p, \rho u w, v(e + p) \end{pmatrix}^T \\
\mathbf{h} &= \begin{pmatrix} \rho w u, \rho w^2 + p, \rho w v, w(e + p) \end{pmatrix}^T
\end{align*}
\]

(2)

where the superscript \( T \) denotes the transpose operation. The vectors of \( \mathbf{r}, \mathbf{s}, \) and \( \mathbf{t} \) represent the viscous components which are described as

\[
\begin{align*}
\mathbf{r} &= \begin{pmatrix} \tau_{xx}, \tau_{xy}, \tau_{xz}, u\tau_{xx} + v\tau_{xy} + w\tau_{xz} + kT \end{pmatrix}^T \\
\mathbf{s} &= \begin{pmatrix} \tau_{xy}, \tau_{yy}, \tau_{yz}, u\tau_{xy} + v\tau_{yy} + w\tau_{yz} + kT \end{pmatrix}^T \\
\mathbf{t} &= \begin{pmatrix} \tau_{xz}, \tau_{zy}, \tau_{zz}, u\tau_{xz} + v\tau_{zy} + w\tau_{zz} + kT \end{pmatrix}^T
\end{align*}
\]

(3)

where \( \tau_{ij} \) are the viscous stress tensor components, \( T \) is the temperature, and \( k \) is the thermal conductivity. The ideal gas law and Sutherland’s law closes the system of equations and the entire equation set is nondimensionalized by free stream density and speed of sound.\textsuperscript{16} The Navier-Stokes equations are discretised on arbitrary grid topologies using a cell-centered finite volume method. Second-order accuracy in space is achieved using the exact Riemann solver of Gottlieb and Groth,\textsuperscript{18} and least squares gradient calculations
using QR factorization. To accelerate the solution of discretized system, a point-implicit method using analytic first-order inviscid and viscous Jacobians. A Newtonian sub-iteration method is used to improve time accuracy of the point-implicit method. Tomaro et al.\textsuperscript{19} converted the code from explicit to implicit, enabling Courant-Friedrichs-Lewy numbers as high as 10\textsuperscript{6}. The Cobalt solver has been used at the Air Force Seek Eagle Office (AFSEO) and the United States Air Force Academy (USAF) for a variety of unsteady nonlinear aerodynamic problems of maneuvering aircraft.\textsuperscript{20,21,22,23,24}

B. ROMs Based on Indicial Functions

The indicial response functions are used as a fundamental approach to represent the unsteady aerodynamic loads. The mathematical models are detailed by Tobak et al.\textsuperscript{2,3} and Reisenthel et al.\textsuperscript{4,5} In this paper, the lift and pitch moment of an aircraft with a flap are considered. If the time responses in aerodynamic coefficients due to the step changes in angle of attack, $\alpha$, angular velocity, $q$ are known, then the total produced lift and pitch moment at time $t$ can be obtained using Eq. (4):

$$C_j(t) = C_{j0}(M) + \frac{d}{dt} \int_{0}^{t} C_{jq}(t - \tau, \alpha, M)q(\tau) d\tau$$

where $j = L, m$ representing lift and pitch moment respectively; $C_{j0}$ denote the zero angle of attack aerodynamic coefficients and are found from static calculations; $M$ denotes the free-stream Mach number. The response function due to pitch rate, i.e. $C_{jq}(\alpha, M)$ can be estimated using a time-dependent interpolation scheme from the observed responses. This value is next used to estimate the second integral in Eq. 4, however, the estimation of the integral with respect to the angle of attack needs more explanation. Assuming a set of angle of attack samples of $\alpha = [\alpha_1, \alpha_2, ..., \alpha_n]$ at free-stream Mach numbers of $M = [M_1, M_2, ..., M_m]$, the pitch moment response to each angle of $\alpha_i$, $i = 1, 2, ..., n$ at Mach number of $M_j, j = 1, 2, ..., m$ is denoted as $A_\alpha(t, \alpha_i, M_j)$. In these response simulations, $\alpha(t) = 0$ at $t = 0$ and is held constant at $\alpha_i$ for all $t > 0$. For a new angle of attack $\alpha^* > 0$ at a new free-stream Mach number of $M^*$, the responses of $A_\alpha(t, \alpha_k, M^*)$ are being interpolated at $\alpha_k = [\alpha_1, \alpha_2, ..., \alpha_k]$, such that $0 < \alpha_1 < \alpha_2 < ... < \alpha_s$ and $\alpha_s = \alpha^*$. These angles can have a uniform or non-uniform spacing. The indicial functions of $C_{j\alpha_k}$ for $k = 1, ..., s$ at each interval of $[\alpha_{k-1}, \alpha_k]$ are defined as

$$C_{j\alpha_1} = \frac{A_\alpha(t, \alpha_1, M^*) - C_{j0}}{\alpha_1}$$

$$C_{j\alpha_k} = \frac{A_\alpha(t, \alpha_k, M^*) - A_\alpha(t, \alpha_{k-1}, M^*)}{\alpha_k - \alpha_{k-1}}$$

where $C_{j0}$ denotes the zero angle of attack pitch moment coefficient. The interval indicial functions are then used to estimate the values of first integral in Eq. 4. These steps can easily be followed for a negative angle of attack, i.e. $\alpha^* < 0$. Once this model is created, the aerodynamic response to a wide range of motions can be predicted on the order of few seconds, however, the above model still requires a special time-dependent surrogate model to predict response functions at each $\alpha^*$ and $M^*$ from some available samples.

C. Surrogate-Based Modeling of Indicial Functions

Having a ROM to predict the aerodynamic responses to any arbitrary motion over a wide flight regime could become a very expensive approach because a large number of indicial functions need to be computed. In order to achieve a reasonable computational cost, a special time-dependent surrogate-based modeling approach is adapted to predict indicial responses for a new point from available (observed) responses. These observed responses are viewed as a set of time-correlated spatial processes where the output is considered a time-dependent function. Romero et al.\textsuperscript{25} developed a framework for multi-stage Bayesian surrogate models for the design of time dependent systems and tested their model for free vibrations of a mass-spring-damper system assuming the input parameters of stiffness and damping factor at different initial conditions. This framework is examined for reduced order modeling of nonlinear and unsteady aerodynamic loads. Assume an input vector of $x(t) = (x_1(t), x_2(t), ..., x_n(t))$ where $n$ represents the dimensionality of the input vector. To construct a surrogate model for fitting the input-output relationship, the unsteady aerodynamic responses corresponding to a limited number of input parameters (training parameters or samples) need to
be generated. Design of Experiment methods, for example, can be used to select \( m \) samples from the input space. The input matrix \( D(m \times n) \) is then defined as:

\[
D = \begin{bmatrix}
x_{11} & x_{12} & \cdots & x_{1n} \\
x_{21} & x_{22} & \cdots & x_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}
\]  
(7)

where rows correspond to different combinations of the design parameters. For each row in the input matrix, a time-dependent response was calculated at \( p \) discrete values of time, and this information is summarized in the output matrix of \( Z(m \times p) \) as:

\[
Z = \begin{bmatrix}
y_{11} & y_{12} & \cdots & y_{1p} \\
y_{21} & y_{22} & \cdots & y_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
y_{m1} & y_{m2} & \cdots & y_{mp}
\end{bmatrix}
\]  
(8)

where for aerodynamic loads modeling, \( p \) equals the number of iterations used in time-marching CFD calculations. The objective of surrogate modeling is to develop a model that allows predicting the aerodynamic response of \( y(x_0) = (y_{01}, y_{02}, \ldots, y_{0p}) \) at a new combination of input parameter of \( x_0 \). To construct this surrogate model, the responses at each time step are assumed as a separate set, such that each column of the output matrix is a partial realization of the total response. In this sense, \( p \) surrogate models are created; they are denoted as \( Z_i(D) \) for \( i = 1, 2, \ldots, p \). A universal-type Kriging function\(^{15}\) is used to approximate these models. Each \( Z_i(D) \) function can be approximated as the sum of a deterministic mean (trend), \( \mu \) and a zero-mean spatial random process, \( \epsilon \) with a given covariance structure of \( \sigma^2 \), therefore each function value at the new sample of \( x_0 \) is

\[
\tilde{Z}_i(x_0) = \mu + \epsilon
\]  
(9)

where, the tilde accent shows that surrogate model is an approximation of the actual function. Universal kriging, which is used in this paper, assumes that the mean value \( \mu \) is a linear combination of known regression functions of \( f_0(x), f_1(x), \ldots, f_n(x) \). In this paper, the linear functions are used, therefore, \( f_0(x) = 1 \) and \( f_j(x) = x_j \) for \( j = 1, 2, \ldots, n \). This changes Eq. (9) as:

\[
\tilde{Z}_i(x_0) = \sum_{j=0}^{n} \beta_{ij} f_j(x_0) + \epsilon
\]  
(10)

where \( \beta_{ij} \) represent the regression coefficient for the \( j \)-th regression function of response function at time step \( i \), \( i = 1, 2, \ldots, p \). To estimate the spatial random process of \( \epsilon \), a spatially weighted distance formula is defined between samples given in matrix \( D \) such that for sample \( x_i \) and \( x_j \), the distance is written as:

\[
d(x_i, x_j) = \sum_{h=1}^{n} \theta_h \left| x_{ih} - x_{jh} \right|^{p_h} \quad (\theta_h \geq 0 \text{ and } p_h \in [0, 1])
\]  
(11)

where \( |.| \) shows the Euclidean distance; the parameter \( \theta_h \) expresses the importance of the \( h \)-th component of the input vector, and the exponent \( p_h \) is related to the smoothness of the function in coordinate direction \( h \). A correlation matrix \( R(m \times m) \) with a Gaussian spatial random process is then defined as:

\[
R = \begin{bmatrix}
\exp \left[ -\frac{d(x_1,x_1)}{\sigma^2} \right] & \exp \left[ -\frac{d(x_1,x_2)}{\sigma^2} \right] & \cdots & \exp \left[ -\frac{d(x_1,x_m)}{\sigma^2} \right] \\
\vdots & \vdots & \ddots & \vdots \\
\exp \left[ -\frac{d(x_m,x_1)}{\sigma^2} \right] & \exp \left[ -\frac{d(x_m,x_2)}{\sigma^2} \right] & \cdots & \exp \left[ -\frac{d(x_m,x_m)}{\sigma^2} \right]
\end{bmatrix}
\]  
(12)
To compute the Kriging model, values must be estimated for $\beta_{ij}$, $\sigma$, $\theta_h$, and $p_h$. These parameters can be quantified using the maximum likelihood estimator, as described by Jones et al.\textsuperscript{26} Next the vector of $R(m \times 1)$ is defined from correlations between the new design parameter $x_0$ and the $m$ sample points, based on the distance formula in Eq. (11), i.e.

$$r = \begin{bmatrix}
\exp\left[-\frac{d(x_1,x_0)}{\sigma^2}\right] \\
\exp\left[-\frac{d(x_2,x_0)}{\sigma^2}\right] \\
\vdots \\
\exp\left[-\frac{d(x_m,x_0)}{\sigma^2}\right]
\end{bmatrix}$$

and now $\tilde{Z}_i(x_0)$ can be estimated as

$$\tilde{Z}_i(x_0) = \sum_{j=0}^{n} \beta_{ij} f_j(x_0) + r^T R^{-1} (Z_i(D) - F \beta)$$

where, $\beta$ is the $n + 1$ dimensional vector of regression coefficients; $Z_i(D)$ is the observed responses at time step $i$, $i = 1, 2, ..., p$ and matrix $F$ is

$$F = \begin{bmatrix}
f_0(x_1) & f_1(x_1) & \cdots & f_n(x_1) \\
: & : & \cdots & : \\
f_0(x_m) & f_1(x_m) & \cdots & f_n(x_m)
\end{bmatrix}$$

The total response at $x_0$ is then combination of predicted values of each surrogate model, i.e.

$$\tilde{Z}(x_0) = \left(\tilde{Z}_1(x_0), \tilde{Z}_2(x_0), ..., \tilde{Z}_p(x_0)\right)$$

D. CFD Calculation of Indicial Functions

The indicial functions can be derived from analytical, CFD, or experimental results.\textsuperscript{7} Limited analytical expressions of indicial functions exist for two-dimensional airfoils.\textsuperscript{8} For incompressible flows, Wagner\textsuperscript{9} was the first who detailed the analytical unsteady lift of a thin airfoil undergoing a plunging motion using a single indicial function (the so-called Wagner’s function) with its exact values defined in terms of Bessel functions. His function was approximated in non-dimensional time by a two-pole exponential function as\textsuperscript{27}

$$C_{La} = 2\pi(1 - 0.165 \exp(-0.0455s) - 0.33 \exp(-0.3s))$$

where $s = 2Vt/c$ is the normalized time. For unsteady, compressible flow past two-dimensional airfoils, Biplinghoff et al.\textsuperscript{10} also described an exponential approximation to the exact solutions of the indicial functions at different Mach numbers. However, these analytical expressions are not valid for aircraft configurations due to the three-dimensional mechanism of tip vortices.

Direct and indirect methods are provided to estimate indicial functions. Leishman\textsuperscript{11} has presented an indirect technique for identifying indicial functions from aerodynamic responses due to harmonic motions. However, the derived indicial functions using indirect methods depend largely on the quality of motion, e.g. amplitude and frequency. Experimental tests are practically nonexistent for direct indicial function measurements due to wind tunnel constraints. An alternative is to use CFD, but special considerations are required to simulate step responses in CFD. Singh and Baeder\textsuperscript{12} used a surface transpiration approach to directly calculate the angle of attack indicial response using CFD. Ghoreyshi et al.\textsuperscript{13} also described an approach based on a grid motion technique for CFD-type calculation of linear and nonlinear indicial functions with respect to angle of attack and pitch rate. In this paper, the indicial functions with respect to angle of attack, pitch rate, and surface deflections are calculated using CFD and the grid motion approach.

**Cobalt** uses an arbitrary Lagrangian-Eulerian formulation and hence allows all translational and rotational degrees of freedom.\textsuperscript{13} The code can simulate both free and specified six degree of freedom (6DoF) motions.
The rigid motion is specified from a motion input file. For the rigid motion the location of a reference point on the aircraft is specified at each time step. In addition the rotation of the aircraft about this reference point is also defined using the rotation angles of yaw, pitch and bank. The aircraft reference point velocity, $v_a$, in an inertial frame is then calculated to achieve the required angles of attack and sideslip, and the forward speed. The velocity is then used to calculate the location. The initial aircraft velocity, $v_0$, is specified in terms of Mach number, angle of attack and side-slip angle in the main file. The instantaneous aircraft location for the motion file is then defined from the relative velocity vector, $v_a - v_0$. For CFD-type calculation of a step change in angle of attack, the grid immediately starts to move at $t = 0$ to the right and downward. The translation continues over time with a constant velocity vector. Since there is no rotation, all the effects in aerodynamic loads are from changes in the angle of attack. For a unit step change in pitch rate, the grid moves and rotates simultaneously. The grid starts to rotate with a unit pitch rate at $t = 0$. To hold the angle of attack zero during the rotation, the grid moves right and upward.

III. Test Case

The X-31 aircraft is considered in this paper. The aircraft geometry and wind tunnel data were provided to the participants in NATO RTO Task Group AVT-161 (Assessment of Stability and Control prediction Methods for NATO Air and Sea Vehicles). The objective of this task group is to evaluate CFD codes against the wind tunnel results. A three-view drawing of the vehicle is shown in Fig. 1.

The computational mesh was generated in two steps. In the first step, the inviscid tetrahedral mesh was generated using the ICEMCFD code. This mesh was then used as a background mesh by TRITET, which builds prism layers using a frontal technique. TRITET rebuilds the inviscid mesh while respecting the size of the original inviscid mesh from ICEMCFD. The mesh overview is shown in Fig. 2. The grid is a symmetric configuration and contains 4.9 million points and 11.7 million cells. Three boundary conditions were imposed to the surfaces: a far-field, symmetry, and solid wall. The low-speed experiments are available from the DLR, German Aerospace Center. The wind tunnel model has a closed inlet and fitted with moving lift and control surfaces. The experiments are composed of two setups. The first setup uses a belly-mounted sting attached to the model directly under the main wing. This setup allows six degree of freedom motions. The second setup uses an aft mounted sting connected to an arm in the wind tunnel. The values of lift, drag, and pitch moment of second setup are used to validate CFD predictions. CFD simulations were run on the Cray XE6 machine at the Engineering Research Development Center (ERDC); machine name is Garnet with 2.7GHz core speed.

Some features of aerodynamic characteristics from the SARC-DDES turbulence model predictions are explained. There is an emanating vortex from the canard tip at small angles of attack. This vortex is the source of the small non-linearity in the pitch moment at low angles of attack. As angle of attack is increased, the canard vortex becomes stronger, resulting in a negative pressure on the upper surface and forward movement of the aerodynamic center. Therefore, the pitch moment slope suddenly increases from the slope value at zero degrees angle of attack. This vortex is shown in Fig. 3(a) for 10 degrees angle of attack. Around an angle of 14 degrees, the canard vortex starts to breakdown and the wing vortex is formed as shown in Fig. 3(b). The wing vortex helps to further forward movement of aerodynamic center and increase of pitch moment. At 18 degrees angle of attack, the canard vortex breakdown point is nearly moved to the leading edge and then the wing vortex starts to breakdown as shown in Fig. 3(c). This results in an aftward movement of aerodynamic center and a change in the pitch moment slope sign. The wing vortex breakdown point moves towards the leading edge by increasing angle of attack (Fig. 3(d)-(e)). The canard vortex is fully separated at these angles. As the vortex breakdown point becomes close to the wing leading edge (Fig. 3(f)), the pitch moment starts to rise again. Note that the purpose of this work is not to validate CFD codes or turbulence models, but rather to validate various reduced-order modeling approaches. Therefore, only one code (Cobalt) and only one turbulence model (SARC-DDES) have been used throughout the study based on our experience with these tools in predicting unsteady aerodynamics. Since the primary result of the work is to validate the modeling approaches, only comparisons will be made between the model results and the original CFD simulations.
IV. Results and Discussion

The indicial response functions in this paper are interpolated from some available samples in the angle of attack and free-stream Mach number space. Note that these functions only need to have dependency on angle of attack and Mach number, and once they have been calculated they could be used to predict the aerodynamic response to any frequency of interest. The samples could be generated using methods of factorial designs, Latin hypercube sampling, low discrepancy sequences, and designs based on statistical optimality criteria (A-, D- and G- optimal designs). The results of a factorial design, LHS, and expected improvement functions are shown. The step motions considered encompass $\alpha$ and $M_0$ values in the range of $[-5^\circ, 10^\circ]$ and $[0.5, 0.9]$, respectively. Ghoreyshi et al.\textsuperscript{14} showed that the X-31 aircraft has different pitch moment slopes in negative and positive angles of attack. Therefore, indicial functions were generated for negative and positive angles of attack.

A set of samples including 144 points is defined on the $\alpha$ and $M$ space using factorial design; the points are shown in Fig. 4. In the present paper, the response functions are directly calculated from unsteady RANS simulations and using a grid motion tool. All computations started from a steady-state solution and then advanced in time using second-order accuracy. The motion files were generated for step changes in aircraft forcing parameters (angle of attack, side-slip angle, and angular rates). These files define the rotations and displacements at discrete time instants and Cobalt then interpolates motion data using cubic-splines and moves the grid for each computational time step. The grid undergoes only translation motion for $\alpha$ responses, where the relative velocity between grid and flow at each instant defines the angle of attack. For the pitch rate responses, the motions start from a steady-state solution with zero degrees angle of attack and side-slip angle. The grid then rotates and translates simultaneously. The rotation corresponds to a unit step change in the pitch rate, while the translation motion is used in order to keep angles of attack zero during rotations.

The response functions with respect to the angle of attack are calculated using the CFD and grid motion approach for each sample conditions. In these simulations, the solution starts from a steady-state condition at angle of attack of $\alpha_i$ and Mach number of $M_i$, and then performing a small step in the angle of attack for all $t > 0$. In these calculations, $M_i$ and $\alpha_i$ values correspond to the samples shown in Fig 4, and the side-slip angle is zero degrees at all times and the grid does not rotate at any time. The response functions are then computed by taking the differences between time-varying forces and moments occurring after the step and the steady-state solution at $\alpha = \alpha_i$ degrees, and dividing them by the magnitude of the step ($\Delta \alpha$). For a weakly nonlinear system, the response will be nearly independent of the step magnitude (assuming that $\alpha_i + \Delta \alpha \leq \alpha_{i+1}$). The step value used in this study is a unit step. The pitch moment indicial responses to a unit step change in the angle of attack from $\alpha_i = 0$ are shown in Fig. 5 for Mach numbers in range of 0.5 to 0.9. The pitch moment are plotted against the nondimensional time $s = 2Vt/c$, where $V$ is the frees-stream velocity, $t$ is the response time, and $c$ is the reference length. Figure 5 shows that the pitch moment responses have a peak at $s = 0$. As the steady flow around the vehicle is disturbed by the grid motion, a compression wave and an expansion wave are formed on the lower and upper surface of the vehicle that cause a sharp peak in the responses.\textsuperscript{13} As the response time progresses, the waves begin to move away from the vehicle, the pitch moment starts to fall, and then responses asymptotically reach the steady-state values. Figures 5 shows that the initial peak becomes smaller for compressible flow. An explanation is given by Leishman;\textsuperscript{11} this is due to the propagation of pressure disturbances at the speed of sound, compared to the incompressible case, where the disturbances propagate at infinite speed.

Figure 6 shows the pitch moment responses for different angles of attack at Mach number of 0.5 and 0.9. These figures show that the initial values of responses are invariant with angle of attack, but the transient trend and steady state values change depending on the angle of attack. Note that slopes are different in negative and positive angles of attack.

Typically, the angle of attack effects are negligible for the responses due to the angular rates at low to moderate angles of attacks. Figures 7 shows the pitch moment indicial responses respectively with a unit step change in pitch rate for Mach numbers in range of 0.5 to 0.9. Again there is an initial jump in pitch moment as grid starts to rotate which its value decreases as Mach number increases. The pitch moment responses start to fall a short time after initial excitation and then it reaches asymptotically a steady-state value, the so called pitch dynamic derivative. Figures 7 show that increasing Mach number results in the slight decrease of pitch damping derivatives.

The validity of ROM and effects of sample design is tested for several motions in the angle of attack/frequency/Mach number space and compared with time-accurate CFD simulations in Figs. 8 and 9.
These figures show that the ROM predictions agree well with the CFD data. Small discrepancies are found in the high speed motions (root mean square error is 0.002151). This is likely due to sudden pitch moment slope changes from Mach number of 0.85 to 0.9. The models even accurately predict the X-31 responses to high frequency pitch oscillations as shown in Fig. 9.

Next sample design includes 40 points which are shown in Fig. 10. Reduced order models were created from the responses at these points. The predictions are compared with full-order data and predictions from 144-point design in Fig. 11. The results show that the ROM using 40-point sample fail to predict the trends in most of motions.

V. Conclusions

The aircraft stability and control analysis requires a very large number of CFD simulations to determine appropriate forcing parameters within the frequency/amplitude/Mach number space. Typically, the time-accurate CFD simulations start from a steady state solution and are marched (iterated) in pseudo time within each physical time step using a dual-time stepping scheme. Also, to have a free decay response to the initial grid perturbation, it is often necessary to march time-accurate solution for several oscillations. Also, the configuration used in this work, the X-31 aircraft, has highly swept slender wings resulting in complex vortical flow under various conditions. A highly refined mesh, small time step, and the use of hybrid turbulence models such as Detached-Eddy Simulation and Delayed Detached-Eddy Simulation are required to accurately resolve the unsteady flow around the aircraft in space and time. Because of the combination of large grids, small time steps, hybrid turbulence models, and a large number of simulations, the full-order modeling approach is too expensive for stability and control analysis of aircraft. This paper investigates the use of response function with a time-dependent surrogate approach to reduce the CFD simulation time required to create a full aerodynamics database, making it possible to accurately model aircraft static and dynamic characteristics from a limited number of time-accurate CFD simulations.

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References


Figure 1. The X-31 aircraft geometry

(a) Half-Model Mesh  
(b) Surface mesh around LEX and canard

Figure 2. The X-31 aircraft mesh model
Figure 3. The X-31 vortical flows using DDES with SARC turbulence model. The conditions are: $M_0=0.18$ and $Re = 2 \times 10^6$. 
Figure 4. Samples designs with 144 points.
Figure 5. Linear $\alpha$ indicial functions with a unit step change in angle of attack.

Figure 6. Non-linear response functions at $M_0 = 0.5$ and $M_0 = 0.9$. 
Figure 7. Pitch-rate indicial functions.
Figure 8. Pitch moment modeling in Mach number/angle of attack/frequency space. The ROM is based on a time-dependent surrogate model using sample design of FF 16 x 9. Motions are defined as $\alpha = 3.0 + 7 \sin(2\pi f t)$ where $f=1.687$ Hz.
Figure 9. Pitch moment modeling in Mach number/angle of attack/frequency space. The ROM is based on a time-dependent surrogate model using sample design of FF 16 x 9. Motions are defined as $\alpha = 3.0 + 7\sin(2\pi ft)$ where $f=5.35$ Hz.
Figure 10. Sample design 2 consisting of $8 \times 5$ points.
Figure 11. Pitch moment modeling using sample design 2 with 40 points. Motions are defined as $\alpha = 3.0 + 7\sin(2\pi ft)$ where $f=1.687$ Hz.
Figure 12. Sample design 3 consisting of 144 points generated by LHS method.