In this paper, we present a novel SOC tracking algorithm for Li-ion batteries. The proposed approach employs a voltage drop model that avoids the need for modeling the hysteresis effect in the battery. Our proposed model results in a novel reduced order (single state) filtering for SOC tracking where no additional variables need to be tracked regardless of the level.
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ABSTRACT

In this paper, we present a novel SOC tracking algorithm for Li-ion batteries. The proposed approach employs a voltage drop model that avoid the need for modeling the hysteresis effect in the battery. Our proposed model results in a novel reduced order (single state) filtering for SOC tracking where no additional variables need to be tracked regardless of the level of complexity of the battery equivalent model. We identify the presence of correlated noise that has been so far ignored in the literature and use this for improved SOC tracking. The proposed approach performs within 1% or better SOC tracking accuracy based on both simulated as well as HIL evaluations.

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Robust Battery Fuel Gauge Algorithm Development, Part 3: State of Charge Tracking

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Abstract—In this paper, we present a novel SOC tracking algorithm for Li-ion batteries. The proposed approach employs a voltage drop model that avoid the need for modeling the hysteresis effect in the battery. Our proposed model results in a novel reduced order (single state) filtering for SOC tracking where no additional variables need to be tracked regardless of the level of complexity of the battery equivalent model. We identify the presence of correlated noise that has been so far ignored in the literature and use this for improved SOC tracking. The proposed approach performs within 1% or better SOC tracking accuracy based on both simulated as well as HIL evaluations.

Index Terms—Battery management system (BMS), Battery fuel gauge (BFG), state of charge (SOC), online system identification, adaptive nonlinear filtering, extended Kalman filter (EKF), reduced order filtering.

I. INTRODUCTION

The battery SOC can be estimated through two distinct approaches: The first method, termed Coulomb counting, assumes knowledge of the initial state of charge and perfect knowledge of battery capacity to compute the remaining state of charge after accounting for the amount of Coulombs transferred from/into the battery. This approach has the following sources of error:

(A) Uncertainty in the knowledge of initial SOC;
(B) Uncertainty in the knowledge of battery capacity;
(C) Errors in measured Coulombs as a result of errors in the measured current and errors in time differences due to timing oscillator inaccuracies/drifts.

The second method of SOC estimation exploits the unique and stable relationship between the open circuit voltage (OCV) and SOC of the battery and allows one to compute the SOC for a measured OCV. However, it is possible to directly measure the OCV only when the battery is at rest. When the battery is in use, the dynamic relationship between the battery voltage and current has to be accounted for through parameter and state estimation approaches. The OCV-SOC based state of charge estimation approach has the following sources of error:

(a) Uncertainty in the modeling and parameter estimation of the dynamic electrical equivalent model of the battery
(b) Errors in measured voltage and current.

In [25]–[27], Plett reported several battery equivalent models and an extended Kalman filter (EKF) based approach to SOC tracking. Later, the same models were used in [28], [29] that reported an unscented Kalman filter (UKF) filter based approach to SOC tracking. Similar EKF/UKF based SOC tracking approaches with varying enhancements were reported in [11], [12], [14], [15], [18]–[22], [30]. Some other notable approaches for SOC tracking are presented in [10], [16], [17].

In this paper, based on the normalized OCV modeling approach of [24], robust battery equivalent circuit model (ECM) parameter estimation approach of [6] and robust online battery capacity estimation approach of [7] we develop a robust SOC tracking algorithm. Other aspects of battery fuel gauge algorithm development, OCV modeling, equivalent circuit model (ECM) parameter estimation and battery capacity estimation, can be found in the same conference proceedings in [24], [6] and [7], respectively. The robustness of the proposed algorithm is rooted in the normalized OCV modeling approach presented in [24]; robustness in all other components of the BFG, parameter estimation [6], capacity estimation [7] and SOC tracking (present paper) are all based on normalized OCV modeling which yields unchanged OCV-SOC curve over temperature changes and battery aging. Extended details and other related aspects of the robust BFG algorithm will appear in [1]–[5], [23].

The rest of the paper is organized as follows. The system model and a formal statement of the battery SOC tracking problem is provided in Section II. Section III details the proposed online SOC tracking algorithm. The performance of the proposed SOC tracking approach is demonstrated through experimental results in Section IV and the paper is concluded in Section V.

II. SYSTEM MODEL AND PROBLEM STATEMENT

The battery equivalent circuit model considered in this paper is shown in Figure 1. The terminal voltage $v[k]$ in terms of the components of the battery equivalent circuit is given by

$$v[k] = V_0(s[k]) + i[k]R_0 + i_1[k]R_1 + i_2[k]R_2 + h[k]$$

where $V_0(s[k])$ represents the open circuit voltage (in volts) of the battery at time $k$ which is written here as a function of SOC at time $k$, $s[k] \in [0, 1]$; $h[k]$ accounts for the hysteresis in the battery voltage; $i_1[k]$ and $i_2[k]$ are the currents through $R_1$ and $R_2$, respectively.

There are several nonlinear representations that approximate the OCV as a function of SOC. For example, the well known combined model is given in [25]. In this paper, we adopt a novel inverse polynomial-log-linear model for the
The state equation (1) can be rewritten by substituting \( z_i[k] \) for \( i[k] \) according to (6) as follows

\[
x_s[k + 1] = x_s[k] + c_h \Delta z_i[k] - c_h \Delta n_i[k]
\]  

(7)

The currents through the resistors \( R_1 \) and \( R_2 \) can be written in the following form

\[
x_{i1}[k + 1] \triangleq i_1[k + 1] = \alpha_1 i_1[k] + (1 - \alpha_1) i[k]
\]

(8)

\[
x_{i2}[k + 1] \triangleq i_2[k + 1] = \alpha_2 i_2[k] + (1 - \alpha_2) i[k]
\]

(9)

where

\[
\alpha_1 = e^{-\frac{\pi^2}{\eta_1}}
\]

(10)

\[
\alpha_2 = e^{-\frac{\pi^2}{\eta_2}}
\]

(11)

By substituting the measured current \( z_i[k] \) for \( i[k] \), the currents in (8) and (9) can be rewritten as follows

\[
x_{i1}[k + 1] = \alpha_1 x_{i1}[k] + (1 - \alpha_1) z_i[k] - (1 - \alpha_1) n_i[k]
\]

(12)

\[
x_{i2}[k + 1] = \alpha_2 x_{i2}[k] + (1 - \alpha_2) z_i[k] - (1 - \alpha_2) n_i[k]
\]

(13)

The hysteresis voltage \( h[k] \) is a nonlinear function of the load current and SOC of the battery\(^{[25]}\). The hysteresis process can be written as

\[
x_h[k] \triangleq h[k] = f_h(x_s[k], i[k]) x_h[k] + n_h[k]
\]

(14)

where \( n_h[k] \) is the process noise of the hysteresis model which is assumed white, zero-mean Gaussian and with s.d. \( \sigma_h \). In this paper, we model hysteresis as the error in the predicted OCV of the BFG. More details can be found in Section III.

The voltage in (1) is a measured quantity and the measured voltage \( v_s[k] \) is subject to errors. The measured voltage is written as

\[
z_v[k] = v[k] + n_v[k] = V_0(s[k]) + i[k] R_0 + i_1[k] R_1 + i_2[k] R_2 + h[k] + n_v[k]
\]

(15)

where \( n_v[k] \) is assumed to be white Gaussian noise with zero mean and s.d. \( \sigma_v \).

Now, by substituting (6), (8), (9) and (14), in (15), the following measurement model is derived

\[
z_v[k] = V_0(x_s[k]) + z_i[k] R_0 + x_{i1}[k] R_1 + x_{i2}[k] R_2 + x_h[k] + n_{z_v}[k]
\]

(16)

where

\[
n_{z_v}[k] = n_v[k] - R_0 n_i[k]
\]

(17)

Now, given the instantaneous voltage and current measurements, \( z_v[k] \) and \( z_i[k] \), the objective of BFG is to track the instantaneous SOC of the battery \( x_s[k] \). The presence of “nuisance” variables \( x_{i1}[k], x_{i2}[k], \) and \( x_h[k] \) in the observation model (16) makes it a joint estimation problem, i.e., SOC and these variables have to be jointly estimated.

In addition, the following model parameters need to be estimated through system identification techniques:

- Battery capacity: \( C_{\text{batt}} \)

representation of OCV in terms of SOC:

\[
V_0(s[k]) = K_0 + \frac{K_1}{s[k]} + \frac{K_2}{s^2[k]} + \frac{K_3}{s^3[k]} + \frac{K_4}{s^4[k]} + K_5 s[k]
\]

\[ + K_6 \ln(s[k]) + K_7 \ln(1 - s[k]) \]  

(2)

where \( K_0, K_1, K_2, K_3, K_4, K_5, K_6 \) and \( K_7 \) can be estimated offline through OCV-SOC characterization.

The instantaneous change in SOC in SOC can be written in the following form (the subscript of \( x \) is introduced to indicate a state component)

\[
x_s[k + 1] \triangleq s[k + 1] = s[k] + c_h \Delta i[k]
\]

(3)

where \( i[k] \) is in Amps,

\[
c_h = \frac{\eta}{3600 C_{\text{batt}}}
\]

(4)

is the Coulomb counting coefficient in Amp\(^{-1}\)sec\(^{-1}\) (the factor 3600 indicates \( C_{\text{batt}} \) is in Ah (the commonly used unit for capacity) rather than Coulombs). \( C_{\text{batt}} \) is the capacity of the battery in Ampere hours (Ah), \( \Delta \) is the sampling interval in seconds and \( \eta \) is a constant that depends on whether the battery is being charged or discharged, i.e.,

\[
\eta = \begin{cases} 
\eta_c & i[k] > 0 \\
\eta_d & i[k] < 0
\end{cases}
\]

(5)

It must be noted that (3) yields the instantaneous SOC of the battery. This way of computing the SOC is known in the literature as Coulomb counting. We also refer to this as the “predicted SOC” in this paper.

The current \( i[k] \) is measured and the current measurement is subject to errors. The measured current \( z_i[k] \) is written as

\[
z_i[k] = i[k] + n_i[k]
\]

(6)

where \( n_i[k] \) is the current measurement noise, which is considered white, zero-mean and with known standard deviation (s.d.) \( \sigma_i \).
• Open circuit voltage model parameters: 
  \( K_0, K_1, K_2, K_3, K_4, K_5, K_6, K_7 \)
• Dynamic equivalent circuit model parameters: 
  \( R_0, R_1, C_1, R_2, C_2 \)
• Charging and discharging efficiencies: \( \eta_c, \eta_d \)
• Process noise variances: \( \sigma_v^2, \sigma_i^2 \)
• Measurement noise variances: \( \sigma_{vD}^2, \sigma_i^2 \)

The requirement to estimate all the model parameters makes 
the SOC tracking problem more challenging. Further, the 
chemical properties of the battery change as a result of 
temperature changes, aging, and usage patterns and as a result 
these model parameters are subject to change over time. Hence 
the model parameters have to be re-estimated over time.

By exploiting the expectation maximization (EM) algorithm 
\([13], [31]\), the authors developed a strategy in \([8]\) for SOC 
tracking by jointly estimating many of the model parameters 
listed above. The EM approach involves the iteration of 
parameter estimation and SOC tracking over a large data 
window which results in significant computational and mem-
ory requirements. This is not desirable in portable mobile 
applications where the power requirement and compactness 
of the BFG is a major concern.

In this paper we assume that the OCV parameters 
\( K_0, K_1, \ldots, K_7 \) of the battery are estimated offline. 
The voltage and current measurement error standard deviations, 
\( \sigma_v \) and \( \sigma_i \), respectively, are assumed to be available from the 
measurement circuitry design. The charging and discharging 
efficiencies, \( \eta_c \) and \( \eta_d \), respectively, are assumed to be known 
through calibration. Hence, our objective in this paper is 
to develop an online SOC tracking algorithm by assuming 
the knowledge of battery capacity \( \text{C}_{\text{batt}} \) and the electrical 
equivalent model parameters \( R_0, R_1, R_2, C_1 \) and \( C_2 \) of 
the battery.

III. SOC TRACKING APPROACH

The objective of reduced order filtering is to track \( x_s[k] \) 
while avoiding the need to track the redundant variables 
\( x_i[k], x_{\Delta} [k] \) and \( x_h[k] \). First, let us rewrite (7) in the following form:

\[
x_s[k + 1] = x_s[k] + c_h \Delta z_i[k] + w_s[k]
\]

where

\[
w_s[k] = -c_h \Delta n_i[k]
\]

is the process noise which is white with s.d.

\[
\sigma_s = c_h \Delta \sigma_i
\]

Now, let us rewrite the voltage measurement (16) as

\[
z_e[k] = V_0(x_s[k]) + a[k]^T b + n_D[k]
\]

where

\[
a[k]^T = \begin{bmatrix} v_D[k - 1] & v_D[k - 2] & z_i[k] & z_i[k - 1] \\
               & -z_i[k - 2] & 1
\end{bmatrix}
\]

and the voltage drop is given by

\[
v_D[k] = z_e[k] - V_0(x_s[k])
\]

where \( b \) is a parameter vector to be estimated and \( n_D[k] \) is the measurement noise.

It must be noted that \( a[k] \) in (22) is defined in terms of the voltage drops \( v_D[k - 1] \) and \( v_D[k - 2] \). The estimated parameters derived in terms of the parameters of the battery equivalent model in Figure 1 as

\[
b(1) = \alpha(k) = \alpha_1 + \alpha_2 \\
b(2) = \beta(k) = \alpha_1 \alpha_2 \\
b(3) = R_0 \\
b(4) = R_1 = (\alpha_1 + \alpha_2)R_0 - (1 - \alpha_1)R_1 - (1 - \alpha_2)R_2 \\
b(5) = R_2 = \alpha_1 \alpha_2 R_0 - \alpha_2 (1 - \alpha_1) R_1 - \alpha_1 (1 - \alpha_2) R_2, \\
b(6) = b[k] = x_h[k] - \alpha(k) x_h[k - 1] + \beta(k) x_h[k - 2]
\]

The measurement noise \( n_D[k] \) is zero mean and with 
autocorrelation \( \mathcal{R}_{n_D}(l) \).

Next, we discuss the significance of the voltage drop in 
estimating the parameters \( b \) in (21). Using (21), the voltage 
drop (25) can be written as

\[
v_D[k] = a[k]^T b + n_D[k]
\]

Given the voltage drop observations, the above model (25) can 
be used to linearly estimate \( b \). However, in order to get 
the voltage drop as an observation, one needs the knowledge of 
the SOC \( x_s[k] \) for which a predicted value of SOC, \( \hat{x}_s[k - 1] \) 
or the updated SOC estimate \( \hat{x}_s[k] \) can be used, i.e.,

\[
v_D[k] = z_e[k] - V_0(\hat{x}_s[k])
\]

Later, we discuss how the predicted SOC \( \hat{x}_s[k - 1] \) and 
updated SOC \( \hat{x}_s[k] \) are obtained (see (51) and (38), 
respectively.)

Let us now discuss the advantage of the voltage drop model. 
All the existing approaches for BFG use the voltage and 
current observations, \( z_e[k] \) and \( z_i[k] \), for model identification 
and SOC tracking.

Consider the conventional voltage observation model (16). 
The terms in \( x_h[k] \) denote the hysteresis voltage which, as 
shown in (14), is a function of current \( i[k] \), SOC \( x_s[k] \) and 
time \( k \). For example, when a battery experiences a load of 
1A (which is heavy in mobile applications) for a few seconds, 
the magnitude of the resulting hysteresis is small compared to 
when the load was 1A continuously for 30 min. In addition, 
the magnitude of the hysteresis is a function of the SOC at 
that time as well.

When one uses the voltage observation (16) across battery 
terminals for model identification, the hysteresis \( x_h[k] \) also 
needs to be modeled and the model parameters have to be 
estimated. A model for hysteresis in terms of SOC, current 
and time is nonlinear and not yet fully understood.

Another disadvantage of trying to model and estimate 
hysteresis is that it makes online model identification almost...
impossible: Since hysteresis is a function of SOC, model identification requires the data spanning the entire range of SOC. This might be impossible at times since some applications might never use the battery from full to empty. Since hysteresis is also a function of current, model identification requires usage data spanning the possible load currents applied for various durations. Hence, complete modeling and model identification of hysteresis becomes impractical.

It is also important to note that estimating model parameters offline using sample batteries and then using those parameters in fuel gauging may not be satisfactory; some of the battery parameters are known to change based on usage patterns.

In this paper, we entirely avoid hysteresis modeling by introducing the voltage drop model described above. The voltage drop \( \nu_D[k] \) represents the voltage across the internal battery model components \( R_0, R_1, R_2 \) and \( x_h[k] \). The term \( x_h[k] \) is purposely introduced in order to account for the errors in predicted SOC \( \hat{x}_s[k|k-1] \) that is used to derive the voltage drop “measurement” \( \hat{\nu}_D[k] \). One could think of \( x_h[k] \) as an “instantaneous hysteresis” which must be corrected to zero by adjusting SOC estimate \( \hat{x}_s[k|k] \).

\[
\hat{x}_s[k+1|k] = \hat{x}_s[k|k] + c_h[k] \Delta z_i[k]
\]

\[
P_s[k+1|k] = P_s[k|k] + \sigma^2_i
\]

\[
H[k+1] = \frac{d\hat{z}_e[k]}{dx_i[k]} \hat{x}_s[k+1|k]
\]

\[
\hat{z}_i[k+1] = V_0(\hat{x}_s[k+1|k]) + \tilde{\alpha}[k] \hat{b}[k]
\]

\[
S[k+1] = \frac{P[k+1|k]H[k+1|k]H[k+1|k]^T + U[k]|} + \mathcal{R}_{n_D}(0)
\]

\[
G[k+1] = \frac{P[k+1|k]H[k+1|k]^T + U[k]|}{S[k+1]}
\]

\[
\hat{x}_s[k+1|k] = \hat{x}_s[k+1|k] + G[k+1] (z_v[k+1] - \hat{z}_v[k+1])
\]

\[
P_s[k+1|k+1] = (1 - G[k+1] H[k+1|k]) P_s[k+1|k]
\]

where \( \tilde{\alpha}[k] = a[k]^T \) and \( \tilde{\beta} \) in the state-space model \((18) - (27)\) for SOC tracking. Hysteresis can be thought of as an error in the OCV-SOC characteristic curve (see Figure 2). It is quite difficult to model and accurately estimate the hysteresis because it varies with the previous current and SOC (see [2]). However, the true OCV-SOC relationship can be easily estimated. Indeed, the
$V_0(x_s[k])$ in (27) is based on the true OCV-SOC model. Let us assume that the estimated hysteresis is $\hat{h}[k] = 10$ mV. This means the "perceived OCV" by the filter is $10$ mV away from the actual OCV of the battery. For the BFG algorithm, the perceived OCV, $V_0(x_s[k])$, is directly (and monotonically) related to the SOC estimate $\hat{x}_s[k]$. In other words, if the perceived OCV of the filter is different from the actual OCV, so is the filter estimate $\hat{x}_s[k]$ from the true SOC of the battery. Hence, when the filter sees a drop of $10$ mV in its predicted terminal voltage $\hat{z}_s[k+1]$ in (34), it adjusts its SOC estimate $\hat{x}_s[k+1]=\hat{x}_s[k]+1$ in (38) such that the "perceived OCV error" (or estimated hysteresis $H$) is (gradually) adjusted to zero. Hence, a good indication of proper functioning of the proposed approach is the estimated $H[k]$ being always close to zero. This summarizes our novel hysteresis modeling approach of this paper where we do not actually try to model the hysteresis in terms of its current and SOC dependent paths, but rather, we force the OCV error “hysteresis” to be zero through a combination of (i) true OCV modeling, (ii) modeling for voltage drop, and (iii) filtering.

IV. Simulation Results

In this section we provide performance analysis of the proposed BFG. First, we assess the performance using simulated battery data. Then, we use real data collected through hardware in the loop (HIL) experiments for performance assessment.

Typical battery usage data was simulated using the dynamic equivalent model which consists of a series resistance, the hysteresis component and a single RC circuit. The dynamic model parameters, assumed constant over the entire simulation, are selected as follows: $R_0 = 0.2 \Omega$, $R_t = 0.1 \Omega$, $C_1 = 50 \text{ F}$, $h[0] = 0$ V. It is assumed that the data is sampled at a constant $\Delta = 0.1$ seconds. Hence, the online model parameters $b = [\alpha R_0 R_t H]^T$ can be written as $\alpha = 0.9802$ (see (10)), $R_0 = 0.2$, $R_t = 0.1941$, $H = h[k] - \alpha h[k-1]$ where $h[k]$ is simulated as a constant bias. It is assumed that the true battery capacity is $C_{\text{batt}} = 1.5$ Ah.

The OCV parameters are obtained by performing an OCV test on a Samsung battery (with serial number EB575152). This resulted in the following values for the OCV parameters: $K_0 = -3.0927$, $K_1 = 43.3102$, $K_2 = -7.4126$, $K_3 = 0.7908$, $K_4 = -0.0359$, $K_5 = -30.3610$, $K_6 = 59.0316$, $K_7 = -0.3932$.

A random white noise with s.d. $\sigma_z = 10^{-3}$ A was added to $i[k]$ (see (5)) in order to simulate the measured current $z_i[k]$. The current through $R_1$ is simulated for reference (shown in the second plot from the top of Figure 3). The true, noiseless, SOC is computed by Coulomb counting (15) by assuming that $x_i[0] = 1$ and assuming the knowledge of $i[k]$, i.e., by making $z_i[k] = i[k]$ and $\omega_s[k] = 0$ in (15), the true value of SOC, $x_s[k]$, is then computed. Now, assuming that the initial SOC, $x_s[0] = 1$, the measured battery terminal voltage was simulated by making use of the state-space model (15), (27) with the voltage measurement noise standard deviation $\sigma_v = 10^{-3}$ V. In Figure 3 we show these simulated quantities: true current profile $i[k]$, the current $x_1[k]$ through $R_1$, true SOC $x_s[k]$ and the (simulated) noisy battery terminal voltage $z_i[k]$, respectively, are shown from top to bottom in Figure 3. The simulated voltage $z_i[k]$ and current information $z_i[k]$ are fed to the BFG and the estimated parameters, capacity and SOC outputs of the BFG were recorded for analysis.

Next, we analyze the SOC tracking performances of the algorithm. First, we define the SOC tracking error as

$$\epsilon_{x_s}[k] = 100 \sqrt{\frac{1}{M} \sum_{m=1}^{M} (x_s[k] - \hat{x}_{s,m}[k])^2}$$

(40)

where $\epsilon_{x_s}[k]$ is in percentage and $\hat{x}_{s,m}[k]$ is the estimated SOC at time $k$ of the $m$th Monte-Carlo run.

Figure 4 shows the estimated SOCs as well as the SOC tracking errors. We also use this figure to demonstrate our novel state space model and the linear parameter estimation strategy for SOC tracking: First, we demonstrate SOC tracking based on model parameters estimated through conventional, unweighted LS method explained. The plots corresponding to this are labeled as LSI. Then, we demonstrate SOC tracking based on the new, weighted LS parameter estimation which is more accurate. The plots corresponding to this second approach are labeled as LS2.

V. Conclusions

This paper presented a novel state of charge tracking approach for Li-ion batteries. The proposed approach employs linear approach for parameter estimation and a robust online method for capacity estimation.

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Validation data. From top to bottom: Simulated load profile $i[k]$, computed current $x_1[k]$ through $R_1$, true SOC $x_{true}[k]$ and noisy battery terminal voltage $z_{true}[k]$. 

Fig. 3. Validation data. From top to bottom: Simulated load profile $i[k]$, computed current $x_1[k]$ through $R_1$, true SOC $x_{true}[k]$ and noisy battery terminal voltage $z_{true}[k]$. 

Fig. 4. SOC tracking performance.