PURPOSE: This Coastal and Hydraulics Engineering Technical Note (CHETN) describes two methods to quickly assess sediment mobility relative to water depth as a function of local waves and currents. Estimating dredged sediment mobility relative to placement depth and grain size is valuable for preliminary or reconnaissance engineering studies to compare multiple placement sites. Presently, a Matlab script is available to perform the calculations using both methods. Future development will include an interactive web tool with automated data uploads for engineers to utilize.

BACKGROUND: Nearshore berms are commonly used for placement of dredged sediment that may contain more fine silts and clays than are allowed for placement directly on the beach. The United States Army Corps of Engineers (USACE) produced several Dredging Research Technical Notes in the Dredging Research Program (DRP) Technical Area 5, Management of Dredging Projects, for nearshore berm design guidance from 1990 to 1993. The majority of these technical notes used various numerical models for design guidance. A brief summary of the applicable design guidance obtained from the technical notes is shown in Table 1.

<table>
<thead>
<tr>
<th>Technical Note</th>
<th>Key Design Guidance</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRP-5-01 (McLellan 1990)</td>
<td>Recommended a linear berm rather than a conical berm to prevent wave focusing.</td>
</tr>
<tr>
<td>DRP-5-02 (McLellan et al. 1990)</td>
<td>Discussed methods to determine the seaward limit of the littoral zone for active berm placement.</td>
</tr>
<tr>
<td>DRP-5-06 (Burke and Allison 1992)</td>
<td>Recommended minimum longshore length of 610 m (2,000 ft) with end slopes of 1V:125H to prevent wave focusing.</td>
</tr>
<tr>
<td>DRP-5-08 (Pollock and Allison 1993)</td>
<td>Recommended minimum berm crest width, $b_c$, 61 m (200 ft) to reduce wave height.</td>
</tr>
</tbody>
</table>

Although the practice of placing dredged sediment in the nearshore region is commonplace, there is not a simple web tool application available that can rapidly produce a preliminary assessment of the sediment mobility at prospective placement sites and aid with the evaluation of what portion of the placed sediment is likely to be mobilized. Estimating sediment mobility is the first step for engineers and planners to estimate the potential volume of sediment that a placement operation might yield to beneficially nourish a wetland or nearshore region, thereby allowing stakeholders to gain support for a project.
METHODS FOR ESTIMATING SEDIMENT MOBILITY: Two methods are applied to estimate the sediment mobility for the selected location, water depth, and sediment profile of the sediment to be placed. Method 1 analyzes the bed shear stress from local wave and current conditions and compares it with the critical thresholds for various median grain size diameters. The bed shear stress is a function of the wave orbital velocity and is calculated using linear wave theory. Method 2 analyzes the near-bed velocity and compares the critical near-bed velocity to locally generated velocity data. Method 2 uses the nonlinear stream function wave theory to calculate the near-bed wave orbital velocity, which generally produces larger velocities than linear wave theory. Both methods are applied to 10 years of offshore wave hindcasts from the closest Wave Information Studies (WIS) station to the study site that are transformed to a single nearshore depth. Additionally, both methods are applied to the 10-year offshore significant wave height and period data that is then transformed to a range of nearshore depths. Both Methods 1 and 2 are explained within this technical note and an example scenario at Duck, NC, is provided. A Matlab script is available to automate both methods.

Method 1 – Bed Shear Stress. This method imports 10 years (1 January 1990–1 January 2000) of wave hindcast data from the closest WIS station to the proposed area of evaluation (http://wis.usace.army.mil/) and saves it as a .mat file. The user must input the offshore water depth of the WIS station, the depth and shoreline orientation of the nearshore placement location of interest, median sediment diameter of the potential nearshore berm sediment, assumed current velocity 1 meter (m) above the bed, water temperature, and salinity. The temperature and salinity data are used in the density and viscosity calculations.

For this technical note an example study site is selected and the sediment mobility indexes are calculated using the local shoreline orientation and WIS Station 63218 near Duck, NC. The assumed current 1 m above the bed is 0.1 meter per second (m/s). The coordinate system of the shoreline angle is shown in Figure 1.
The measured wave angle and height from the WIS station are transformed from offshore to the nearshore region using conservation of energy flux and Snell’s Law, which are given as

\[ H_2 = H_o \frac{C_{go}}{C_{g2}} \frac{\cos \theta_o}{\cos \theta_2} \]  

(1)

and

\[ \frac{\sin \theta_2}{C_2} = \frac{\sin \theta_o}{C_o} \]  

(2)

respectively, where \( H_2 \) and \( H_o \) are the nearshore and offshore wave heights, \( C_g \) is the group velocity, \( \theta \) is the angle from shore normal, \( C \) is the wave celerity, and the subscripts “2” and “o” indicate calculations in the nearshore and offshore, respectively. The polar coordinate system used for \( \theta \) is shown in Figure 2.

![Figure 2. The polar coordinate system used in the sediment transportation calculations.](image)

The critical shear stress is estimated from Shields diagram following a procedure given by Soulsby (1997) and Soulsby and Whitehouse (1997) as

\[ D_* = d_{50} \left( \frac{g \left( \frac{\rho_s}{\rho} - 1 \right)}{\nu^2} \right)^{1/3} \]  

(3)

\[ \theta_{cr} = \frac{0.30}{1 + 1.2 D_*} + 0.055 \left[ 1 - \exp(-0.02D_*) \right] \]  

(4)

and
\[ \tau_{cr} = \theta_{cr} g (\rho_s - \rho) d_{50} \]  

(5)

where \( D \) is the dimensionless grain size, \( d_{50} \) is the median grain size, \( g \) is the acceleration due to gravity, \( \rho_s \) is the sediment density, \( \rho \) is the water density, \( v \) is the kinematic viscosity of water, \( \theta_{cr} \) is the Shields parameter, and \( \tau_{cr} \) is the critical shear stress. The critical shear stress is the threshold stress for which the sediment can be expected to be dislodged from the seabed for all greater shear stresses.

The bottom skin shear stress is calculated using a method described by Soulsby (1997) and Myrhaug (1989) for currents and waves. Form shear stress is not included in the calculations. The current-induced shear stress, \( \tau_c \), is calculated as

\[ \tau_c = \rho \left( \frac{\bar{U} \kappa}{\ln \left( \frac{z}{z_0} \right)} \right)^2 \]  

(6)

where \( \bar{U} \) is the assumed mean current velocity, \( \kappa \) is the von Karman’s constant (\( \kappa = 0.4 \)), \( z_0 \) is the bed roughness length (\( z_0 = d_{50} / 12 \) for flat sand), and \( z \) is the height of the current velocity assumed by the user (\( z = 1 \) m). The wave-induced shear stress, \( \tau_w \), is given as

\[ \tau_w = \frac{1}{2} \rho f_w U_w^2 \]  

(7)

where \( f_w \) is the wave friction factor, and \( U_w \) is the wave orbital velocity determined by Soulsby’s (1997) orbital bottom velocity figure which integrates the linear wave theory orbital velocity across the Joint North Sea Wave Observation Project (JONSWAP) wave spectra. Linear wave theory is appropriate when the wave steepness (height/wavelength) is very small and the magnitude of the orbital velocity is the same beneath the crest and the trough. The friction factor, \( f_w \), is calculated using the Reynold’s number, \( R_w \), relative roughness, \( r \), and the semi-orbital excursion, \( A \), which are given as

\[ R_w = \frac{U_w A}{v} \]  

(8)

\[ r = \frac{A}{k_s} \]  

(9)

and

\[ A = \frac{U_w T}{2\pi} \]  

(10)
where \( T \) is the wave period, and \( k_s \) is the Nikuradse equivalent sand grain roughness \((k_s = 30 \, z_o)\). The single wave period, \( T \), is applied to Equation 10 for monochromatic waves, but the peak wave period, \( T_p \), is appropriate for spectral wave conditions from WIS stations. The friction factor, \( f_w \), is calculated as the greater of \( f_{wr} \) and \( f_{ws} \), which are given as

\[
f_{wr} = 0.237 r^{-0.52} \quad \text{(rough turbulent flow)}
\]

and

\[
f_{ws} = B R_w^{-N} \quad \text{(laminar and smooth turbulent flow)}
\]

where

\[
B = 2, \, N = 0.5 \quad \text{for } R_w \leq 5 \times 10^5 \quad \text{(laminar flow)}
\]

and

\[
B = 0.0521, \, N = 0.187 \quad \text{for } R_w > 5 \times 10^5 \quad \text{(smooth turbulent flow)}
\]

The maximum shear stress, \( \tau_{max} \), from the waves and currents shear stresses is calculated as

\[
\tau_m = \tau_c \left[ 1 + 1.2 \left( \frac{\tau_w}{\tau_c + \tau_w} \right)^{3.2} \right]
\]

and

\[
\tau_{max} = \left[ (\tau_m + \tau_w \cos \phi)^2 + (\tau_w \sin \phi)^2 \right]^{1/2}
\]

where \( \tau_m \) is the mean bed shear stress and \( \phi \) is the angle between the wave and current directions.

Figure 3 shows the histogram of maximum bed shear stresses using transformed wave hindcasts from WIS Station 63218 to a nearshore depth of 8 m near Duck, NC. The program analyzed approximately 88,000 waves and required 112 seconds (s) to run after the WIS data were imported into a .mat file. The legend shows the median grain size, critical shear stress, frequency of mobility during the 10-year period, \( fM \), and mean mobility score which is given as

\[
M = \frac{\tau_{max} - \tau_{cr}}{\tau_{cr}}
\]

The frequency of mobility applies to the specified median grain size diameter. Sediments with smaller grain size diameter will be mobilized more frequently. The mean mobility score allows
the user another way to analyze the predictive sediment mobility by quickly assessing how much the maximum bottom stress exceeds the critical stress on average, and can be particularly useful when comparing sites with similar frequencies of mobility. The mean mobility score can be negative in sites where the average maximum bottom stress is less than the critical bottom stress. Sites with negative mobility scores will experience little mobility.

In the Duck, NC, example shown in Figure 3, the critical shear stress at a depth of 8 m ranged from 0.14 to 0.26 Pa, resulting in 71% to 84% of the waves moving the sediment, and a mean mobility score ranging from 1.7 to 2.0. The finer sediment has an increased frequency of mobility and increased mean mobility score, as expected. For this example, if the user were placing a nearshore berm with a median grain size of 0.2 millimeters (mm), it could be estimated that approximately 73% of the waves would be effective in transporting sediment. However, a berm with median grain size greater than 0.5 mm would be less likely to be transported.

![Figure 3. Histogram of the calculated maximum bed shear stress at a depth of 8 m with the critical shear stress for the respective median grain sizes noted with the vertical dashed lines. N is the number of waves during the 10-year period in each shear stress bin.](image)

In addition to transforming tens of thousands of waves to a single nearshore point, the offshore significant wave height, period, and direction are calculated, and the single offshore wave is transformed to a range of nearshore depths to estimate sediment mobility. The range of nearshore depths can be determined by the user based on the depths in the area of the potential berm location.

The Matlab code imports 10 years of wave measurements from the closest WIS station, calculates the significant wave height and period, and applies Equations 1–16 to calculate the maximum bed
shear stress for the range of nearshore depths. Figure 4 shows the maximum bed shear stress induced by the offshore significant wave height $H_0 = 1.82$ m, peak offshore wave period $T_p = 8.89$ s, and shoreline angle of 160°. The maximum bed shear stress induced by this wave height and period exceeded the critical thresholds for mobility at all nearshore depths from 3 to 12 m.

Figure 4. The total maximum bed shear stress for nearshore depths from 3 to 12 m for the significant offshore wave height and period from WIS Station 63218.

**Method 2 – Near-Bottom Velocity.** This method imports 10 years (1 January 1990–1 January 2000) of wave data from a WIS station, but determines the sediment mobility using the near-bottom critical velocity instead of the bed shear stress. The near-bottom velocity is calculated using nonlinear, stream function wave theory. By also calculating the sediment mobility using this method as well as Method 1, a range of predicted mobility is provided. Stream function wave theory is appropriate for water depths in the range $0.006 gT^2$ to $0.016 gT^2$ (Soulsby, 1997), or for the Duck, NC, example in the range 4.7 to 12.6 m.

The critical near-bottom velocity, $u_{cr}$ as given by Ahrens and Hands (1998) is based on research by Hallermeier (1980) and Komar and Miller (1974), is given as

$$u_{cr} = \sqrt{8g\gamma d_{50}} \quad \text{for} \quad d_{50} \leq 2.0 \text{mm} \quad (18)$$

and

$$u_{cr} = \left[0.46\gamma g T^{1/4} \left(\pi d_{50}^{3/4}\right)^{4/7}\right] \quad \text{for} \quad d_{50} > 2.0 \text{mm} \quad (19)$$
where $\gamma$ is defined as $\gamma = (\rho_s - \rho) / \rho$, where $\rho_s$ is the sediment density and $\rho$ is the water density. Ahrens and Hands (1998) used Dean’s (1974) stream function wave theory table (SFWT) to derive the following equations for the near-bottom wave induced velocity based on stream function wave theory for the wave crest, $u_{\text{max crest}}$ and trough, $u_{\text{max trough}}$, as

$$u_{\text{max crest}} = \left(\frac{H}{T}\right) \left(\frac{h}{L_0}\right)^{-0.579} \exp\left[0.289 - 0.491 \left(\frac{H}{h}\right) - 2.97 \left(\frac{h}{L_0}\right)\right] \quad (20)$$

and

$$u_{\text{max trough}} = -\left(\frac{H}{T}\right) \exp\left[1.966 - 6.70 \left(\frac{h}{L_0}\right) - 1.73 \left(\frac{H}{h}\right) + 5.58 \left(\frac{H}{L_0}\right)\right] \quad (21)$$

where $h$ is the water depth, $H$ is the wave height in the placement site, and $L_0$ is the offshore wave length given by $L_0 = (gT^2)/2\pi$. The maximum near-bottom velocity was taken as $u_{\text{max}} = \max(\left|u_{\text{max crest}}\right|, \left|u_{\text{max trough}}\right|)$.

Figure 5 shows a histogram of the near-bottom velocity as a function of the number of waves in each shear stress bin. The critical velocities per median grain size are shown with dashed lines. The program required 6 seconds to analyze 88,000 waves after the WIS data were imported into a .mat file. The frequency of mobility ranged from 84.5% to 98.7% depending on the median grain size. The mean mobility score using the near-bottom velocity is calculated as

$$M_u = \frac{u_{\text{max}} - u_{cr}}{u_{cr}} \quad (22)$$

Figure 5 is a great example of the importance of the mean mobility score. For median grain sizes of 0.1 and 0.2 mm, the frequency of mobility is within 3%, but the mean mobility scores are significantly different (2.6 vs. 4.1). The greater mobility score indicates that the smaller grain size will move farther when mobilized and most likely winnow the finer sediment from the site.

The near-bottom maximum and critical velocities (Equations 18–21) were applied to multiple depth contours with the single offshore significant wave height and period calculated from 10 years of offshore WIS wave measurements. Figure 6 shows the near-bottom velocities for a range of nearshore depths. The critical velocities for several median grain sizes are shown with the vertical dashed lines. The near-bottom velocities for the range of depths shown exceed the critical velocity for all median grain sizes, so the user would anticipate that berms with these median diameters would be mobilized.
Figure 5. Histogram of the calculated maximum near-bottom velocity with the critical velocity for the respective median grain size noted with the vertical dashed lines.

Figure 6. The total maximum near-bottom velocities for nearshore depths from 3 to 12 m from the offshore significant wave height and period. The critical velocity for each median grain size is shown with vertical dashed lines.
**SUMMARY:** This technical note discusses two methods to estimate sediment mobility. Both methods are applied to 10 years of offshore wave hindcasts which are transformed to a single nearshore depth, and the offshore significant wave height and period are transformed to a range of nearshore depths. Figures are created that show the critical shear stress and critical near-bed velocity for a range of median grain sizes, the frequency of sediment mobility with relation to grain sizes, and a general mobility index. Method 1 analyzes the bed shear stress and Method 2 analyzes the near-bed velocity. The bed shear stress analyzed in Method 1 is a function of the near-bed velocity. The near-bed wave orbital velocity for Method 1 is calculated using linear wave theory, which is appropriate when the wave steepness (height/wave length) is small. Method 2 calculated the near-bed orbital wave velocity using stream function wave theory, which is appropriate for water depths in the range $0.006 gT^2$ to $0.016 gT^2$, or for the Duck, NC, example in the range 4.7 to 12.6 m. The orbital wave velocity is larger using the nonlinear stream function wave theory compared to linear wave theory, hence the 85% to 99% frequency of mobility shown in Figure 5 compared to the 71% to 84% frequency of mobility shown in Figure 3. Waves become more nonlinear as they transform into the nearshore, making Method 2 more appropriate and Method 1 more conservative for the example herein. By using both methods, a range of mobility is shown.

**FUTURE DEVELOPMENT:** The two methods described in this technical note will be developed into a web application to provide engineers a rapid reconnaissance tool for the preliminary assessment of nearshore berm locations. If these methods are applied to a web tool, the user would select the WIS station, and a default value for the temperature, salinity, and current 1 m above the bed would appear for the user. The web tool would include direct data uploads for the selected WIS station. The depth of closure calculation could be included in this tool using the method and program described by Brutsché et al. (in prep.). The information provided from the tool could potentially be used to create a map with colored contours to show the sediment mobility at various depths.

**ADDITIONAL INFORMATION:** This Coastal and Hydraulics Engineering Technical Note (CHETN) was prepared as part of the USACE Coastal Inlets Research Program (CIRP) by Dr. Brian C. McFall, Dr. S. Jarrell Smith, Cheryl E. Pollock, James Rosati, III, and Dr. Katherine E. Brutsché, U.S. Army Engineer Research and Development Center (ERDC), Coastal and Hydraulics Laboratory (CHL), Vicksburg, MS. Questions pertaining to this CHETN may be directed to Brian McFall (Brian.C.McFall@usace.army.mil) or to the USACE CIRP Program Manager, Julie Dean Rosati (Julie.D.Rosati@usace.army.mil). Additional information regarding CIRP may be obtained from the CIRP web site [http://cirp.usace.army.mil/](http://cirp.usace.army.mil/).

This ERDC/CHL CHETN-IV-108 should be cited as follows:

REFERENCES


Burke, C. E., and M. C. Allison. 1992. Length and end slope considerations, interim design guidance update for nearshore berm construction DRP-5-06. Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station.


McLellan, T. 1990. Engineering design considerations for nearshore berms. DRP-5-01. Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station.


Pollock, C. B., and M. C. Allison. 1993. Berm crest width considerations, interim design guidance update for nearshore berm construction. DRP-5-08. Vicksburg, MS: U.S. Army Engineer Waterways Experiment Station.


NOTE: The contents of this technical note are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such products.