**Title and Subtitle:** How Many Grid Points are Required for Time Accurate Simulations? Scheme Selection and Scale-Discriminant Stabilization

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**Abstract:** Briefing Charts/Viewgraphs

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<table>
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<th>Subject Terms</th>
<th>Unclassified</th>
</tr>
</thead>
</table>

**Limitation of Abstract:** SAR

**Number of Pages:** 16

**Telephone Number:** N/A
How Many Grid Points are Required for Time Accurate Simulations?

Scheme Selection and Scale-Discriminant Stabilization

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Boston, MA
Motivation

- computational efficiency
  - coarser grids
  - larger time steps

spatial concerns:

- how well are gradients captured? (resolution requirement)

spatial/temporal concerns:

- dispersion and dissipation error
  - better characteristics for a broader range of wavenumbers

scheme stabilization concerns:

- balancing accuracy with stability
  - artificial dissipation, filtering

Distribution Statement A: Approved for public release; distribution is unlimited.
Spectral Representation

Fourier basis:

\[ u_j = \sum_k \hat{u}(k)e^{ikx_j} \]

\[ u_{j+1} = \sum_k \hat{u}(k)e^{ikx_j}e^{ik\Delta x} \quad \text{with} \quad \theta = (k\Delta x) \in [-\pi, \pi] \]

\[ \delta_x u \rightarrow a_0 \dot{u}_j + \sum_{l=1} a_l (u_{j+l} - u_{j-l}) = \frac{1}{\Delta x} \left[ b_0 u_j + \sum_{r=1} b_r (u_{j+r} - u_{j-r}) \right] \]

\[ Z_{\text{conv.spec}} = i \odot k \quad \rightarrow Z_{\text{conv}}(\theta) = b_0 + i \odot \left[ \frac{2 \sum_{r=1}^R b_r \sin(r\theta)}{1 + 2 \sum_{l=1}^L a_l \sin(l\theta)} \right] \]

modified wavenumber
Modified Wavenumber

\[ \text{error}_k = \frac{k - k_{\text{mod}}}{k} \]

**goal:** proper representation of derivative
Gradient Capture vs. Resolution: Single Mode

Solution/Derivative:

\[ f(x) = \sin(x) \quad \text{with} \quad x \in [0, 2\pi] \]
\[ \frac{df}{dx} = \cos(x) \]

Convergence:

FFT:
Gradient Capture vs. Resolution: Multiple Modes

Solution/Derivative:

\[ f(x) = \sum_{m=1}^{4} \sin(mx) \text{ with } x \in [0, 2\pi] \]

\[ \frac{df}{dx} = \sum_{m=1}^{4} m \cos(mx) \]

Convergence:

FFT:

Distribution Statement A: Approved for public release; distribution is unlimited.
Isentropic Vortex: no Stabilization

\[ M_\infty = 0.5 \]

\[ CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1 \]

CD04 (11pt B&B)

need to stabilize accumulation of high frequency error

Distribution Statement A: Approved for public release; distribution is unlimited.
Stabilization: Artificial Dissipation and Filtering

Artificial Dissipation:

\[
\frac{\partial Q}{\partial t} = - \frac{\partial E}{\partial x} + \sum_{m} (-1)^{m-1} (\Delta x)^{2m-1} \epsilon_{2m} \lambda_{u+c} \left| \frac{\partial^{2m} Q}{\partial x^{2m}} \right|
\]

damping strongly dependent on base scheme (couples with temporal scheme)

Filtering:

\[
\frac{\partial Q^*}{\partial t} = - \frac{\partial E}{\partial x}
\]

\[
\left[ 1 + \sum_{m} (-1)^{m+1} \epsilon_{IF,2m} \left( \frac{\Delta x}{2} \right)^{2m} \frac{\partial^{2m} Q}{\partial x^{2m}} \right] Q = \left[ 1 + \sum_{n} (-1)^{n+1} \epsilon_{EF,2n} \left( \frac{\Delta x}{2} \right)^{2n} \frac{\partial^{2n} Q}{\partial x^{2n}} \right] Q^*
\]

adds consistent amount of damping to base scheme (decoupled from temporal integration)
Isentropic Vortex: Traditional Stabilization

\[ Q = \left[ 1 + \left( \frac{\Delta x}{2} \right)^{10} \frac{\partial^{10}}{\partial x^{10}} \right] Q^* \]

\[ CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1 \]

distribution statement A: Approved for public release; distribution is unlimited.

effectiveness of stabilization strategy dependent on spectral content
Damping Characteristics: Growth Factor

- need scale-discriminant, tunable formulations
- strong preservation of resolvable modes
Scale-Discriminant Stabilization

goal: preserve accurately resolved frequencies and remove error-prone content

scale-discriminant dissipation

\[
\text{error}_k = \frac{k - k_{\text{mod}}}{k} = 0.01
\]

<table>
<thead>
<tr>
<th>FD stencil</th>
<th>((k\Delta x)_{\text{cutoff}} / \pi)</th>
<th>~PPW</th>
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<tbody>
<tr>
<td>CD02</td>
<td>0.08</td>
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</tr>
<tr>
<td>CD04</td>
<td>0.24</td>
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<tr>
<td><strong>CD06</strong></td>
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</tr>
<tr>
<td>CD10</td>
<td>0.46</td>
<td>5</td>
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<td><strong>CD04 (7pt Tam)</strong></td>
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<td>5</td>
</tr>
<tr>
<td>CD04 (11pt B&amp;B)</td>
<td>0.55</td>
<td>4</td>
</tr>
</tbody>
</table>

for the isentropic vortex…

- maintain coherence of vortex
- preserve vortex intensity
Isentropic Vortex: Scale-Discriminant Stabilization

\[ \text{CFL}_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1 \]

CD04 (11pt B&B) + IF10 (2/3Pi)

- traditional dissipation is sufficient when spectrum is well resolved

16 points across vortex

distance traveled: 10 widths

- distance traveled: 50 widths

- distance traveled: 100 widths

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Isentropic Vortex: Scale-Discriminant Stabilization

\[ CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1 \]

CD04 (11pt B&B) + IF10 (2/3Pi)

__ref__ IF10 (2/3Pi) __EF10__

- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme
Isentropic Vortex: Scale-Discriminant Stabilization

\[ CFL_{u,1D} \approx 0.01 \quad \alpha = 1, \phi = 1 \]

CD04 (11pt B&B) + IF48 (2/3Pi)

- scale-discriminant dissipation preserves structure
- robustness requires tuning to scheme resolvability
- efficacy limited by dissipation scheme

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Conclusions/Going Forward

- non-linear instabilities instigated by high frequency error (i.e.: dispersion, aliasing)
- remove error with respect to overall scheme resolvability – a robust strategy
- scale-discriminant dissipation provides stability while minimizing dissipation error

**going forward:**
- non-linear stability (residual filtering, skew-symmetric forms etc...)
- incorporating temporal error
- applications to explicit LES