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High Fidelity Modeling of Field-Reversed Configuration (FRC) Thrusters

Space Propulsion and Power Portfolio Review
28 September – 02 October 2015

Justin Koo
AFRL/RQRS
Overview

• **Background**
  – Branch-level strategy
  – Status of experimental efforts

• **Project Description**
  – Goals/Objectives/Technical Challenge/Approach

• **Progress Update**
  – Low fidelity (Hugrass) status
  – High fidelity (Multifluid) status
  – 2-fluid FRC simulation

• **Conclusions**
Background
FRC propulsion

- **Field Reversed Configuration (FRC) is offshoot of fusion research**
  - Ionization by rotating B-field
  - Pulsed inductive \( j \times B \) acceleration
  - Magnetically insulated, plasmoid accelerated downstream

- **Key attributes**
  - Very low mass (estimate \( \sim 1-2 \) kg/kW)
  - Efficiency comparable to or higher than Hall thrusters (predicted)
  - Operates on diverse propellants; potential for multi-mode application
  - Pulsed operation provides near-constant efficiency over wide power range

\[
F_z = j \theta \times B_r
\]
Electrodeless Lorentz Force (ELF) Rotating Magnetic Field (RMF) thruster design appears well suited for low power operation; different set of computational challenges than theta-pinch RMF simulation.

Branch-level FRC development strategy

- **In-house**
  - Computational 6.1 program to develop numerical tools (this program)
  - Experimental 6.2 program to generate validation data
    - Briefly attempted theta-pincho configuration, failed to make sufficient progress (limited amount of data)
    - Switched to rotating magnetic field (RMF) configuration
    - Simultaneous development of PI/PPI systems

- **External**
  - Leveraging SBIRs to develop engineering-level FRC models for testing
  - Testing thrusters at MSNW LLC (Redmond, WA) and AFRL/RQRES (Edwards AFB, CA)
Experimental efforts (1)

PPI (liquid vaporization)

PI (RF ionization at 15 MHz)
Experimental efforts (2)

Custom circuit for PI stage
Project Description
Justification

• Some computational models exist
  – Neutral flow dynamics
  – Fully-ionized FRC translation
  – Collisional kinetic of charge-exchange

• The right integration of numerical models does not exist
  – Partially ionized magnetized plasmas are difficult to simulate
  – Modern diagnostics include internal/external probes and spectroscopy
  – Simulation of realistic FRCs requires specific physic modules such as coil-plasma interaction models

Fundamental question is how to scale FRCs to run efficiently at low power levels – difficult/impossible without simulation
Goals

• **Pre-ionization** – need to generate some amount of seed plasma; for multi-plasmoid operation, first pulse is the most difficult; possible to consider many different configurations (e.g. liquid phase propellants)
  – Very specific to configuration
  – Introduction of multiphase and chemistry greatly complicate physics

• **Formation** – go from seed plasma to strongly ionized closed-field plasmoid; kinetic effects expected in first stage; interaction between coil/plasma drive process

• **Translation** – plasma remain almost fully ionized; interested primarily in collisional effects as plasma impact neutral background
  – Fairly straightforward to model with existing fluid tools (high enough collisionality for Maxwellian assumption)

**Focus on Formation →** develop hierarchy of tools to understand plasmoid formation, test spacecraft / integration and design new thrusters
Objectives

• **Multi-scale / multi-physics capability** – emphasis is on high order methods and consistent collision modules

• **Formation and acceleration dynamics** – validation with experimental test campaign

• **FRC stability and turbulence onset** – fundamental physics of current sheets in plasma

• **Collisional-radiative characteristics** – detailed spectral signatures to reduce need to perturb plasma

Well-developed software framework will support not only FRC simulation, but also development of future HET / plume / general partially ionized, magnetized plasma devices
Technical Challenges

• **Field/plasma model** — Plasmas can have very complex phase space configurations and self-induced electromagnetic forces. Fundamental multiscale problem since electrostatic forces are relevant in this problem and force very small timescales.

• **Collisional Physics** — kinetic of ionization (and especially energy loss) are very sensitive to details of electron energy distribution function

• **Multi-Scale Effects** — diffusive behavior in region of separatrix is not modeled correctly in collisionless code — artificial diffusion makes computational phenomena look real.

• **System Complexity** — realistic simulations need to couple circuit models to plasma, represent three-dimensional / end effects, address radiation loss to impurities, etc.
Approach

• **Hierarchy of Plasma Models**
  – Continuum: Hugrass – Ideal MHD – Hall/Resistive MHD – MF
  – Kinetic: Explicit – Implicit Particle-in-cell (PIC)

• **Collisional Physics** — Consistent derivations from underlying cross-sections for both C-R and MF coupling terms

• **Code acceleration** — Development of physics modules in next-generation simulation framework at AFRL/RQRS; leveraging both software (algorithmic) and hardware (GPU) acceleration
Plasma Description

- 3-Dimensions + 3-Velocities
- Evolve the particles position and velocity
- e.g. Particle-In-Cell models

- Ensemble average of particles distribution, $f_s(x,v,t)$
- Evolve the distribution function
- e.g. Vlasov-Maxwell models
Kinetic (non-Maxwellian)

- The Boltzmann eqn:
  \[
  \frac{\partial f_s}{\partial t} + v \cdot \frac{\partial f_s}{\partial x} + \frac{q_s}{m_s} (E + v \times B) \cdot \frac{\partial f_s}{\partial v} = \frac{\partial f_s}{\partial t} \bigg|_c
  \]

- Take the 0th, 1st, 2nd moments of the Boltzmann Eqn.

  \[
  m_s \int v^n \frac{\partial f_s}{\partial t} dv + m_s \int v^{n+1} \cdot \frac{\partial f_s}{\partial x} dv + q_s \int v^n (E + v \times B) \cdot \frac{\partial f_s}{\partial v} dv = m_s \int v^n \frac{\partial f_s}{\partial t} \bigg|_c dv
  \]

- Each moment of the Boltzmann eqn gives an equation for the moment variable, and introduces the next higher moment variable

- This process can go on indefinitely

Can implement Boltzmann equation multiple ways:
- **PIC** (high dimensionality – 3D3V; high statistical noise)
- **Vlasov** (low dimensionality – 2D2V hero runs; smooth solution)

Leverage 6.2 program to provide codes and expertise
Moments of the distribution

- Modeling each particle velocity and position is not practical.
- Instead an average is performed to give a statistical description.
- Calculate the number of particles per unit volume having approximately the velocity $v$ near the position $x$ and at time $t$, distribution function $f(v, x, t)$

$$\rho_s = m_s \int f_s(v) dv$$

$$\rho_s u_s = m_s \int v f_s(v) dv$$

$$p_s = \frac{1}{3} m_s \int w^2 f_s(v) dv$$

$$h_s = \frac{1}{2} m_s \int w^2 f_s(v) dv$$

$$w = v - u_s$$

No assumptions on shape of distribution function yet….still a general fluid!
Multifluid (MF) Plasma Model

**Fluid Description (3 equations for each specie)**

- **Mass**
  \[
  \frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s u_s) = S_s
  \]

- **Mom.**
  \[
  \frac{\partial \rho_s u_s}{\partial t} + \nabla \cdot \left( \rho_s u_s u_s + p_s \vec{I} \right) = \frac{\rho_s q_s}{m_s} (E + u_s \times B) + \sum_r R_{rs} - \nabla \cdot \vec{\Pi}_s
  \]

- **Energy**
  \[
  \frac{\partial \varepsilon_s}{\partial t} + \nabla \cdot ((\varepsilon_s + p_s) u_s) = \left( \frac{\rho_s q_s}{m_s} E + \sum_r R_{rs} \right) \cdot u_s + \vec{\Pi}_s : \nabla u_s - \nabla \cdot q_s + \sum_r Q_{rs}
  \]

**Electromagnetic Fields:**

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0, \quad \nabla \cdot \mathbf{B} = 0
\]

\[
\frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_0 \sum_s \frac{q_s}{m_s} \rho_s u_s, \quad \varepsilon_0 \nabla \cdot \mathbf{E} = \sum_s \frac{q_s}{m_s} \rho_s
\]
Further Simplification

• If there are no neutral species, sum all charged species to get a single charged “fluid” equation

• Connect with electromagnetism through Faraday’s law and suitable version of Ohm’s Law

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0
\]  

(Faraday’s Law)

MHD variants of Ohm’s Law

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<thead>
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<th>Type</th>
<th>Equation</th>
<th>Description</th>
</tr>
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<td>Ideal</td>
<td>( \mathbf{j} = \mathbf{E} + \mathbf{u} \times \mathbf{B} )</td>
<td>Magnetic field frozen in flow (no diffusion)</td>
</tr>
<tr>
<td>Resistive</td>
<td>( \mathbf{j} = \sigma (\mathbf{E} + \mathbf{u} \times \mathbf{B}) )</td>
<td>Magnetic field can diffuse into plasma</td>
</tr>
<tr>
<td>Hall</td>
<td>( \mathbf{j} = \mathbf{s} \mathbf{E} )</td>
<td>Incorporates Hall currents</td>
</tr>
<tr>
<td>Resistive Hall</td>
<td>( \mathbf{j} = \mathbf{s} (\mathbf{E} + \mathbf{u} \times \mathbf{B}) )</td>
<td>Hall currents and magnetic field diffusion</td>
</tr>
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• Remove fluid equations altogether and simply represent plasma as finite resistivity affecting Maxwell’s equations – Hugrass RMF model

Need all models in same framework to facilitate simulation at multiply levels of fidelity → can identify role of missing physics
Detailed C-R

• Require spectral signatures to interrogate many regions of the plasma without significantly perturbing plasma
  — Probes don’t function well in RMF environment

• Leveraging 6.2 and 6.1 work to access detailed Ar models
  — Based on theoretical and experimental databases from NIST, LXCAT, etc.
  — 6.2 work supports detailed model development / 6.1 work has supported intelligent model reduction for computational tractability

• Groundwork has been laid to develop self-consistent inelastic transport coefficients from same cross-section databases
Progress Update
Numerical Model Development

• Tackling both sides of plasma hierarchy
  — Low-fidelity Hugrass model
    • Quick turnaround so more useful for interfacing with experimentalists
    • Potential low-order model for time-parallel acceleration strategy
  — High-fidelity Multifluid MHD model
    • Bringing SoA multifluid capability to FRC modeling
      — High order DG formulation
    • Working on extension to three-fluid
    • Maintaining consistency with C-R rates

• Computational machinery for high-fidelity will be used to build MHD (single fluid) system
Hugrass model

- Implementation of Maxwell’s equations in non-vacuum
- Based on numerical model originally published in 1981 by Hugrass (J. Plasma Physics, 26, 455-464)
- Radial-azimuthal code (infinitely long cylinder)
- Coupled equations for axial magnetic field and axial component of magnetic vector potential; solved spectrally

\[
\frac{\partial A_z}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 A_z + \frac{1}{ne \mu_0 r} \left[ \left( \frac{\partial A_z}{\partial r} \right) \left( \frac{\partial B_z}{\partial \theta} \right) - \left( \frac{\partial A_z}{\partial \theta} \right) \left( \frac{\partial B_z}{\partial r} \right) \right]
\]

\[
\frac{\partial B_z}{\partial t} = \frac{\eta}{\mu_0} \nabla^2 B_z + \frac{1}{ne \mu_0 r} \left[ \left( \frac{\partial A_z}{\partial \theta} \right) \frac{\partial}{\partial r} \nabla^2 A_z - \left( \frac{\partial A_z}{\partial r} \right) \frac{\partial}{\partial \theta} \nabla^2 A_z \right]
\]

- No self-consistent plasma transport – can still define spatial/temporal dependence of resistivity (\(\eta\))
Vacuum field of RMF FRC

\[ r_{A,B} = l(t) \cos(\omega_{RMF} t + \varphi) \]
\[ r_{C,D} = l(t) \sin(\omega_{RMF} t) \]
\[ r_{E,F} = l(t) \cos(\omega_{RMF} t + \varphi) \]
\[ r_{G,H} = -l(t) \sin(\omega_{RMF} t) \]
### Development of MFPM

<table>
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<tr>
<th>Kinetic</th>
<th>LTE, velocity moments</th>
<th>MFPM</th>
<th>$\epsilon_o \to 0$, $m_e \to 0$</th>
<th>MHD</th>
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#### Ideal MHD Model is Valid When:

- High collisionality, $\tau_{ii}/\tau \ll 1$
- Small Larmor radius, $r_{Li}/L \ll 1$
- Low Resistivity, $\left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{r_{Li}}{L}\right)^2 \frac{\tau}{\tau_{ii}} \ll 1$

#### Multi-Fluid Plasma Model

- Less computationally expensive than kinetic models
- Multi-fluid effects become relevant at small spacial and temporal scales
- Finite electron mass and speed-of-light effects are included
- There is charge separation is modeled
- Displacement current effects are resolved in the MFPM
Discontinuous Galerkin (1)

- Conservation Laws are given by \( \frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = S \)

- Multiply by basis functions and integrate over the volume

\[
\int_{\Omega} v_h \frac{\partial Q}{\partial t} dV + \int_{\partial \Omega} v_h F \cdot dA - \int_{\Omega} F \cdot \nabla v_h dV = \int_{\Omega} v_h S dV
\]

- That is an ODE of the form \( \frac{dQ_h}{dt} = L_r(Q) \), solved using 2nd, 3rd or 4th order TVD Runge-Kutta

Spatial order of 2nd-16th is common.
Discontinuous Galerkin (2)

- Riemann problems are solved at each interface to compute fluxes
- The source of dissipation / dispersion depends on the Riemann solver
- Variables are allowed to be discontinuous at the cell interfaces

**Advantages**

- Method is conservative
- Compact stencil for flux evaluation (easily parallelizable)
- Provides good flux-source coupling (no source splitting is needed)
- Resolves fast oscillations (e.g. plasma frequency) and large gradients (e.g. shocks)

**Disadvantages**

- Explicit Runge-Kutta Discontinuous Galerkin CFL limit is $< 1/(2p-1)$ where high order schemes have higher $p$
Multiscale issues

**Characteristic speeds:**

\[ v_{cs} = \sqrt{\frac{P_s}{\gamma \rho_s}}, \quad c \]

**Characteristic frequencies:**

\[ \omega_{ps} = \sqrt{\frac{n_s q_s^2}{\epsilon_0 m_s}}, \]

\[ \omega_{cs} = \frac{q_s B}{m_s} \]

\[ \nu_{sr} = \frac{4\sqrt{\pi} e^4 Z^4 n_r \log \Lambda}{3 \sqrt{m_s T_s^{3/2}}} \]

- Example lab plasma (seconds):
  \[ \frac{1}{\omega_{pe}} = 10^{-14}, \quad \frac{L}{c} = 10^{-9}, \quad \frac{1}{\omega_{ci}} = 10^{-8}, \quad \frac{L}{v_{ci}} = 10^{-5} \]
Availability

**Structured DG:**
- WARPX code developed at Washington
- Validated and with proven results
- Can be used to verify the unstructured case

**Unstructured DG:**
- Currently being developed at AFRL
- For complex geometries/experiments
- Will include different physics modules
Verification

**FORWARD FACING STEP**

```
- Unstructured grid readers
- Unstructured data structures
- PETC's library for data manipulation and time integration
- XML input file reader
- Quadrature rules integration
- Different Riemann solvers for numerical flux (e.g. Roe, HLL, HLLE)
- Flux Limiters for shock capturing
- Initial/boundary conditions
```
RMF startup (1)

Exploring collisionless limit for efficient FRC formation
RMF startup (2)

Electron mass density

Ion mass density

4x later, stable FRC?

Magnetic field

Current

Distribution A – Approved for public release; distribution unlimited
Next Steps

• Numerical
  – Develop/run continuum plasma hierarchy
  – Run kinetic plasma codes
  – Integrate with collisional physics
  – Code acceleration

• Physics
  – FRC formation validation
    • Magnetic reconnection
    • Instability modeling
Conclusions
Project Highlights

- **Leverages 6.1 and 6.2 efforts**
  - Explicit Particle-In-Cell (PIC) and implicit PIC development for Hall Effect Thrusters (6.2, AFRL/RQRS)
  - C-R model reduction (6.1, Luginsland/Marshall)
  - Algorithm acceleration (6.1, Farhoo)

- **Integration of plasma modules into framework code provides tools for investigation of numerous plasma phenomena**
  - Adapt Multifluid description to study electrostatic anomalous electron transport due to fluid instabilities
  - Use Multifluid / MHD description as preconditioner for implicit PIC
  - Detailed C-R necessary to evaluate utility of reduced models (such as Quasi-Steady State (QSS) approximation)

**Common simulation framework has potential to greatly increase transition opportunities by decreasing integration time**
Summary

• Research plan directly supports R&D objectives for advanced plasma propulsion (i.e. FRCs)
• Progress made in various areas:
  – Moving towards consistent plasma hierarchy
  – Integrating additional physics modules for real engineering utility
  – Need more algorithm work (accuracy/stiffness)
• Fundamental work, can be applied to other areas
• Challenging, exciting project….  
• ….but off to a slow start