Exotic optical beam classes for free-space communication

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Final Report

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Exotic optical beam classes for free-space communication

The goal of this project was to study and optimize special classes of optical beams, with particular attention paid to beams with partial coherence or nested vortex structures, or combinations of the two. For intensity-based communication and remote sensing, partially coherent beams were studied with the goal of minimizing intensity fluctuations induced by random media. More recently, researchers have been interested in using the discrete topological phase core of optical vortex beams as an alternate carrier of information. Properties of vortex beams, both coherent and partially coherent, were investigated, and properties of partially coherent vortex beams were characterized for the first time. The efficacy of vortex detection by diffraction was also evaluated. Relations between different types of optical singularities were discovered and analyzed, and a surprising connection between vortex evolution and infinite mathematics was uncovered.

atmospheric propagation, optical vortices, partial coherence

Greg Gbur

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Exotic optical beam classes for free-space communication and sensing applications

Final Performance Report
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Award: FA9550-13-1-0009

1. Summary

The goal of this project was to study and optimize special classes of optical beams, with particular attention paid to beams with partial coherence or nested vortex structures, or combinations of the two. For intensity-based communication and remote sensing, partially coherent beams were studied with the goal of minimizing intensity fluctuations induced by random media. More recently, researchers have been interested in using the discrete topological phase core of optical vortex beams as an alternate carrier of information. Properties of vortex beams, both coherent and partially coherent, were investigated, and properties of partially coherent vortex beams were characterized for the first time. The efficacy of vortex detection by diffraction was also evaluated. Relations between different types of optical singularities were discovered and analyzed, and a surprising connection between vortex evolution and infinite mathematics was uncovered.

2. Nonuniformly correlated beam propagation in atmospheric turbulence

Data is typically transmitted in free-space optical (FSO) communications systems through variations in the intensity of light. Unlike longer-wavelength fields like radio waves, however, visible light fields can be distorted on propagation by turbulence in the atmosphere, even in clear air. These distortions usually effect the phase of light; on propagation, these phase variations manifest as fluctuations in intensity, known as scintillations. Such scintillations can produce unacceptable errors in FSO systems even over a distance of several kilometers at ground level.
It is now well-accepted that partially coherent beams can have lower scintillation than their fully coherent counterparts in many situations; this has been demonstrated in large part due to the efforts of the PI in previous grant periods (see, for instance, [1-3]). A partially coherent beam sends energy through multiple incoherent paths through the atmosphere, while a coherent beam sends energy along a single path. The net result is that a partially coherent beam will, on average, be more likely to hit the detector and have a more uniform intensity at that detector.

More recent research on such beams has turned the focus to optimization of the effect, a generally difficult task due to the large variety of parameters that must be taken into account, both in the beam and the atmosphere. The PI noted several years ago [3] that so-called Schell-model beams appear to be well-approximated by a finite incoherent array of beams. This puts a quantitative limit on the minimum scintillation achievable for this class of beams as well as the average intensity, and it became natural to ask whether non-Schell-model beams could show even further improvement.

An important class of such beams, now referred to as nonuniformly correlated (NUC) beams, were introduced several years ago by Gori and Santarsiero [4]. They demonstrated how a wide variety of beams with previously unknown correlations could be generated with a simple analytic model amenable to atmospheric propagation studies. In this research period, the PI investigated such beams in turbulence; their work appeared in [Publication 1], and an illustration of their results is shown in Fig. 1. Remarkably, it is found that the NUC beams can have both lower scintillation and higher on-axis intensity for certain propagation distances. This suggests that such beams may be ideal for FSO communications applications.

Based on his research for AFOSR, the PI was invited in 2014 to write a review of the current state of progress of research in partially coherent beam propagation in the atmosphere. This is [Publication 2] of the grant period.
Figure 1. (a) On-axis scintillation index for a typical nonuniformly correlated beam (NUC), a Gaussian Schell-model beam (GSM), and a coherent Gaussian beam, as a function of propagation distance through moderate turbulence.  (b) Average intensity as a function of propagation distance.

3. Coherent and partially coherent vortex beam propagation and detection

The use of optical vortices as alternative carriers of information in FSO communications has become a very active area of research in recent years; see, for example, Refs. [5-6]. In 2008, the PI coauthored a thorough numerical study showing the robustness of vortices in turbulence as well as overall limitations of the approach [7], including the wandering of optical vortices from the detector aperture and pair production of new vortices.

The aforementioned disadvantages of vortex beams in turbulence are essentially related to the interference of the vortex beam with itself. It became natural to wonder if the deleterious effects might be lessened or eliminated altogether by using vortex beams that are also partially coherent. However, there has been relatively little work done on partially coherent vortex beams (PCVBs), and almost all of that has been limited to first-order vortices located in the beam’s waist plane; the PI and collaborator summarized current research in a review article several years ago [8].

A first step to better understanding PCVBs is the study of the evolution of a beam with a first-order vortex; this work was released in [Publication 3]. Building on earlier work studying the structure of phase singularities in the waist plane of the beam [9], the PI and student evaluated the full structure of a PCVB as it propagates in space. Some results are shown in Fig. 2.
Figure 2. Zero manifolds of a PCVB on propagation, showing (a) radial positions for high coherence and (b) low coherence. (c) shows how the angular position of the vortices evolves on propagation.

It had been found earlier that, in the waist plane of the beam, the zero manifolds with respect to the two radial variables of the spatial correlation function form hyperbolas, and only at points appearing on opposite sides of the origin. The radial behavior remains hyperbolic, though with respect to combined radial variables $\rho_+ \text{ and } \rho_-$. Surprisingly, though, it was found that the angular orientation of the correlation singularities varies with propagation distance, roughly speaking forming a “kink” in their orientation with respect to the beam axis. This angular shift may provide a noninterferometric means to determine the vortex charge of a PCVB.

The determination of vortex charge is essential to the use of vortex beams as information carriers; however, because a vortex is primarily a phase object, this determination is nontrivial. Early methods of vortex detection involved interferometry, which is not easy to employ in compact and durable devices for field use. In recent years, it has been noted that the diffraction of a vortex beam by a triangular aperture produces an interference pattern with a simple relation to the vortex charge [10]; in particular, a vortex of charge N produces a diffraction pattern in the form of a triangle with N+1 lobes on a side. Studies of such aperture diffraction have though been limited to ideal cases in which a pure vortex mode is detected directly on axis; in the presence of turbulence, beam wander and mode mixing will result in the beam at the detector being far from ideal. In [Publication 6], the PI and student have developed a simple analytic model for the diffraction of vortex beams of any order by a triangular (or other shape) aperture. They used this model to study the effect of beam wander and mode mixing on the detected signal.

An example of their results is shown in Fig. 3. A vortex of charge 1 is moved horizontally away from the center of the aperture. When it is perfectly oriented, one can
see that there are 3 bright lobes forming a triangle, indicating a charge 1 vortex; as it is moved, however, the lobes change in brightness, eventually forming a single bright spot when the vortex is moved completely out of the aperture.

Figure 3. The diffraction pattern for an \( m = +1 \) vortex, with the vortex displaced horizontally by a fraction of the aperture size \( a \). (a) \( x/a = 0.0 \), (b) \( x/a = 0.318 \), (c) \( x/a = 0.636 \), (d) \( x/a = 2 \).

The analytic model predicts that the diffraction pattern loses its distinctive shape when \( x/a \) is roughly 1/3, independent of the wavelength of light; this was found to be in agreement with computational results. Disappointingly, mixed-mode states were not generally identifiable, as the lower-order mode typically dominated the pattern over the higher one. However, it was noted that the change in pattern for a displaced vortex is highly directionally dependent, suggesting that one could use the diffraction pattern not only to identify a pure vortex state but correct for beam wander. Other methods of vortex detection are currently being investigated, as well.

In order to use PCVBs for multiplexing of data in FSO communications, it is necessary to have a good analytic model and understanding of beams with higher-order vortices nested in them. We have recently determined such an analytic model, and are preparing the results for publication in the next grant period. The next step will be to assess both the
scintillation characteristics and the vortex characteristics of such beams on propagation in atmospheric turbulence.

4. Relationship between optical singularities of different classes

The vortices produced in partially coherent beams at first glance appear to be of a fundamentally different character than those found in fully coherent beams. A fully coherent beam, for instance, has a well-defined phase, while a partially coherent beam does not. Work by the PI and others has, however, demonstrated that coherent vortices may be considered a special case and a subset of the more general phenomenon of correlation singularities.

Other classes of optical singularities exist as well, including singularities of the state of polarization and correlation singularities of partially coherent electromagnetic fields. Though there have been a scattered collection of papers describing the relationships between various pairs of singularity types, very few have considered the relationships as a whole.

The PI and collaborators introduced a simple 3-pinhole diffraction experiment to demonstrate this relationship in [Publication 4]. By varying the polarization or spatial coherence of light coming from one or more pinholes, they were able to demonstrate a continuous cycle of phase, polarization and coherence singularities in the interference pattern. Strikingly, they noted that a decrease in spatial coherence can actually result in the creation of electromagnetic coherence singularities, a phenomenon not previously described. These results may prove useful in designing new methods of generating and detecting optical vortices for applications.

5. Construction of arbitrary vortex and superoscillatory fields

We have noted that the wandering of an optical vortex is one of the significant problems with the application of vortex beams to FSO applications. From a geometrical optics perspective, one may envision the zero line of the vortex being deflected on propagation through the atmosphere. The use of an Nth-order vortex does not provide significant improvement, as it breaks into N first-order vortices which are highly correlated in their
propagation path. In other words: if one wanders from the detector, the others are very like to wander from the detector as well.

The PI and collaborator have tried, with some success, to solve this problem by constructing a beam as an incoherent, spatially separated array of $N$ first-order vortices, rather than a single $N$th order vortex. An example of the average vortex charge received at the detector is shown in Fig. 4. A spatially separated array of first-order beams loses its topological charge at a much slower rate than a single high-order vortex. These numerical results suggest that vortex communications may be much more robust in such an array configuration. These results are being studied further and will be reported on in more detail in the next grant period.

![Figure 4. Average topological charge (vortex order) as a function of propagation distance in moderate turbulence. Work done by PI and postdoctoral researcher Yalong Gu.](image)

A natural question that has arisen in such work: how does one efficiently generate arrays of vortices? We desire the simplest source possible, which has the flexibility to put vortices in a wavefield at any position and any orientation.

Recent investigations by the PI on the phenomenon of superoscillation has provided a potential answer. “Superoscillations,” introduced in the 1990s [11], are oscillations of a field at a rate much faster than the bandlimit of the field itself. Much more recently, such
superoscillations have been shown to be connected with the spacing of zeros in a wave, which in turn has shown that closely-spaced optical vortices result in superoscillations in a transverse plane of a propagating wave [12]. In exploring such superoscillations, the PI and student have developed a simple way to produce superoscillatory vortex patterns in a transverse plane, or in fact any distribution of vortices in such a plane. Such a method can be simply implemented with a spatial light modulator and lens system; these results are being prepared for publication as [Publication 7].

An example of the results is shown in Fig. 5. The system was designed to place 5 first-order vortices extremely close together in a line near the central axis. The close spacing of the vortices comes at the cost of an extremely low intensity in the neighborhood of the zeros. For this case, the oscillations along the line of zeros are roughly twice as fast as the bandlimited function would traditionally allow.

Figure 5. Magnitude (a), (b) and phase (c), (d) of a designed superoscillatory vortex field. (b) and (d) show a zoomed in region at the center of (a), (c).

This analytic method is very simple to use, giving new flexibility to the design of complicated vortex fields.
6. Transfinite mathematics in optical vortex beams

The investigation of other vortex generation techniques has led to a surprising relation between optical vortices and the mathematics of infinite sets, which has now been released as [Publication 5].

A traditional method for generating vortex beams is the use of a spiral phase plate, a transparent piece of glass given a ramp-like shape. If the height of the ramp is chosen correctly, it can be used to produce a vortex of any integer order. However, there is obviously no physical constraint on how one designs such a spiral plate; what then happens if the plate is designed to produce a half-integer vortex, or any other fractional vortex? This problem was investigated by Berry some time ago [13], and it was found that the result is a complicated production of an in principle infinite number of vortex pairs, as shown in Fig. 6.

![Figure 6](image_url)

Figure 6. The creation of a new vortex as the “charge” $\alpha$ of the spiral phase plate is increase from 4 to 5. 
(a) $\alpha = 4.4$, (b) $\alpha = 4.47$, (c) $\alpha = 4.5$, (d) $\alpha = 4.55$, (e) $\alpha = 4.65$, (f) $\alpha = 4.995$.

In short, as $\alpha$ approaches 4.5, pairs of vortices are created along a horizontal line stretching to the right; at $\alpha = 4.5$, it can be shown that the number of vortex pairs is infinite. As $\alpha$ is made greater than 4.5, the vortices annihilate from infinity inwards, but each negative charge annihilating with its neighbor to the right. The result is a remaining extra unbalanced positive charge.
What seems to have been previously unappreciated is that this behavior exactly mimics a classic paradox of infinite mathematics known as Hilbert’s Hotel. A hotel with an infinite number of rooms, all occupied, can always create a vacancy by asking each guest to move to the next higher room. Similarly, this vortex system creates an extra unbalanced vortex by creating an infinite number of vortex pairs, then reshuffling them like guests in Hilbert’s Hotel. This is one of the first overt examples of infinite mathematics manifesting in a physical system known.

As Hilbert’s Hotel can, in principle, accommodate an infinite number of new guests, the PI investigated whether multiple vortex charges could be created at the same time. The results are illustrated in Fig. 7. By using a multi-ramp phase plate with N ramps, one can get the charge of the transmitted beam to jump by N discontinuously. Though seemingly an abstract concept, Fig. 7(b) suggests that it is possible to design vortex systems in which the vortex charge can be changed by an almost arbitrarily large amount with a very small change in the system itself. This has practical implications for the design of vortex-based optical communications systems, in which a fast switching vortex source will be necessary.

![Diagram of vortex beam]

Figure 7. Showing (a) a multi-ramp spiral phase plate and (b) the topological charge of the transmitted vortex beam as a function of ramp height. For the 5-ramp plate shown here, the charge jumps discontinuously in steps of 5.

7. Future work

The research done in this performance period was focused primarily on the properties of vortex and partially coherent beams, and their synthesis, as well as the detection and generation of vortex beams. In the next grant period, we propose to study the
propagation of these hybrid PCVBs in atmospheric turbulence in detail, taking into account also their generation and detection.

References


**Publications produced from grant**


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Abstract
The goal of this project was to study and optimize special classes of optical beams, with particular attention paid to beams with partial coherence or nested vortex structures, or combinations of the two. For intensity-based communication and remote sensing, partially coherent beams were studied with the goal of minimizing intensity fluctuations induced by random media. More recently, researchers have been interested in using the discrete topological phase core of optical vortex beams as an alternate carrier of information. Properties of vortex beams, both coherent and partially coherent, were investigated, and properties of partially coherent vortex beams were characterized for the first time. The efficacy of vortex detection by diffraction was also evaluated. Relations between different types of optical singularities were discovered and analyzed, and a surprising connection between vortex evolution and infinite mathematics was uncovered.

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