Scientific Letter

The PORTFOLIO CREATION MODEL developed for the Capital Investment Program Plan Review (CIPPR)

To inform senior management about CIPPR decision support, this scientific letter has been prepared upon request [1] to clarify some of the key concepts about the portfolio creation model that has been developed to produce project portfolios for the Capital Investment Program Plan Review (CIPPR).

The portfolio creation model is one element of the portfolio approach that has been envisioned for CIPPR in order to enable better decisions concerning the inclusion or exclusion of projects within an enhanced strategic investment planning process. While another letter is being prepared to explain the overall concept and benefits of the portfolio approach, this letter provides a brief and accessible description of the portfolio creation model to enhance understanding of what might seem to be a “black-box” among decision makers and their staffs. A mathematical description of the portfolio creation model is contained at Annex A.

Background

The Capital Investment Program Plan Review (CIPPR) was formally initiated through a directive [2] from the Vice Chief of Defence Staff (VCDS) and the Associate Deputy Minister Finance and Corporate Services (ADM FIN CS). The aims of the first iteration of CIPPR were articulated as follows:

1. Undertake a rationalization of all investments at the Identification (ID), pre-Identification (pre-ID) and Options Analysis (OA) stages\(^1\) that have an acquisition cost of greater than $5 million.
2. Produce a consolidated balanced portfolio consisting of critical, viable, and affordable capabilities representing best value for money.
3. Institutionalize a process that will be transparent, repeatable, rigorous and coherent against which all present and future investment will be assessed.

Under the umbrella of what has been called the portfolio approach, the first iteration of CIPPR is currently being executed by the CIPPR working group, its staff and participating organizations from across Defence, through execution of the following elements:

1. a value framework for assessing the relative merits of individual projects;
2. an optimization model for creation of portfolios;
3. an enhanced toolset for collection of project data;
4. an interactive visualization application;
5. a tool for comparative portfolio analysis;
6. alternate portfolios for consideration by senior Defence leaders;
7. selection of a preferred portfolio;
8. integration of an enhanced Capital Investment Review within ongoing Strategic Planning processes; and
9. effective project governance;

\(^1\)These stages are part of the project delivery process as laid out in the Defence Project Approval Directive (PAD).
With respect to the list above, the subject of this scientific letter is element (2) — the optimization model developed to create viable portfolios containing individual capital projects. This optimization model, known as the CIPPR \textit{portfolio creation model}, allows for the \textit{simultaneous} consideration of:

1. the merit or “value” of each project, and
2. the need to satisfy a set of practical constraints and preferences – including fiscal and capacity constraints that limit Defence’s ability to realize projects.

\textbf{Result: The portfolio creation model}

The problem of selecting a portfolio of projects from a large set of candidate projects is a combinatorial optimization problem commonly known as a 0-1 knapsack problem. The generic 0-1 knapsack problem is described as follows. Suppose there is a knapsack that can carry a limited weight and there is a set of items. Each item has an associated weight and a quantified estimate of merit or “value”. The objective is to fill the knapsack with a subset of items so that it attains a maximum value, while at the same time ensuring that, in aggregate, the items within it do not exceed its weight limit.

An illustration of the problem is shown in Figure 1, where the knapsack has a weight limitation of 10 lbs. and there are 10 candidate items. The optimal solution consists of items that would be put in the knapsack to give it maximum value and whose collective weight is less than 10 lbs. are shown in green. The items that would not be included in the knapsack are shown in red. In this example, the decision to select an item is binary, i.e., yes or no, and therefore the knapsack problem is described as a 0-1 knapsack problem.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Figure1.png}
\caption{Example knapsack problem. Green items are selected to be in the knapsack and red items are not selected.}
\end{figure}

This basic knapsack model can be extended to include additional constraints. For example, the knapsack may have volume as well as weight limitations. There may also be interdependencies between the items which must be taken into account.

Note: For the first iteration of CIPPR context the term \textit{value} does not refer to a monetary value, rather it corresponds to a “degree of goodness” or “merit”.


account (e.g., if one item is chosen then another item can not be chosen), or there may be multiple knapsacks to fill. When extensions such as these are included, the problem becomes known as a 0-1 multiple-knapsack problem with multiple constraints. The CIPPR portfolio creation model was developed based on this class of problem.

The objective of the CIPPR portfolio creation model is to select an “optimal” set of projects from a larger set of potential projects in order to fill two knapsacks.

1. The first knapsack corresponds to the portion of Defence’s accrual funding envelope that is projected to be available to pay for capital projects.
2. The second knapsack corresponds to the expected amount of cash or “A-base” funding against which capital projects may be expensed.

Both of these knapsacks extend beyond a 20 year time horizon, and both effectively limit the amount of funding that is available for projects in every fiscal year. The aim is to select projects for inclusion in the portfolio that, in aggregate, provide the maximum value to Defence (as defined via a predefined project value framework\textsuperscript{3}). The CIPPR portfolio creation model incorporates several constraints. Due to the complexity of the constraints, the portfolio creation model is implemented as a mathematical optimization \cite{3, 4}. The mathematical formulation of this optimization is presented at Annex A.

The fundamental aspects of the CIPPR portfolio creation model are outlined as follows:

- **Projects are indivisible and project selection is binary**: Projects can not be divided into segments; either a project is selected for inclusion in the portfolio or it is not. Additionally, there can not be duplicate copies of a project in the portfolio.

- **Projects typically demand funding over several years**: Each project's projected annual financial demand (between an estimated start-year and an estimated end-year) is used to determine the viability of including the project against constraints that limit the available fiscal supply every year.

- **Projects may be funded from one of two sources of financial supply**: Two financial demand profiles are identified for each project - a cash-based expenditure profile and an accrual-based expenditure profile. Only one funding source is be selected by the model to fund a project.

- **There are multiple knapsacks, each with a different capacity profile**: There are two knapsacks. The first knapsack corresponds to the portion of the accrual funding envelope that is projected to be available to pay for projects in the foreseeable future. The second knapsack corresponds to the expected amount of cash or “A-base” funding against which projects could be expensed.

- **Project value is determined based on three overarching criteria**: Three criteria are used to determine the overall merit or “value” of each individual project are: 1) alignment with National Policy, 2) alignment with Capability/Institutional View, and 3) Sponsor Priority. The value computation for each project is based on a linear weighted summation equation wherein the score achieved against each criteria is multiplied by an a weight factor associated with that criteria. The higher the associated weight factor the greater the influence of that criteria on the overall value of each project.

- **The value of a knapsack is computed as a function of the values of the individual projects contained within it**: How the aggregate value of a portfolio is computed can significantly affect the set of projects that are included in the optimal portfolio. Within the CIPPR portfolio creation model the value of the knapsack is computed by adding the values of the individual projects within it.

- **The quantity to be maximized by the portfolio of projects is the aggregate value of all knapsacks**: The aggregate value of the accrual and cash-based knapsacks is maximized by the portfolio creation model in order to arrive at the optimal set of projects to include in the portfolio. This aggregate value is computed using a linear sum.

\textsuperscript{3}A letter on the subject of the value framework developed for CIPPR is currently being prepared.
The portfolio is constrained by the practical capacity to deliver projects: The model accounts for the limited capacity of the three main organizations within Defence that deliver capital projects (i.e., ADM(Mat), ADM(IE), and ADM(IM)) by way of three separate fiscal proxies. In lieu of having information about: 1) the available supply of human capacity to delivery projects, or 2) the demand each project is expected to make against the available supply, these proxies effectively set upper limits on the amount of money that can be spent on materiel, infrastructure, and information systems projects in each fiscal year.

The entire capacity of the knapsack need not be utilized: By design, the portfolio creation model does not require that all of the space in the cash and accrual based envelopes be used. Rather, the model allows for under- or over-programming. In each year the model will ensure that projects can only utilize up to a specified percentage of the available space in each fiscal envelope.

Interdependencies between projects can be explicitly accounted for: The portfolio creation model is designed to explicitly enforce one- and two-way dependencies between projects if these dependencies are specified within the project dataset. A one-way project dependency indicates that the inclusion of a particular project within the portfolio will necessitate the inclusion of another project, but not vice-versa. A two-way dependency indicates that, for two dependent projects, both must either be included or excluded from the portfolio (i.e., it is not acceptable that one project is included while the other is excluded and vice versa).

Individual projects can be “forced” into or out of the portfolio: The model is designed to explicitly allow for the manual inclusion or exclusion of projects in the portfolio. Pre-commitments made by Government, sudden requirements that arise due to military operations, or unexpected opportunities afforded by industry or our alliance partners are just three examples which may necessitate the manual inclusion or exclusion of projects in the portfolio — irrespective of the framework being used to determine the value of projects.

Integer programming techniques are used to solve for the set of projects to be included in the portfolio: Integer programming techniques are appropriate to solve optimization models where all decision variables must be integers. The portfolio creation model uses a branch and bound algorithm to traverse the space of potential portfolios before arriving at what is deemed to be the optimal portfolio. With the set of projects currently being considered by CIPPR, an optimal portfolio is typically found in minutes (or in some cases seconds). If required a run-time limit can be set to limit the amount of time available for the branch and bound algorithm to find the optimal solution. When a run-time limit is set, the algorithm will return the best solution that it is able to find as well as an estimate of the “distance” between this solution and the theoretical optimum solution. Typically after less than 1 min there is only a small (i.e., 5%) difference between the solution found by the branch and bound algorithm and the theoretical optimum.

Some of the main assumptions and limitations of the portfolio creation model are outlined below.

Project data are certain: The attributes of each project (e.g., values, funding requirements, organizational capacity requirements) are deemed to be fixed point estimates without error or uncertainty. A consequence of this assumption is that the model can not be used presently to assess risks related to cost or delivery capacity.

Available budgets and organizational capacities are certain: Similar to project data, these are assumed to be fixed point estimates. Therefore, at present, the model can not be used to assess risks related to uncertain budgets.

Synergistic effects between projects are not considered: The value of a project does not change when other projects are included or excluded from the portfolio. For example, the value of a radar system project is not affected by the inclusion or exclusion of a missile system project.

Interdependencies between candidate projects and projects currently in the Definition and Implementation stages are not significant: Only linkages between candidate projects that are in the pre-ID, ID and OA phases

---

4The theoretical optimum is computed via a linear relaxation of the knapsack problem.
are taken into account. Therefore, if a radar system project is required to support a missile system project that is already in the Definition or Implementation stage, then this linkage can not be automatically accounted for within the model. However, it can be manually accounted for by forcing the inclusion of the radar system project.

- **Only project acquisition costs are of interest:** Operations and maintenance, national procurement and other lifecycle or sustainment costs outside of acquisition are not directly accounted for within the model.

### Discussion

The output of the portfolio creation model should be seen as a starting point for discussion amongst decision-makers rather than be seen as a final product. This intention is amplified by the fact that the authors have developed a complementary visual analytics software package called the Visual Investment Plan Optimization and Revision (VIPOR) that allows decision makers to iteratively and interactively add and remove projects to the portfolio and visually assess the impacts of their decisions.

In preparation for future iterations of CIPPR, exploration of the following aspects is recommended in order to improve the utility of the portfolio creation model when supporting strategic capital investment decisions.

- **More explicit control over the substitution effect:** The portfolio approach explicitly recognizes that maximizing the aggregate value of a portfolio that is subject to several constraints does not necessarily require that the highest ranked projects be included in the portfolio. In some cases the aggregate value of the portfolio may be higher if several lower value projects are selected rather than a single higher value project. This concept is called the “substitution effect”. Currently, the portfolio creation model tends to favour the replacement of extremely expensive projects with many smaller less expensive projects when arriving at a portfolio. This is not necessarily a drawback because it forces careful consideration of large projects. If a very large project is desired by a decision maker, it can be forced into the portfolio. However, future versions of the portfolio creation model could be configured to provide more explicit control of the substitution effect. This would provide decision makers the ability to incrementally alter the degree with which this effect impacts construction of the portfolio.

- **Automatic re-profiling:** In practice Defence sometimes has the ability to influence the profile of the available accrual envelope in order to mitigate so-called pinch-points or bottlenecks that would otherwise put undue restriction on delivery of projects in certain fiscal years and leave financial resources unused in other fiscal years. Therefore, in addition to having the model determine the set of projects to include in the portfolio, it could also be configured to automatically re-profile the available accrual or cash-based envelopes according to a set of rules or constraints. Alternative portfolios could then be created with the auto-profiling feature turned on or off in order to demonstrate the projected impacts of re-profiling.

- **Alternative project timelines:** Currently the model assumes that the start and end dates for each project are fixed. However, these point estimates could be specified to fall within a band centred around a particular year. The portfolio creation model could then be configured to determine how best to schedule projects in accordance with these bands. It is expected that through this kind of enhancement it would be easier to make optimal use of available resources and it would allow portfolios to reach even higher levels of aggregate value.

- **Project alternatives:** Currently each project exists only once within the list of potential projects that could be included in the portfolio. However, considering that there are a multitude of fiscal and capacity constraints, in some cases it may be prudent to configure the model to consider alternate versions of the same project. In each version of the project, the project's value, organizational capacity, financial demand, etc., could be different. The portfolio creation model could then be configured to determine which version of the project, if any, to include in the portfolio.

*Interdependencies with projects already in progress:* Decision interdependencies between projects in pre-ID, ID and OA phases of the project delivery process can be accommodated by the model. If data about existing projects is made available, dependencies with projects already in progress could also be incorporated.

\[^{5}\text{Computational load would increase however.}\]
• **Project synergies**: The portfolio creation model is based on the assumption that synergistic effects between projects are not significant. As such, the value of two or more projects, when delivered together, is not greater than (or less than) the sum of the individual project values. This assumption could be challenged, and future instances of the portfolio creation model could benefit from exploring how project synergies might best be identified, characterized, and integrated.

• **Explicit inclusion of uncertainty**: The model presumes that project data and the data associated with each portfolio constraint are point estimates that are known with complete certainty. As such, the model does not explicitly incorporate concepts related to things like “confidence intervals” or “error bands” which recognize that not all data may be known with the same degree of certainty. Developments for future instances of the portfolio creation model would benefit from exploring approaches to better account for uncertainty when arriving at the optimal portfolio.

**Conclusion**

A portfolio creation model has been designed and implemented to enable the CIPPR portfolio approach. The portfolio creation model takes into account project value, annual financial constraints, annual organizational capacity limitations, funding source restrictions, and project interdependencies. This model represents a significant improvement over previous Departmental approaches used to respond to strategic investment planning problems. The portfolio creation model, in conjunction with the portfolio-based approach and the set of data and tools developed for CIPPR, are expected to provide the basis for a transparent, repeatable, rigorous, and coherent decision-making process that is agile, iterative, interactive, and explicitly facilities the incorporation of professional judgement.

**References**


Prepared by: **M. Rempel and C. Young, DRDC – Centre for Operational Research and Analysis.**

**Attachments**

Annex A: Mathematical representation of the portfolio creation model
Annex A: Mathematical representation of the portfolio creation model

The portfolio creation model is a multiple-knapsack multi-criteria problem. The objective of the problem is to select the subset of projects, from a set of candidate projects $P$, that collectively provide the maximum value to the DND/CAF (as defined by the project value framework), subject to a set of annual financial constraints in the set of funding sources $F$, annual organizational capacity constraints in the set of organizations $O$, project funding source restrictions, and project interdependencies. The value that each project $p$ to the DND/CAF is modelled as a weighted linear sum of criterion values, where the criteria are the set $C$: national policy alignment, capability/ institutional alignment, and sponsor priority.

The portfolio creation model is stated as:

$$\text{max} \sum_{f \in F} \sum_{p \in P} \sum_{c \in C} w_c \cdot v_{p,c} \cdot s_{p,f}$$  \hspace{1cm} (A.1)

subject to:

$$\sum_{p \in P} r_{p,y,f} \cdot s_{p,f} \leq \omega_{f,y} \cdot b_{f,y}, \ \forall f \in F, \ y \in Y \hspace{1cm} (A.2)$$

$$\sum_{p \in P} \sum_{f \in F} \alpha_{p,o,y} \cdot s_{p,f} \leq k_{o,y}, \ \forall o \in O, \ y \in Y \hspace{1cm} (A.3)$$

$$\sum_{f \in F} s_{p,f} \leq 1, \ \forall p \in P \hspace{1cm} (A.4)$$

$$\gamma_{p,f} - s_{p,f} \geq 0, \ \forall p \in P, \ f \in F \hspace{1cm} (A.5)$$

$$\sum_{f \in F} (s_{i,f} - s_{j,f}) \geq 0, \ \forall (i,j) \in I \hspace{1cm} (A.6)$$
where:

- $p, P$ is the index and set of projects;
- $y, Y$ is the index and set of years;
- $c, C$ is the index and set of criteria;
- $o, O$ is the index and set of organizations with capacity limitations regarding implementation of projects;
- $(i, j), I$ is the set of pairs of project interdependencies $(i, j)$, where $i, j \in P$;
- $w_c$ is the weight of criteria $c$; all weights must be greater than 0 and the sum of the weights must be 1;
- $f, F$ is the index and set of funding sources;
- $\gamma_{p,f}$ is 1, 0; 1 if project $p$ can be funded by funding source $f$, 0 otherwise;
- $r_{p,f,y}$ is the requested funding by project $p$ in funding source $f$ in year $y$;
- $\alpha_{p,o,y}$ is the capacity consumed by project $p$ in organization $o$ in year $y$;
- $b_{f,y}$ is the funding limit in funding source $f$ in year $y$;
- $k_{o,y}$ is the capacity limit in organization $o$ in year $y$;
- $v_{p,c}$ is project $p$’s value for criteria $c$;
- $s_{p,f}$ is 1, 0; 1 if project $p$ is selected to be funded by funding source $f$, 0 otherwise;
- $\omega_{f,y}$ is the maximum percentage of the available funding in funding source $f$ in year $y$ that should be filled.

The equations in the model are interpreted as described below.

- The objective in Equation A.1 maximizes the portfolio’s value.
- Equation A.2 is the financial constraint that ensures that the sum of the selected projects’ funding does not exceed the funding sources’ budgets in each year. The parameter $\omega_{f,y}$ controls the available funding in funding source $f$ in year $y$.
- Equation A.3 is the capacity constraint that ensures that the sum of the selected projects’ required capacity in each year does not exceed the available capacity in the relevant organizations.
- Equation A.4 ensures that each project is only funded by a single source.
- Equation A.5 ensures that each project can only be funded from the appropriate funding sources.
- Equation A.6 ensures that the projects’ interdependencies are respected; for a pair of projects $(i, j)$, $j$ can only be selected if $i$ is selected, however $i$ can be selected without $j$. 