A method is presented for determining wave runup on composite slopes of different roughnesses. This method is one of successive approximations and involves replacement of the actual rough composite slope with a hypothetical single slope extended from the breaking depth and an estimated runup value.

Design data on the height of runup are needed to determine crest elevations of protective structures subject to wave action such as seawalls, revetments, and dam embankments. These elevations are important from both functional and structural considerations.

The suggested procedures for determining the crest elevation to prevent overtopping of a riprap revetment fronted by a beach (Figure 1) follows:

The water depth limits the maximum wave reaching the structure. The maximum wave runup is not necessarily limited to the maximum wave breaking directly on the structure but may be controlled by waves breaking seaward of the structure and running up on a composite-slope (compound-slope) composed of two or more materials with different roughnesses.
There is no clear cut tested procedure to serve as a guide in developing a solution of the problem. The following methodology, which is discussed in Chapter 7, Section II la (SPM 1984) may serve as a guide until more definitive results are available.

First it is important to recognize that although wave forecasting procedures define a significant wave height and period for a particular set of conditions, i.e. wind velocity, fetch, and water-depth, waves with a particular defined wave height can occur over a range of periods centered around the forecasted period. For design purposes, this range of wave periods is generally assumed to be 0.5T to 1.9T. When determining both breaking wave heights and wave runup on a structure, it is important to consider this variation in wave period.

The following problem illustrates the suggested methodology for estimating the maximum wave runup on a revetment type structure with a compound slope and complex roughness.

**GIVEN:**
A) Quarystone revetment as shown in Figure 2.
B) Design water depth at toe of structure, \( d_s \), is 5.0 ft.
C) Wave heights are depth limited.
D) Wave periods can vary up to 12 sec.

**FIND:**
A) Maximum wave runup on the 1 on 2 structure slope for waves breaking at the toe of the revetment.
B) Maximum wave runup for waves breaking seaward of the structure.

**SOLUTION PROBLEM A:** The waves that break at the toe of the structure only run up on the rough 1 on 2 slope.
With $d_s = 5.0$ ft and wave period, $T = 6$ sec, find the value of $H'_o$, for the breaking wave height, $H_b$, and the resultant wave runup, $R$ (see Figure 1).

From Figure 7-4 (SPM, 1984) with $m = 0.05$ (nearshore slope 1:20) and

$$\frac{d_s}{gT^2} = \frac{5.0}{32.2(6)^2} = 0.00431$$

$$\frac{H_b}{d_s} = 1.20 \text{ and } H_b = 5.0 \cdot 1.20 = 6.0 \text{ ft}$$

From Figure 7-5 (SPM) with $m = 0.05$ (1:20) and

$$\frac{H_b}{H_o} \approx \frac{6}{1.31} \text{ and } H_o = 6.0 \cdot \frac{1}{1.31} = 4.58 \text{ ft}$$

$$\frac{H'_o}{H_o} = \frac{4.58}{32.2(6)^2} = 0.00395$$

$$\frac{d_s}{H'_o} = \frac{5}{4.58} = 1.09$$

Using the procedure described by Stoa (1978), determine the wave runup on a smooth slope and then reduce the runup by the composite roughness ratio, $r_c$. The roughness ratio, $r_r$, from Table 7-2 (SPM) for a two layer (impermeable foundation) would be in the order of $r_r = 0.60$.

Using Figure 7-10 ($d_s/H'_o \approx 0.80$) and Figure 7-11 ($d_s/H'_o \approx 2.0$) the relative, $R/H'_o$ for $d_s/H'_o = 1.09$ and $H'_o/gT^2 = 0.00395$ is obtained as follows:

<table>
<thead>
<tr>
<th>$d_s$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H'_o$</td>
<td>$H'_o$</td>
</tr>
<tr>
<td>Figure 7-10</td>
<td>0.80</td>
</tr>
<tr>
<td>1.09</td>
<td>2.82$^*$ By interpolation</td>
</tr>
<tr>
<td>Figure 7-11</td>
<td>2.00</td>
</tr>
</tbody>
</table>

3
where \( k \approx 1.15 \) from Figure 7-13 (for \( H = 1.5 \) to 4.5 ft and 1 on 2 slope).

\[
R_{\text{smooth}} = H_k \left( \frac{R}{H_k} \right) = 4.58(1.15)2.82
\]

\( R_{\text{smooth}} = 14.85 \text{ ft} \)

\[
R_{\text{rough}} = r \times R_{\text{smooth}} = 0.60(14.85) = 8.91 \text{ ft}
\]

Repeat these calculations for various wave periods. The summary results are as follows:

**TABLE 1 - Wave Runup on Revetment Slope**

<table>
<thead>
<tr>
<th>( T ) (sec)</th>
<th>( H_h ) (ft)</th>
<th>( H'_{o} ) (ft)</th>
<th>( R_{\text{rough}} ) (ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6.00</td>
<td>4.58</td>
<td>8.91</td>
</tr>
<tr>
<td>8</td>
<td>6.40</td>
<td>4.13</td>
<td>10.37</td>
</tr>
<tr>
<td>10</td>
<td>6.65</td>
<td>3.74</td>
<td>10.71</td>
</tr>
<tr>
<td>12</td>
<td>6.75</td>
<td>3.29</td>
<td>9.94</td>
</tr>
</tbody>
</table>

The maximum wave runup on the structure from waves breaking at the toe of the structures is 10.7 ft resulting from a 10-sec period wave.

**SOLUTION PROBLEM B:** Table 1 (from Problem A) shows the wave height values, \( H'_o \), for several wave periods that just break at the toe of the structure. To solve Problem B, a similar process must be carried out for each wave period using assumed wave height \( H'_o \) greater than those found in Table 1. For example, for the 6-sec wave assume \( H'_o = 5.0 \text{ ft} \).

From Figure 7-3 (SPM) for \( m = 0.05(1:20) \) and

\[
\frac{H'_o}{H'_o} = \frac{5.0}{32.2(6)} = 0.00431 ,
\]

\[
\frac{H'_b}{H'_o} = 1.26 \text{ and } ,
\]

\( H'_b = 1.26 \times H'_o = 1.26(5) = 6.30 \text{ ft}. \)
From Figure 7-2 (SPM) for \( m = 0.05(1:20) \) and

\[
\frac{H_b}{gT^2} = \frac{6.30}{32.2(6)} = 0.0054,
\]

\[
\frac{d_b}{H_b} = 1.0,
\]

\[
d_b = 1.0(H_b) = 6.30 \text{ ft.}
\]

In order to obtain a hypothetical composite slope, a value of runup, \( R \), must be assumed. Assuming \( R = 8.0 \) ft, using Figure 3, calculate \( \alpha \) (angle between composite slope and horizontal), and slope lengths \( l_s, l_r, \) and \( l_c \).

\[
\alpha = \arctan \left( \frac{(R + d_s) \cot \theta + (d_b - d_s) \cot \beta}{(R + d_b) \cot \theta + (d_b - d_s) \cot \beta} \right)
\]

Figure 3 - Problem "B"

\[
\alpha = \arctan \left( \frac{R + d_b}{(R + d_s) \cot \theta + (d_b - d_s) \cot \beta} \right)
\]

\[
R = 8.0, \ d_b = 6.3, \ d_s = 5.0, \ \theta = 26.565^\circ (1.2 \text{ slope}), \ \text{and} \ \beta = 2.862^\circ (1.20 \text{ slope}).
\]

\[
\alpha = \arctan \left( \frac{8.0 + 6.3}{(8.0 + 5.0) \cot 26.565^\circ + (6.3 - 5.0) \cot 2.862^\circ} \right)
\]

5
\[ l_s = \frac{d_b - d_s}{\sin \beta} \cos (\alpha - \beta) - \frac{6.3 - 5.0}{\cos (15.376^\circ - 2.862^\circ)} = 25.418 \text{ ft} \]

\[ l_r = \frac{R + d_s}{\sin \theta} \cos (\theta - \alpha) = \frac{8.0 + 5.0}{\cos (26.656^\circ - 15.376^\circ)} = 28.517 \text{ ft} \]

\[ l_c = l_s + l_r = 53.93 \text{ ft}, \frac{l_s}{l_c} = \frac{25.42}{53.93} = 0.47, \frac{l_r}{l_c} = \frac{28.52}{53.92} = 0.53. \]

From the preceding calculation, the composite roughness, \( r_c \), can be determined with \( r_r = 0.6 \) and \( r_s = 1.0 \):

\[ r_c = r_s \frac{l_s}{l_c} + r_r \frac{l_r}{l_c}, \]

\[ r_c = 1.00(0.47) + 0.60(0.53) = 0.788, \text{ say 0.79.} \]

Next determine the wave runup on the composite 1:3.64 slope \( (\alpha = 15.376^\circ) \) for

\[ \frac{d_b}{H_o} = \frac{6.30}{5.0} = 1.26. \]

Using Figures 7-10 and 7-11 (SPM, 1984) to determine the relative runup on a smooth slope results in the following value for a slope of 1:3.64 and

\[ \frac{H'}{H_o} = \frac{5.0}{32.2(6)^2} = 0.00431 \]

\[ \frac{d_s}{H_o} = 0.80 \]

\[ 1.26 \]

\[ 2.00 \]

\[ 1.74 \]

\[ 1.755 \]

\[ 1.78 \]

\[ + \text{ By interpolation} \]

From Figure 7-13 for a 1:3.64 slope and \( H = 4 \) to 12-ft curve, \( k = 1.126 \)

therefore:
\[ R_{\text{smooth}} = H' k(R/H') \]
\[ (\text{smooth}) = 5.0 (1.126) 1.755 = 9.88 \text{ ft} , \]

thus:

\[ R_{\text{composite}} = r \times R_{\text{smooth}} = 0.79(9.88) = 7.80 \text{ ft} \]

Since the calculated value of \( R_{\text{composite}} \) (7.8 ft) does not agree with the assumed value (8.0 ft), the calculation for a different composite slope must be repeated until closer agreement is achieved (~0.05 ft). In this case, the calculated runup value did not reach the estimated runup value, therefore, a new estimated value equal to or slightly less than the calculated value is an appropriate approximation. In the case of underestimating the calculated wave runup, an estimated value slightly (say about 5 percent) greater than the calculated value should be assumed.

Table 2 summarizes the iterative matrix calculations for various wave heights and periods. Figure 4 is a plot of the values from Tables 1 and 2. From Figure 4, it can be seen that the 10-sec wave produces the greatest breaking wave runup on the revetment \( (R = 10.7 \text{ ft}) \) whereas the maximum breaking wave runup on the composite slope \( (R = 13.9 \text{ ft}) \) is from the 12-sec wave.

<table>
<thead>
<tr>
<th>TABLE 2 - Wave Runup on Composition Slope, R(ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) (sec)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>
Figure 4 - Wave Runup on Composite Slope

REFERENCES:
