### Optimal battery charging, Part I: Minimizing time-to-charge, energy loss, and temperature rise for OCV-resistance battery model

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### Abstract

In this paper we present a closed-form solution to the problem of optimally charging a Li-ion battery. A combination of three cost functions is considered as the objective function: time-to-charge (TTC), energy losses (EL), and a temperature rise index (TRI). First, we consider the cost function of the optimization problem as a weighted sum of TTC and EL. We show that the optimal charging strategy in this case is the well-known Constant CurrentConstant Voltage (CCeCV) policy with the value of the current in the CC stage being a function of the ratio of weighting on TTC and EL and of the resistance of the battery. Then, the views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.

### Subject Terms

Battery charging, Optimal charging, Time to charge, Open circuit voltage, State of charge

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Optimal battery charging: Minimizing time-to-charge, energy loss, and temperature rise for OCV-resistance battery model

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HIGHLIGHTS

• Closed form solution for minimizing weighted sum of time-to-charge and energy loss.
• Semi-closed form solution by adding the temperature rise index to the cost function.
• Approximating temperature rise effect as a constant heating equivalent resistance.
• Analysis of efficiency performance of commercial batteries.

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1. Introduction

Battery charging is a problem of significant interest, especially as the battery-dependent smart devices proliferate. The literature abounds with different strategies for charging batteries. Among the traditional methods of charging, the simplest is the constant trickle current charge strategy, which, due to its low charging current, requires a long charging time (around 10 h) [9]; constant current strategy with higher rates of current requires shorter charging time.

The most widely-used traditional strategy is the constant-current constant-voltage (CC–CV) [9] strategy, in which a constant current is applied to the battery until the terminal voltage reaches a specified value, and afterward the charging current decreases by applying a constant voltage to the terminals of the battery. In Refs. [21,22], a multi-step constant-current charging is devised for shortening the charging time and prolonging the cycle life of the battery. Using orthogonal arrays, Taguchi-based methods for battery charging [12,23] present a systematic method to find the optimal solution with guidelines for choosing the design parameters. In Ref. [13], a boost charging strategy is proposed by applying very high currents to close-to-fully discharged batteries. In pulse-charging methods [6,7,17,20,27], the battery is exposed to very short rest or even deliberate discharging periods during the charging process. Soft-computing approaches are also used in the optimization of battery charging profile. In Ref. [24], the charging...
problem is viewed as an optimization problem with the objective function of maximizing the charge within 30 min using a multi-stage constant current charging algorithm whose optimal solution is obtained via an ant-colony approach. In Ref. [25], a universal voltage protocol is proposed to improve charging efficiency and cycle life by applying a charging profile depending on the state-of-health (SOH) of the battery, using SOH estimation approaches [26] in the optimization process. Recently, in Ref. [11], battery charging is considered as an optimization problem with cost function of time-to-charge and energy loss (as we do in this paper), but they have not solved the problem analytically; rather they have presented a numerical solution to the problem. Other approaches, such as genetic algorithm and neural network based strategies [16], data mining [2,10], and Grey-predicted charging system [8] have also been used for charging batteries.

In this paper we look at the charging problem from a fresh perspective using optimal control theory, and our goal is to find the optimal current profile that minimizes a specific cost function. In this sense, different objectives may be embedded in the cost function. One obvious cost function is the time-to-charge (TTC). We prefer to minimize the charging time as much as possible, as TTC reduction contributes to user satisfaction. Another important objective is the energy loss (EL) during charging. Reducing the energy loss increases the charging efficiency. In this paper, first we use an integrated cost function that includes both the TTC and EL. Then, we also include the effects of temperature into account, and the cost function is selected as a linear combination of three criteria: time-to-charge, energy loss, and temperature rise index (TRI). In both cases, analytical solutions of the optimal charging problem are derived.

The paper is organized as follows. In Section 2, we derive an analytical solution for the optimal charging current profile to minimize TTC and EL. In Section 3, we extend this approach to the case where temperature rise is considered as well. Section 4 is devoted to simulation results and finally we conclude the paper in Section 5.

2. Analytic solution for optimal charging current profile

We consider a simplified equivalent circuit model of the battery as shown in Fig. 1. The theory extends naturally to more complex models involving parallel RC elements (shown in Fig. 2), but analytical closed form solutions are not possible in the latter case. The model consists of a voltage source corresponding to the open-circuit voltage (OCV), which is dependent on the state of charge (SOC), and a resistance $R_0$. The SOC and OCV are represented respectively by $s$ and $V_0$. The OCV is a nonlinear function of SOC and is denoted by $V_0(s[k])$.

The state of charge is zero when the battery is totally discharged and it is one if it is completely charged. The sampling time is denoted by $\Delta$ (in seconds). We assume that the initial and final SOC are known: $s[0] = s_0$, $s[k_f] = s_f$, where $k_f$ is the charging time. We also assume that the maximum allowed value of the terminal charging voltage is $v_c$, that is, $v[k] < v_c$ for all $k$.

The SOC dynamics for the battery considering the foregoing model are as follows:

$$s[k + 1] = s[k] + c_h i[k]$$

where $c_h$ (in 1/Ampere) is the parameter in Coulomb counting, given by

$$c_h = \frac{\Delta}{3600 c_{bat}}$$

where $c_{bat}$ (in Ah) is the battery capacity, assumed to be known.

Let the objective function be a combination of TTC and EL. In other words,

$$J_{IE} = w_t J_t + w_E J_E = w_t k_f \Delta + w_E \sum_{k=0}^{k_f-1} R_0 i^2[k] \Delta$$

where $J_t$ is the TTC cost function, $J_E$ is the EL cost function; $w_t$ and $w_E$ are weights on the TTC and EL cost functions, respectively. The resistance of the battery, i.e., $R_0$, is assumed to be known.

The charging problem then could be formulated as follows:

Minimize $J_{IE}$ subject to:

$$s[k + 1] = s[k] + c_h i[k] \quad s[0] = s_0 \quad s[k_f] = s_f$$

$$V_0(s[k]) + R_0 i[k] \leq v_{max}$$

$$i[k] \leq i_{max}$$

It is important to note that only the ratio of weights affects the optimal current profile of $i[k]$. Therefore, by dividing (3) by $w_E$, we redefine the cost function as follows:

$$J_{IE} = J_{IE} / w_E = \rho_t J_t + J_E = \rho_t k_f \Delta + \sum_{k=0}^{k_f-1} R_0 i^2[k] \Delta$$

where $\rho_t = w_t / w_E$. Also note that when the current is injected into the battery, the $V_0$ starts to increase and this, in turn, causes the terminal voltage to rise, until it reaches $v_{max}$, which is the maximum allowed terminal voltage. During the whole charging process the current should not exceed $i_{max}$, which is the maximum allowed charging current. In this paper, we use $v_c$ for $v_{max}$, where $v_c$ is the voltage corresponding to SOC of 1; that is

$$v_c = V_0(1)$$
Assume that at time $k_1$, the terminal voltage $v[k_1]$ reaches $v_t$ and let us denote the state of charge at time $k_1$ as $s_1$. After time $k_1$, the terminal voltage should be fixed at the constant voltage (CV) $v_t$; hence, for $k = k_1, k_1 + 1, \ldots, k_f - 1$, the dynamics of the system are as follows:

$$i[k] = \frac{1}{R_0} (v_t - V_0(s[k])) \quad (9)$$

$$s[k+1] = s[k] + c_h i[k] \quad (10)$$

$$s[k_1] = s_1, \quad s[k_f] = s_f \quad (11)$$

Before going further, let us define a new equivalent problem as follows:

Minimize

$$J_{IE} = \rho_1 H + J_E = \rho_1 k_1 \Delta + \sum_{k=0}^{k_1-1} R_0 i^2[k] \Delta \quad (12)$$

subject to:

$$s[k+1] = s[k] + c_h i[k] \quad (13)$$

This problem is in fact the minimization in the stage where the terminal voltage is below $v_t$ and therefore here the condition $V_0(s[k]) + R_0 i[k] \leq v_t$ is not shown as we know that it holds.

Inspired by Refs. [18] and [1], we solve the problem in three steps as described below:

1. Given $k_1$ (when the terminal voltage constraint becomes active), find the optimal current profile that minimizes the energy losses, and calculate the corresponding energy losses as a function of $k_1$.

2. Generate a new equivalent cost function $J_{IE}$ consisting of the weighted TTC plus the $k_1$-dependent minimum energy loss obtained in step 1, and find the optimal $k_1$ based on this cost function.

3. Given the optimal $k_1$ from step 2, evaluate the optimal current obtained in step 1.

In the first step, assuming $k_1$ is known, we find the optimal current $i^*[k|k_1]$ that minimizes the energy loss. Having this optimal current profile, we can calculate the minimum EL cost function $J_{IE}(k_1)$, which is a function of $k_1$. In the second step, we use the partially optimized cost function $J_{IE} = \rho_1 k_1 \Delta + J_E(k_1)$ and we find the optimum value for $k_1$, say $k_1^*$. In the third step, we insert the optimal final time $k_1^*$ into the current $i^*[k|k_1]$ (obtained in step 1) to find the optimal current $i^*[k]$. The final result will be $J_{IE}(k_1^*)$.

The first stage is formulated as follows:

Minimize

$$J_{IE}(k_1) = \sum_{k=0}^{k_1-1} R_0 i^2[k] \Delta \quad (14)$$

subject to:

$$s[k+1] = s[k] + c_h i[k] \quad s[0] = s_0 \quad s[k_1] = s_1 \quad (15)$$

The Hamiltonian function for this problem is

$$H[k] = R_0 i^2[k] \Delta + \lambda[k+1] (s[k] + c_h i[k]) \quad (16)$$

The following equations must hold for the optimal solution [5]:

$$\frac{\partial H[k]}{\partial i[k]} = 0 \quad (17)$$

$$\lambda[k] = \frac{\partial H[k]}{\partial s[k]} \quad (18)$$

$$s[k+1] = \frac{\partial H[k]}{\partial \lambda[k+1]} \quad (19)$$

From (17) we have

$$\lambda[k] = \lambda[k+1] k = k_1 - 1, \ldots, 0 \quad \lambda[k_1] = \nu \quad (21)$$

where $\nu$ is the Lagrange multiplier associated with the constraint $s[k_1] = s_1$. Equation (21) implies that all co-states are equal; therefore, we can write

$$\lambda[k] = \nu \quad k = 0, 1, \ldots, k_1 \quad (22)$$

Based on (22), equation (20) can be written as

$$i^*[k] = -\frac{c_h \nu}{2 R_0 \Delta} \quad k = 0, 1, \ldots, k_1 - 1 \quad (23)$$

Note that equation (23) states that the optimal current is constant. From (19), we can write

$$s[k+1] = s[k] + c_h i[k] \quad (24)$$

which is actually the dynamics of the system. Knowing the initial state of charge ($s_0$), and noting the optimal current in (23) is constant, we have

$$s[k] = s_0 + c_h \sum_{l=0}^{k-1} i[l] = s_0 - \frac{k c_h^2 \nu}{2 R_0 \Delta} \quad (25)$$

Since for $k = k_1$ we have $s[k_1] = s_1$, therefore

$$s_1 = s_0 - \frac{k_1 c_h^2 \nu}{2 R_0 \Delta} \quad (26)$$

Solving for $\nu$, we have

$$\nu = -\frac{2 R_0 \Delta (s_1 - s_0)}{k_1 c_h^2} \quad (27)$$

Inserting (27) into (23), we have

$$i^*[k] = \frac{s_1 - s_0}{k_1 c_h^2} \quad k = 0, 1, \ldots, k_1 - 1 \quad (28)$$

Inserting (28) into the cost function, the optimal cost function, given $k_1$ is:

$$J_{IE}(k_1) = \sum_{k=0}^{k_1-1} R_0 \left( \frac{s_1 - s_0}{k_1 c_h^2} \right) \Delta = \frac{R_0 \Delta (s_1 - s_0)^2}{k_1 c_h^2} \quad (29)$$

Now, consider step 2 and define the cost function as

$$J_{IE} = \rho_1 k_1 \Delta + J_{IE}(k_1) = \rho_1 k_1 \Delta + \frac{R_0 \Delta (s_1 - s_0)^2}{k_1 c_h^2} \quad (30)$$

To find the optimum $k_1$, the following relations should hold:
simplification of the dynamics of the system into the condensed condition of (40) will be useful in the next section where we derive an analytical solution when the cost function includes the summation of temperature rises as well.

It should be noted that the practical meaning of the parameters of optimization problem (e.g., \( w_k \) and \( w_E \) in (33) and \( \rho_i \) in (12)) is to use them in an iterative design procedure to reach the desired performance. For example, if the maximum allowed energy loss is \( E_{\text{max}} \) and the maximum acceptable time-to-charge is \( T_{\text{TTC}} \), then in the design procedure, \( w_k \) and \( w_E \) should be selected inversely proportional to \( T_{\text{TTC}} \) and \( E_{\text{max}} \), respectively; that is \( w_k \approx 1/T_{\text{TTC}} \) and \( w_E \approx 1/E_{\text{max}} \) and then iterate. Or equivalently, \( \rho_i \) should be selected proportional to \( E_{\text{max}}/T_{\text{TTC}} \); that is \( \rho_i \approx E_{\text{max}}/T_{\text{TTC}} \) and then iterate on the proportionality factor. As \( E_{\text{max}}/T_{\text{TTC}} \) is approximately, the allowable average power loss, \( \rho_i \) should be selected proportional to allowable average power loss.

3. Optimal charging problem considering temperature

In this section, we will extend the cost function to include the battery temperature via temperature rise index (TRI, to be defined) as well as TTC and EL. To this end, we need a temperature model for the battery. Refs [15] and [19] describe the temperature model of the battery as a linear system with two states, namely, \( T_{\text{core}} \) and \( T_{\text{air}} \), and reference [14] uses the nonlinear heat transfer equation with a single state. Simulations show that the dynamics of \( T_{\text{air}} \) have negligible fluctuations around the ambient temperature. Therefore, the temperature model, considered below, can be simplified to the linear part of the heat transfer equation:

\[
T[k+1] = T[k] - a(T[k] - T_{\text{amb}}) + b\beta^2[k]
\]

where

\[
a = \frac{\Delta}{m_{\text{batt}}C_{\text{h,batt}}E_{\text{Eff}}}
\]

is the cooling coefficient and

\[
b = \frac{R_0\Delta}{m_{\text{batt}}C_{\text{h,batt}}}
\]

Here \( T \) is the battery core temperature in kelvin (K), \( T_{\text{amb}} \) is the ambient temperature in K, \( m_{\text{batt}} \) is the battery mass in kg, \( C_{\text{h,batt}} \) is the heat capacity of the battery in J/(Kg K), and \( E_{\text{Eff}} \) is the effective thermal resistance in K/W (kelvin/watt).

Defining temperature rise (TR) as \( \bar{T}[k] = T[k] - T_{\text{amb}} \) and assuming \( T[0] = T_{\text{amb}} \), we can write

\[
\bar{T}[k+1] = (1-a)\bar{T}[k] + b\beta^2[k], \quad \bar{T}[0] = 0
\]

The solution of (44) is

\[
\bar{T}[k] = b\sum_{l=0}^{k-1} (1-a)^{k-1-l}\beta^2[l]
\]

Equation (45) states that the temperature rise at any time is the integral of the square of current, from time zero up to that time with a “forgetting factor” of \((1-a)\) and the scaling factor \(b\).

Since \( \bar{T}[k] \) is positive for any \( k \), the cost function including TTC, EL and TR can be written as

\[
J_{\text{opt}} = \rho_{\text{d}}t + J_{\text{E}} + \rho_{\text{f}}t
\]

where \( J_{\text{d}} \) and \( J_{\text{E}} \) are TTC and EL as before and \( J_{\text{f}} \) is the temperature rise index (TRI) defined as follows:
\[ J_T = \Delta \sum_{k=0}^{k_1-1} T[k] \]  \quad (47)

Since \( T[0] = 0 \), the TRI can be written as
\[ J_T = \Delta \sum_{k=0}^{k_1-1} T[k+1] \]  \quad (48)

Using (45) and (48), we can write (46) as follows
\[ J_{\text{ET}} = \rho_t k_f \Delta + \sum_{k=0}^{k_1-1} R_T^2 |k| \Delta \]  \quad (49)

\[ + \rho_f b \Delta \sum_{k=0}^{k_1-1} \sum_{l=0}^{k_1-1} (1-a)^{k_1-l} \frac{1}{|l|} \frac{1}{\Delta} \]

which can be simplified as follows
\[ J_{\text{ET}} = \rho_t k_f \Delta + \Delta \sum_{k=0}^{k_1-1} \left( R_0 + \rho_f b \sum_{l=0}^{k_1-1} (1-a)^{l} s \right)^2 |k| \]  \quad (50)

Simplifying the inner summation and noting that \( b/a = R_0 R_{\text{Eff}} \), we can write
\[ J_{\text{ET}} = \rho_t k_f \Delta + \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}}[k]^2 |k| \]  \quad (51)

\[ R_{\text{eq}}[k] = R_0 + R_T[k] \]  \quad (52)

\[ R_T[k] = \rho_f R_0 R_{\text{Eff}} \left( 1 - (1-a)^{k} \right) \]  \quad (53)

where \( R_T[k] \) is the equivalent resistance. Assume, as before, that at time \( k_1 \), the terminal voltage \( V \) reaches its maximum allowable value of \( V_{\text{cc}} \), and SOC reaches \( s_1 \). Given \( s_1 \) and \( k_1 \), we can write the cost function as
\[ J(s_{1}, k_{1}) = \Delta \sum_{k=0}^{k_1-1} R_{\text{eq}}[k] \frac{1}{|k|} \]  \quad (54)

Note that we discarded the contributions of \( il[k], \ldots, il[k_1-1] \), because when the terminal voltage reaches \( V_c \) the current is already determined by the constrained dynamics of the system in (9); we also discarded the contribution of \( k_f \), i.e. \( \rho_t k_f \Delta \), because: firstly, \( k_f \) is given; secondly, given \( s_1 \), \( k_f - k_1 \) is also known, which means \( k_f \) is known. An important point to note is that, while the upper bound of the summation in (54) is \( k_1 - 1 \), the formulation for \( R_{\text{eq}} \), i.e., (52), considers the effect of the whole charging time and it contains \( k_1 \) rather than \( k_1 - 1 \).

Now, given \( s_1 \) and \( k_1 \), we can state the optimal charging problem as follows: Minimize (54) subject to (40), or equivalently
\[ \text{Minimize : } L = J(s_1, k_1) + \lambda \left( \sum_{l=0}^{k_1-1} il[l] - \frac{S_1 - S_0}{\epsilon_h} \right) \]  \quad (55)

Taking the derivative of Lagrangian \( L \) with respect to \( il[k] \) for \( k = 0, 1, \ldots, k_1 - 1 \) and equating it to zero, we have:
\[ \frac{\lambda}{2 R_{\text{eq}}[k] \Delta} = \frac{\lambda G_{\text{eq}}[k]}{2 \Delta} \]  \quad (56)

where \( G_{\text{eq}}[k] = 1/R_{\text{eq}}[k] \) is the conductance. Taking the derivative of \( L \) with respect to \( \lambda \), and using (56) we find the optimal current profile in the first stage as follows:
\[ i^*[k] = \frac{G_{\text{eq}}[k](S_1 - S_0)}{c_0 \sum_{k=0}^{k_1-1} G_{\text{eq}}[k]} \quad k = 0, 1, \ldots, k_1 - 1 \]  \quad (57)

We refer to the current profile in (57) as the optimal time-to-charge, energy loss, and temperature rise (OTET) policy. Note that (57) is similar to what we obtained for the OtE case. In particular, if \( \rho_f = 0 \), then (57) will be the same as (28). Also, comparing (56) with the OtE case and noting that for \( k = 0, 1, \ldots, k_1 - 1 \) we can use the approximation of \( R_{\text{eq}}[k] \approx R_0(1 + \rho_f R_{\text{Eff}}) \), analogous to the optimal current profile of (37), we can write
\[ i^*[k] = \frac{\rho_t}{R_0(1 + \rho_f R_{\text{Eff}})} \quad k = 0, 1, \ldots, k_1 - 1 \]  \quad (58)

We refer to the current profile of (58) as the near-optimal time-to-charge, energy loss, and temperature rise (NOTET) policy.

4. Simulations

In this section, we present simulations based on the theoretical foundations of the previous section.

4.1. Verification of the optimal solution

Here, we apply different levels of current and the simulation is run until the terminal voltage reaches \( V_c \) and after that a constant voltage of \( V_c \) is applied until the battery is charged to \( S_{0} \). Five different current profiles are chosen including the optimal current profile (Fig. 3). The optimal current profile as mentioned before has the value of \( \sqrt{\rho_t/k_0} \) in the CC stage. The battery parameters of Nokia BP-4L (Cell #3), given in the appendix, are used. The following simulation parameters are used: \( \rho = 1 \), \( \Delta = 1(s), S_0 = 0, S_{0} = 1 \).

The appendix also shows the parameters of the OCV curve (calculated based on [3]). The OCV is a function of SOC as in Ref. [3].

\[ z(s) = E + s(1 - 2E) \]  \quad (59)

\[ OCV(z) = K_0 + K_1 z^{-1} + K_2 z^{-2} + K_3 z^{-3} + K_4 z^{-4} + K_5 z + K_6 \ln(z) + K_7 \ln(1-z) \]  \quad (60)

and \( E = 0.15 \). Fig. 3 shows the current profiles with different levels of current in the CC stage. As seen from Fig. 3, at lower levels of current, the CC stage will take a longer time and the terminal voltage reaches the threshold voltage of \( V_c \) at a later time. At higher levels of current, however, the OCV grows more rapidly. As the terminal voltage is \( v[k] = V_0(s[k]) + R_0[k]i[k] \), at higher levels of current, the threshold voltage of \( v_c \) is reached in a shorter time.

Fig. 4(a) shows the cost function \( J_{\text{ET}} \) for the five current profiles of Fig. 3 and Fig. 4(b) shows the corresponding current levels in the CC stage. It is seen that the optimal current profile (i.e., profile 3) has the lowest cost function. Deviating from this profile, either by increasing or decreasing the current in the CC stage, results in an increase in the cost function. For the lower current levels (profiles 1–2), the rise in the cost function is due to a rise in TTC and for higher current levels (profiles 4–5) the rise in cost function is due to rise in EL.

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4.2. Effect of weights

In this subsection, we use different cost functions and find the corresponding optimal profiles. Different values of $\rho_t$ from 0.1 to 0.5 are chosen. Figs. 5–7, respectively, show the profiles of current, state of charge and terminal voltage. Fig. 5 shows that low values of $\rho_t$ result in low values of current in the CC stage. In other words, a low $\rho_t$ puts less emphasis on charging time and more emphasis on the energy losses; hence, it results in low level of current which provides low energy losses. On the other hand, by increasing $\rho_t$, more emphasis is placed on the charging time. Consequently, the level of current is increased proportionally to $\sqrt{\rho_t}$ to reduce the TTC.

Fig. 3. Five different current profiles (including the optimal profile).

Fig. 4. Five different current profiles: a) cost functions b) current levels in CC stage.

Fig. 5. Current profiles for different values of $\rho_t$. 
Fig. 6 shows the state-of-charge profiles for different values of \( r_t \). It is seen that by increasing \( r_t \), more emphasis is placed on charging time and the SOC reaches the final value in a shorter time.

Fig. 7 shows the terminal voltage profiles for different values of \( r_t \). Note that for low values of \( r_t \), as the emphasis on energy loss is high, the corresponding current level in the CC stage is low, and consequently, the terminal voltage reaches the threshold value of \( v_c \) at a later time. Hence, the duration of the CC stage is high and the charging time is high as well.

Fig. 8 shows the time-to-charge, energy losses and efficiency as functions of \( r_t \). As expected, high values of \( r_t \) result in lower TTC. The low TTC, however, is obtained by increasing the current level; as EL is proportional to the square of current, thus the high values of \( r_t \) result in high values of EL. The high values of EL mean that a higher fraction of input power is wasted; hence it is equivalent to a decline in efficiency.

Fig. 9 shows the time-to-charge versus efficiency (ratio of effective to total energy) curve. TTC and efficiency are two counteracting objectives. For low values of \( r_t \), as less emphasis is put on TTC, the TTC is high; however, high TTC is the result of low current...
values, which incur low energy losses and hence higher efficiency. For example at \( \rho_t = 0.1 \), the TTC is 195 min, but the efficiency is as high as 95.82%. On the other hand, for high values of \( \rho_t \) which place more emphasis on TTC, the TTC is reduced dramatically; however, low TTC is achieved by increasing the current values, which results in high energy losses and hence lower efficiency. For example, at \( \rho_t = 0.5 \), the TTC is as low as 148 min, but the efficiency decreases to 92.87%.

### 4.3. Temperature effect

In this section, we consider the effect of temperature rise index (TRI) on optimal charging. The cost function is a weighted sum of TTC (seconds), EL (Joules) and TRI (Kelvin seconds), given by

\[
J_{\text{IE}} = \rho_T \times \text{TTC} + EL + \rho_T \times \text{TRI}
\]

We used two sets of thermal parameters, shown in Table 1. Parameter set “A” is adopted from Ref. [19]. Parameter set “B” is a scaled version of parameter set “A” with \( m_{\text{batt}} \) set as the weight of Nokia BP-4L. For each set of thermal parameters (“A” or “B”), the weights of the cost function are chosen as \( \rho_t = 1 \), \( \rho_T = 1 \) and \( \rho_t = 1 \), \( \rho_T = 4 \). Three schemes are used: OtE (equations (37) and (38)), OtET (equations (38) and (57)), and NOtET (equations (38) and (58)). The cost function in (61) or (46) is calculated for the three schemes. Table 2 shows the cost functions of the three schemes for different weightings. As seen from this table, the cost function for the OtE has the highest value. Also the difference between the cost function of OtET and NOtET is negligible with the OtET being slightly smaller when thermal parameter set “A” is used. For thermal parameter set of “B”, there is visually no difference between NOtET and OtET. Due to this negligible difference in the cost function and also since the calculation of NOtET profile is much easier than that of the OtET, it is reasonable to use NOtET rather than the OtET scheme. Also note that the weight on TRI results in a reduction of current, as can be seen from Fig. 10. This reduction in current level results in a lower temperature rise (see Fig. 11). In other words, energy losses with \( R_{eq} \) instead of \( R_0 \) can be used as a surrogate cost function for the TRI.

### 4.4. Analysis of different commercial batteries

In this section, we discuss the behavior of different commercial batteries. The parameters of the investigated batteries are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter set</th>
<th>( m_{\text{batt}} ) (kg)</th>
<th>( R_{\text{Eff}} ) (K/W)</th>
<th>( C_{\text{batt}} ); ( \text{batt} ) (J/(kg K))</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.37824</td>
<td>7.8146</td>
<td>795</td>
</tr>
<tr>
<td>B</td>
<td>0.080</td>
<td>1.6528</td>
<td>168.15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \rho_T )</th>
<th>( \rho_T )</th>
<th>Thermal parameters</th>
<th>( J_{\text{IE}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
<td>OtE</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
<td>NOtET</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>A</td>
<td>OtET</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>A</td>
<td>OtE</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>A</td>
<td>NOtET</td>
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<tr>
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<td>A</td>
<td>OtET</td>
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<tr>
<td>1</td>
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<td>OtE</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>B</td>
<td>NOtET</td>
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<td>1</td>
<td>1</td>
<td>B</td>
<td>OtET</td>
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<td>4</td>
<td>B</td>
<td>OtE</td>
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<td>1</td>
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<td>B</td>
<td>NOtET</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>B</td>
<td>OtET</td>
</tr>
</tbody>
</table>

Table 1: Battery thermal parameters.

Table 2: Cost function for different schemes.

![Fig. 9. TTC versus efficiency.](image1)

![Fig. 10. Current profiles for \( \rho_t = 1, \rho_T = 1 \) and temperature parameter set “A”.](image2)

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Note that the equivalent electrical circuit parameters given in Table 3 are for model III (see Fig. 2). In simulations, we use the summation of \( R_0 + R_1 \) of model III as an estimate of resistance \( R_0 \) in model I. The batteries are Samsung EB575152 (four cells), Samsung EB504465 (four cells), Samsung AB463651 (two cells), Nokia BP-4L (four cells), LG LGIP (two cells).

Next, we apply the OtE algorithm with \( \rho_t = 0.5 \) to 16 commercial batteries to investigate the times-to-charge and efficiencies of the batteries. The parameters of the batteries, i.e., the electrical parameters of the models in Figs. 1 and 2, and the parameters of the OCV function in (60), were calculated using experimental data and by applying the BFG algorithms in Refs. [3,4]. These parameters are listed in Tables 3 and 4 in the Appendix. Fig. 12 shows the TTC versus efficiency for different types of batteries. Among all batteries, Sam-EB575152 (Cell 3) has the lowest efficiency (90.73%). This can be attributed to the high resistance of this battery, which might be due to aging. Sam-EB504465 (Cell 4) has the highest TTC (102 min) and Nokia BP-4L (Cell 4) has the highest efficiency. Note...
that the cells of the same battery are close to each other in terms of efficiency and TTC. Considering all the cells of a battery, we can say that LG-LGIP cells (circle markers) have the highest efficiency (91.4%). Fig. 13 shows the cost function values of $J_E(k_t)$ when TTC is weighted with weight value of $\rho_1 = 0.5$, Samsung EB575152 (Cell 2) has the best performance.

5. Conclusion

The optimal charging problem involving a weighted combination of time-to-charge (TTC), energy loss (EL) and temperature rise index (TRI) was considered. The optimal TTC and EL solution (OtE) is found to be the well-known CC−CV strategy with the value of current in the CC stage being a function of the ratio of weighting on TTC and EL and also the resistance of battery. To the best of our knowledge, this is the first time that it is proved that the well-known CC−CV charging profile is the optimal solution of a particular optimization problem, namely, the problem of minimizing the weighted sum of time-to-charge and energy loss. In addition, an analytical solution for the optimal TTC, EL and TRI, referred to as OtET, was developed. Due to similarity of the structure of the OtE and OtET solutions, a near-optimal version of OtET was developed (referred to as NOtET). The NOtET is a CC−CV strategy with the value of current in the CC stage being a function of the ratio of weighting on TTC and EL, the resistance of the battery and the effective thermal resistance. A number of simulations were conducted to evaluate the effect of weighting parameters. Finally, extensive results on industrial batteries from LG, Nokia and Samsung were presented.

Acknowledgments

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a, b$</td>
<td>coefficients in temperature model, (42,43)</td>
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<tr>
<td>$C_{batt}$</td>
<td>capacity, (2)</td>
</tr>
<tr>
<td>$C_h$</td>
<td>parameter in coulomb counting, (1)</td>
</tr>
<tr>
<td>$C_{h, batt}$</td>
<td>heat capacity of the battery in J/(kg·K), (42)</td>
</tr>
<tr>
<td>$CC$</td>
<td>constant current</td>
</tr>
<tr>
<td>$CV$</td>
<td>constant voltage</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>sampling time, (2)</td>
</tr>
<tr>
<td>$EL$</td>
<td>energy loss</td>
</tr>
<tr>
<td>$\eta$</td>
<td>charging efficiency</td>
</tr>
<tr>
<td>$C_{eq}[k]$</td>
<td>conductance equal to reciprocal of $R_{eq}[k]$, (56)</td>
</tr>
<tr>
<td>$H[k]$</td>
<td>Hamiltonian function, (16)</td>
</tr>
<tr>
<td>$f[k]$</td>
<td>charging current, (1)</td>
</tr>
<tr>
<td>$f'[k]$, $f'[k_1]$</td>
<td>optimal current, (28)</td>
</tr>
<tr>
<td>$J_E$</td>
<td>energy loss cost function, (3)</td>
</tr>
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<td>$J_T$</td>
<td>temperature rise cost function, (46)</td>
</tr>
<tr>
<td>$J_t$</td>
<td>time to charge cost function, (3)</td>
</tr>
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<td>$\bar{J}_E$</td>
<td>objective function as a combination of TTC and EL cost functions, (34)</td>
</tr>
<tr>
<td>$\bar{J}_{ET}$</td>
<td>partially optimized cost function, (30)</td>
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<td>$J_{ET}$</td>
<td>cost function including time-to-charge (TTC), energy loss (EL), and temperature rise index (TRI), (46)</td>
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</table>

Appendix

The following tables show the parameters of the equivalent electrical circuit model III for different commercial batteries.

<table>
<thead>
<tr>
<th>Make</th>
<th>Model</th>
<th>Cell#</th>
<th>$R_0$ (mΩ)</th>
<th>$R_1$ (mΩ)</th>
<th>$C_1$ (F)</th>
<th>$a$</th>
<th>$C_{batt}$ (Ah)</th>
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<tr>
<td>Samsung</td>
<td>EB575152</td>
<td>1</td>
<td>253</td>
<td>106</td>
<td>4581</td>
<td>0.997934</td>
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<td>Samsung</td>
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<td>2</td>
<td>209</td>
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<td>0.997962</td>
<td>1.2187</td>
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<td>Samsung</td>
<td>EB575152</td>
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<td>418</td>
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<td>Samsung</td>
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<td>Samsung</td>
<td>EB504465</td>
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<td>198</td>
<td>2100</td>
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</tr>
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<td>214</td>
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<td>0.997602</td>
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<td>100</td>
<td>5031</td>
<td>0.988012</td>
<td>1.5514</td>
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<td>64</td>
<td>8141</td>
<td>0.98808</td>
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<tr>
<td>Samsung</td>
<td>BP-4L</td>
<td>3</td>
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<td>0.988028</td>
<td>1.5612</td>
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