TECHNICAL REPORT ON A PROJECT ON
LOW-SPEED SLIDING

by
E. Rabinowicz & B.G. Rightmire

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Supervisor: B.G. Rightmire
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Lubrication Laboratory
Department of Mechanical Engineering
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Technical Report on a Project on

LOW-SPEED SLIDING

being carried out for the
Office of Naval Research
under Contract NR 065-335

Introduction

Three different aspects of the project were studied during the past year. They were (1) Design and operation of an apparatus to measure stick-slip amplitudes at various velocities and using springs of different stiffness, (2) Calculation of the size distribution of asperities through the autocorrelation technique described in earlier reports. (3) Studies of the frictional properties of rubber at low speeds and with controlled atmosphere and temperature. These experiments are described in the three sections of the report. In addition work has continued on the design and operation of a very low-speed friction apparatus which records velocities as well as friction forces, readings being taken automatically for short periods of time at regular intervals to conserve paper during an extended experiment. The various parts of the apparatus have functioned separately and the completed equipment will be described in a subsequent report.

Two publications connected with the project have appeared since the last report. They are

A copy of each is appended to this report.

Two theses have been written by students working on the project.
"The Influence of Spring Stiffness on Stick-Slip" by Donald Clavin, submitted for the Bachelor of Science degree in the Mechanical Engineering Department. The results of this thesis are summarized in Section I.

"Experimental Study of Rubber Friction" by Kwan-lok So, submitted for the Master of Science degree in the Mechanical Engineering Department. The results of this thesis are summarized in Section III.
I. Relation between stick-slip and spring stiffness

The apparatus used for these experiments is shown in Figure 1. It employs a \( \frac{3}{8} \)-inch diameter rider having a hemispherically shaped end which contacts a flat specimen mounted on a steel turntable. The rider, which is loaded by a dead weight, is held by a supporting arm that is free to deflect with the friction forces acting on the rider and in turn these forces act on the two strain rings in series; on one of these are mounted 4 strain gages. Both the change in resistance of the strain gages due to the friction force, and the change in resistance of an interruptor mechanism on the rotating disk are displayed on a Sanborn two-channel recorder.

One of the two strain rings is removable, and the stiffness of the friction arm may be changed by replacing it by one of different thickness. Four rings were used in all

<table>
<thead>
<tr>
<th>#</th>
<th>produced a deflexion of</th>
<th>cm/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0.226</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>0.068</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.034</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>0.008</td>
</tr>
</tbody>
</table>

Experiments were carried out of copper on copper lubricated by cetane and steel on steel unlubricated, two combinations of materials that are known readily to give rise to stick-slip. Friction runs were carried out over a range of velocities with all four rings in turn mounted on the friction arm, and the amplitude of the stick-slip, namely the difference between the spring force at the beginning and end of the slip, was measured in each case and is shown plotted in Figs. 2 (copper), and 3 (steel). It was found necessary to test all
four springs within a short period of time (say 1-2 hours) as
otherwise the results were not strictly comparable.

A number of results emerge from these figures.

1. At any sliding velocity, the amplitude of stick-slip is
greater if the spring is made less stiff. If the spring is too stiff
stick-slip is completely eliminated.

2. For any spring there is a limiting velocity above which
stick-slip does not occur. The smaller the stiffness of the spring
the greater this upper limiting velocity.

3. For any spring there seems to be a limiting velocity below
which stick-slip does not occur. The smaller the stiffness of the
spring, the lower this lower limiting velocity.

These results are readily explicable in terms of the probable
f - v curve (Fig. 4). It may be assumed that with any spring there is
a limiting negative slope in the f - v curve, such that for any steeper
slope stick-slip can occur, and for any less steep slope it cannot. Also
as the steepness of the slope is increased, the amplitude of the stick-
slip increases correspondingly. If a stiffer spring is used, we will
expect that a greater steepness of slope is required before stick-slip
commences. (c.f. Fig. 5). For the stiffest springs used in these tests
stick-slip did not take place since the steepness of the slope obtained
in our tests was nowhere great enough.

Further experiments are planned which should enable us to
formulate a quantitative expression for stick-slip amplitudes.
2. Calculation of Size-Distribution of Asperities.

Previous calculations of the autocorrelation function (see Figs. 6 & 7, Appendix A) have been carried out assuming that the force-displacement curve of a single junction is triangular or rectangular. Greenwood and Tabor in a recent paper give experimental evidence based on large-scale models suggesting that the function is parabolic (as shown above). In attempting to calculate a hypothetical friction curve based on such a law, it is necessary to find simple integral numbers that approximate closely to a parabolic sequence. After some trial and error, it was found possible to fit a parabola surprisingly well, as shown in Table I.

<table>
<thead>
<tr>
<th>Station</th>
<th>Weight of parabola assumed in computation</th>
<th>Actual height of parabola</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>x - 5</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x - 4</td>
<td>2</td>
<td>2.19</td>
<td>+.19</td>
</tr>
<tr>
<td>x - 3</td>
<td>4</td>
<td>3.80</td>
<td>-.10</td>
</tr>
<tr>
<td>x - 2</td>
<td>5</td>
<td>5.12</td>
<td>+.12</td>
</tr>
<tr>
<td>x - 1</td>
<td>6</td>
<td>6.16</td>
<td>-.14</td>
</tr>
<tr>
<td>x</td>
<td>6</td>
<td>6.10</td>
<td>+.10</td>
</tr>
</tbody>
</table>

Then to calculate the hypothetical friction trace, calculate terms of the series

\[ F_x = \frac{a}{6} \left[ 2(S_{x-4} + S_{x+4}) + 4(S_{x-2} + S_{x+2}) + 5(S_{x-1} + S_{x+1}) + 6(S_{x-0} + S_{x+0}) \right] \]

where \( S_{x-4} \) etc. correspond to random numbers in the range 4 to 6.

2000 points of such a trace were calculated and the correlogram for these points determined. From the values of \( r_k \) the corresponding
Values of \( R_k^2 = (1 - r_k)^2 \) were calculated and plotted on Fig. 6.

Values of \( R_k^2 \) for 110 points of an experimental friction trace are also plotted on this curve, with the scales suitably adjusted. It will be seen that \( R_k^2 \) as experimentally determined, is greater at low values of \( k \), and as has been shown in an earlier report, this difference is caused by variations in the sizes of the junctions for the experimental friction trace.

The next step was to determine the size distribution of junctions which gave the best fit with the observed data assuming for simplicity that only 4 different sizes of junction were present, of diameter \( d \), \( d/2 \), \( d/4 \) and \( d/8 \). The amplitudes of the contributions of the four sizes is then given by matching as closely as possible, the calculated and experimented \( R-k \) plots. As finally determined, the amplitudes \( a, b, c, d \) of the four sizes are given by the equations.

<table>
<thead>
<tr>
<th>Effect of junction of diameter ( d )</th>
<th>dits but diameter ( d/2 )</th>
<th>dits but diameter ( d/4 )</th>
<th>dits but diameter ( d/8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( .0853 \ a )</td>
<td>( +.0853 \ b )</td>
<td>( +.261 \ c )</td>
<td>( +.694 \ d ) = 1.37 ( (1) )</td>
</tr>
<tr>
<td>( .0853 \ a )</td>
<td>( +.261 \ b )</td>
<td>( +.694 \ c )</td>
<td>( + \ d ) = 3.76 ( (2) )</td>
</tr>
<tr>
<td>( .166 \ a )</td>
<td>( +.473 \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 6.28 ( (3) )</td>
</tr>
<tr>
<td>( .261 \ a )</td>
<td>( +.694 \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 9.3 ( (4) )</td>
</tr>
<tr>
<td>( .365 \ a )</td>
<td>( +.884 \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 11.8 ( (5) )</td>
</tr>
<tr>
<td>( .473 \ a )</td>
<td>( + \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 14.9 ( (6) )</td>
</tr>
<tr>
<td>( .588 \ a )</td>
<td>( + \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 17.9 ( (7) )</td>
</tr>
<tr>
<td>( .694 \ a )</td>
<td>( + \ b )</td>
<td>( + \ e )</td>
<td>( + \ d ) = 20.8 ( (8) )</td>
</tr>
<tr>
<td>( .797 \ a )</td>
<td>( + \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 23.3 ( (9) )</td>
</tr>
<tr>
<td>( .884 \ a )</td>
<td>( + \ b )</td>
<td>( + \ e )</td>
<td>( + \ d ) = 26.4 ( (10) )</td>
</tr>
<tr>
<td>( .980 \ a )</td>
<td>( + \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 29.2 ( (11) )</td>
</tr>
<tr>
<td>( a )</td>
<td>( + \ b )</td>
<td>( + \ c )</td>
<td>( + \ d ) = 31.8 ( (12) )</td>
</tr>
</tbody>
</table>
This set of equations is of course over-determined.

a may be calculated from eqns. 8-12, by subtracting 6 from 9 etc.
and then the values of a subtracted from expressions 5-6 and
b determined etc. The final values obtained were

\[
\begin{align*}
a &= 28.0 \\
b &= 3.1 \\
c &= 2.4 \\
d &= 4.1 \\
\end{align*}
\]

To get from the values a, b, c, and d the importance
of the corresponding junctions to the whole contact area, we have
to allow for the fact that the statistical uncertainty, comparatively,
for n junctions is proportional to \( n^{\frac{1}{2}} \), and thus multiply a, b, c
and d by 1, 2, 4, and 8 respectively.

The final contribution to the total sliding area of each
category of junction is shown in Fig. 7, as is a distribution taken
from earlier work based on wear measurements. These two entirely
independent methods both suggest that most of the sliding area consists
of junctions fairly homogeneous in size.

Thus according to the wear data some 55% of the total sliding
area consists of junctions whose length does not differ by a factor
of more than 2. For the autocorrelation data, the corresponding
value is 75%.
III. Velocity and temperature dependence on rubber friction

A year ago the results of friction-velocity tests on several materials at room temperature were reported. For each of these materials a maximum was found in the friction-velocity curve. During the past year, the region to the left of the maximum, in which friction increases with velocity, has been studied for rubber sliding on smooth glass. These tests have been carried out over a velocity range from about $10^{-6}$ to $10^{-3}$ cm/sec and a temperature range from about 20°C to 60°C.

A thin sheet of gum rubber (dental dam) cemented to a spherical metal surface was slid over a glass microscope slide in the low-speed friction machine. The machine was arranged inside a plywood box heated by electric lamps thermostatically controlled to keep the temperature constant to within ±0.1°C. An atmosphere of dry air from a commercial cylinder was maintained inside the box.

The rubber surfaces of the riders were cleaned after every test by first rubbing lightly with surgical gauze moistened in reagent acetone and then rinsing with a few drops of the acetone. The glass plate was cleaned in warm "Lakeseal" solution, rinsed under running water, then in distilled water, and allowed to drain and dry naturally. Care was exercised to prevent contamination from the hands of the operator. All experiments were performed in an air-conditioned room.

In order to check the reproducibility of results with different riders, three riders as nearly alike as possible were used at random during the tests. A different path on the glass plate was used for every test and the plate itself was cleaned after every 5 or 8 experiments.
Typical results are given in Fig. 8. Curves showing the apparent trend of the experimental points for riders of three different radii are given in Fig. 9.

These curves are similar to those found here previously for neoprene sliding on ground glass. Schallamach (1) has also obtained this type of result for rubber blocks sliding on ground glass and on silicon-carbide paper. The following conclusions can be drawn from Figs. 8 and 9, in spite of the scatter of the test points.

1. At high values of tangential force $F$, the logarithm of the sliding velocity $\log V$ tends to increase linearly with $F$.

2. The $\log V$ vs $F$ curves become concave downward at low values of $F$; that is, $\log V$ decreases more and more rapidly as $F$ becomes small.

3. The dependence of $\log V$ on temperature $T$ tends to be linear at high values of $F$. At low values of $F$, the relation between $\log V$ and $T$ is masked by the strong dependence of $\log V$ on $F$.

The results here presented are inconclusive regarding the effect of temperature, but corroborate the trend shown by previous work. All the data available indicate that the low-speed sliding of rubber is a rate process. The following mechanism for this process is suggested.

The glass-rubber interface, which is naturally quite inhomogeneous, is assumed to consist of small regions of high resistance to sliding ("islands") surrounded by regions of low resistance. More precisely, as one goes from the center of an island toward the shore,
the sliding resistance is assumed to drop from a high to a low value, as suggested in Fig. 10. An external tangential force will thus produce shearing deformation and stress at these islands, while over the rest of the interface, comprising the "ocean" of low resistance, slip will occur. In the absence of any further action, a small tangential force would thus produce an initial relative displacement of rubber and glass, but no continued movement would take place. At temperatures above the absolute zero, however, the energy of thermal vibration of the rubber molecules may be sufficient to cause occasional slip at an island, so that in a certain range of temperature and applied tangential force, continuous relative movement will result. At high values of applied force, only the central parts of the islands need thermal help to slip; while for small applied force, the area over which thermal help is needed becomes large.

This rate process is governed by an equation of the form

\[ V = C \exp \left( -\frac{A}{kT} \right) \]

or

\[ \ln V = \ln C - \frac{A}{kT} \]

where in the present case

- \( V \) is the steady velocity of sliding (cm/sec)
- \( C \) is a frequency factor (cm/sec)
- \( A \) is the thermal energy required to produce slip of an island (ergs)
- \( k \) is Boltzmann's constant \( (1.38 \times 10^{-16} \text{ erg/deg K}) \)
- \( T \) is the absolute temperature (deg K)
By reasoning similar to that of Becker (2), $A$ is expressed as:

$$A = \int \frac{\ell(t_0 - \tau)^2}{20} ds \quad (\tau > \tau_0) \quad (14)$$

where

- $a$ is the area of an island (cm²)
- $\ell$ is the unstretched length of the interfacial bonds (cm)
- $\tau_0$ is the tangential stress at any point required to cause slip (dyne/cm²)
- $\tau$ is the tangential stress at any point due to the applied force (dyne/cm²)
- $G$ is the elastic shear modulus (dyne/cm²)

At high values of $F$ it is seen from Fig. 10 that $\tau_0$ can be much larger than $\tau$ over most of the area of an island. Equation (14) thus yields a practically linear relation between $A$ and $\tau$.

Since $\tau$ is proportional to $F$ when the areas of the islands do not change appreciably, Eq. (13) gives a linear variation of $\ln V$ with $F$, as found experimentally in this range of $F$.

As $F$ decreases, $\tau$ tends to decrease more rapidly than $F$, on account of the increasing area of the islands (Fig. 10), while from Eq. 14, $A$ is seen to increase. The theory thus predicts a rapid drop in $\ln V$ as $F$ becomes small, in agreement with the experimental evidence.

Quantitative conclusions are questionable on account of the scatter of the data, but from the straight portion of the curves for the specimen of $\frac{1}{8}$-inch radius (Fig. 9) it is found that

---

$A = 1.6 \times 10^{12}$ ergs

$c = 10^{12}$ cm/sec

Estimates of $\Theta_0$, $\Theta$, and the size of the islands are being made with the aid of the theory. Further tests are in progress to obtain additional data on the effect of temperature.
FIGURE 1

SCHEMATIC DIAGRAM OF APPARATUS USED
AMPLITUDE OF STICK-SLIP VS. VELOCITY OF SPECIMEN

COPPER ON COPPER
LUBRICANT = CETANE
LOAD = 1750 gms

FIGURE 2
AMPLITUDE OF STICK-SLIP VS. VELOCITY OF SPECIMEN

STEEL ON STEEL
NO LUBRICANT
LOAD = 1750 gm

FIGURE 3
SPHERICAL DENTAL DAM SPECIMENS
OF 1" IN DIAMETER (STEEL BALL)
SLIDING ON SMOOTH GLASS PLATE

Figure 8
SPHERICAL DENTAL DAM SPECIMENS
SLIDING ON A SMOOTH GLASS PLATE

Figure 9
Schematic diagram of variation of $\tau_0$ over a typical "island" of contact between rubber and glass. $\tau_0$ is the tangential stress required to cause slip. The shearing stress caused by the applied tangential force $F$ is denoted by $\tau$.  

Figure 10
Autocorrelation Analysis of the Sliding Process

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Lubrication Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received September 2, 1955)

A simple model of the sliding process is developed in which the junctions are of the same size, but have different shear strengths, and, using an artificially obtained friction trace, it is shown that the size of the junctions may be deduced through a simple autocorrelation analysis. Applied to real friction traces obtained at slow sliding speeds, the technique gives an average junction diameter of $9.10^{-4}$ cm, in good agreement with previous estimates, while a different statistical method gives a value of $5.10^{-4}$ cm.

I. INTRODUCTION

It is generally accepted that two surfaces pressed together under an applied load make effective contact over only a few patches on their apparent contact area. A study of these patches, generally called "junctions," provides important information as to the nature of the sliding process, in particular, problems connected with metal transfer and wear and the transition between boundary and full-fluid lubrication. Investigations of the junctions have been carried out in the past using electric resistance measurements, optical and electron microscopy, taper sections, and measurements of wear fragments produced by the breaking of these junctions. In this paper is described a new method of studying the junctions which shows promise of becoming an important addition to presently available techniques, namely, statistical analysis of friction measurements obtained during a sliding experiment.

The calculations involve the carrying out of an autocorrelation analysis in which the instantaneous value of a variable, in this case the friction force, is compared with corresponding values obtained during the same experiment at progressively further removed intervals. In this way, a reliable assessment may be made as to how rapidly the variable is changing, and this in turn can give information as to the underlying factors, in our case the junctions, that give rise to the friction force and its fluctuations.

AUTOCORRELATION ANALYSIS

Many experimenters have produced friction traces in which the friction force is shown as a function of sliding distance at low speeds of sliding, and some examples are shown later in this paper in Fig. 8. A feature of such traces is that points taken close together on the trace have nearly the same friction coefficient, while points a large distance apart may have coefficients that are nearly the same or that differ widely; in fact, the two values are independent of each other. The change from

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1 R. Holm, Electric Contacts (Almquist and Wiksells, Stockholm, 1948), Part I.
5 A. C. West, Lubrication Eng. 9, 211 (1953).
SIMPLE MODEL OF THE FRICTION PROCESS

In order to see the information that may be obtained by an autocorrelation analysis, it is of interest to carry out a theoretical analysis based on a simple model. It will be assumed that all asperities are circular cones and make contact with other similar cones over a flat region of contact. Figure 1 shows that the real area of contact changes during sliding, reaching a maximum value of \( \pi d^2/4 \) which is denoted by \( a \). Figure 2 is a plot of the area of contact as a function of sliding distance. The actual junction is in existence for a sliding distance \( 2d \), but the area-displacement curve approximates closely to the triangle of base \( \pi d/2 \), and it is the latter function which is used in our calculations. It is assumed initially that all the junctions are of the same size.

FLUCTUATIONS OF FRICTION FORCE PRODUCED BY VARIATIONS OF SHEAR STRENGTH

If we assume that all the junctions have the same flow pressure \( p \) and shear strength \( s \), a constant friction coefficient may be expected, since the real area of contact is given by the expression

\[
L = pA,
\]

where \( L \) is the load, and the friction force \( F \) by the relation

\[
F = sA,
\]

where

\[
f = F/L = s/p = \text{constant}.
\]

However, the shear strength may be expected to be a very variable quantity for metal surfaces sliding in air. In the case of some of the junctions, the oxide layers are unbroken and the shear strength is that of the oxide. For some junctions the oxide layer is broken up and complete metal-to-metal contact is made so that the shear strength of the metal is the determining factor. Many junctions are probably part metal and part oxide. As an approximation we shall assume that the same junction maintains the same shear strength while being made and broken. In the case of the all-oxide and all-metallic junctions this assumption is very

\[
F = sA.
\]

\[
f = F/L = s/p = \text{constant}.
\]

Note that for normally distributed friction values the mean deviation is about 0.7956 where \( s \) is the root mean square deviation, so that the denominator of the expression in Eq. (2) becomes 1.128. This is because 1.128 is the denominator of the expression in Eq. (2) becomes 1.128.

\[
1.128
\]

This is because 1.128 is the denominator of the expression in Eq. (2) becomes 1.128.

See reference 2, Chap. 5.

AUTOCORRELATION ANALYSIS OF SLIDING

During sliding at constant load the real area of contact remains constant [Eq. (3)]. Assuming all junctions to be of the same size, and with an area-displacement function as shown in Fig. 2, then as soon as one junction reaches its maximum area and starts diminishing in size another must start up to maintain the total area at a constant value. In general, a number of such junction systems act simultaneously. We shall consider a simple case in which one junction is started at every unit interval of length and persists for a total distance of 10 units. Thus 10 junctions are present at any moment in different stages of growth.

Figure 3 has been drawn to illustrate this point, and represents an area-displacement plot both of the individual junctions and of their sum, the total contact area. This area is the sum of the intercepts on some specified vertical line (corresponding to one definite displacement) of all the sloping lines and is shown at the top of the figure. As sliding continues the individual junctions change in size, but the total area remains constant. Since the flow pressure is assumed constant we may multiply the abscissa by \( p \) and then Fig. 3 becomes a plot of the total normal force as a function of sliding distance. As postulated earlier, this force remains constant during sliding.

However the friction force is not constant, since the various junctions have different shear strengths. If we assume for our sliding combination that the shear strength of the oxide is half that of the metal, then the friction force at some instant at which all the junctions happened to be in intimate metallic contact would be twice that of some later instant during which all the junctions were separated by an oxide layer. In general, the friction force will have some intermediate value, and it will fluctuate as one set of junctions is replaced by another.

**CALCULATION OF HYPOTHETICAL FRICTION TRACES**

We shall assume that a junction may have any shear strength in the range \( s_m/2 \) to \( s_m \) (where \( s_m \) is the shear strength for metallic contact) and that all such values are equally likely. To simplify the calculation we shall confine the strength to the rational ratios \( s_m/8 \), \( 4s_m/8 \), \( 5s_m/8 \), \( 6s_m/8 \), etc. Each of these five possible values of the shear strength is assumed equally likely. To find successive values of shear strength of our hypothetical junctions we may read off numbers in the range 4 to 8 from a table of random numbers, e.g., 4, 7, 8, 4, 5, 8, 6, 6, etc.

The effect of this on our junction model will be to multiply the heights of the triangles in Fig. 3 by the various ratios \( 4s_m/8 \), \( 7s_m/8 \), \( 8s_m/8 \), etc. and the total friction force will be the sum of the individual triangles. Figure 4 shows a typical trace obtained in this way. It will be seen that the total frictional force is now a variable quantity. As a matter of great importance we note that although a new junction with different shear strength is formed at every unit interval of distance, the variation of the total friction force is much more gradual, the curve showing persistence, and it is this feature that may be analyzed by autocorrelation analysis.

Five-hundred consecutive points of a trace such as is shown in Fig. 4 were calculated from 510 random numbers in the range 4 to 8, and the total friction force determined by an arithmetical procedure equivalent to adding up for each "station" the shear resistance at that point of each junction. Thus, at an arbitrary station \( x \), the friction force will be given by

\[
F_x = \left( a/5 \right) \left( (s_{x-4} + s_{x+4}) + 2(s_{x-3} + s_{x+3}) + 3(s_{x-2} + s_{x+2}) + 4(s_{x-1} + s_{x+1}) + 5s_x \right),
\]

where

\[
a = \left( s_m/2 \right) - \left( s_m \right).
\]


Fig. 7, and the correlogram for all the 500 points as the dashed curve of Fig. 6. The two curves are fairly similar.

These two junction models are likely to represent the possible extremes for different materials. In one case the junctions slide over each other, and in the other case one junction penetrates through the other. In practice both mechanisms will operate and an actual $r-k$ curve will lie between the extremes of Fig. 7. To simplify, we may take $r_k=0.5$ at one-quarter the range of action of the junction. Having established this relationship, we are in a position to take a trace whose junctions are of unknown diameter and from the correlogram make deductions as to the size of the junctions. It may be noted that the shape of the correlogram does not depend on the amplitude of the fluctuations of the friction force but only on their variation with distance. Thus, deductions may be made from the correlogram even when the variation in shear strength of the junctions is unknown.

Fig. 7. A plot similar to Fig. 5 but assuming the force-displacement effect of a junction to be rectangular as shown.

EXPERIMENTAL MEASUREMENTS

Friction traces suitable for autocorrelation analysis were obtained with the slow-speed friction apparatus described elsewhere. In this arrangement, a hemispherically ended rider of one material is kept stationary with respect to a flat, slowly moving plate of another material. The friction force produces deflection in a strain ring on which are mounted four strain gauges, and a permanent record is obtained using a Sanborn recorder.

Three typical traces obtained with the same rider and plate, but at different sliding velocities, are shown in Fig. 8. It will be noted that as a function of distance on the recording paper, fluctuations in the friction force are much more pronounced at the higher speeds. An autocorrelation was carried out using 100 points on each trace and Fig. 9 plots the autocorrelation obtained as a function of distance, not on the recording paper but on the sliding surface. The results suggest that the three surfaces are autocorrelated in the same way, subject to statistical fluctuations, which are rather severe as the average length of track analyzed amounts to only 10 complete junctions. In particular, the value

$$F_s = \sigma \cdot (s_{n-4} + s_{n-3} + s_{n-2} + s_{n-1} + s_n + 1 + s_{n+2} + s_{n+3}).$$

(7)

Two-hundred points of this trace based on the same numbers as were used in obtaining Fig. 5 are shown in Fig. 6, and the correlogram for all the 500 points as the dashed curve of Fig. 6. The two curves are fairly similar.

A recent paper by Green\(^4\) suggests that the force-displacement curve of a junction may approximate to a rectangle rather than to a triangle. To see the implications of this model, a new friction trace was constructed using the appropriate formula

$$F_s = \sigma \cdot (s_{n-4} + s_{n-3} + s_{n-2} + s_{n-1} + s_n + 1 + s_{n+2} + s_{n+3}).$$

Two-hundred points of this trace based on the same

---

of $r_0$ reaches a value of 0.5 at a distance of about $3.8 \times 10^{-4}$ cm, so that the hypothetical junctions appear to be $7.0 \times 10^{-4}$ cm in average length, and the real junctions (see Fig. 2) some $9.7 \times 10^{-4}$ cm. This value may be compared to estimates of $7.10^{-4}$ and $17.10^{-4}$ cm obtained previously, for similar sliding conditions, by other methods.\textsuperscript{16}

**DISCUSSION**

In this paper only one number has been extracted from the correlograms, namely, the average junction size, but in principle it is possible, making assumptions as to the shape of the force-displacement curves of the junctions, to obtain information as to the actual size distribution of the junctions. Before this can be done profitably, more investigation of the force-displacement relations is needed.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig8.png}
\caption{Friction traces obtained at slow sliding speeds. For the top two graphs only a portion of the full length of trace is shown.}
\end{figure}

Estimates of the number of junctions may be obtained from another statistical calculation, involving the standard deviation of the friction values, namely, the quantity $\sigma/f$. If there are $n$ junctions present at any instant, of equal size but of different shear strength, then simple statistical reasoning suggests that $\sigma/f$ will vary as $n^{-1}$. The appropriate constant will depend on the range of shear strengths encountered and the shape of the force-displacement function. For the conditions of Figs. 5 and 7, we have found by actual calculation that $\sigma/f = 0.27 n^{-1}$ for the triangular model and $0.24 n^{-1}$ for the rectangular.

This provides an independent way of calculating the average junction diameter. For the runs shown in Fig. 8, $\sigma/f$ is found to be 0.022. Hence, assuming the triangular model, there are 150 junctions present. The total real area of contact may be calculated from Eq. (3), $L$ being 0.1 kg and $\phi$ being 6000 kg/cm$^2$. Hence $A = 1.67 \times 10^{-4}$ cm$^2$. The average size of a junction is $a/2$, and hence $a$ is $2.2 \times 10^{-7}$ cm$^2$ and $d = 5.3 \times 10^{-4}$ cm. This estimate is of the same order of magnitude as the previous one, but lower, and suggests that the actual range in shear strength of the junctions is less than our postulated factor of 2:1.

**SUMMARY**

The autocorrelation method previously described appears to provide a new and powerful tool for the study of frictional and related phenomena, and in particular it is capable of providing information about the size of the junctions formed between sliding surfaces. The experimental technique is simple and general.

**ACKNOWLEDGMENTS**

The author is indebted to Professor Brandon G. Rightmire for many helpful and clarifying discussions, and to the Office of Naval Research for financial support under project number NR 065–335.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig9.png}
\caption{Correlogram for the friction traces shown in Fig. 8 as a function of distance, not on the recording paper, but on the sliding surface.}
\end{figure}

Stick and Slip

When two substances rub against each other, they frequently stick and then slip. The phenomenon accounts for the squeak of bearings, the music of violins and many other sounds of our daily experience

by Ernest Rabinowicz

The three laws describing the force of friction say that when one solid body slides over another, the frictional force (1) is proportional to the load, or pressure of one against the other, (2) is independent of the area of contact, and (3) is independent of the sliding velocity. The first two laws were stated by Leonardo da Vinci and rediscovered in the 1690s by Guillaume Amontons, a French engineer working under the sponsorship of the French Royal Academy of Sciences. The third law was first expressed in 1785 by Charles Augustin de Coulomb, the French physicist better known for his researches in electrostatics.

If the three laws are correct, friction depends only on the applied load, and the coefficient of friction (the ratio friction-force-to-load) for any given materials should be constant under all conditions. The first two laws generally hold true, with no more than 10 per cent deviation. But it has been known for some time that friction is not independent of sliding speed. The coefficient of friction between two bodies may vary as much as 30 to 50 per cent according to the speed of motion. In 1835 A. Morin of France proposed that, since the frictional force resisting the start of sliding for two bodies at rest was obviously greater than the resistance after they were in motion, there should be two coefficients of friction: a static one, for surfaces at rest, and a kinetic one, for surfaces in motion. Today, as a result of work by a number of investigators, we know that both the static and the kinetic coefficients themselves vary. The kinetic coefficient drops off as the sliding speed increases. And the static coefficient depends to some extent on the length of time the surfaces have been in contact—a fact which can be attested by anyone who has ever had occasion to loosen a stubborn screw or nut that has been in place for a considerable period. Thus the only satisfactory way to represent the friction coefficient for any pair of surfaces is by two plots, one of the static

![Chalk marks on a blackboard demonstrate stick-slip. The top mark was made by a piece of chalk held at an acute angle to the direction of motion, the marks below it, by pieces of chalk held at an obtuse angle to this direction. In the latter marks the chalk stuck to the blackboard, then slipped, then stuck again and so on. The more tightly the chalk is held, the smaller the distance of slip.](image)
It is the breakdown of the third "law" of friction—the variation of frictional force with velocity—that is responsible for stick-slip, the phenomenon we shall now consider. Suppose we attach a block to an anchored spring and place it on a longer slab which we set in motion at a slow speed. At first the block is dragged along on the moving slab; it will not be held back by the spring, i.e., slide on the slab, until the spring's pull is equal to the static coefficient of friction. The pull of the stretched spring reaches that value when the block arrives at the point A [see drawing at bottom of page 114]. Now the block begins to slip on the moving surface. As soon as it does, the lower kinetic coefficient of friction takes over, and the block slides rapidly toward the left. When it has moved back to point C, it comes to rest. Here the higher static coefficient takes charge, and the block again sticks to the surface and is dragged to A. Then it slips back to C. This is a simple laboratory demonstration of the stick-slip phenomenon, so named in 1939 by F. P. Bowden and L. L. Leben, physical chemists at the University of Cambridge, who built an apparatus to study the process.

At the point B on the scale, halfway between points A and C, the pull of the spring is equal to the kinetic coefficient of friction. If the static coefficient were the same as the kinetic, the block would be dragged to this point and then stay there, sliding on the moving slab beneath it. As it is, the block oscillates about this position, sticking and slipping by turns. The situation is complicated by the fact that during motion the friction coefficient varies with changes in the sliding velocity, but whether stick-slip may occur can be determined in any given situation simply from the direction in which this relation is changing [see chart at lower right on page 112].

What does all this have to do with machinery? Few mechanisms in common use contain sliding surfaces attached to springs. The answer is that whenever solid bodies are pressed together, there is some elastic displacement or deformation of the material, resulting in an effect like the operation of the spring in the foregoing laboratory demonstration.

Common examples of stick-slip are: the creaking of doors, the chattering of window sashes, the violent shuddering...
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**EVOLUTION OF THE FRICTION CONCEPT** is illustrated. In the late 18th century it was thought that the coefficient of friction remained constant as the relative velocity of the sliding substances was increased (upper left). In the early 19th century it was postulated that there were two kinds of friction: static and kinetic (upper right). Friction was greatest when two substances were moved from a state of rest, and fell off immediately when they began to slide. Around 1940 it was shown that friction fell off gradually with the increase of velocity (lower left). Today it is known that friction first increases with velocity and then falls off (lower right). When the changing relationship between friction and velocity has the slope to the left of the peak in this curve, substances slide steadily. When it has the form of the steeper part of the slope to the right of the peak, stick-slip occurs.

Of drawbridges, the squeaking of bicycle wheels and the squealing of automobile tires. Stick-slip has its uses. Without it a violinist could produce no music, and he takes good care to promote it by rosining his bow. But in most situations stick-slip is a nuisance or worse. A tool cutting metal should slide smoothly into the material; when its slide is interrupted by stick-slip, the cut will be rough and uneven [see photographs on page 110]. In the driving mechanism of a phonograph turntable stick-slip would ruin the sound. And during World War II the problem of stick-slip in one delicate situation was a matter of life and death. The turning of a submarine's propeller shaft produces stick-slip noise which can be detected with sonic listening gear. Since the war the Office of Naval Research has sponsored research on stick-slip at the Massachusetts Institute of Technology.

Friction, most investigators now agree, arises from the adhesion of molecules in the surfaces in contact with each other. The bond between the surfaces may be so strong at some points that tiny fragments are torn off one and stick to the other. Experiments with radioactive tracer material have proved this. If the end of a radioactive rod is rubbed along a flat surface, small particles are transferred and make the surface radioactive. This is an excellent experiment for showing the stick-slip phenomenon. A piece of photographic film is laid on the surface that has been rubbed with the rod. After it has been exposed for several hours to the radioactive track left by the rod, the film is developed. The image of the track turns out to be not a continuous line but a series of spots [see photographs on page 118]. The sliding rod end stuck and slipped, leaving considerable material where it stuck and very little where it slipped. Exactly the same happens when you rub a piece of chalk, tilted in the direction of motion, over a blackboard: you will get a stuttering line of dots.

In any adhesive process the bond becomes stronger the longer it is left undisturbed. This is why the static coefficient of friction increases with time of
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contact. In the case of sliding surfaces, the period of contact between points on the two surfaces is, of course, longer when the surfaces slide slowly than when they move rapidly. Consequently if the slide of one surface over another slows down, friction increases. This is the situation that favors stick-slip. However, laboratory tests have developed the unexpected finding that at extremely slow speeds the situation is reversed: as friction increases the sliding velocity also increases. The most plausible explanation seems to lie in the phenomenon called creep. All materials slowly change shape ("creep") even under moderate forces. An increase in force will increase the rate of creep. Thus in the case of surfaces sliding very slowly over each other, an increase in frictional force may produce a perceptible acceleration of the slide in the form of creep of one surface past the other. The limit of speed attained by the creep mechanism varies with the material, because soft materials creep faster than hard ones. The creep of steel is so slight that it cannot be observed. Lead can be made to slide by creep at speeds up to a millionth of a centimeter per second (about one foot per year); soap up to 10 centimeters per second.

These considerations present us with the paradoxical conclusion that there is really no such thing as a static coefficient of friction for most materials. Any frictional force applied to them will produce some creep, i.e., motion.

Studies of sliding at very low speeds are important because they yield systematic information on friction-velocity relations which will enable designers of machines to select materials that will be immune from stick-slip over the range of speeds at which the mechanism is to operate. We also need a great deal more data on the friction coefficients of metals. It seems odd that in this age of metals, tables of coefficients listed in handbooks still have little to say about metals and apply largely to various woods, leather and stones—engineering materials of long ago.

Three main methods are available for curing stick-slip where it is not wanted. Firstly, we can alter the sliding speed. Sometimes this means slowing

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**Experimental Apparatus** is used to show the principle of stick-slip. A block is attached to a spring. The slab on which the block rests is moved (arrows). If the static coefficient of friction were the same as the kinetic coefficient of friction, the block would simply move with the slab from 0 to B and stay there. Because the static coefficient is greater than the kinetic, the block moves with the slab to A and then slips back to C. If the movement of the slab were continued at the same speed, the block would oscillate between A and C.
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**Diagram:**

Lubricated Surfaces may be subject to stick-slip. This chart represents one piece of steel slid over another with a film of lubricant between them. When the lubricant is first applied, it covers 100 per cent of the area between the two surfaces. This area is steadily reduced as the surfaces are rubbed together. When 90 per cent of the film remains, the curve is still almost horizontal and no stick-slip occurs. When only 75 per cent remains, the slope of curve is down (see curve at lower right on page 112) and stick-slip can begin.
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