Primary Calibration of Hydrophones in the Frequency Range of 250 Hz to 500 kHz Using Three-Transducer Spherical Wave Reciprocity

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PREFACE

This report was prepared under Project No. SS1517, “Acoustic Facilities and Standards,” principal investigator Steven E. Crocker (Code 1531). The sponsoring activity is the Underwater Sound Reference Division (Code 1531).

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Reviewed and Approved: 1 August 2016

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Two commercial hydrophones were calibrated using the method of three-transducer, spherical wave reciprocity over a calibration frequency band of 250 Hz to 500 kHz. The measurements were performed at two different acoustic test facilities operated by the Underwater Sound Reference Division (USRD) for an international Key Comparison organized by the Consultative Committee for Acoustics, Ultrasound and Vibration at the International Bureau of Weights and Measures (Sèvres, France). A Brüel & Kjaer Model 8104 hydrophone was calibrated over the frequency band of 250 Hz to 120 kHz where the combined standard uncertainty ranged from 1.2% to 6.7% with an average value of 1.9%. A Teledyne Reson model TC4034 was calibrated over the frequency band of 100 kHz to 500 kHz where the combined standard uncertainty ranged from 1.9% to 3.5% with an average value of 2.4%. The measurements were performed to establish the international equivalence of hydrophone calibrations performed in the United States. Participating nations included the United States, United Kingdom, Russia, China, Turkey, South Korea, and Brazil. The comparison was coordinated by the National Physical Laboratory of the United Kingdom.

Hydrophone calibration, spherical wave reciprocity, underwater acoustic measurement, measurement uncertainty
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LIST OF ABBREVIATIONS AND ACRONYMS

ADC     Analog-to-Digital Converter
BIPM    International Bureau of Weights and Measures
B&K     Bruel & Kjaer
BNC     Bayonet Neill–Concelman
CC      Consultative Committee
CCAUUV  Consultative Committee for Acoustics, Ultrasound, and Vibration
CIPM    International Committee of Weights and Measures
DAC     Digital-to-Analog Converter
DALE    Data Acquisition Leesburg
DAQ     Data Acquisition System
DOF     Degrees of Freedom
FFVS    Free-Field Voltage Sensitivity
KC      Key Comparison
LCP     Local Calibration Procedure
LEFAC   Leesburg Acoustic Test Facility
LSE     Least Squares Estimator
METCAL  Metrology and Calibration Program
LIST OF ABBREVIATIONS AND ACRONYMS (Cont’d)

MRA Mutual Recognition Arrangement
NI National Instruments, Inc.
NIST National Institute of Standards and Technology
NMI National Metrology Institute
NPL National Physical Laboratory, United Kingdom
NUWC Naval Undersea Warfare Center
OTF Open Tank Facility
PZT Lead Zirconate Titanate
QA Quality Assurance
SI International System of Units
S/N Serial Number
THAMES V2 Transducer and Hydrophone Acoustic Measurement and Evaluation System, Version 2
TCR Transmit Current Response
UK United Kingdom
USRD Underwater Sound Reference Division

LIST OF MATHEMATICAL SYMBOLS AND UNITS

\( \text{dB} \) Decibel
\( d_{PH} \) Distance between acoustic centers of projector and hydrophone
\( d_{PT} \) Distance between acoustic centers of projector and transducer
\( d_{TH} \) Distance between acoustic centers of transducer and hydrophone
\( d_{TP} \) Distance between acoustic centers of transducer and projector
\( e_H \) Open circuit voltage at hydrophone terminals
\( e_{PH} \) Open circuit voltage at hydrophone terminals, projector source
\( e_{PT} \) Open circuit voltage at transducer terminals, projector source
\( e_{TH} \) Open circuit voltage at hydrophone terminals, transducer source
\( e_{TP} \) Open circuit voltage at projector terminals, transducer source
\( f \) Frequency
\( i_{PH} \) Drive current for projector-hydrophone impedance measurement
\( i_{PT} \) Drive current for projector-transducer impedance measurement
\( i_{TH} \) Drive current for transducer-hydrophone measurement
\( i_{TP} \) Drive current for transducer-projector measurement
\( J \) Spherical wave reciprocity parameter
\( M_H \) Open circuit voltage sensitivity of hydrophone
\( M_T \) Open circuit voltage sensitivity of transducer
\( \rho \) Density of water
\( S_P \) Transmit current response of projector
\( S_T \) Transmit current response of transducer
\( Z_{PH} \) Acoustic transfer impedance, projector to hydrophone
\( Z_{PT} \) Acoustic transfer impedance, projector to transducer
\( Z_{TH} \) Acoustic transfer impedance, transducer to hydrophone
\( Z_{TP} \) Acoustic transfer impedance, transducer to projector
1. INTRODUCTION

This document describes the primary calibration of two hydrophones over the frequency range of 250 Hz to 500 kHz. The calibrations were performed in support of an international effort to verify the equivalence of hydrophone calibrations performed by national laboratories of several nations including the United States, United Kingdom, Russia, China, Turkey, South Korea, and Brazil.

The International Bureau of Weights and Measures (BIPM) is the diplomatic treaty organization with the mission to provide a coherent international system of units and to promote the global comparability of measurements. The International Committee of Weights and Measures (CIPM) is the particular body within the BIPM that works to ensure the international equivalence of measurement standards through the framework of the Mutual Recognition Arrangement (MRA). In particular, “the CIPM MRA responds to the need for an open, transparent and comprehensive scheme to give users reliable quantitative information on the comparability of national metrology services and to provide the technical basis for wider agreements negotiated for international trade, commerce and regulatory affairs.”

The CIPM ensures the international equivalency of measurement standards by the performance of Key Comparisons (KC), whereby an artifact representing a particular physical quantity is circulated among the laboratories of participating National Metrology Institutes (NMI) and the resultant measurement results are compared in round-robin fashion. The comparisons are organized and executed by one of several Consultative Committees (CC), each of which operates under the auspices of the CIPM and focuses on the measurement of particular physical quantities within the international system of units (SI). The Consultative Committee for Acoustics, Ultrasound, and Vibration (CCAUUV) coordinates, among other things, the demonstration of international equivalence of measurements in underwater sound through the performance of appropriate comparisons among the participating NMI.

The United States is represented at the BIPM by the National Institute of Standards and Technology (NIST) and is a signatory to the CIPM MRA. The Underwater Sound Reference Division (USRD) at the Naval Undersea Warfare Center (NUWC) supports NIST by providing subject matter expertise in underwater sound, and serves a critical national role in the establishment and dissemination of standards for the measurement of underwater sound. The USRD also supports NIST through direct participation in CCAUV Key Comparisons for the measurement of underwater sound.

This document describes the USRD measurements and results for Key Comparison CCAUV-W.K2 for the primary calibration of hydrophones over the frequency range of 250 Hz to 500 kHz. The National Physical Laboratory (NPL) in the United Kingdom (UK) was the pilot laboratory that executed this comparison in coordination with the laboratories of participating NMI, including NIST with the support of USRD.
2. TECHNICAL PROTOCOL

2.1 DEVICES FOR CALIBRATION

Two commercial hydrophones were supplied by the National Physical Laboratory (UK) to serve as test artifacts for this comparison. Referring to figure 1, the tested hydrophones were (a) Brüel & Kjaer (B&K) Model 8104 (Serial No. 2935850) and (b) Teledyne Reson TC4034 (Serial No. 0514046). Both are passive acoustic devices that together span the frequency range defined for the comparison.

Specifications for the B&K 8104 are provided in table 1, where the nominal voltage sensitivity is provided as $-205 \text{ dB re } 1\text{V/} \mu \text{Pa}$ at the end of a 10-meter cable. The frequency range for the calibration was 250 Hz to 120 kHz.

Specifications for the Reson TC4034 are provided in table 2, where the nominal voltage sensitivity is provided as $-218 \text{ dB re } 1\text{V/} \mu \text{Pa}$ at the end of a 10-meter cable. The frequency range for the calibration was 100 kHz to 500 kHz.

(a) B&K 8104

(b) Reson TC4034

Figure 1. B&K 8104 and Reson TC4034 Hydrophones
### Table 1. B&K 8104 Hydrophone Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency range for calibration</td>
<td>250 Hz to 120 kHz</td>
</tr>
<tr>
<td>Nominal voltage sensitivity (at 250 Hz)</td>
<td>$-205$ dB re $1 \text{V/µPa}$</td>
</tr>
<tr>
<td>Nominal capacitance</td>
<td>7800 pF</td>
</tr>
<tr>
<td>Active element type</td>
<td>4 PZT rings (12-millimeter diameter)</td>
</tr>
<tr>
<td>Position of acoustic center</td>
<td>16 mm from the boot end</td>
</tr>
<tr>
<td>Horizontal directivity (at 100 kHz)</td>
<td>$\pm 2$ dB (typical)</td>
</tr>
<tr>
<td>Vertical directivity (over 270° at 50 kHz)</td>
<td>$\pm 2$ dB (typical)</td>
</tr>
<tr>
<td>Cable</td>
<td>Twin conductor, shielded</td>
</tr>
<tr>
<td>Connector</td>
<td>BNC</td>
</tr>
<tr>
<td>Length of cable</td>
<td>10 m</td>
</tr>
<tr>
<td>Weight with cable (in air)</td>
<td>1.6 kg</td>
</tr>
</tbody>
</table>

### Table 2. ResonTC4034 Hydrophone Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency range for calibration</td>
<td>100 kHz to 500 kHz</td>
</tr>
<tr>
<td>Nominal voltage sensitivity (at 250 Hz)</td>
<td>$-218$ dB re $1 \text{V/µPa}$</td>
</tr>
<tr>
<td>Nominal capacitance</td>
<td>3000 pF</td>
</tr>
<tr>
<td>Active element type</td>
<td>PZT sphere (6-millimeter diameter)</td>
</tr>
<tr>
<td>Position of acoustic center</td>
<td>5.5 mm from the boot end</td>
</tr>
<tr>
<td>Horizontal directivity (at 100 kHz)</td>
<td>$\pm 2$ dB (typical)</td>
</tr>
<tr>
<td>Vertical directivity (over 270° at 50 kHz)</td>
<td>$\pm 2$ dB (typical)</td>
</tr>
<tr>
<td>Cable</td>
<td>Twin conductor, shielded</td>
</tr>
<tr>
<td>Connector</td>
<td>BNC</td>
</tr>
<tr>
<td>Length of cable</td>
<td>10 m</td>
</tr>
<tr>
<td>Weight with cable (in air)</td>
<td>1.6 kg</td>
</tr>
</tbody>
</table>
2.2 PARAMETER TO BE DETERMINED

The parameter determined for each of the hydrophones was the magnitude of the end-of-cable, open-circuit, free-field voltage sensitivity $M_H$ (FFVS). This is defined as the quotient of the open-circuit voltage at the output terminals of the hydrophone $e_H$ to the sound pressure $p$ in the undisturbed free-field of a plane wave that would exist at the position of the hydrophone’s acoustic center, if the hydrophone were removed from the field.\(^5\) The expression for the hydrophone’s sensitivity is provided as equation (1)

$$M_H = \frac{e_H}{p}.$$  (1)

2.3 CONVENTIONAL RECIPROCITY THEORY

A conventional reciprocity method, performed in accordance with ANSI-ASA S1.20-2012\(^6\), was used to calibrate each hydrophone. In particular, the calibrations were carried out using the method of three-transducer, spherical wave reciprocity.

This conventional reciprocity calibration method employs three transducers where one is a hydrophone that receives acoustic waveforms, one is a projector that transmits acoustic waveforms, and one is a reciprocal transducer that both transmits and receives acoustic waveforms. In general, this method provides for calibration of any of the three transducers. Since the objective of this comparison was to calibrate the hydrophone, derivation of the calibration equation is briefly summarized as follows.

Measurements performed during a conventional reciprocity calibration are indicated in figure 2, where (P) represents the projector, (H) represents the hydrophone, and (T) represents the transducer. The minimum set of measurements required to support the calibration include only the first three (i.e., a, b, c). The fourth (i.e., d) may be used to verify the measurement apparatus.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Conventional Reciprocity Measurements}
\end{figure}
The first measurement (see figure 2a) evaluates the acoustic transfer impedance between the projector (P) and the hydrophone (H) by observation of the open-circuit voltage \( e_{PH} \) at the hydrophone output when ensonified by the projector driven with input current \( i_{PH} \), such that the acoustic pressure developed at the hydrophone is \( p_{PH} \) and

\[
e_{PH} = M_H \ p_{PH} = M_H \ \frac{S_p \ i_{PH}}{d_{PH}}, \quad (2)
\]

where \( S_p \) is the transmit current response (TCR) of the projector referred to a distance of 1 meter from its acoustic center, and \( d_{PH} \) is distance from the projector’s acoustic center to the hydrophone. Rearrangement of equation (2) provides the acoustic transfer impedance \( Z_{PH} \) as

\[
Z_{PH} = \frac{e_{PH}}{i_{PH}} = \frac{M_H \ S_p}{d_{PH}}. \quad (3)
\]

Referring to figure 2b, the second measurement replaces the hydrophone (H) with the reciprocal transducer (T), such that the open-circuit voltage observed across the transducer’s terminals is \( e_{PT} \) when the projector is driven with current \( i_{PT} \) and the acoustic centers of the two devices are separated by a distance \( d_{PT} \), providing an acoustic transfer impedance of

\[
Z_{PT} = \frac{e_{PT}}{i_{PT}} = \frac{M_T \ S_p}{d_{PT}}, \quad (4)
\]

where \( M_T \) is the FFVS of the reciprocal transducer.

The third measurement (see figure 2c) replaces the projector (P) with the hydrophone (H). The reciprocal transducer (T) is used to ensonify the hydrophone, such that the acoustic transfer impedance becomes

\[
Z_{TH} = \frac{e_{TH}}{i_{TH}} = \frac{M_H \ S_T}{d_{TH}}, \quad (5)
\]

where \( S_T \) is the TCR of the reciprocal transducer, \( i_{TH} \) is the transducer drive current, \( e_{TH} \) is the voltage at the hydrophone output, and \( d_{TH} \) is the distance between the acoustic centers of the two devices.

Equations (3) through (5) compose a set of three equations in the four unknowns \( M_H, M_T, S_T, \) and \( S_p \), that when combined yield the square of the hydrophone sensitivity

\[
M_H^2 = \frac{M_T}{S_T} \ \frac{Z_{PH} \ Z_{TH}}{Z_{PT}} \ \frac{d_{PH} \ d_{TH}}{d_{PT}}, \quad (6)
\]

where the second and third terms on the right-hand side are provided by measurements illustrated in figure 2, and the first term is the ratio of the receive response (i.e., FFVS) \( M_T \) to transmit response (i.e., TCR) \( S_T \) of the reciprocal transducer.
The term $M_T/S_T$ is a general reciprocity parameter representing an acoustic transfer admittance between the volume velocity emanating from a moving surface represented by the TCR $S_T$ and the resultant pressure at some distance, represented by the FFVS $M_T$. The reciprocity parameter $J$ for the special case of a spherically-divergent wave field is

$$J = \frac{M_T}{S_T} = \frac{2 d_0}{\rho f}, \quad (6)$$

where $\rho$ is the density of water, $f$ is frequency, and $d_0$ is the same reference distance used to define the TCR (i.e., $d_0 \equiv 1$).\(^7\) Thus, the open-circuit, free-field voltage sensitivity (i.e., FFVS) $M_H$ of the hydrophone is computed from measurements of distance, transmit current, and receive voltage as

$$M_H = \left( j \frac{Z_{PH} Z_{TH}}{Z_{PT}} \frac{d_{PH} d_{TH}}{d_{PT}} \right)^{1/2}. \quad (7)$$

If both the projector (P) and transducer (T) are reciprocal devices, then the measurements depicted in figures 2b and 2d can be used to verify the assumption of reciprocal behavior described in equation (6). The acoustic transfer impedance $Z_{TP}$ is determined from the transmit current $i_{TP}$ provided to the transducer (T) and the received voltage $e_{TP}$ at the projector (P) as

$$Z_{TP} = \frac{e_{TP}}{i_{TP}} = \frac{M_p S_T}{d_{TP}}, \quad (8)$$

where $M_p$ is the projector FFVS, $S_T$ is the transducer TCR, and $d_{TP}$ is the distance between the acoustic centers of the two devices. The assumption of reciprocity is valid when $Z_{PT} = Z_{TP}$.

### 2.4 MEASUREMENT PROCEDURES

All acoustic measurements were performed in accordance with ANSI-ASA S1.20-2012, “Procedures for Calibration of Underwater Electroacoustic Transducers.”\(^6\) In certain cases, local calibration procedures (LCPs) specified by the U.S. Navy Metrology and Calibration (METCAL) program\(^8\) were used. Where applicable, LCPs provide amplifying information such as specific equipment settings for calibrations performed in a particular U.S. Navy laboratory as needed to implement the applicable national or international standard, while ensuring that relevant quality assurance (QA) provisions\(^9\) are used in practice. The LCP\(^{10}\) used for the majority of the measurements reported in this report was developed specifically for primary calibration of hydrophones using the three-transducer, spherical-wave reciprocity method in the open tank facility (OTF) located at NUWC Division Newport, Newport, RI, USA. The low-frequency limit of the OTF, and thus the associated LCP, is 1000 Hz.

Calibration at frequencies less than 1000 Hz were performed at the USRD Leesburg Acoustic Test Facility (LEFAC) located in Leesburg, FL, USA. Measurements were performed...
in accordance with ANSI-ASA S1.20-2012. The B&K 8104 and hydrophone mount were hand carried on commercial flights between the two facilities.

2.4.1 Calibration of B&K 8104 at Frequencies Less than 1000 Hz

The Leesburg acoustic test facility is an open-water facility located in Bugg Spring near Leesburg, FL, USA. Bugg Spring (figure 3a) has a mean discharge of 0.32 m³/s and discharges into a run that flows about 2.4 km north and east into Helena Canal. The spring has a deep circular pool about 120 meters in diameter and 50 meters deep. A sub-bottom profile taken beneath the test facility barge showed the bottom is covered with about 3 meters of soft sediment. A multibeam sonar scan of the spring (figure 3b) shows the nearly vertical limestone walls on all sides except the western shoreline. No boil is evident on the surface due to the depth of the spring vent and large pool volume. Except for algae, there is little aquatic vegetation.

Calibration measurements were performed on 22 June 2016. Environmental conditions during the calibration measurements were favorable, with the exception of water temperature that exceeded the 17°C to 21°C bounds established by the technical protocol. Table 3 summarizes the environmental conditions present during measurements, where the observed water temperature at the measurement depth of 7.77 meters was 1.5°C greater than the upper limit specified in the technical protocol.

![Figure 3. Leesburg Acoustic Test Facility, Leesburg, FL, USA](image-url)
Table 3. LEFAC Environmental Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>Water Temperature</td>
<td>22.5°C</td>
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<tr>
<td>Sound Speed</td>
<td>1489 m/s</td>
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<tr>
<td>Water Density</td>
<td>998 kg/m³</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>33°C</td>
</tr>
<tr>
<td>Dew Point</td>
<td>15°C</td>
</tr>
<tr>
<td>Wind Speed</td>
<td>3.1 m/s</td>
</tr>
<tr>
<td>Atmospheric Pressure</td>
<td>765.6 mmHg</td>
</tr>
<tr>
<td>Precipitation</td>
<td>none</td>
</tr>
</tbody>
</table>

Figure 4 illustrates the geometry used to measure the FFVS of the B&K 8104 at frequencies less than 1000 Hz. As shown in figure 4, the measurements were performed at a depth of 7.77 meters, a limit established by the 10-meter length of the hydrophone cable. The horizontal separation for all of the acoustic transfer impedance measurements was 2 meters.

The transducers were deployed from free-flooding aluminum tubes with interlocking fittings that facilitate a wide variety of deployment depths. While this deployment arrangement provides somewhat greater rigidity than the use of simple cables, it does not entirely prevent the relative motion between the transducers, particularly when the test barge is compelled to move in response to wind impinging on it. Prior estimates for variations in test distances achieved in this measurement facility indicated that such displacements were generally limited to about ±4 centimeters when testing was conducted in favorable weather (e.g., low wind speeds), as were present during these calibration measurements.

As shown in figure 4, both the acoustic projector (P) and the reciprocal transducer (T) used to perform the calibrations were Navy Type F56 transducers composed of 15-centimeter-diameter, lead-zirconate-titanate (PZT) spherical shells encapsulated in polyurethane with a nominal resonance frequency of 12 kHz. Nominal performance parameters for the Navy Type F56 transducer are provided in table 4. All transducers were cleaned with a surfactant prior to submergence and allowed to acclimate overnight prior to the start of data collection.

Calibration data were collected with the Data Acquisition Leesburg (DALE) acoustic measurement system developed and maintained by the USRD. As illustrated in figure 5, the system includes a National Instruments, Inc. (NI) MIO-16E Multifunction Data Acquisition System (DAQ) that includes multiple analog input and output channels with 12-bit resolution operating at 5×10⁵ samples per second. Measurement control and data collection are provided by a special-purpose software system that supports a variety of acoustic measurement tasks, including primary calibration of hydrophones.

Table 4. Nominal Performance of the Navy Type F56 Transducer

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Navy Type F56</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>1 Hz to 15 kHz (as hydrophone)</td>
</tr>
<tr>
<td>TVR</td>
<td>108 dB re 1μPa/V@1m (1 kHz)</td>
</tr>
<tr>
<td>FFVS</td>
<td>-185 dB re 1V/μPa</td>
</tr>
</tbody>
</table>
Figure 4. LEFAC Measurement Geometry for the B&K 8104

Figure 5. Leesburg Acoustic Data Collection
Transmit waveforms were computed by the software based on operator selections, and output to a Krohn-Hite Model 7500 power amplifier through a 12-bit, digital-to-analog converter (DAC). Output of the amplifier was routed through a USRD-developed voltage and current (e&i) sensor, prior to input to the acoustic projector. The e&i sensor includes a voltage probe that provides 46 dB of attenuation to the transmit voltage \( e_p \) and an inductive coil for measurement of transmit current \( i_p \), with signal voltages representing each (i.e., \( e(e_p), e(i_p) \)) routed to the data acquisition system’s analog-to-digital converter (ADC). While the system was configured to record both the transmit voltage and current, only the latter was used in calculations of hydrophone sensitivity.

Both the B&K 8104 and Type F56 transducers were terminated at the input of a Stanford Research Model 560 preamplifier set for 20 dB gain prior to the input of the data acquisition system’s 12-bit ADC. Gains were removed from the respective signals prior to hydrophone sensitivity calculations.

Acoustic data consisted of gated, continuous wave signals transmitted at calibration frequencies less than 1000 Hz, as identified in the technical protocol and listed in table 5. Waveform pulse widths were 9 ms as required to resolve the trailing edge of the acoustic direct path from the leading edge of the surface reflection arrival at 10.5 ms. Only the last 2 ms of each acoustic waveform was used to compute the complex representations of the hydrophone (or reciprocal transducer) output voltages. Thus, data used to calculate the complex signal representations were selected to prevent contamination from the projector’s start transients and surface reflections, while also reducing the potential for cross-talk between the transmitted and received electrical signals. Signal amplitudes were computed as the magnitude of the complex signal representations calculated using a least squares estimation (LSE) method, details of which are provided in the appendix.

Figure 6 provides an example of the projector current \( i_p \) and hydrophone voltage \( e_H \) observed at 500 Hz. Also shown in figure 6 is that part of each signal that was used to compute the complex signal representations, as well as comparisons of the observed time series with the time series computed using the magnitudes and phases of the respective complex representations.

Collection of the data required to compute the hydrophone FFVS was repeated 10 times, where the submerged equipment was dismounted and reinstalled to facilitate estimation of the Type A uncertainty for the overall uncertainty budget.

**Table 5. B&K 8104 Calibration Frequencies (Hz) – LEFAC**

<table>
<thead>
<tr>
<th>250</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
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</thead>
</table>

11
2.4.2 Calibration of B&K 8104 at Frequencies of 1000 Hz and Greater

Acoustic calibrations at frequencies of 1000 Hz and above were performed in accordance with LCP-NRA-001, “Hydrophones Using Reciprocity Calibration in the Open Tank Facility.” As discussed previously, this LCP implements ANSI/ASA S1.20-2012 to include specific equipment selections and settings applicable to the OTF.

The OTF (located at NUWC Division Newport, Newport, RI, USA) is a climate-controlled test facility for the evaluation of underwater acoustic devices. The facility features a large open-water tank, automated data acquisition systems, and associated mechanical support equipment. The test tank measures 9.1 meters long, 4.6 meters wide, and 4.6 meters deep (see figure 7). The sides and bottom are concrete. The mechanical support equipment is rated for a maximum test article mass of 450 kg. The facility includes working deck for personnel access and a precision rotator with an angular resolution of ±0.1°. The minimum operational frequency for the facility is 1000 Hz.

Test conditions present during the OTF calibration measurements are provided in table 6. While the water temperature in the tank is generally stable, it does include a gradient such that the temperature at the typical test depth of 2.28 meters is 1.3°C less than at the surface, as shown in figure 8.
Figure 7. Open Tank Facility (OTF)

Figure 8. OTF Depth versus Sound Speed

Table 6. OTF Environmental Test Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
<td>Water Temperature</td>
<td>19.9°C</td>
</tr>
<tr>
<td>Sound Speed</td>
<td>1481 m/s</td>
</tr>
<tr>
<td>Water Density</td>
<td>998 kg/m³</td>
</tr>
<tr>
<td>Air Temperature</td>
<td>22°C</td>
</tr>
<tr>
<td>Dew Point</td>
<td>11°C</td>
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</table>
Calibration measurements were performed on 28 and 29 June 2016, and required two distinct measurement configurations to provide coverage for the 1-kHz to 120-kHz measurement frequency band. In particular, calibration measurements in the 1-kHz to 20-kHz band were performed using two ITC-1007 transducers manufactured by Channel Industries. The ITC-1007 is composed of a PZT sphere with a diameter of 14.6 centimeters, including the polyurethane encapsulation. Nominal performance parameters for the transducer are provided in table 7.

The measurement geometry for this frequency band is illustrated in figure 9a. As shown in figure 9a, measurements were performed at a depth of 2.28 meters where the transducer-to-hydrophone and projector-to-hydrophone distances were 1.50 meters and the projector-to-transducer distance was 3 meters.

Calibration measurements in the 22-kHz to 120-kHz band were performed using a Navy Type F83 as the projector and a Navy Type F41 as the reciprocal transducer. The Navy Type F83 is a USRD measurement standard based on a single 1-3 piezocomposite transducer. The Type F41 is a general purpose transducer consisting of 12 PZT disks with a 1.27-centimeter diameter. Nominal performance parameters for both transducers are provided in table 8.

The measurement geometry for the 22-kHz to 120-kHz band frequency band is illustrated in figure 9b. As shown in figure 9b, the measurement depth was 2.28 meters and the transducer-to-hydrophone and projector-to-hydrophone distances were 2 meters, the projector-to-transducer distance was 4 meters.

All of the transducers were washed with a surfactant prior to installing them in the tank. Following initial setup, the transducers were left in the tank to acclimate overnight, prior to commencement of calibration measurements the following day.

### Table 7. Nominal Performance of Channel Industries ITC-1007 Transducer

<table>
<thead>
<tr>
<th>Transducer</th>
<th>ITC 1007</th>
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<tbody>
<tr>
<td>Frequency Range</td>
<td>100 Hz to 20 kHz</td>
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<tr>
<td>TVR</td>
<td>149 dB re 1µPa/V@1m (11 kHz)</td>
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<tr>
<td>FFVS</td>
<td>−188 dB re 1V/µPa</td>
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</tbody>
</table>

### Table 8. Nominal Performance of Navy Type F83 and F41 Transducers

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Navy Type F83</th>
<th>Navy Type F41</th>
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</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>10 kHz to 200 kHz</td>
<td>15 kHz to 150 kHz</td>
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<tr>
<td>TVR</td>
<td>156 dB re 1µPa/V@1m (50 kHz)</td>
<td>131 dB re 1µPa/V@1m (50 kHz)</td>
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<tr>
<td>FFVS</td>
<td>−194 dB re 1 V/µPa</td>
<td>−205 dB re 1 V/µPa</td>
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</table>
Figure 9. OTF Measurement Geometry for B&K 8104

Calibration data in the 1-kHz to 120-kHz frequency band were collected with the Transducer and Hydrophone Acoustic Measurement and Evaluation System Version 2 (THAMES V2) developed by NUWC Division Newport. As illustrated in figure 10, the system includes a National Instruments PXI-5105 analog-to-digital converter with 12-bit precision operating at a 5-MHz sample rate. Measurement control and data acquisition are provided by a special purpose software application that supports a variety of acoustic measurement tasks, including primary calibration of hydrophones.

Transmit waveforms were computed by the software application based on operator selections, and output to a class AB linear power amplifier. Output of the amplifier was routed
through an output sense-and-select voltage and current sensor (via Instruments Inc.) prior to input to the acoustic projector.

The B&K 8104 hydrophone was terminated at the input of a Stanford Research Model 560 Preamplifier set for 20-dB gain prior to the input of the THAMES V2 measurement system. The preamplifier gain was removed from the hydrophone signal data prior to calculation of the sensitivity.

![Figure 10. OTF Acoustic Measurement System](image)

The acoustic data consisted of gated, continuous wave signals transmitted at calibration frequencies ranging from 1 kHz to 120 kHz, as identified in the technical protocol and listed in table 9. Waveform pulse widths were 1.5 ms for frequencies ranging from 1 kHz to 20 kHz and 1 ms for frequencies above 20 kHz. Each individual measurement was computed from the average of three individual waveforms.

The collection of the data required to compute the hydrophone FFVS was repeated ten times, where the submerged equipment was dismounted and reinstalled to facilitate estimation of the Type A component of the overall uncertainty budget.
Table 9. B&K 8104 Calibration Frequencies (kHz) – OTF

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<tr>
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<td></td>
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<td></td>
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</tbody>
</table>

2.4.3 Calibration of the Reson TC4034

Acoustic calibration of the Reson TC4034 hydrophone was performed in accordance with LCP-NRA-001, “Hydrophones Using Reciprocity Calibration in the Open Tank Facility”. As noted previously, LCP-NRA-001 implements ANSI/ASA S2.20-2012 to include specific equipment selections and settings applicable to the OTF. Test conditions present during these calibration measurements were the same those for the B&K 8104 and are provided in table 6.

Calibration measurements were performed on 6 July 2016 using two Navy Type E27 transducers. The Type E27 transducer is composed of seven PZT disks in a hexagonal arrangement, cemented to a butyl rubber acoustic window. Nominal performance parameters for the E27 transducer are provided in table 10.

The measurement geometry for the Reson TC4034 is illustrated in figure 11. As shown in the figure, measurements were performed at a depth of 2.28 meters where the source to receiver distance was 1 meter for all measurements. All of the transducers were washed with a surfactant prior to installing them in the tank. Following initial setup, the transducers were left in the tank to acclimate overnight, prior to commencement of calibration measurements the following day.

Calibration data were collected with the THAMES V2 measurement system. The Reson TC4034 hydrophone was terminated at the input of an Ithaco Model 568 Preamplifier set for 20-dB gain prior to the input of the acoustic measurement system. The preamplifier gain was removed from the hydrophone signal data prior to calculation of the sensitivity.

Acoustic data consisted of gated, continuous wave signals transmitted at calibration frequencies ranging from 100 kHz to 500 kHz with 10-kHz frequency resolution (i.e., 100, 110, 120, and so on up to 500 kHz) as identified in the technical protocol. Waveform pulse widths were 15 μs and each individual measurement was computed from the average of three individual waveforms. Collection of the data required to compute the hydrophone FFVS was repeated ten times, where the submerged equipment was dismounted and reinstalled to facilitate estimation of the Type A component of the overall uncertainty budget.
Table 10. Nominal Performance of Navy Type E27 Transducer

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Navy Type E27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency Range</td>
<td>80 kHz to 700 kHz</td>
</tr>
<tr>
<td>TVR</td>
<td>125 dB re (1\mu Pa/V)@1m (250 kHz)</td>
</tr>
<tr>
<td>FFVS</td>
<td>(-225) dB re (1V/\mu Pa)</td>
</tr>
</tbody>
</table>

Figure 11. OTF Measurement Geometry for Reson TC4034
3. UNCERTAINTY BUDGET

The total uncertainties for the calibrations were estimated in accordance with NIST policy as summarized in reference 12. The result is expressed as the combined standard uncertainty \( u_c \) (in percent), and represents the estimated standard deviation of the result. The estimates include a Type A component that was based on standard statistical analysis of data collected for 10 repeated measurements for each of the configurations needed to cover the full calibration bandwidth of the two hydrophones. The measurement apparatus were disassembled and reassembled between each measurement with the objective of capturing as much of the combined standard uncertainty within the Type A component as was feasible. Sources of uncertainty not represented in the statistical variations of repeated measurements (i.e., Type B components) were estimated separately and are tabulated in tables 11 and 12.

Estimation of the Type B uncertainty components was based on consideration of equation (7) for the open-circuit voltage sensitivity of a hydrophone. As shown by inspection of the equation, the expression is composed of three parts including the reciprocity parameter \( J \), the distances between the acoustic centers of the devices \( d_{PH}, d_{TH}, \) and \( d_{PT} \), and the transfer impedances used to compute the hydrophone sensitivity \( Z_{PH}, Z_{TH}, \) and \( Z_{PT} \). Each of these terms can be treated separately to arrive at the combined uncertainty.

3.1 RECIPROCITY PARAMETER

The reciprocity parameter \( J = 2/\rho f \) for a spherically-divergent wave field contributes to the combined uncertainty through components for water density \( \rho \) and frequency \( f \). The density of air-saturated water for temperatures of 22.5°C and 19.9°C were 997.7 kg/m\(^3\) and 998.2 kg/m\(^3\), respectively.\(^{13}\) The associated uncertainty in the density of air-saturated water for an uncertainty of 1°C in water temperature is less than 0.1%, a negligible value in relation to other sources.

The uncertainty component associated with the measurement frequency \( f \) was estimated from manufacturer’s specifications for the waveform generators used in the measurement systems. The THAMES V2 measurement system includes a NI PXI-5412 arbitrary waveform generator where the onboard clock provides a frequency resolution of 1.06 \( \mu \)Hz and accuracy of \( \pm 25 \) parts per million. The DALE measurement system includes a NI MIO-16E multifunction input-output card where the accuracy of the base clock used for digital-to-analog conversion is \( \pm 0.01\% \). While the uncertainties introduced during waveform generation were negligible, the potential for the received signals to include distortion resulting from the projector impulse response or power amplifier response had the potential to introduce non-negligible uncertainties. An analysis of the received signals suggests that the uncertainty was not in any case greater than 0.2%, a value that was still small with respect to other sources of uncertainty. The Type B contribution for uncertainty in frequency of 0.2% was included in the calculation of the combined uncertainty nonetheless.
3.2 DISTANCE MEASUREMENTS

Distances \((d_{PH}, d_{TH}, \text{and} \ d_{PT})\) were in all cases established using tape measures to record the distance between the acoustic centers of the source and receiver. Locations of the acoustic centers for the larger acoustic devices are well established by the spherical geometry of the active elements (i.e., ITC-1007 and Navy Type F56) and long history of use at USRD. The locations of the acoustic centers of the test artifacts (i.e., B&K 8104 and Reson TC 4034) were taken from the available product literature. Thus, uncertainties in the distances were controlled by positioner locations, the alignments of the hydrophone mounting arrangement back to the water surface, and any relative motion between the source and receiver over the course of the measurements (important only at LEFAC). Estimation of Type B uncertainties included only the first of the above sources (i.e., positioner locations), while the latter two should be adequately represented in the Type A component during the collection of multiple, independent data sets. Note that the positioners were not relocated between measurements, and only the submerged equipment was removed and reset. The nominal precision with which the centerline of the hydrophone mounting pole could be located was taken to be 1 to 2 centimeters, thus its contribution to the combined uncertainty was 1%. Contributions from other uncertainties related to the measurement geometry are represented in the Type A uncertainty component.

3.3 ELECTRICAL TRANSFER IMPEDANCE MEASUREMENTS

Uncertainties in the transfer impedances \((Z_{PH}, Z_{TH}, \text{and} \ Z_{PT})\) were controlled by the respective measurement uncertainties for hydrophone voltage and projector drive current. The maximum amplitude error for the projector drive current sensor in the THAMES V2 measurement system is illustrated in figure 12 where the value ranged from 2\% for frequencies less than 100 kHz to 4\% from 100 kHz to 500 kHz. A rectangular probability distribution was assumed for the specified measurement tolerance, thus the standard uncertainty for the measurement of drive current \(u_i\) was estimated as \(u_i = a/\sqrt{3}\) where \(a\) was the maximum amplitude error provided in figure 12. While the same current measurement uncertainty was assumed for the DALE measurement system, it was generally negligible when compared to the Type A contribution, thus had no effect on the result for this system.

Voltage measurement uncertainties were based on tolerances for the respective data acquisition systems, where the maximum errors were taken to represent rectangular probability distribution functions and the standard uncertainty for received voltage \(u_e\) was estimated as \(u_e = a/\sqrt{3}\) where \(a\) was the maximum amplitude error.
Measurement procedures employed for these calibrations made repeated use of only two signal paths in each measurement system; one for drive current and another for received voltage. Test leads were reconfigured after each of the individual measurements shown in figure 2 (i.e., P→H, P→T, and T→H) such that all measurements of received voltage used a common signal path, including any systematic errors introduced by the measurement components. Measurements of drive current were likewise performed using the same signal path. Thus, Type B uncertainties in measurements of voltage and current required to calculate the transfer impedances were correlated across the separate impedance measurements. As inspection of equation (6) shows, correlated measurement errors would tend to cancel for two of the three transfer impedance measurements. Therefore, standard uncertainty contributions for drive current and received voltage were accounted only once in the calculation of the combined uncertainty, instead of three times as would be the case for random, independent errors.

3.4 OTHER SOURCES OF UNCERTAINTY

Several other sources of measurement uncertainty were estimated to include the effect of non-steady-state transducer response, transducer orientation and alignment, electrical cross-talk, and gain in the preamplifiers used to terminate the received voltage signals.

Standard measurement practice stipulates that an acoustic projector be allowed to achieve steady-state operation prior to the collection of acoustic data to prevent contamination from the projector’s transient response. In certain cases, it was necessary to begin acoustic data collection before several wave periods had elapsed to prevent reflections from the surface entering the acoustic data acquisition window. This was particularly the case for measurements performed at LEFAC and for measurements performed at the lower end of the calibration band in the OTF. In each case, the standard uncertainty associated with the non-steady-state behavior of the acoustic projector was estimated as 1% when operating near the minimum effective frequency for a transducer and 2% when operating below it.
Uncertainties associated with transducer orientation and alignment included a Type B component due to operator interpretation of markings for the acoustic center of the test artifacts, and a Type A component due to variation in the measurement setup and geometry. The Type B contribution was considered to be negligible where the acoustic wavelength was relatively large with respect to the hydrophone elements, but was estimated as 1% to 2% as the size of the wavelength approached that of the hydrophone ceramic, and its effective acoustic center. No attempt was made to identify the maximum receive response of the test articles other than visual identification of markings on the hydrophones themselves.

The DALE measurement system exhibits a non-negligible level of electrical cross-talk between the transmit and receive signal paths as illustrated in figure 13. The upper trace shows the hydrophone receive voltage \( e_H \) at 500 Hz, while the lower trace shows the envelope of the received voltage computed as the magnitude of the analytic representation of the hydrophone signal (i.e., the Hilbert Transform). The effect of cross-talk is seen in the envelope of the received voltage as undulations that followed the projector start transient at about 2 ms and ended at 9 ms when the projector drive was terminated. While the acquisition window was selected to minimize the impact of cross-talk by collecting data near and beyond the end of the transmit signal, it was not feasible to fully resolve the transmit and receive signals in time. The estimated standard uncertainty component accounting for electrical cross-talk was 1% for the DALE measurement system. Electrical cross-talk in the THAMES V2 measurement system was negligible.

Measurement uncertainty associated with the gain provided by preamplifiers used to terminate received voltages were based on manufacturer data and confirmed by measurements performed on the particular amplifiers used for these calibrations.

![Figure 13. Electrical Cross-Talk in the DALE Measurement System](image-url)
### Table 11. B&K 8104 Uncertainty Budget

<table>
<thead>
<tr>
<th>UNCERTAINTY SOURCE</th>
<th>LEFAC (DALE)</th>
<th>OTF (THAMES V2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Navy Type F56 (x2)</td>
<td>ITC 1007 (x2)</td>
</tr>
<tr>
<td>Frequency (kHz)</td>
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</tbody>
</table>

#### Type B Uncertainty

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#### Transfer Impedances

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#### Type B (Subtotal)

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<th>60.0</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type B (Subtotal)</td>
<td>2.2</td>
<td>1.7</td>
<td>1.5</td>
<td>1.4</td>
<td>1.6</td>
<td>1.6</td>
<td>1.3</td>
<td>1.9</td>
<td></td>
</tr>
</tbody>
</table>

#### Type A (DOF = 9)

<table>
<thead>
<tr>
<th>Source</th>
<th>0.25</th>
<th>0.50</th>
<th>0.90</th>
<th>1.00</th>
<th>10.0</th>
<th>20.0</th>
<th>22.0</th>
<th>60.0</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A (DOF = 9)</td>
<td>6.3</td>
<td>1.6</td>
<td>0.7</td>
<td>0.6</td>
<td>1.2</td>
<td>1.4</td>
<td>0.7</td>
<td>1.5</td>
<td>0.7</td>
</tr>
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</table>

#### Standard Uncertainty uc (%)

<table>
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<th>0.25</th>
<th>0.50</th>
<th>0.90</th>
<th>1.00</th>
<th>10.0</th>
<th>20.0</th>
<th>22.0</th>
<th>60.0</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Uncertainty uc (%)</td>
<td>6.7</td>
<td>2.4</td>
<td>1.7</td>
<td>1.5</td>
<td>1.5</td>
<td>2.2</td>
<td>1.6</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

† Errors correlated among three transfer impedance measurements (P→H, P→T, and T→H)

†† Errors correlated between two transfer impedance measurements (P→H, and P→T)

### Table 12. Reson TC 4034 Uncertainty Budget

<table>
<thead>
<tr>
<th>UNCERTAINTY SOURCE</th>
<th>OTF (THAMES V2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Navy Type E27 (x2)</td>
</tr>
<tr>
<td>Frequency (kHz)</td>
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</tr>
</tbody>
</table>

#### Type B Uncertainty

<table>
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<th>Source</th>
<th>Reciprocity Parameter</th>
</tr>
</thead>
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<td>Acoustic Frequency</td>
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</tr>
<tr>
<td>Water Density</td>
<td>—</td>
</tr>
<tr>
<td>Non-Reciprocal Behavior</td>
<td>1.0</td>
</tr>
</tbody>
</table>

#### Transfer Impedances

<table>
<thead>
<tr>
<th>Source</th>
<th>Separation Distance</th>
<th>Current Measurement †</th>
<th>Voltage Measurement †</th>
<th>Non-Steady State ††</th>
<th>Orientation and Alignment</th>
<th>Preamplifier Gain †</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.0</td>
<td>2.3</td>
<td>0.7</td>
<td>—</td>
<td>—</td>
<td>0.3</td>
</tr>
</tbody>
</table>

#### Type B (Subtotal)

| Source                        | 1.6  | 1.6  | 1.7  |

#### Type A (DOF = 9)

| Source                        | 3.1  | 1.6  | 1.9  |

#### Standard Uncertainty uc (%)

| Source                        | 3.5  | 2.2  | 2.5  |

† Errors correlated among three transfer impedance measurements (P→H, P→T, and T→H)

†† Errors correlated between two transfer impedance measurements (P→H, and P→T)

**Note:** In tables 11 and 12, all numerical values are a percentage (%).
4. RESULTS

4.1 B&K 8104 SERIAL NO. 2935850

The B&K 8104 hydrophone was calibrated over a range of frequencies spanning 250 Hz to 120 kHz using measurements at two different facilities in Leesburg, FL, and Newport, RI, USA. The calibrations were performed in accordance with ANSI-ASA S1.20-2012, with additional, facility specific information provided by LCP-NRA-00110 for measurements performed in the OTF.

Calibration results are illustrated in figure 14 where the error bars span the combined standard uncertainty associated with each measurement. Calibration results are also tabulated in table 13 where the results are expressed as both the sensitivity level in dB re 1V/μPa and sensitivity in μV/Pa. Here, the combined standard uncertainties are expressed in percent.

Measurement uncertainties ranged from a high of 6.7% at 250 Hz to a low of 1.2% at 14 kHz with a mean of 1.9%. The relatively large uncertainties at the lowest measurement frequencies were controlled by the Type A component, where low signal-to-noise ratios contributed to greater measurement variance. It should be noted that measurement uncertainties for primary calibrations of USRD standards at low frequency are considerably less when performed in the USRD reciprocity coupler; however, use of this device was not possible for the B&K 8104 because it is not mechanically compatible with the coupler.

![Figure 14. B&K 8104 Serial No. 2935850 Sensitivity Level](image)
Table 13. B&K 8104 Serial No. 2935850 Calibration Results

<table>
<thead>
<tr>
<th>Frequency kHz</th>
<th>Sensitivity Level</th>
<th>Sensitivity µV/µPa</th>
<th>Uncertainty %</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>-206.81</td>
<td>45.67</td>
<td>6.7</td>
</tr>
<tr>
<td>0.3</td>
<td>-205.92</td>
<td>50.57</td>
<td>4.7</td>
</tr>
<tr>
<td>0.4</td>
<td>-206.34</td>
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<td>3.2</td>
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<td>0.5</td>
<td>-206.68</td>
<td>46.34</td>
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<td>47.97</td>
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<td>-206.06</td>
<td>49.77</td>
<td>1.9</td>
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<td>-206.30</td>
<td>48.41</td>
<td>1.6</td>
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<tr>
<td>0.9</td>
<td>-206.49</td>
<td>47.36</td>
<td>1.7</td>
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<tr>
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<td>-206.25</td>
<td>48.72</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>-206.31</td>
<td>48.39</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
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<td>-207.04</td>
<td>44.46</td>
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</table>

<table>
<thead>
<tr>
<th>Frequency kHz</th>
<th>Sensitivity Level</th>
<th>Sensitivity µV/µPa</th>
<th>Uncertainty %</th>
</tr>
</thead>
<tbody>
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</table>

4.2 RESON TC4034, SERIAL NO. 0514046

The Reson TC4034 hydrophone was calibrated over a range of frequencies spanning 100 kHz to 500 kHz using measurements performed in accordance with ANSI-ASA S1.20-2012, and LCP-NRA-001 in the OTF.

Calibration results are illustrated in figure 15 where the error bars span the combined standard uncertainty associated with each measurement. Calibration results are also tabulated in table 14 where the results are expressed as both the sensitivity level in dB re 1V/µPa and sensitivity in µV/µPa. Here, the combined standard uncertainties are expressed in percent where they ranged from 1.9% to 3.5% with a mean of 2.4%.
Figure 15. Reson TC4034 Serial No. 0514046 Sensitivity Level

Table 14. Reson TC4034 Serial No. 0514046 Calibration Results

<table>
<thead>
<tr>
<th>Frequency kHz</th>
<th>Sensitivity level dB re 1V/µPa</th>
<th>Sensitivity µV/Pa</th>
<th>Uncertainty %</th>
</tr>
</thead>
<tbody>
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<td>10.67</td>
<td>3.5</td>
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<td>10.60</td>
<td>2.9</td>
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<td>3.1</td>
</tr>
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</tr>
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</table>
5. CONCLUSION

In conclusion, two commercial hydrophones were calibrated using the method of three-transducer, spherical wave reciprocity over a calibration frequency band of 250 Hz to 500 kHz. The measurements were performed at two different acoustic test facilities operated by the USRD for Key Comparison CCAUV.W-K2.

Calibration measurements at frequencies of 1000 Hz and above were performed in the OTF. These measurements were performed in accordance with ANSI-ASA S1.20-2012, and LCP-NRA-00110 using the same standards and measurement systems that are used to provide primary calibrations of the USRD measurement standards that are used by the Navy and distributed through a leasing program to various other government, academic, and commercial customers. In addition, the USRD provides both primary and secondary calibration services for customer owned hydrophones.

Calibration measurements at frequencies less than 1000 Hz were performed in Bugg Spring at the USRD Leesburg Acoustic Test Facility in Leesburg, FL, USA. These measurements were performed in accordance with ANSI-ASA S1.20-2012. While the USRD provided primary calibrations at low frequencies using a conventional reciprocity method for this Key Comparison, this is not the standard method for primary calibration of USRD measurement standards at low frequency. Instead, two USRD measurement standards (i.e., Navy Type H48 and Navy Type A47 hydrophones) are calibrated in a reciprocity coupler that provides lower measurement uncertainties than those reported here. However, the coupler and measurement standards form a closed system that does not accommodate other hydrophone types. Instead, these measurement standards are used in the USRD Low-Frequency Acoustic Test Facilities where they serve as the calibrated reference standards for comparison calibrations performed in both standing and travelling wave tubes operated and maintained by the USRD. Nonetheless, any request for primary calibration of a customer-owned hydrophone at low frequency would be calibrated as described in this report.
REFERENCES


5. Underwater acoustics – Hydrophones – Calibration in the frequency range 0.01 Hz to 1 MHz, IEC Standard 60565:2006.


APPENDIX: LEAST SQUARES SPECTRUM ESTIMATION

The complex, frequency domain representation of a signal $x$ is calculated using a least squares estimator (LSE) to arrive at the magnitude and phase of the signal at frequency $f$. Derivation of the LSE begins by assuming a complex, sinusoidal model for the time series waveform. The assumed signal model is given by the real part of the complex exponential

$$x(t_n) = \beta_0 + \Re\{Xe^{j2\pi ft_n}\}; \forall n = 0, 1, 2, \ldots, N - 1$$  \hspace{1cm} (A-1)

where $x$ is the time series signal observed by the sensor at times $t_n$ and $n$ is an index into the $N$ samples of time series data. The real constant $\beta_0$ accounts for any direct-current bias in the measurement. The complex, frequency domain representation $\bar{X}$ associated with the transmit frequency $f$ is characterized by a magnitude $X$ and phase $\phi$ as shown in equation (A-2)

$$\bar{X} = X e^{j\phi}.$$  \hspace{1cm} (A-2)

Thus, the model of the signal is given by equation (A-3) as

$$x(t_n) = \beta_0 + X \cos(2\pi ft_n + \phi).$$  \hspace{1cm} (A-3)

The objective is to estimate the magnitude $X$ and phase $\phi$ of the signal from the observed time series $x(t_n)$. The LSE is obtained by minimizing the error $\epsilon$ such that

$$\epsilon = \sum_{n=0}^{N-1} (x(t_n) - \beta_0 - X \cos(2\pi ft_n + \phi))^2.$$  \hspace{1cm} (A-4)

Equation (A-4) presents a non-linear, least squares minimization problem, which in many cases requires an iterative search method using a gradient descent or other approach not dependent on continuous derivatives (e.g., simulated annealing, evolutionary algorithms, etc.). However, the present least squares minimization problem is linearized by recognizing that

$$X \cos(2\pi ft_n + \phi) = X \cos(\phi) \cos(2\pi ft_n) - X \sin(\phi) \sin(2\pi ft_n).$$  \hspace{1cm} (A-5)

The constant coefficients of the time varying sinusoids are defined as

$$\beta_1 = X \cos \phi$$
$$\beta_2 = -X \sin \phi$$  \hspace{1cm} (A-6)

and the transformed signal model is given by equation (A-7)

$$x(t_n) = \beta_0 + \beta_1 \cos(2\pi ft_n) + \beta_2 \sin(2\pi ft_n).$$  \hspace{1cm} (A-7)

The transformed signal model is given in matrix form as

$$\begin{bmatrix} x \end{bmatrix} = \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} \beta \end{bmatrix}$$  \hspace{1cm} (A-8)

where the kernel $H$ of the model is given by equation (A-9)
Having linearized the signal model, the least squares estimate for the parameter vector $\beta$ is provided by the Moore-Penrose generalized inverse of equation (A-8) to yield

$$\hat{\beta} = (H^T H)^{-1} H^T x$$  \hspace{1cm} (A-10)$$

Given the parameter vector $\hat{\beta}$, least squares estimates for the magnitude and phase of the complex signal are provided as equations (A-11) and (A-12), respectively.

$$\hat{X} = \sqrt{\hat{\beta}_1^2 + \hat{\beta}_2^2}$$  \hspace{1cm} (A-11)$$

$$\hat{\phi} = \tan^{-1}\left(\frac{-\hat{\beta}_2}{\hat{\beta}_1}\right)$$  \hspace{1cm} (A-12)$$

Finally, the estimate for the complex signal in equation (A-2) is

$$\hat{X} = \hat{X} e^{i\hat{\phi}}$$  \hspace{1cm} (A-13)$$

and the estimate of DC bias in the measurement is $\hat{\beta}_0$.

A Matlab function to implement the LSE is as follows:

```matlab
function e = lse(t,x,f)
% function e = lse(t,x,f)
% Lease Squares Spectrum Estimate
% Inputs
%   x is an input time series
%   t is a vector of times for the x observations
%   f is the frequency at which the LSE is computed
% Output
%   e is the complex representation of x

H = [ones(size(x,1),1),cos(2*pi*f*t(:)),sin(2*pi*f*t(:))];
e = H\x;
e = e(2,:)-1i*e(3,:);
```

A-2
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