On the Mitigation of Solar Index Variability for High Precision Orbit Determination in Low Earth Orbit

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Effective satellite mission operations are directly impacted by the ability to generate accurate and precise orbit predictions. High precision orbit determination processes rely on detailed force models to propagate an orbit solution and predict future orbit behavior. While gravity forces are typically well understood, the modeling of non-conservative forces is often more challenging, causing increased difficulty in achieving and maintaining high precision orbit predictions for satellites operating in low Earth orbit. In particular, the atmosphere models used to predict the drag force experienced by a satellite may rely on input parameters such as solar flux and geomagnetic indices, which are often difficult to predict. Multiple methods of selecting the solar flux and geomagnetic index parameters are examined in combination with a number of current and historically recommended atmospheric density models to assess the impact of uncertainty in the predicted index values. Geodetic satellites with high precision satellite laser ranging data are used as test cases for the Naval Research Laboratory's Orbit Covariance Estimation and ANalysis (OCEAN) tool to evaluate solution accuracy and predictive capabilities of each combination. In all test cases examined, using either the Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar 2000 or Jacchia-Bowman 2008 atmospheric density model with solar flux and geomagnetic index values held constant, rather than using the predicted index values, provided the most accurate orbit predictions. Surprisingly, the exponential atmospheric density model, which does not take into account atmospheric parameters, yielded more accurate orbit predictions than any model using predicted solar flux and geomagnetic indices.

Nomenclature

$F10.7$	Solar flux at 10.7 cm wavelength
A_p	Geomagnetic index measuring global averaged geomagnetic activity
$S10.7$	Solar flux index measuring irradiance between 26-34 nm
$M10.7$	Solar flux index measuring Mg II core to wing ratio
$Y10.7$	Solar flux index composite of Lyman α and X-ray irradiance
DST	Disturbance Storm Time geomagnetic index

I. Introduction

It is often of critical importance for mission operations to accurately predict a satellite's orbit. High precision orbit determination methods require precise modeling of all forces imparted upon a satellite. For satellites in Low Earth Orbit (LEO), atmospheric drag forces are typically the largest source of force modeling error.[1] This is due, in large part, the high level of uncertainty

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in atmospheric density models and variability in the atmosphere itself, which proportionally cause error in drag force modeling. This error source may become more critical as a larger number of small satellite missions, which typically experience relatively higher drag forces, are flown.

A number of atmospheric density models have been implemented in the Naval Research Laboratory's (NRL) Orbit Covariance Estimation and ANalysis (OCEAN) high precision orbit determination tool in order to determine the suitability of each model for precision orbit determination applications. This work has been detailed in Reference 2. It is critical, however, to quantify the impact of predicted atmosphere model parameters on orbit determination performance, which was identified as future work in Reference 2. Solar and geomagnetic index parameters, which feed many atmospheric density models, are often difficult to predict, causing a corresponding error in satellite drag force modeling. The typical error associated with predicting the A_p geomagnetic indices is regularly the same order of magnitude as the index itself.[3] Errors are additionally associated with $F10.7$ solar flux index. References 3 and 4 give detailed error reports on the predictive accuracy of these indices as calculated by the National Oceanic and Atmospheric Administration (NOAA) Space Weather Prediction Center (SWPC).

There is an extensive body of work studying the relationship between atmospheric drag and satellite orbit determination. Reference 5 provides an in depth discussion to the numerous solar and geomagnetic indices as they pertain the satellite orbit determination. General studies of the accuracy of satellite drag modeling are found in References 6, 7, and 8. The relationship between atmospheric drag modeling and the orbits of several geodetic satellites is examined in Reference 9. However, much of the prior work focuses on atmospheric density model performance if solar flux and geomagnetic indices are well known, which is not the case with orbit predictions.

To quantify the effects of predicted solar and geomagnetic indices on predicted orbits, definitive orbit solutions were calculated for several geodetic satellites using laser ranging data and several common atmospheric density models seeded with the observed solar and geomagnetic indices. The results were compared to test cases using either the as-predicted indices, seasonal average index data, or arbitrarily fixed index data. The primary focus is to use high precision measurements to geodetic satellites to better understand how uncertainty in predicted atmosphere model input parameters impacts the ability to accurately predict a satellite's orbit.

II. Atmospheric Density Models

Historically, several atmospheric density models have been used in orbit determination algorithms to quantity the effects of atmospheric drag on a satellite's orbit. One of several atmospheric density models may be used within OCEAN during the orbit determination process. The exponential atmospheric model, the Naval Research Laboratory Mass Spectrometer and Incoherent Scatter Radar 2000 (NRLMSISE-00) model, and the Jacchia-Bowman 2008 (JB08) model are examined as they are a representative set of atmospheric density models.

The exponential atmospheric density model is the simplest model examined, and provides a test case that removes all effects of solar flux and geomagnetic index uncertainty, as the model does not take any of the indices as inputs. The atmospheric density is modeled as an exponential function, and does not account for diurnal, semi-diurnal, solar cyclical variations, or the variations due to the rotating atmosphere.[1]

Researchers at the NRL developed the NRLMSISE-00 model in 2002 to better calculate atmospheric temperature and density profiles for a number of atmospheric constituent components. This semi-empirical model is based on mass spectrometer and incoherent scatter data from a linage of a number of models, such as those discussed in References 10,11, 12, and 13. The NRLMSISE-00 model was developed to improve upon the Jacchia 1970 as well as the MSISE-90 models by incorporating data from additional satellite observation and on-orbit accelerometer data. This model includes diurnal and semi-diurnal effects and is driven by observed solar and geomagnetic activity indices.[14]

While the JB08 model is based on earlier Jacchia atmospheric density models, it makes use of an expanded set of solar flux and geomagnetic indices. This empirical model makes use of solar indices in the Ultraviolet spectrum, which is currently recognized as having an appreciable impact on atmospheric heating.[15] The model additionally makes use of not only the A_p geomagnetic index, but also an additional geomagnetic storm index to better capture atmospheric behavior during periods of high solar activity.[16]

Table 1 shows the solar flux and geomagnetic indices which are input parameters to the atmospheric density models.

Table 1. Solar Flux and Geomagnetic Index Inputs to Atmospheric Density Models

Atmospheric Density Model	F10.7	A_p	S10.7	M10.7	Y10.7	DST
Exponential						
Jacchia 1970	•	•				
NRLMSISE-00	•	•				
Jacchia-Bowman 2008	•	•	•	•	•	•

III. Solar Flux and Geomagnetic Indices

Several solar flux and geomagnetic indices have been used historically to incorporate effects due to solar flux variability and resulting geomagnetic behavior. Reference 17 gives an excellent overview of the indices commonly used in atmospheric density models.

The $F10.7$ index represents the solar flux at 10.7 cm. This index is used in many atmospheric density models as a proxy to account for atmospheric heating due to changing solar conditions.[1]

The A_p geomagnetic index has similarly been used in a number of atmospheric density models. This index is derived from a scaling of globally averaged magnetic measurements and is used to represent to amplitude of geomagnetic activity.[17]

The JB08 model makes use of an expanded set of solar flux and geomagnetic indices. The $S10.7$ index a proxy for solar flux in the 26-34 nm range, and corresponds to the flux in the Extreme Ultraviolet (EUV) range, which has been found to be highly correlated to thermospheric heating. The $M10.7$ is derived from measurements of the Mg II core to wing ratio in spectrometer measurement near 280 nm. This index has been found to be a good proxy for emissions in the EUV and Far Ultraviolet (FUV) ranges. The $Y10.7$ is an index of composite measurements of solar Lyman-α emissions and X-ray emissions. This index was newly incorporated in the JB08 model, and is used to model energy transfer to the atmosphere between 85km and 200km altitude. Last, the Disturbance Storm Time (DST) geomagnetic index is used to as an indicator of the strength of the storm-time ring current in the inner magnetosphere.[17]

The SWPC produces observed solar and geomagnetic indices using ground-based measurements from various locations around the world as noted in References 3 and 4. The SWPC also provides forecast data for $F10.7$ and geomagnetic indices up to 45 days in the future. When testing the as-predicted indices, the NRLMSISE-00 model in OCEAN uses the SWPC forecast data. When testing the seasonal average index data, index files in the SWPC format were generated to match the long-term solar cycle mean values given in Reference 16 for the appropriate point of the solar cycle. To test a set of arbitrarily fixed indices, values observed during the SLR data span were held constant throughout estimation and prediction.

The indices required by the JB08 model can be obtained through a commercial service provided by Space Environment Technologies via their Solar Irradiance Platform application, or for free if only historical data is required. When testing the predicted indices, the JB08 model in OCEAN uses the predictions provided in this index set. However, the obtained predictions for the DST geomagnetic indices did not extend over the full prediction time span; predicted values were not available beyond the first 5 days. As a best approximation of operational procedures, the DST index predictions were extended over the full orbit prediction interval by assuming that the last available predicted index values were held constant for dates farther in the future. As in the NRLMSISE-00 model case, index files were generated to match the long-term solar cycle mean values given in Reference 16 in order to test the impact of seasonal average index data. Similarly, to test a set of arbitrarily fixed indices, values observed during the SLR data span were held constant throughout propagation.

IV. Orbit Determination Methodology

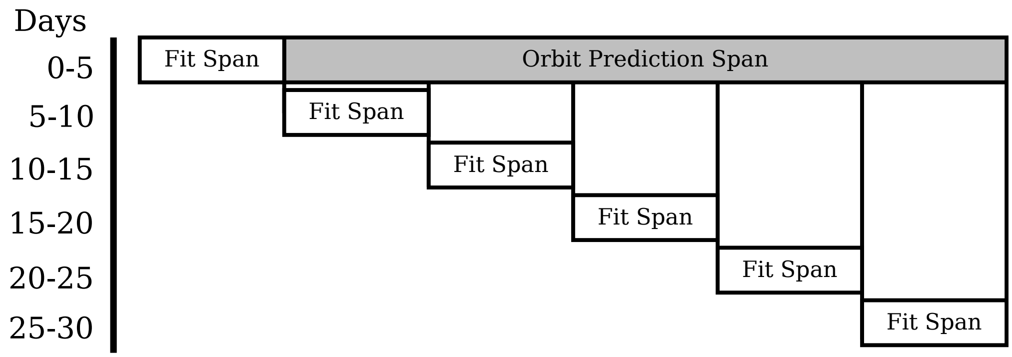
The Orbit/Covariance Estimation and ANalysis (OCEAN) software was used to preform high precision orbit estimations. OCEAN is a highly configurable, database driven software tool that enables precision orbit determination for a range of satellite missions. OCEAN allows users to simulate data, propagate a spacecraft state, or solve for an orbit using a Kalman Filter-Smoother (KFS) or Weighted Least Squares Orbit Determination (WLS-OD) process. Early history of OCEAN is given in Reference 18, while references 19, 20, 21, 22, 23, 24 and 25 discuss further developments.

The International Laser Ranging Service (ILRS) is responsible for coordinating and archiving Satellite Laser Ranging (SLR) data to support numerous geodetic research activities.[26] Several geodetic satellites have been designed and launched to facilitate the study of Earth's gravity by incorporating laser retro-reflectors for SLR measurements. These high precision SLR data are well suited to estimate highly precise orbits. For example, ILRS guidelines call for a precision of the normal point laser range measurement to the geodetic satellite LAGEOS-1 of under one centimeter.[27] In this study, OCEAN is used with SLR data to calculate precision orbits for the geodetic satellites LARETS, STARLETTE, and STELLA. Due to the availability of SLR data, these satellites provide an opportunity to identify performance differences for various atmosphere model and input parameter configurations. SLR data and atmosphere model parameters (both observed and predicted) are taken from February, 2016, which represents conditions near the middle of the current solar cycle.

Three atmospheric density models are examined: the exponential model, the NRLMSISE-00 model, and the JB08 model. As previously noted, three sets of indices (as-predicted, constant, and seasonal) were examined with these models to determine the impact of these input parameters on orbit prediction performance. Each model and input parameter configuration was used to produce an orbit solution and a twenty-five day prediction using five days of SLR data. The twenty-five day prediction time span was chosen to demonstrate the longer term variation in predictive accuracy.

By comparing predicted orbits to fitted orbits for each test case, the predictive accuracy of the underlying models including solar and geomagnetic variability is evaluated. Fitted orbits are determined with the OCEAN WLS-OD methodology using observed atmospheric model indices and successive five day increments of SLR data. The RMS residual error was tabulated to better assess the quality of the orbit solutions. The primary metric used to evaluate performance was the comparison between an orbit prediction and subsequent orbit fit solutions; The degree to which the orbit prediction agreed with the post-fit solution was used to evaluate the overall suitability of each atmospheric density model and input parameter combination for precision orbit prediction. The performance of the orbit predictions was chosen as a metric, as this is often critical in satellite mission operations. The definitive orbits for the JB08 and NRLMSISE-00 test runs are calculated using the JB08 and NRLMSISE-00 models using the true observed solar flux and geomagnetic indices, respectively. The exponential test cases use the JB08 definitive orbits, as prior work has shown higher quality orbit fits using the JB08 atmospheric density model. However, it has also been shown that the variation between definitive orbits using various atmospheric density model is on the order of tens of centimeters. This is several order of magnitude lower than the typical orbit prediction error observed, so the particular method used for generating definitive orbit.[2] The orbit comparison methodology is depicted in Figure 1.

Figure 1. Depiction of Orbit Solution Comparison Methodology



Several satellites were chosen so that the effects due to satellite altitude and inclination may be examined. For each test case, the orbital position and velocity, as well as the coefficients of drag and solar radiation pressure are estimated. Station specific biases as well as empirical accelerations are not estimated. This allows the effects of the dynamical models to be evaluated, rather than the ability to estimate unmodeled error sources. ILRS data from February and March 2016 are used. This represents a moderate level of solar activity.

The EGM2008 geopotential to degree and order 90 is used. Forces due to solid earth tides, ocean tides, and pole tides are modeled. OCEAN implements current International Earth Rotation and Reference Systems Service (IERS) conventions as specified in Reference 28.

A. LARETS

The Russian Space Agency (RSA) launched LARETS in 2003 as a follow on the WESTPAC satellite to study geodynamics.[29] The lower altitude is also well suited for studying the variability of atmospheric density models on orbit determination and prediction performance. Likewise, this satellite's passive operations and known cross-sectional area allow it to be easily used to study force and environment models including the impact of solar flux and geomagnetic variability[30].[31]

The nominal orbital elements for the LARETS satellite are given in Table 2.

Table 2. Nominal Orbital Elements for LARETS

Element	Nominal Value
Semi-major Axis	$7,068$ km
Eccentricity	0.0023
Inclination	97.7^o

B. STARLETTE

STARLETTE was launched by the Centre Nationale d'Etudes Spatiales (CNES) in 1975. Passive SLR measurements are possible given the numerous laser retroreflectors covering its surface.[32] The

STARLETTE satellite was designed to provide improved knowledge of the Earth's geopotential and to study solid Earth tides, ocean tides, and polar motion.[33] This satellite's passive operations, known cross-sectional area, and low altitude cause it to be well suited to investigate the effects of solar flux and geomagnetic variability on atmospheric density models and orbit prediction performance.

The nominal orbital elements for STARLETTE are given in Table 3.

Table 3. Nominal Orbital Elements for STARLETTE

Element	Nominal Value
Semi-major Axis	$7,190$ km
Eccentricity	0.0206
Inclination	49.83^o

C. STELLA

CNES launched STARLETTE in 1993 as nearly identical follow-on to STARLETTE. However, this satellite was placed in a higher inclination orbit to better capture Earth's gravitational variations at different latitudes.[34] STELLA is similarly well suited to study the effects of solar flux and geomagnetic variability on atmospheric density models and orbit prediction performance.

The nominal orbital elements for the STELLA satellite are given in Table 4.

Table 4. Nominal Orbital Elements for STELLA

Element	Nominal Value
Semi-major Axis	$7,178$ km
Eccentricity	0.0206
Inclination	98.6^o

V. Testing Results

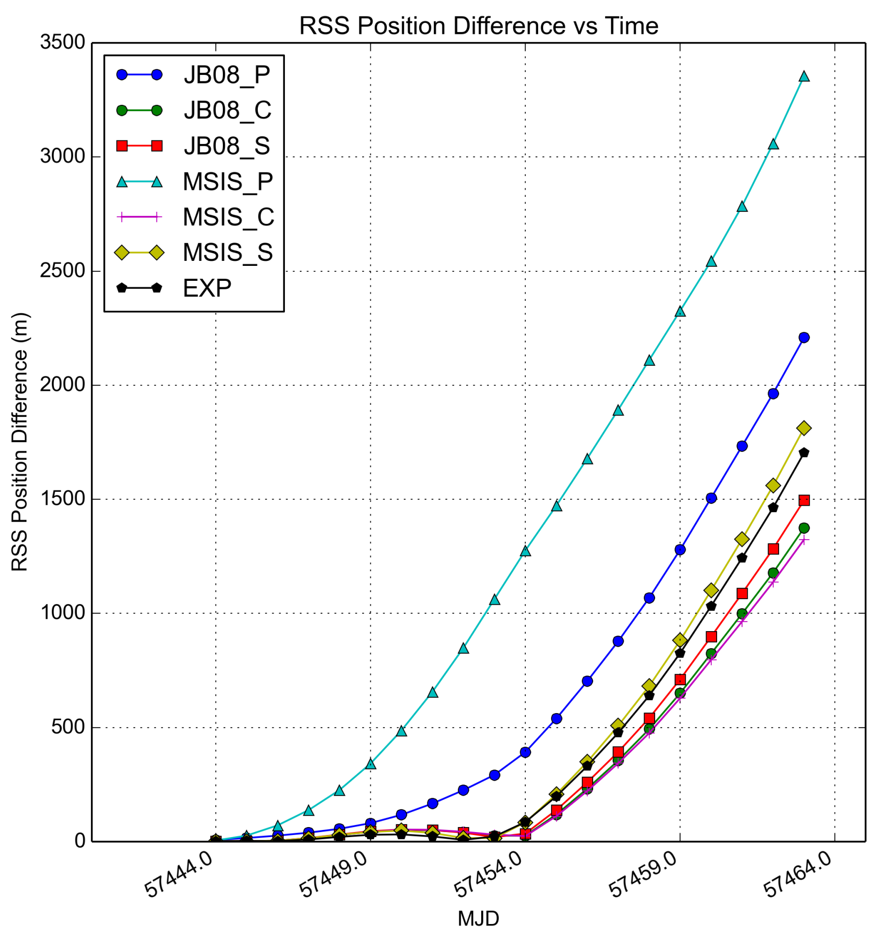
Separate results are presented for each satellite considered. For each satellite, Root Mean Square (RMS) residual errors are given in the Appendix for each fit span as a measure of orbit solution quality. While RMS residual error can indicate the quality of the orbit fit to the measurement data, it often does not provide guidance as to the predictive accuracy of the orbit solution.[22] Thus, Root Sum Square (RSS) position errors between the initial interval orbit prediction and the current interval orbit solution are given as the primary metric of the predictive accuracy for a specific atmospheric density model, solar flux, and geomagnetic index combination. Each fit interval is five days long. Plots of average daily RSS error are provided to visualize the relative performance of the underlying model and input parameter combinations for precision orbit determination.

A. LARETS Results

Results for the LARETS satellite are presented in Figure 2, which shows the average daily RSS position difference between the orbit prediction and subsequent orbit fits using the three strategies for mitigating uncertainties in solar and geomagnetic index prediction accuracy. Here, the Jacchia-Bowman 2008 models are labeled as JB08, the NRLMSISE-00 models are labeled as MSIS, and the exponential model is labeled as EXP. The suffix "P" denotes the use of the predicted index values as of the first day of the initial fit span, the suffix "C" denotes the use of constant index values throughout the initial fit and prediction span, and the suffix "S" denotes the use of average seasonal values throughout the initial fit and prediction span. The observed index values are always used during the definitive orbit fit so that the most precise orbit solution is used as a basis for comparison.

As can be seen, the use of the JB08 or NRLMSISE-00 model performs substantially better over the longer term when using the constant solar flux and geomagnetic index values. Both the JB08 and NRLMSISE-00 models yield more accurate orbit predictions with seasonal index values than with predicted index values. Interestingly, the exponential model yields the most accurate orbit predictions over the first eight days.

Figure 2. LARETS Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit

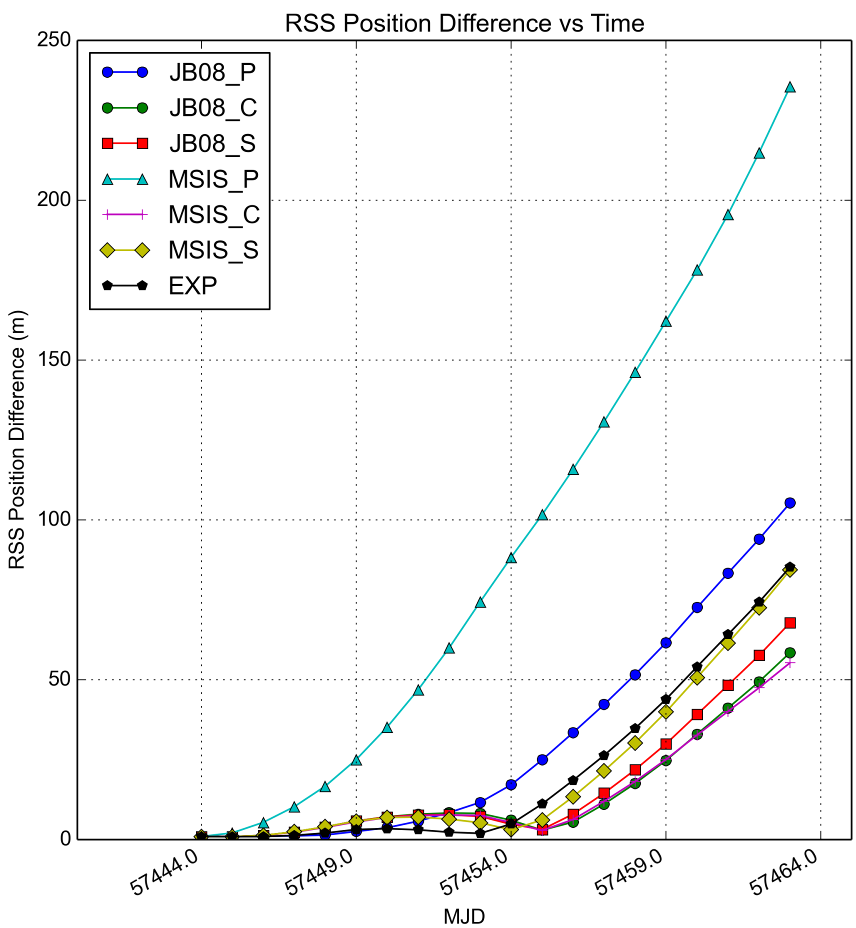


B. STARLETTE Results

Results for the STARLETTE satellite are presented in Figure 3, which shows the average daily RSS position error between the predicted orbit and subsequent fit orbits for each atmospheric density model and input parameter configuration. The same nomenclature used in Figure 2 applies.

The results for the STARLETTE satellite are consistent with results from the LARETS satellite. As can be seen in Figure 3, the JB08 and NRLMSISE-00 models again yield the most accurate longer term orbit predictions when used with constant index values. Use of the seasonal average index values also yields more accurate orbit predictions than use of the predicted index values. Again, the exponential model provides the most accurate orbit predictions over shorter orbit prediction spans. Here the exponential model yields the most accurate orbit prediction in the first nine days.

Figure 3. STARLETTE Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit

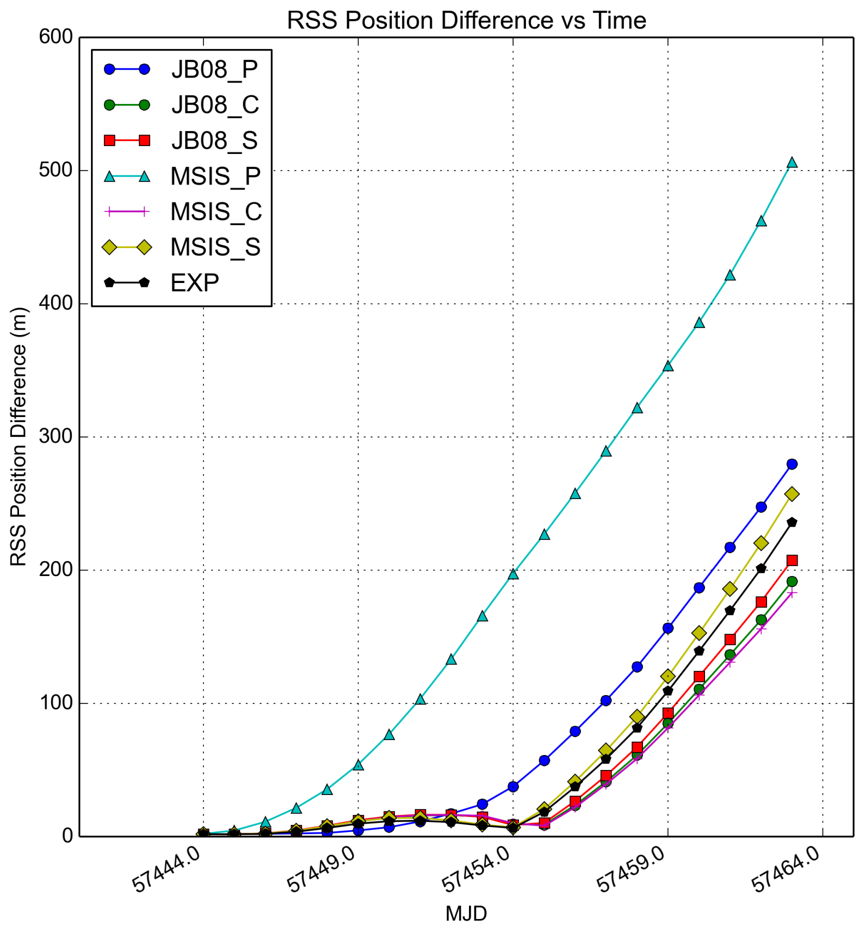


C. STELLA Results

The results from the STELLA test cases are presented below. The same nomenclature used in Figures 2 and 3 applies.

The plot of average daily RSS position error between the predicted orbit and fit orbits for each atmospheric density model is given in Figure 4. As in the previous results, the greatest consistency between the predicted and fitted orbits over the long term is achieved when using either the JB08 or NRLMSISE-00 models with constant index values. Here, the exponential model does not outperform the other models over the first week of prediction, but does still provide strong performance relative to using the predicted solar and geomagnetic indices.

Figure 4. STELLA Average Daily RSS Orbit Difference Between Predicted Orbit and Fit Orbit



VI. Conclusion

The effects of errors in the prediction of solar and geomagnetic indices are examined in the orbit determination processing for several geodetic satellites. Several strategies were examined for mitigating these errors, including use of as-predicted indices, constant indices, and seasonal average indices. For the test cases examined, the most accurate long term orbit predictions are achieved when using either the JB08 or NRLMSISE-00 atmospheric density models with solar flux and geomagnetic index values held constant over the fit and prediction time span. Interestingly, using the exponential atmospheric density model, which is not driven by the solar flux or geomagnetic indices, provides accurate shorter term predictions in majority of cases. This is perhaps due to the fact that by estimating a coefficient of drag, one is effectively estimating the average observed drag force, which is proportional to atmospheric density. Also interestingly, the test cases show that more accurate orbit predictions are found when using seasonal average flux and geomagnetic index values, rather than predicted flux and geomagnetic index values.

This work raises several questions informing future research. A more thorough examination of conditions throughout the solar cycle is needed to fully characterize the interaction between solar flux and geomagnetic index error and the impact on orbit prediction. Also, expanding the range of atmospheric density models may yield interesting results, since the exponential density model performed well. For example, a more complex model that does not make use of solar flux or geomagnetic indices might yield better orbit predicts. The Harris-Priester atmospheric density model includes diurnal effects, but does not use $F10.7$ or A_p indices as inputs.[35] Expanding the study to include additional satellites, would also provide an opportunity to assess the performance over a greater range of altitudes and inclinations.

Ultimately, it has been shown that the variability in the solar flux and geomagnetic indices can indeed cause large accumulated errors in orbit prediction accuracy for satellites in LEO, and there exist methods for mitigating these uncertainties.

Appendix A: Orbit Solution Statistical Summary

The Root Mean Square (RMS) error residuals for each fit are provided here to demonstrate the precision of the orbit solution. The averaged differences between the predicted orbit solution and definitive fit span are tabulated here as well.

Table 5 shows the RMS residual error for each of the fit intervals used as truth. The definitive interval fits used the as-observed solar flux and geomagnetic index data to provide the most accurate orbit solution possible. Thus, there is only one definitive fit table per satellite test case. The exponential model used the JB08 definitive orbits. Table 6 gives the RMS residual error for the STELLA definitive fit intervals, while Table 7 give the RMS residual error for the STARLETTE definitive fit intervals.

Table 5. RMS Error Residuals in Meters for LARETS Definitive Fit Span

Fit Span	NRLMSISE-00 Model	JB08 Model
Interval 1	0.6334	0.5296
Interval 2	0.4227	1.2486
Interval 3	0.4625	0.7351
Interval 4	0.8187	0.4803
Interval 5	0.6287	0.2707

Table 6. RMS Error Residuals in Meters for STELLA Definitive Fit Span

Fit Span	NRLMSISE-00 Model	JB08 Model
Interval 1	0.7602	0.7589
Interval 2	0.4921	0.5383
Interval 3	0.6959	0.7110
Interval 4	0.8252	0.7968
Interval 5	0.1818	0.1839

The RMS residual error for each orbit solution is given in Tables 8, 9, and 10. These are tabulated in three separate tables corresponding to which solar flux and geomagnetic index values were used.

Table 7. RMS Error Residuals in Meters for STARLETTE Definitive Fit Span

Fit Span	NRLMSISE-00 Model	JB08 Model
Interval 1	0.3773	0.3716
Interval 2	0.2973	0.3060
Interval 3	0.3832	0.4172
Interval 4	0.4555	0.4518
Interval 5	0.1557	0.1557

Table 8. RMS Error Residuals in Meters for Orbit Solution Span Using Predicted Index Values

Satellite	NRLMSISE-00 Model	JB08 Model	Exponential
LARETS	0.5258	1.2950	0.8928
STELLA	0.6062	0.6167	0.5863
STARLETTE	0.2340	0.2504	0.2384

Appendix B: Orbit Comparison Statistical Summary

This section summarizes the statistics comparing the predicted ephemeris to the definitive ephemeris. The RMS position difference between the predicted orbit and fit orbit is given for each time interval and for each combination of atmospheric density models, solar flux, and geomagnetic index values. Table 11 summarizes the results for the LARETS test case. Table 12 summarizes the results for the STELLA test case. The results for the STARLETTE test case are found in Table 13. The lowest RMS position difference for each time span is highlighted in bold text. The Jacchia-Bowman 2008 models are labeled as JB08, the NRLMSISE-00 models are labeled as MSIS, and the exponential model is labeled as EXP.

Table 9. RMS Error Residuals in Meters for Orbit Solution Span Using Constant Index Values

Satellite	NRLMSISE-00 Model	JB08 Model	Exponential
LARETS	0.7964	0.7764	0.8928
STELLA	0.5944	0.5972	0.5863
STARLETTE	0.2357	0.2362	0.2384

Table 10. RMS Error Residuals in Meters for Orbit Solution Span Using Seasonal Index Values

Satellite	NRLMSISE-00 Model	JB08 Model	Exponential
LARETS	0.7960	0.7763	0.8928
STELLA	0.5935	0.6493	0.5863
STARLETTE	0.2361	0.2330	0.2384

Table 11. RMS of Position Difference in Meters Between Orbit Prediction and Definitive Fit for LARETS

Predict Span	JB08_P	JB08_C	JB08_S	MSIS_P	MSIS_C	MSIS_S	EXP
Interval 1	34.08	14.98	16.02	124.49	15.26	15.40	**10.79**
Interval 2	192.41	43.29	43.72	726.42	45.22	35.50	**25.18**
Interval 3	754.98	296.85	328.11	1709.94	**286.05**	423.90	398.92
Interval 4	1766.87	**1036.23**	1128.54	2834.12	1766.87	1374.27	1290.20
Interval 5	3026.63	**2059.85**	2239.86	4388.24	3026.63	2701.90	2560.94

Table 12. RMS of Position Difference in Meters Between Orbit Prediction and Definitive Fit for STELLA

Predict Span	JB08_P	JB08_C	JB08_S	MSIS_P	MSIS_C	MSIS_S	EXP
Interval 1	**2.32**	4.35	4.60	19.59	4.36	4.39	3.72
Interval 2	14.83	14.76	15.14	114.01	15.00	12.37	**10.64**
Interval 3	86.81	35.32	38.98	262.36	**33.78**	53.96	48.81
Interval 4	221.61	142.23	154.22	429.14	**136.20**	193.17	176.64
Interval 5	393.15	299.76	323.07	650.07	**288.20**	398.56	365.17

Table 13. RMS of Position Difference in Meters Between Orbit Prediction and Definitive Fit for STARLETTE

Predict Span	JB08_P	JB08_C	JB08_S	MSIS_P	MSIS_C	MSIS_S	EXP
Interval 1	**1.21**	2.29	2.27	9.25	2.22	2.31	1.36
Interval 2	7.31	7.60	7.21	51.39	7.14	6.39	**2.95**
Interval 3	36.05	**10.24**	12.67	118.22	10.64	18.04	21.99
Interval 4	84.73	43.07	50.39	198.73	**41.57**	63.74	65.95
Interval 5	141.23	89.30	102.37	306.95	**81.25**	124.95	121.66

References

[1]Vallado, D. A., *Fundamentals of astrodynamics and applications*, Vol. 12, Springer Science & Business Media, 2001.

[2]Warner, J. G. and Lemm, K. M., "On Comparing Precision Orbit Solutions of Geodetic Satellites Given Several Atmospheric Density Models," *AAS/AIAA Astrodynamics Specialist Conference*, No. AAS 15-752, American Astronautical Society, 2015.

[3]Center, N. S. W. P., "Estimated Ap Forecast Verification," http://www.swpc.noaa.gov/sites/default/files/images/u30/Estimated%20Ap.pdf.

[4]Center, N. S. W. P., "10.7cm Solar Flux Forecast Verification," http://www.swpc.noaa.gov/sites/default/files/images/u30/F10.7%20Solar%20Flux.pdf.

[5]Vallado, D. A. and Finkleman, D., "A critical assessment of satellite drag and atmospheric density modeling," *AIAA/AAS astrodynamics specialist conference and exhibit*, 2008, pp. 18–21.

[6]Marcos, F. A., "Accuracy of atmospheric drag models at low satellite altitudes," *Advances in Space Research*, Vol. 10, No. 3, 1990, pp. 417–422.

[7]Mance, S., *An investigation into satellite drag modeling performance*, Ph.D. thesis, University of Kansas, 2010.

[8]Bruinsma, S., Exertier, P., and Biancale, R., "An assessment of new satellite total density data for improving upper atmosphere models," *Planetary and Space Science*, Vol. 47, No. 12, 1999, pp. 1465–1473.

[9]Krzysztof, S., "Impact of the Atmospheric Drag on Starlette, Stella, Ajisai, and Lares Orbits," *Artificial Satellites*, Vol. 50, No. 1, 2015, pp. 1–18.

[10]Hedin, A., Salah, J., Evans, J., Reber, C., Newton, G., Spencer, N., Kayser, D. C., Alcayde, D., Bauer, P., Cogger, L., et al., "A global thermospheric model based on mass spectrometer and incoherent scatter data MSIS, 1. N2 density and temperature," *Journal of Geophysical Research*, Vol. 82, No. 16, 1977, pp. 2139–2147.

[11]Hedin, A., "A Revised thermospheric model based on mass spectrometer and incoherent scatter data: MSIS-83," *Journal of Geophysical Research: Space Physics (1978–2012)*, Vol. 88, No. A12, 1983, pp. 10170–10188.

[12]Rees, D., Barnett, J., and Labitzke, K., "COSPAR International Reference Atmosphere: 1986. Pt. 2: Middle atmosphere models," *Advances in space research*, Vol. 10, 1990.

[13]Hedin, A. E., "Extension of the MSIS thermosphere model into the middle and lower atmosphere," *Journal of Geophysical Research: Space Physics (1978–2012)*, Vol. 96, No. A2, 1991, pp. 1159–1172.

[14]Picone, J., Hedin, A., Drob, D. P., and Aikin, A., "NRLMSISE-00 empirical model of the atmosphere: Statistical comparisons and scientific issues," *Journal of Geophysical Research: Space Physics (1978–2012)*, Vol. 107, No. A12, 2002, pp. SIA–15.

[15]Bowman, B. R., Tobiska, W. K., Marcos, F. A., et al., "A new empirical thermospheric density model JB2006 using new solar indices," *AIAA*, Vol. 6166, 2006, pp. 2006.

[16]Bowman, B. R., Tobiska, W. K., Marcos, F. A., Huang, C. Y., Lin, C. S., and Burke, W. J., "A new empirical thermospheric density model JB2008 using new solar and geomagnetic indices," *AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Honolulu, Hawaii*, 2008.

[17]Tobiska, W. K., Bowman, B. R., and Bouwer, S. D., "Solar and Geomagnetic Indices for Thermospheric Density Models," http://spaceweather.usu.edu/files/uploads/PDF/Ch4_solar_geomag_indices.pdf.

[18]Soyka, M. T., Middour, J. W., Binning, P. W., Pickard, H., and Fein, J., "The Naval Research Laboratory's orbit/covariance estimation and analysis software- OCEAN," *Astrodynamics 1997*, 1997, pp. 1567–1586.

[19]Soyka, M. T., Middour, J. W., and Fein, J., "Simultaneous orbit determination of large satellite constellations," *Spaceflight mechanics 1998*, 1998, pp. 1275–1293.

[20]Binning, P. W., Soyka, M. T., and Middour, J. W., "Orbit determination using space to ground Differential GPS in NRL's OCEAN package," *Astrodynamics 1999*, 2000, pp. 421–434.

[21]Seago, J. H., Davis, M. A., Smith, W., Fein, J., Brown, B., Middour, J., Soyka, M., and Lydick, E., "More results of naval space surveillance system calibration using satellite laser ranging." *Advances in the Astronautical Sciences*, Vol. 112, 2002, pp. 1177–1196.

[22]Warner, J. G. and Lum, A., "On Comparing Precision Orbit Solutions of Geodetic Satellites Given Several Ocean Tide and Geopotential Models," *AIAA/AAS Astrodynamics Specialist Conference*, American Institute of Aeronautics and Astronautics, 2014, pp. 1–12.

[23]Warner, J. G. and Lemm, K., "Effects of Full Order Geopotential Hessian on Precision Orbit Determination of Geodetic Satellites," *AIAA/AAS Astrodynamics Specialist Conference*, American Institute of Aeronautics and Astronautics, 2014, pp. 1–11.

[24]Eiler, E. and Warner, J. G., "Characterizing the Effects of Low Order Perturbations on Geodetic Satellite Precision Orbit Determination," *AAS/AIAA Astrodynamics Specialist Conference*, No. AAS 15-778, American Astronautical Society, 2015.

[25]Ward, E. M., Warner, J. G., and Maisonobe, L., "Do Open Source Tools Rival Heritage Systems? A comparison of tide models in OCEAN and Orekit," *AIAA/AAS Astrodynamics Specialist Conference*, American Institute of Aeronautics and Astronautics, 2014, pp. 1–8.

[26]Pearlman, M., Degnan, J., and Bosworth, J., "The International Laser Ranging Service," *Advances in Space Research*, Vol. 30, No. 2, 2002, pp. 135 – 143.

[27]"ILRS System Performance Standards," http://ilrs.gsfc.nasa.gov/network/system_performance/index.html, 2014.

[28]Petit, G. and Luzum, B., "IERS conventions (2010)," Tech. rep., DTIC Document, 2010.

[29]Shargorodsky, V., "WESTPAC Satellite, the Scientific-technical Note for User," *Science Research Institute for Precision Device Engineering, Moscow, Russia*, 1997.

[30]Rutkowska, M., "The orbit estimation for Larets satellite," *Journées 2005-systèmes de référence spatio-temporels*, Vol. 1, 2006, p. 87.

[31]Rutkowska, M., "The accuracy of orbit estimation for the low-orbit satellites LARETS and WESTPAC," *Advances in Space Research*, Vol. 36, No. 3, 2005, pp. 498–503.

[32]Kramer, H. J., *Observation of the Earth and Its Environment: Survey of Missions and Sensors*, Springer, 2002.

[33]Kanner, L. and Associates, "Translation of 'Le satellite de geodesie 'Starlette',' Groupe de Recherches de Geodesie Spatiale, Centre National d'Etudes Spatiales, Bretigny-sur-Orge, France, Report, 1974, 25 pp," National Aeronautics and Space Administration, Washington, D. C., July 1974.

[34]Lefebvre, M., "New Satellite Missions for Solid Earth Studies-Status and Preparations," *CSTG Bulletin*, , No. 11, 1989, pp. 25–32.

[35]Harris, I. and Priester, W., "Of the diurnal variation of the upper atmosphere," *Journal of the Atmospheric Sciences*, Vol. 22, No. 1, 1965, pp. 3–10.