Coding Instead of Splitting - Algebraic Combinations in Time and Space

Muriel Medard
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Final Report
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Our accomplishments fall into several categories. In [1] and [2], we consider different aspects of the problem of distributed function computation across a network and propose a network flow approach to characterize trade-offs between computation and communication costs. Moreover, we demonstrate an application of this framework in cloud computing. In [3], we consider a network coded distributed storage problem in highly dynamic environments where the nodes and the communication links availability is volatile. We propose a robust decentralized network coded approach that requires a small number of repair nodes for node recovery. In [4], we propose a reliable, multi-path protocol called Multi-Path TCP with Network Coding (MPTCP/NC) and show that it can provide users in mobile environments a higher quality of service by enabling the use of multiple network technologies and the capability to overcome packet losses. In [5], we introduce inflatable sparse network coding (TSNC), a scheme in which the density of network coded packets varies during a transmission session. We also propose a family of inflatable sparse codes for multicast erasure networks with a controllable trade-off between completion time performance and decoding complexity. In [6] and [7], we investigate different aspects of coding over a two unicast network such as the tightness of the generalized network sharing bound. We also develop a new linear network coding algorithm for two-unicast networks over directed acyclic graphs. In [8], we analyze the matched filter decoding error probability in random binary and Gaussian coding setups and show that the performance in the two cases is surprisingly similar without explicit adaptation of the codeword construction to the modulation. In [9], we study an adaptive sampling scheme where sampling times do not need to be stored/transmitted since they can be computed using a function of previously taken samples. Therefore the energy consumption of the sampling process is efficiently reduced even when the signal is varying slowly. In [10], we present a general method for inferring direct effects from an observed correlation matrix containing both direct and indirect effects, while in [11], we introduce a method to solve the inverse problem of identifying the source of the propagated signal for large, complex real-world networks.

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Final Report for AFOSR Contract AFOSR Grant FA9550-13-1-0023: Coding Instead of Splitting - Algebraic Combinations in Time and Space

1 Basic Information

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- Organization Name: MIT
- Program Manager: James H. Lawton, PhD

2 Report Abstract

Our accomplishments fall into several categories. In [1] and [2], we consider different aspects of the problem of distributed function computation across a network and propose a network flow approach to characterize trade-offs between computation and communication costs. Moreover, we demonstrate an application of this framework in cloud computing. In [3], we consider a network coded distributed storage problem in highly dynamic environments where the nodes and the communication links availability is volatile. We propose a robust decentralized network coded approach that requires a small number of repair nodes for node recovery. In [4], we propose a reliable, multi-path protocol called Multi-Path TCP with Network Coding (MPTCP/NC) and show that it can provide users in mobile environments a higher quality of service by enabling the use of multiple network technologies and the capability to overcome packet losses. In [5], we introduce tunable sparse network coding (TSNC), a scheme in which the density of network coded packets varies during a transmission session. We also propose a family of tunable sparse codes for multicast erasure networks with a controllable trade-off between completion time performance and decoding complexity. In [6] and [7], we investigate different aspects of coding over a two unicast network such as the tightness of the generalized network sharing bound. We also develop a new linear network coding algorithm for two-unicast-Z networks over directed acyclic graphs. In [8], we analyze the matched filter decoding error probability in random binary and Gaussian coding setups and show that the performance in the two cases is surprisingly similar without explicit
adaptation of the codeword construction to the modulation. In [9], we study an adaptive sampling scheme where sampling times do not need to be stored/transmitted since they can be computed using a function of previously taken samples. Therefore the energy consumption of the sampling process is efficient specially when the signal is varying slowly. In [10], we present a general method for inferring direct effects from an observed correlation matrix containing both direct and indirect effects, while in [11], we introduce a method to solve the inverse problem of identifying the source of the propagated signal for large, complex real world networks.

3 Report Abstract

Our accomplishments fall into several categories.

Distributed Functional Computation over Networks. In [1], we consider different aspects of the problem of compressing for function computation across a network, which we call network functional compression. In network functional compression, computation of a function (or, some functions) of sources located at certain nodes in a network is desired at receiver(s). The rate region of this problem has been considered in the literature under certain restrictive assumptions, particularly in terms of the network topology, the functions, and the characteristics of the sources. In [1], we present results that significantly relax these assumptions. For a one-stage tree network, we characterize a rate region by introducing a necessary and sufficient condition for any achievable coloring-based coding scheme called coloring connectivity condition. We also propose a modularized coding scheme based on graph colorings to perform arbitrarily closely to rate lower bounds. For a general tree network, we provide a rate lower bound based on graph entropies and show that, this bound is tight in the case of having independent sources. In particular, we show that, in a general tree network case with independent sources, to achieve the rate lower bound, intermediate nodes should perform computations. However, for a family of functions and random variables, which we call chain rule proper sets, it is sufficient to have no computations at intermediate nodes to perform arbitrarily closely to the rate lower bound. In addition, we consider practical issues of coloring-based coding schemes and propose an efficient algorithm to compute a minimum entropy coloring of a characteristic graph under some conditions on source distributions and/or the desired function. Finally, extensions of these results for cases of having feedback and lossy function computations are discussed.

In [2], by using network flow principles, we propose algorithms to address various challenges in cloud computing. One of the main challenges is to consider both communication and computation constraints in the network. In the proposed network flow framework, we model the amount of computation in each node of the network as a function of its total self-loop flows. We consider two computation cost models: a linear computation cost model and a maximum computation cost model. We show that, our network flow framework can be used as a systematic technique of balancing computation loads over different nodes of the network. This network flow framework can also be used for cloud network design. A network
topology is optimal for certain computations if it maximizes the total computation rate under communication/computation constraints. We propose a greedy algorithm to design a cloud network with a certain network characteristics in terms of communication and computation costs. We provide simulation results to illustrate the performance of our algorithms.

**Network Coded Distributed Storage:** In distributed cloud storages fault tolerance is achieved by regenerating the lost data from the surviving clouds. Recent studies suggest using maximum distance separable (MDS) network codes in cloud storage systems to allow efficient and reliable recovery after node faults. MDS codes are designed to use a substantial number of repair nodes and rely on centralized management and a static fully connected network between the nodes. However, in highly dynamic environments, like edge caching in communication networks or peer-to-peer networks, the nodes and the communication links availability is very volatile. In these scenarios MDS codes functionality is limited. In [3], we study a non-MDS network coded approach, which operates in a decentralized manner and requires a small number of repair nodes for node recovery. We investigate long-term behavior of the modeled system and demonstrate, analytically and numerically, the durability gains over uncoded storage.

**Multi-Path TCP with Network Coding:** Existing mobile devices have the capability to use multiple network technologies simultaneously to help increase performance; but they rarely, if at all, effectively use these technologies in parallel. In [4], we first present empirical data to help understand the mobile environment when three heterogeneous networks are available to the mobile device (i.e., a WiFi network, WiMax network, and an Iridium satellite network). We then propose a reliable, multi-path protocol called Multi-Path TCP with Network Coding (MPTCP/NC) that utilizes each of these networks in parallel. An analytical model is developed and a mean-field approximation is derived that gives an estimate of the protocols achievable throughput. Finally, a comparison between MPTCP and MPTCP/NC is presented using both the empirical data and mean-field approximation. Our results show that network coding can provide users in mobile environments a higher quality of service by enabling the use of multiple network technologies and the capability to overcome packet losses due to lossy, wireless network connections.

**Tunable Sparse Network Coding:** In [5], we show the potential and key enabling mechanisms for tunable sparse network coding, a scheme in which the density of network coded packets varies during a transmission session. At the beginning of a transmission session, sparsely coded packets are transmitted, which benefits decoding complexity. As the transmission continues and the receivers have accumulated coded packets, the coding density is increased. We propose a family of tunable sparse network codes (TSNCs) for multicast erasure networks with a controllable trade-off between completion time performance to decoding complexity. Coding density tuning can be performed by designing time dependent coding matrices. In multicast networks, this tuning can be performed within the network by designing time-dependent pre-coding and network coding matrices with mild conditions on the network structure for specific densities. We present a mechanism to perform efficient
Gaussian elimination over sparse matrices going beyond belief propagation but maintaining low decoding complexity. Supporting implementation results are provided showing the trade-off between decoding complexity and completion time.

Coding Algorithms for Two Unicast Network: In [6], we study two-unicast-Z networks—two-source two destination (two-unicast) wireline networks over directed acyclic graphs, where one of the two destinations (say the second destination) is apriori aware of the interfering (first) sources message. For certain classes of two-unicast-Z networks, we show that the rate-tuple \((N, 1)\) is achievable as long as the individual source-destination cuts for the two source-destination pairs are respectively at least as large as \(N\) and 1, and the generalized network sharing cut—a bound previously defined by Kamath et. al. - is at least as large as \(N + 1\). We show this through a novel achievable scheme which is based on random linear coding at all the edges in the network, except at the GNS-cut set edges, where the linear coding coefficients are chosen in a structured manner to cancel interference at the receiver first destination.

In [7], we derive a new linear network coding algorithm for two-unicast-Z networks over directed acyclic graphs, that is, for two-unicast networks where one destination has apriori information of the interfering source message. Our algorithm discovers linear network codes for two-unicast-Z networks by combining ideas of random linear network coding and interference neutralization. We show that our algorithm outputs an optimal network code for networks where there is only one edge emanating from each of the two sources. The complexity of our algorithm is polynomial in the number of edges of the graph.

Matched Filter Decoding: In [8], we consider the additive white Gaussian noise channel with an average input power constraint in the power-limited regime. A well-known result in information theory states that the capacity of this channel can be achieved by random Gaussian coding with analog quadrature amplitude modulation (QAM). In practical applications, however, discrete binary channel codes with digital modulation are most often employed. We analyze the matched filter decoding error probability in random binary and Gaussian coding setups in the wide bandwidth regime, and show that the performance in the two cases is surprisingly similar without explicit adaptation of the codeword construction to the modulation. The result also holds for the multiple access and the broadcast Gaussian channels, when signal-to-noise ratio is low. Moreover, the two modulations can be even mixed together in a single codeword resulting in a hybrid modulation with asymptotically close decoding behavior. In this sense the matched filter decoder demonstrates the performance that is largely insensitive to the choice of binary versus Gaussian modulation.

Time-Stampless Adaptive Nonuniform Sampling: Advances in sampling and coding theory have contributed significantly towards lowering power consumption of resource-constrained devices, e.g. battery-operated sensor nodes, enabling them to operate for extended periods of time. In [9], rate and energy efficiency of a recently proposed adaptive nonuniform sampling framework by Feizi et al., called Time-Stampless Adaptive Nonuni-
form Sampling (TANS), is examined and compared against state-of-the-art methods. TANS addresses one of the main limitations of nonuniform sampling schemes: sampling times do not need to be stored/transmitted since they can be computed using a function of previously taken samples. The sampling rate is adapted continuously with the aim of reducing the rate and therefore the energy consumption of the sampling process when the signal is varying slowly. Three TANS methods are proposed for different signal models and sampling requirements: i) TANS by polynomial extrapolation, which only assumes the third derivative of the signal is bounded but requires no other specific knowledge of the signal; ii) TANS by incremental variation, where the sampling time intervals are chosen from a lattice; and iii) TANS constrained to a finite set of sampling rates. Practical implementation details of TANS are discussed, and its rate and energy performance are compared with uniform sampling followed by a transformation-based compression, nonuniform sampling, and compressed sensing. Our results demonstrate that TANS provides significant improvements in terms of both the rate-distortion performance and the energy consumption compared against the other approaches.

**Inference of Direct Relationships over Networks:** Recognizing direct relationships between variables connected in a network is a pervasive problem in biological, social and information sciences as correlation-based networks contain numerous indirect relationships. In [10], we present a general method for inferring direct effects from an observed correlation matrix containing both direct and indirect effects. We formulate the problem as the inverse of network convolution, and introduce an algorithm that removes the combined effect of all indirect paths of arbitrary length in a closed-form solution by exploiting eigen-decomposition and infinite-series sums. We demonstrate the effectiveness of our approach in several network applications: distinguishing direct targets in gene expression regulatory networks; recognizing directly interacting amino-acid residues for protein structure prediction from sequence alignments; and distinguishing strong collaborations in co-authorship social networks using connectivity information alone. In addition to its theoretical impact as a foundational graph theoretic tool, our results suggest network deconvolution is widely applicable for computing direct dependencies in network science across diverse disciplines.

**Source Inference in Networks:** Several models exist for diffusion of signals across biological, social, or engineered networks. However, the inverse problem of identifying the source of such propagated information appears more difficult even in the presence of multiple network snapshots, and especially for the single snapshot case, given the many alternative, often similar, progression of diffusion that may lead to the same observed snapshots. Mathematically, this problem can be undertaken using a diffusion kernel that represents diffusion processes in a given network, but computing this kernel is computationally challenging in general. In [11], we propose a path-based network diffusion kernel which considers edge-disjoint shortest paths among pairs of nodes in the network and can be computed efficiently for both homogeneous and heterogeneous continuous-time diffusion models. We use this network diffusion kernel to solve the inverse diffusion problem, which we term Network Infusion (NI), using
both likelihood maximization and error minimization. The minimum error NI algorithm is based on an asymmetric Hamming premetric function and can balance between false positive and false negative error types. We apply this framework for both single-source and multi-source diffusion, for both single-snapshot and multi-snapshot observations, and using both uninformative and informative prior probabilities for candidate source nodes. We also provide proofs that under a standard susceptible-infected diffusion model, (1) the maximum-likelihood NI is mean-field optimal for tree structures or sufficiently sparse Erdos- Renyi graphs, (2) the minimum-error algorithm is mean-field optimal for regular tree structures, and (3) for sufficiently-distant sources, the multi-source solution is mean-field optimal in the regular tree structure. Moreover, we provide techniques to learn diffusion model parameters such as observation times. We apply NI to several synthetic networks and compare its performance to centrality-based and distance-based methods for Erdos-Renyi graphs, power-law networks, symmetric and asymmetric grids. Moreover, we use NI in two real-world applications. First, we identify the news sources for 3,553 stories in the Digg social news network, and validate our results based on annotated information, that was not provided to our algorithm. Second, we use NI to identify infusion hubs of human diseases, defined as gene candidates that can explain the connectivity pattern of disease-related genes in the human regulatory network. NI identifies infusion hubs of several human diseases including T1D, Parkinson, MS, SLE, Psoriasis and Schizophrenia. We show that, the inferred infusion hubs are biologically relevant and often not identifiable using the raw p-values.

4 Archival Publications


**Other Information** Changes in research objectives: None Change in AFOSR Program Manager: James H. Lawton, PhD Extensions granted or milestones slipped, if any: none.
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James H. Lawton

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Abstract
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Archival Publications (published) during reporting period:

[11] Soheil Feizi, Ken Duy, Manolis Kellis and Muriel Medard, Network Infusion to Infer
Information Sources in Networks, Oral and poster presentations at RECOMB Conference on Systems Biology and Regulatory Genomic, 2014. A draft is available on MIT DSpace https://dspace.mit.edu/handle/1721.1/92031

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No

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Changes in research objectives (if any):
Change in AFOSR Program Manager, if any:
James H. Lawton, PhD

Extensions granted or milestones slipped, if any:
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Laboratory Task Manager
Program Officer
Research Objectives
Technical Summary

Funding Summary by Cost Category (by FY, $K)

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