**Optimal battery charging, Part I: Minimizing time-to-charge, energy loss, and temperature rise for OCV-resistance battery model**

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**Supplementary Notes**

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**Subject Terms**

Battery charging, Optimal charging, Time to charge, Open circuit voltage, State of charge
Report Title
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ABSTRACT
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ARO Report Number 57823.120-CS
Optimal battery charging, Part I: Minimizing time...

Block 13: Supplementary Note
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Optimal battery charging: Minimizing time-to-charge, energy loss, and temperature rise for OCV-resistance battery model

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Abstract

In this paper we present a closed-form solution to the problem of optimally charging a Li-ion battery. A combination of three cost functions is considered as the objective function: time-to-charge (TTC), energy losses (EL), and a temperature rise index (TRI). First, we consider the cost function of the optimization problem as a weighted sum of TTC and EL. We show that the optimal charging strategy in this case is the well-known Constant Current–Constant Voltage (CC–CV) policy with the value of the current in the CC stage being a function of the ratio of weighting on TTC and EL and of the resistance of the battery. Then, we extend the cost function to a weighted sum of TTC, EL and TRI and derive an analytical solution for the problem. It is shown that the analytical solution can be approximated by a CC–CV with the value of current in the CC stage being a function of ratio of weighting on TTC and EL and of the resistance of the battery and the effective thermal resistance.

1. Introduction

Battery charging is a problem of significant interest, especially as the battery-dependent smart devices proliferate. The literature abounds with different strategies for charging batteries. Among the traditional methods of charging, the simplest is the constant trickle current charge strategy, which, due to its low charging current, requires a long charging time (around 10 h) [9]; constant current strategy with higher rates of current requires shorter charging time.

The most widely-used traditional strategy is the constant-current constant-voltage (CC–CV) [9] strategy, in which a constant current is applied to the battery until the terminal voltage reaches a specified value, and afterward the charging current decreases by applying a constant voltage to the terminals of the battery. In Refs. [21,22], a multi-step constant-current charging is devised for shortening the charging time and prolonging the cycle life of the battery. Using orthogonal arrays, Taguchi-based methods for battery charging [12,23] present a systematic method to find the optimal solution with guidelines for choosing the design parameters. In Ref. [13], a boost charging strategy is proposed by applying very high currents to close-to-fully discharged batteries. In pulse-charging methods [6,17,20,27], the battery is exposed to very short rest or even deliberate discharging periods during the charging process. Soft-computing approaches are also used in the optimization of battery charging profile. In Ref. [24], the charging

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http://dx.doi.org/10.1016/j.jpowsour.2015.02.075
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Please cite this article in press as: A. Abdollahi, et al., Journal of Power Sources (2015), http://dx.doi.org/10.1016/j.jpowsour.2015.02.075
The problem is viewed as an optimization problem with the objective function of maximizing the charge within 30 min using a multi-stage constant current charging algorithm whose optimal solution is obtained via an ant-colony approach. In Ref. [25], a universal voltage protocol is proposed to improve charging efficiency and cycle life by applying a charging profile depending on the state-of-health (SOH) of the battery, using SOH estimation approaches [26] in the optimization process. Recently, in Ref. [11], battery charging is considered as an optimization problem with cost function of time-to-charge and energy loss (as we do in this paper), but they have not solved the problem analytically; rather they have presented a numerical solution to the problem. Other approaches, such as genetic algorithm and neural network based strategies [16], data mining [2,10], and Grey-predicted charging system [8] have also been used for charging batteries.

In this paper we look at the charging problem from a fresh perspective using optimal control theory, and our goal is to find the optimal current profile that minimizes a specific cost function. In this sense, different objectives may be embedded in the cost function. One obvious cost function is the time-to-charge (TTC). We prefer to minimize the charging time as much as possible, as TTC reduction contributes to user satisfaction. Another important objective is the energy loss (EL) during charging. Reducing the energy loss increases the charging efficiency. In this paper, first we use an integrated cost function that includes both the TTC and EL. Then, we also include the effects of temperature into account, and the cost function is selected as a linear combination of three criteria: time-to-charge, energy loss, and temperature rise index (TRI). In both cases, analytical solutions of the optimal charging problem are derived.

The paper is organized as follows. In Section 2, we derive an analytical solution for the optimal charging current profile to minimize TTC and EL. In Section 3, we extend this approach to the case where temperature rise is considered as well. Section 4 is devoted to simulation results and finally we conclude the paper in Section 5.

2. Analytic solution for optimal charging current profile

We consider a simplified equivalent circuit model of the battery as shown in Fig. 1. The theory extends naturally to more complex models involving parallel RC elements (shown in Fig. 2), but analytical closed form solutions are not possible in the latter case. The model consists of a voltage source corresponding to the open-circuit voltage (OCV), which is dependent on the state of charge (SOC), and a resistance \( R_0 \). The SOC and OCV are represented respectively by \( s \) and \( V_0 \). The OCV is a nonlinear function of SOC and denoted by \( V_0(s[k]) \).

The state of charge is zero when the battery is totally discharged and it is one if it is completely charged. The sampling time is denoted by \( \Delta \) (in seconds). We assume that the initial and final SOC are known: \( s[0] = s_0, s[k_f] = s_k \), where \( k_f \) is the charging time. We also assume that the maximum allowed value of the terminal charging voltage is \( v_c \), that is,\( v[k] \leq v_c \) for all \( k \).

The SOC dynamics for the battery considering the foregoing model are as follows:

\[
\dot{s}[k + 1] = s[k] + c_h i[k] \\
\dot{v}[k] = \frac{v}{3600} c_{\text{batt}}
\]

where \( c_h \) (in 1/Ampere) is the parameter in Coulomb counting, given by

\[
c_h = \frac{\Delta}{3600 c_{\text{batt}}}
\]

where \( c_{\text{batt}} \) (in Ah) is the battery capacity, assumed to be known.

Let the objective function be a combination of TTC and EL. In other words,

\[
\tilde{J}_T = w_1 J_{\text{TTC}} + w_E J_{\text{EL}} = w_1 k_f \Delta + w_E \left( \sum_{k=0}^{k_f-1} R_0 i[k] \Delta \right)
\]

where \( J_{\text{TTC}} \) is the TTC cost function, \( J_{\text{EL}} \) is the EL cost function; \( w_1 \) and \( w_E \) are weights on the TTC and EL cost functions, respectively. The resistance of the battery, i.e., \( R_0 \), is assumed to be known.

The charging problem then could be formulated as follows:

Minimize \( J_{\text{EL}} \) subject to:

\[
\begin{align*}
\dot{s}[k + 1] &= s[k] + c_h i[k] \quad s[0] = s_0 \quad s[k_f] = s_k \\
V_0(s[k]) + R_0 i[k] &\leq v_{\text{max}} \quad (5) \\
i[k] &\leq i_{\text{max}} \quad (6)
\end{align*}
\]

It is important to note that only the ratio of weights affects the optimal current profile of \( i[k] \). Therefore, by dividing (3) by \( w_E \), we redefine the cost function as follows:

\[
J_{\text{EL}} = \tilde{J}_T / w_E = \rho_1 J_{\text{TTC}} + J_{\text{EL}} = \rho_1 k_f \Delta + \sum_{k=0}^{k_f-1} R_0 i[k] \Delta \quad (7)
\]

where \( \rho_1 = w_1 / w_E \). Also note that when the current is injected into the battery, the \( V_0 \) starts to increase and this, in turn, causes the terminal voltage to rise, until it reaches \( v_{\text{max}} \), which is the maximum allowed terminal voltage. During the whole charging process the current should not exceed \( i_{\text{max}} \), which is the maximum allowed charging current. In this paper, we use \( v_c \) for \( v_{\text{max}} \), where \( v_c \) is the voltage corresponding to SOC of 1; that is

\[
v_c = V_0(1)
\]
Assume that at time $k_1$, the terminal voltage $\nu[k_1]$ reaches $\nu_l$ and let us denote the state of charge at time $k_1$ as $s_1$. After time $k_1$, the terminal voltage should be fixed at the constant voltage (CV) $\nu_l$; hence, for $k = k_1, k_1 + 1, \ldots, k_f - 1$, the dynamics of the system are as follows:

\[
i[k] = \frac{1}{R_0}(\nu_l - V_0(s[k]))
\]

(9)

\[
s[k + 1] = s[k] + c_hi[k]
\]

(10)

\[
s[k_1] = s_1, \quad \dot{s}[k_f] = s_f
\]

(11)

Before going further, let us define a new equivalent problem as follows:

Minimize

\[
J_E = \rho_1\ell + J_E = \rho_1k_1\Delta + \sum_{k=0}^{k_f-1} R_0i^2[k]\Delta
\]

subject to:

\[
s[k + 1] = s[k] + c_hi[k], \quad s[0] = s_0, \quad s[k_1] = s_1
\]

(13)

This problem is in fact the minimization in the stage where the terminal voltage is below $\nu_l$ and therefore here the condition $V_0(s[k]) + R_0i[k] \leq \nu_l$ is not shown as we know that it holds.

In the first step, assuming $k_1$ is known, we find the optimal current profile that minimizes the energy losses for the constant voltages (CV) $\nu_l$. This optimal current profile allows us to calculate the corresponding energy losses as a function of $k_1$. In the second step, we use the partially optimized cost function $J_E = \rho_1k_1\Delta + J_E(k_1)$ and find the optimal value for $k_1$, say $k_*$. In the third step, we insert the optimal final time $k_1$ into the current $i^*[k_1]$ (obtained in step 1) to find the optimal current profile $i^*[k]$. The final result will be $J_E(k_1)$.

The first stage is formulated as follows:

Minimize

\[
J_E(k_1) = \sum_{k=0}^{k_f-1} R_0i^2[k]\Delta
\]

subject to:

\[
s[k + 1] = s[k] + c_hi[k], \quad s[0] = s_0, \quad s[k_1] = s_1
\]

(14)

The Hamiltonian function for this problem is

\[
H[k] = R_0i^2[k]\Delta + \lambda[k+1](s[k] + c_hi[k])
\]

(16)

The following equations must hold for the optimal solution [5]:

\[
\frac{\partial H[k]}{\partial s[k]} = 0
\]

(17)

\[
\lambda[k] = \frac{\partial H[k]}{\partial s[k]}
\]

(18)

\[
s[k + 1] = \frac{\partial H[k]}{\partial \lambda[k+1]}
\]

(19)

From (17) we have

\[
i^*[k] = \frac{c_h\lambda[k+1]}{2R_0}\Delta, \quad k = 0, 1, \ldots, k_1 - 1
\]

(20)

From (18), we can write

\[
\lambda[k] = \lambda[k + 1] \quad k = k_1 - 1, \ldots, 0 \quad \lambda[k_1] = \nu
\]

(21)

Note that equation (23) states that the optimal current is constant. From (19), we can write

\[
s[k + 1] = s[k] + c_hi[k]
\]

(22)

which is actually the dynamics of the system. Knowing the initial state of charge ($s_0$), and noting the optimal current in (23) is constant, we have

\[
s[k] = s_0 + c_h\sum_{k=0}^{k_f-1} i[k], \quad s[0] = s_0 - \frac{k^2c_h^2\nu}{2R_0}\Delta
\]

(23)

(25)

Since for $k = k_1$ we have $s[k_1] = s_1$, therefore

\[
s_1 = s_0 - \frac{k^2c_h^2\nu}{2R_0}\Delta
\]

(24)

(26)

Solving for $\nu$, we have

\[
\nu = \frac{-2R_0\Delta(s_1 - s_0)}{k_1c_h^2}
\]

(27)

Inserting (27) into (23), we have

\[
i^*[k] = \frac{s_1 - s_0}{k_1c_h}, \quad k = 0, 1, \ldots, k_1 - 1
\]

(28)

Inserting (28) into the cost function, the optimal cost function, given $k_1$ is:

\[
J_E(k_1) = \sum_{k=0}^{k_f-1} R_0\left(\frac{s_1 - s_0}{k_1c_h}\right)^2 \Delta = \frac{R_0\Delta(s_1 - s_0)^2}{k_1c_h^2}
\]

(29)

Now, consider step 2 and define the cost function as

\[
J_E = \rho_1k_1\Delta + J_E(k_1) = \rho_1k_1\Delta + \frac{R_0\Delta(s_1 - s_0)^2}{k_1c_h^2}
\]

(30)

To find the optimum $k_1$, the following relations should hold:
$J_{E^c}(k_1 - 1) \geq J_{E^c}(k_1)$

$J_{E^c}(k_1 + 1) \geq J_{E^c}(k_1)$

(31)

(32)

Inserting (30) into (31) and (32) we obtain two second-order equations in term of $k_t$. Solving these equations, we get $k_t$ and $k_t'$, respectively, for relations (31) and (32).

$k_t^* = \frac{\pm 1 + \sqrt{1 + \frac{4R_0(s_1 - s_0)^2}{\rho_2 c_2^2}}}{2}$

(33)

The optimum $k_t$ is ceil($k_t^*$) or floor($k_t^*$). Since $k_t - k_t' = 1$, we have ceil($k_t^*$) = floor($k_t^*$) = round((k_t + k_t')/2). Thus,

$k_t' = \text{round}\left(\sqrt{\frac{1}{4} + \frac{R_0(s_1 - s_0)^2}{\rho_2 c_2^2}}\right)$

(34)

A more convenient way is to treat $k_t$ in (30) as a continuous variable and take derivative of (30) with respect to $k_t$ as follows:

$\frac{\partial J_{E^c}(k_1)}{\partial k_t} = \rho_2 - \frac{R_0(s_1 - s_0)^2}{k_t^2 c_2^2} = 0$

(35)

$k_t^* = \frac{s_1 - s_0}{c_h} \sqrt{\frac{R_0}{\rho_2}}$

(36)

Note that if we neglect 1/4 in (34), the argument of the rounded function in (34) is exactly the same as the one in (36).

Step 3 involves inserting (36) into (28) to find the optimum current

$i_t[k] = \frac{s_1 - s_0}{k_t c_h} \sqrt{\frac{R_0}{\rho_2}}$

(37)

It is seen that the optimal current is constant and is a function of the weight on TTC and the series resistance. Therefore, the solution of optimal time-to-charge and energy loss (OTC) problem is a CC–CV profile with the current of the CC stage given by (37). Following the CC stage, from $k_t$ to $k_t'$, one has the CV stage where

$v_t[k] = v_c \quad k = k_t, \ldots , k_f$

(38)

To the best of our knowledge, this is the first time that it is proved that the well-known CC–CV charging profile is the optimal solution of a particular optimization problem, namely, the problem of minimizing the weighted sum of time-to-charge and energy loss.

In the sequel, this profile is referred to as OTC profile or OTC policy.

Before we close this section, we point out another way of solving the OTC problem of (12) and (13) by condensing (13) for all values of $k_t$ into a single condition. From (13) we can write

$i_t[k] = (s(k + 1) - s[k])/c_h$

(39)

Since (39) holds for $k = 0, 1, \ldots , k_t - 1$, and using the initial and end values of SOC from (13) we can write:

\[ \sum_{i=0}^{k_t-1} i_t = (s_1 - s_0)/c_h \]

(40)

Therefore, the problem of (12) and (13) is equivalent to a quadratic programming problem with the constraint in (40). In this way, we are dealing with currents $i_t[k]$ as our unknowns. It is easy to show that this results in the same solution as (37). This simplification of the dynamics of the system into the condensed condition of (40) will be useful in the next section where we derive an analytical solution when the cost function includes the summation of temperature rises as well.

It should be noted that the practical meaning of the parameters of optimization problem (e.g., $w_t$ and $w_o$ in (3) and $p_t$ in (12)) is to use them in an iterative design procedure to reach the desired performance. For example, if the maximum allowed energy loss is $E_{\text{max}}$ and the maximum acceptable time-to-charge is $TTC_{\text{max}}$, then in the design procedure, $w_t$ and $w_o$ should be selected inversely proportional to $TTC_{\text{max}}$ and $E_{\text{max}}$, respectively; that is $w_t \propto 1/TTC_{\text{max}}$, $w_o \propto 1/E_{\text{max}}$ and then iterate. Or equivalently, $p_t$ should be selected proportional to $E_{\text{max}}/TTC_{\text{max}}$; that is $p_t \propto E_{\text{max}}/TTC_{\text{max}}$ and then iterate on the proportionality factor. As $E_{\text{max}}/TTC_{\text{max}}$ is approximately, the allowable average power loss, $p_t$ should be selected proportional to allowable average power loss.

3. Optimal charging problem considering temperature

In this section, we will extend the cost function to include the battery temperature via temperature rise index (TRI, to be defined) as well as TTC and EL. To this end, we need a temperature model for the battery. Refs. [15] and [19] describe the temperature model of the battery as a linear system with two states, namely, $T_{\text{core}}$ and $T_{\text{air}}$, and reference [14] uses the nonlinear heat transfer equation with a single state. Simulations show that the dynamics of $T_{\text{air}}$ have negligible fluctuations around the ambient temperature. Therefore, the temperature model, considered below, can be simplified to the linear part of the heat transfer equation

\[ T[k + 1] = T[k] - a(T[k] - T_{\text{amb}}) + b_i^2[k] \]

(41)

where

\[ a = \frac{\Delta}{m_{\text{batt}} c_{\text{batt}} R_{\text{Eff}}} \]

(42)

is the cooling coefficient and

\[ b = \frac{R_0 \Delta}{m_{\text{batt}} c_{\text{batt}}} \]

(43)

Here $T$ is the battery core temperature in kelvin (K), $T_{\text{amb}}$ is the ambient temperature in K, $m_{\text{batt}}$ is the battery mass in kg, $c_{\text{batt}}$ is the heat capacity of the battery in J/(Kg K), and $R_{\text{Eff}}$ is the effective thermal resistance in K/W (kelvin/watt).

Defining temperature rise (TR) as $\bar{T}[k] = T[k] - T_{\text{amb}}$ and assuming $\bar{T}[0] = T_{\text{amb}}$, we can write

\[ \bar{T}[k + 1] = (1 - a) \bar{T}[k] + b_i^2[k], \quad \bar{T}[0] = 0 \]

(44)

The solution of (44) is

\[ \bar{T}[k] = b \sum_{i=0}^{k-1} (1 - a)^{k-1-i} i^2 \bar{T} \]

(45)

Equation (45) states that the temperature rise at any time is the integral of the square of current, from time zero up to that time with a “forgetting factor” of $(1 - a)$ and the scaling factor $b$.

Since $\bar{T}[k]$ is positive for any $k$, the cost function including TTC, EL and TR can be written as

\[ J_{\text{TR}} = \rho_d t + J_E + \rho_r H \]

(46)

where $J_E$ and $J_d$ are TTC and EL as before and $H$ is the temperature rise index (TRI) defined as follows:
\[ J_T = \Delta \sum_{k=0}^{k_1} \bar{T}[k] \]  

(47)

Since \( \bar{T}(0) = 0 \), the TRI can be written as

\[ J_T = \Delta \sum_{k=0}^{k_1-1} \bar{T}[k + 1] \]  

(48)

Using (45) and (48), we can write (46) as follows

\[ J_{\text{ET}} = \rho_i k_T \Delta + \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}} k^2 \bar{T}[k] \]  

(49)

which can be simplified as follows

\[ J_{\text{ET}} = \rho_i k_T \Delta + \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}} k \]  

(50)

Simplifying the inner summation and noting that \( b/a = R_0 R_{\text{Eff}} \), we can write

\[ J_{\text{ET}} = \rho_i k_T \Delta + \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}} [k]^2 \]  

(51)

\[ R_{\text{eq}}[k] = R_0 + R_T[k] \]  

(52)

\[ R_T[k] = \rho_T R_0 R_{\text{Eff}} [1 - (1 - a)^{k-1}] \]  

(53)

where \( R_T[k] \) is the heating equivalent resistance. Assume, as before, that at time \( k_1 \), the terminal voltage \( v \) reaches its maximum allowable value of \( v_{\text{max}} \), and SOC reaches \( s_1 \). Given \( s_1 \) and \( k_1 \), we can write the cost function as

\[ J(s_1, k_1) = \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}} [k]^2 \]  

(54)

Note that we discarded the contributions of \( \bar{T}[k], \ldots, \bar{T}[k_1-1] \), because when the terminal voltage reaches \( v_{\text{max}} \) the current is already determined by the constrained dynamics of the system in (9); we also discarded the contribution of \( k_1 \), i.e., \( \rho_i k_T \Delta \), because: firstly, \( k_1 \) is given; secondly, given \( s_1, k_1 - k_1 \) is also known, which means \( k_1 \) is known. An important point to note is that, while the upper bound of the summation in (54) is \( k_1 - 1 \), the formulation for \( R_{\text{eq}} \), i.e., (52), considers the effect of the whole charging time and it contains \( k \) rather than \( k_1 \).

Now, given \( s_1 \) and \( k_1 \), we can state the optimal charging problem as follows: Minimize (54) subject to (40), or equivalently

Minimize : \[ L = J(s_1, k_1) + \lambda \left( \sum_{l=0}^{k_1-1} i[l] - s_1 - s_0 \right) \]  

(55)

Taking the derivative of Lagrangian \( L \) with respect to \( i[k] \) for \( k = 0, 1, \ldots, k_1 - 1 \) and equating it to zero, we have:

\[ i[k] = \frac{\lambda}{2 R_{\text{eq}}[k] \Delta} = \frac{G_{\text{eq}}[k]}{2 \Delta} \]  

(56)

where \( G_{\text{eq}}[k] = 1/R_{\text{eq}}[k] \) is the conductance. Taking the derivative of \( L \) with respect to \( \lambda \), and using (56) we find the optimal current profile in the first stage as follows:

\[ i[k] = \frac{G_{\text{eq}}[k](s_1 - s_0)}{c_0 \sum_{k=0}^{k_1-1} G_{\text{eq}}[k]} \quad k = 0, 1, \ldots, k_1 - 1 \]  

(57)

We refer to the current profile in (57) as the optimal time-to-charge, energy losses and temperature rise (OTET) policy. Note that (57) is similar to what we obtained for the OTE case. In particular, if \( \rho_T = 0 \), then (57) will be the same as (28). Also, comparing (56) with the OTE case and noting that for \( k = 0, 1, \ldots, k_1 - 1 \) we can use the approximation of \( R_{\text{eq}}[k] \approx R_0(1 + \rho_T R_{\text{Eff}}) \), analogous to the optimal current profile of (37), we can write

\[ i[k] = \frac{\rho_T}{R_0(1 + \rho_T R_{\text{Eff}})} \quad k = 0, 1, \ldots, k_1 - 1 \]  

(58)

We refer to the current profile of (58) as the near-optimal time-to-charge, energy loss and temperature rise (NOTET) policy.

4. Simulations

In this section, we present simulations based on the theoretical foundations of the previous section.

4.1. Verification of the optimal solution

Here, we apply different levels of current and the simulation is run until the terminal voltage reaches \( v_T \) and after that a constant voltage of \( v_{\text{const}} \) is applied until the battery is charged to \( s_{\text{max}} \). Five different current profiles are chosen including the optimal current profile (Fig. 3). The optimal current profile as mentioned before has the value of \( \sqrt{\rho_i/R_0} \) in the CC stage. The battery parameters of Nokia BP-4L (Cell \#3) are used, as mentioned in Ref. [3].

\[ z(s) \triangleq E + s(1 - 2E) \]  

(59)

\[ OCV(z) = K_0 + K_1 z^{-1} + K_2 z^{-2} + K_3 z^{-3} + K_4 z^{-4} + K_5 z + K_6 \ln(z) + K_7 \ln(1 - z) \]  

(60)

and \( E = 0.15 \). Fig. 3 shows the current profiles with different levels of current in the CC stage. As seen from Fig. 3, at lower levels of current, the CC stage will take a longer time and the terminal voltage reaches the threshold voltage of \( v_T \) at a later time. At higher levels of current, however, the OCV grows more rapidly. As the terminal voltage is \( v[k] = V_0(s[k]) + R_0[i[k]] \), at higher levels of current the threshold voltage of \( v_T \) is reached in a shorter time.

Fig. 4(a) shows the cost function \( J_{\text{ET}} \) for the five current profiles of Fig. 3 and Fig. 4(b) shows the corresponding current levels in the CC stage. It is seen that the optimal current profile (i.e., profile 3) has the lowest cost function. Deviating from this profile, either by increasing or decreasing the current in the CC stage, results in an increase in the cost function. For the lower current levels (profiles 1–2), the rise in the cost function is due to a rise in TTC and for higher current levels (profiles 4–5) the rise in cost function is due to rise in EL.
4.2. Effect of weights

In this subsection, we use different cost functions and find the corresponding optimal profiles. Different values of $\rho_t$ from 0.1 to 0.5 are chosen. Figs. 5–7, respectively, show the profiles of current, state of charge and terminal voltage. Fig. 5 shows that low values of $\rho_t$ result in low values of current in the CC stage. In other words, a low $\rho_t$ puts less emphasis on charging time and more emphasis on the energy losses; hence, it results in low level of current which provides low energy losses. On the other hand, by increasing $\rho_t$, more emphasis is placed on the charging time. Consequently, the level of current is increased proportionally to $\sqrt{\rho_t}$ to reduce the TTC.
Fig. 6 shows the state-of-charge profiles for different values of $\rho_t$. It is seen that by increasing $\rho_t$, more emphasis is placed on charging time and the SOC reaches the final value in a shorter time.

Fig. 7 shows the terminal voltage profiles for different values of $\rho_t$. Note that for low values of $\rho_t$, as the emphasis on energy loss is high, the corresponding current level in the CC stage is low, and consequently, the terminal voltage reaches the threshold value of $v_c$ at a later time. Hence, the duration of the CC stage is high and the charging time is high as well.

Fig. 8 shows the time-to-charge, energy losses and efficiency as functions of $\rho_t$. As expected, high values of $\rho_t$ result in lower TTC. The low TTC, however, is obtained by increasing the current level; as EL is proportional to the square of current, thus the high values of $\rho_t$ result in high values of EL. The high values of EL mean that a higher fraction of input power is wasted; hence it is equivalent to a decline in efficiency.

Fig. 9 shows the time-to-charge versus efficiency (ratio of effective to total energy) curve. TTC and efficiency are two counteracting objectives. For low values of $\rho_t$, as less emphasis is put on TTC, the TTC is high; however, high TTC is the result of low current.

Please cite this article in press as: A. Abdollahi, et al., Journal of Power Sources (2015), http://dx.doi.org/10.1016/j.jpowsour.2015.02.075
values, which incur low energy losses and hence higher efficiency. For example at $\rho_t = 0.1$, the TTC is 195 min, but the efficiency is as high as 95.82%. On the other hand, for high values of $\rho_t$ which place more emphasis on TTC, the TTC is reduced dramatically; however, low TTC is achieved by increasing the current values, which results in high energy losses and hence lower efficiency. For example, at $\rho_t = 0.5$, the TTC is as low as 148 min, but the efficiency decreases to 92.87%.

### 4.3. Temperature effect

In this section, we consider the effect of temperature rise index (TRI) on optimal charging. The cost function is a weighted sum of TTC (seconds), EL (Joules) and TRI (Kelvin seconds), given by

$$ J_{\text{ET}} = \rho_T \times \text{TTC} + \rho_T \times \text{EL} + \rho_T \times \text{TRI} $$

We used two sets of thermal parameters, shown in Table 1. Parameter set “A” is adopted from Ref. [19]. Parameter set “B” is a scaled version of parameter set “A” with $m_{\text{batter}}$ set as the weight of Nokia BP-4L. For each set of thermal parameters (“A” or “B”), the weights of the cost function are chosen as $\rho_t = 1$, $\rho_T = 1$ and $\rho_T = 1$, $\rho_T = 4$. Three schemes are used: OTE (equations (37) and (38)), OEt (equations (38) and (57)), and NOtET (equations (38) and (58)). The cost function in (61) or (46) is calculated for the three schemes. Table 2 shows the cost functions of the three schemes for different weightings. As seen from this table, the cost function for the OTE has the highest value. Also the difference between the cost function of OEt and NOtET is negligible with the OEt being slightly smaller when thermal parameter set “A” is used. For thermal parameter set of “B”, there is visually no difference between NOtET and OEt. Due to this negligible difference in the cost function and also since the calculation of NOtET profile is much easier than that of the OEt, it is reasonable to use NOtET rather than the OEt scheme. Also note that the weight on TRI results in a reduction of current, as can be seen from Fig. 10. This reduction in current level results in a lower temperature rise (see Fig. 11). In other words, energy losses with $R_{\text{eq}}$ instead of $R_0$ can be used as a surrogate cost function for the TRI.

### 4.4. Analysis of different commercial batteries

In this section, we discuss the behavior of different commercial batteries. The parameters of the investigated batteries are given in

![Fig. 9. TTC versus efficiency.](image_url)

![Fig. 10. Current profiles for $\rho_t = 1, \rho_T = 1$ and temperature parameter set “A”.](image_url)
the equivalent electrical circuit parameters given in Table 3 are for model III (see Fig. 2). In simulations, we use the summation of “R₀ + R₁” of model III as an estimate of resistance R₀ in model I. The batteries are Samsung EB575152 (four cells), Samsung EB504465 (four cells), Samsung AB463651 (two cells), Nokia BP-4L (four cells), LG LGIP (two cells).

Next, we apply the OtE algorithm with ρᵣ = 0.5 to 16 commercial batteries to investigate the times-to-charge and efficiencies of the batteries. The parameters of the batteries, i.e., the electrical parameters of the models in Figs. 1 and 2, and the parameters of the OCV function in (60), were calculated using experimental data and by applying the BFG algorithms in Refs. [3, 4]. These parameters are listed in Tables 3 and 4 in the Appendix. Fig. 12 shows the TTC versus efficiency for different types of batteries. Among all batteries, Sam-EB575152 (Cell 3) has the lowest efficiency (90.73%). This can be attributed to the high resistance of this battery, which might be due to aging. Sam-EB504465 (Cell 4) has the highest TTC (102 min) and Nokia BP-4L (Cell 4) has the highest efficiency. Note
that the cells of the same battery are close to each other in terms of efficiency and TTC. Considering all the cells of a battery, we can say that LG-LGIP cells (circle markers) have the highest efficiency (91.4%). Fig. 13 shows the cost function values of $J_E = \rho_1 + J_H$. When TTC is weighted with weight value of $\rho_1 = 0.5$, Sam-
EB575152 (Cell 2) has the best performance.

5. Conclusion

The optimal charging problem involving a weighted combination of time-to-charge (TTC), energy loss (EL) and temperature rise index (TRI) was considered. The optimal TTC and EL solution (OtE) is found to be the well-known CC–CV strategy with the value of current in the CC stage being a function of the ratio of weighting on TTC and EL and also the resistance of battery. To the best of our knowledge, this is the first time that it is proved that the well-known CC–CV charging profile is the optimal solution of a particular optimization problem, namely, the problem of minimizing the weighted sum of time-to-charge and energy loss. In addition, an analytical solution for the optimal TTC, EL and TRI, referred to as OtET, was developed. Due to similarity of the structure of the OtE and OtET solutions, a near-optimal version of OtET was developed (referred to as NOtET). The NOtET is a CC–CV strategy with the value of current in the CC stage being a function of the ratio of weighting on TTC and EL, the resistance of the battery and the effective thermal resistance. A number of simulations were conducted to evaluate the effect of weighting parameters. Finally, extensive results on industrial batteries from LG, Nokia and Sam-
sung were presented.

Acknowledgments

The work reported in this paper was partially supported by NSF grants ECCS-0931956 (NSF CPS), ECCS-1001445 (NSF GOALI), CCF-1331850 (NSF CyberSEES), ARO grant W911NF-10-1-0369, and ONR grant N00014-10-1-0029. We thank NSF, ARO and ONR for their support of this work. Y. Bar-Shalom was partially supported by Grant ARO W911NF-10-1-0369. Any opinions expressed in this paper are solely those of the authors and do not represent those of the sponsors.

Nomenclature

$\alpha$, $\beta$ coefficients in temperature model, (42,43) $C_{\text{batt}}$ capacity, (2) $C_h$ parameter in coulomb counting, (1) $C_{\text{batt}}$ heat capacity of the battery in J/(kg·K), (42) $C_0$ constant current $C_V$ constant voltage $\Delta$ sampling time, (2) $E_L$ energy loss $\eta$ charging efficiency $G_{eq}[k]$ conductance equal to reciprocal of $R_{eq}[k]$, (56) $H[k]$ Hamiltonian function, (16) $i[k]$ charging current, (1) $f[k]$ optimal current, (28) $f'[k|k_1]$ optimal current given $k_1$ $J_E$ minimum energy loss cost function as a function of $k_1$ (given $k_1$), (29) $J_E(k_1)$ optimized cost function $k$ time index, (1) $k_1$ optimal final time for constant-current charging stage, (34) $k_f$ final value of time index, (3) $L$ Lagrangian function, (55) $m_{\text{batt}}$ battery mass in kg, (42) $\tilde{N}_{\text{OE}}$ near optimal TTC, energy loss, and temperature rise index, (46) $OCV/z$ open circuit voltage, (60) $O_{\text{TE}}$ optimal time-to-charge and energy loss $O_{\text{TE}}$ optimal TTC, energy loss, and temperature rise resistance in model I of battery as shown in Fig. 1, (3) $R_0$ effective thermal resistance in K/W, (53) $R_{eq}[k]$ heating equivalent resistance added to the battery resistance, (52) $R_{eq}$ ratio of weight of TTC cost function to weight of EL cost function, (12) $s$ state of charge, (1) $s_0$ initial SOC, (4) $s_{1}$ state of charge when terminal voltage reaches maximum allowed value, (11) $s_{nf}$ final SOC, (4) $T$ battery core temperature in kelvin, (41) $T_{\text{ambient}}$ ambient temperature in kelvin, (41) $T[k]$ temperature rise, (44) $T_{\tilde{T}[k]}$ time-to-charge $v[k]$ terminal voltage of battery, (38) $V_0$ open-circuit voltage, (5) $V_{E}$ voltage corresponding to SOC of 1, (8) $w_{E}$ weight for energy loss cost function, (3) $w_{T}$ weight for time to charge cost function, (3) $z(s)$ parameter relating OCV to SOC, (59)

Appendix

The following tables show the parameters of the equivalent electrical circuit model III for different commercial batteries.

<table>
<thead>
<tr>
<th>Make</th>
<th>Model</th>
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<th>$R_0$ (mΩ)</th>
<th>$R_1$ (mΩ)</th>
<th>$C_1$ (F)</th>
<th>$a$</th>
<th>$C_{\text{batt}}$ (Ah)</th>
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Table 4
OCV parameters for commercial batteries.

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References