# Comparison Between Predictions and Experimental Measurements for an Eight Pocket Annular Hydrostatic Thrust Bearing

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**Abstract**
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**Supplementary Notes**
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COMPARISON BETWEEN PREDICTIONS AND EXPERIMENTAL MEASUREMENTS FOR AN EIGHT POCKET ANNULAR HYDROSTATIC THRUST BEARING

A Project Report

By

MICHAEL FORSBERG

Submitted
In partial fulfillment of the requirements for the degree of

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Abstract

Comparison Between Predictions and Experimental Measurements for an Eight Pocket Annular Hydrostatic Thrust Bearing

(May 2008)

Michael Forsberg, B.S.M.E., Oklahoma State University

Chair of Advisory Committee: Dr. Dara Childs

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**Nomenclature**

\( A_0 = \text{Area of the thrust bearing inlet orifice (m}^2\)\)

\( C_d = \text{Orifice discharge coefficient} \)

\( D_{\text{Orifice}} = \text{Diameter of orifice (m)} \)

\( E_N = \text{Uncertainty associated with term "N"} \)

\( f_{mn} = \text{Flexibility coefficient for direction "m" from variable "n"} \)

\( F_z = \text{Load on bearing face} \)

\( M_1 = \text{Misalignment in the x direction (mm/\text{mm})} \)

\( M_2 = \text{Misalignment in the y direction (mm/\text{mm})} \)

\( M_a = \text{Moment acting in "a"} \)

\( P_{\text{Atmospheric}} = \text{Atmospheric pressure} \)

\( P_{\text{Exhaust}} = \text{Pressure bearing exhausts into} \)

\( P_{\text{Pocket}} = \text{Pocket pressure} \)

\( P_{\text{Ratio}} = \text{Pressure ratio} \)

\( P_R = \text{Recess (pocket) pressure (Pa)} \)

\( P_{\text{Supply}} = \text{Supply pressure} \)

\( P_S = \text{Supply pressure (Pa)} \)

\( \text{Re}_{\text{Orifice}} = \text{Orifice Reynolds number} \)

\( \text{Re}_{\text{OuterRadius}} = \text{Outer radius Reynolds number} \)

\( V_{\text{Orifice}} = \text{Fluid velocity through orifice (m/\text{s})} \)

\( V_{\text{OuterRadius}} = \text{Fluid velocity through thrust bearing outer radius (m/\text{s})} \)

\( Z_a = \text{Gap reading from proximity probe "a"} \)

\( Z = \text{Center clearance of thrust bearing (m)} \)

\( \mu = \text{Dynamic viscosity (N * s/m}^2\) \)

\( \rho = \text{Working fluid density (kg/m}^3\) \)

\( \Phi_c = \text{Misalignment angle about axis "c"} \)
**Previous Test Rigs**

Numerous test rigs have been designed for experimentation with thrust bearings. These test rigs fall into two categories: single thrust bearing test rigs where the load is applied by pushing the rotor to apply loading to a stationary thrust bearing; and multiple-thrust-bearing test rigs with the load applied through a thrust bearing and transmitted to a separate thrust bearing where the load is reacted. For tests involving shaft rotation, multiple-thrust-bearing rigs are preferred.

New [1] describes a multiple-thrust-bearing test rig for testing hydrodynamic thrust bearings seen in Figure 1.

In this test rig the load is applied to the housing containing the test thrust bearing with a hydrostatic thrust bearing. The test thrust bearing then transmits the load through the rotor, which is supported on a pair of hydrostatic journal bearings, where it is reacted with a hydraulic loading pad. The load is then measured with the hydraulic loading system. To measure the torque exerted on the thrust bearings, as well as to allow free axial movement for loading, the housing containing the thrust bearings is mounted on hydrostatic bearings. This design allows the housing to rotate freely, which loads a spring balance and measures the torque. Additionally, the test bearings are instrumented.
to measure the clearance and temperatures while flow rates are measured outside of the test rig. Gregory [2], Neal [3], and Horner et al [4] describe similar test rigs.

Harada et al [5] uses the multiple thrust bearing test rig in Figure 2.

![Figure 2: Harada test rig for hydrostatic thrust bearing testing [5]](image)

The Harada test rig uses a piston floated on gas bearings to support and load the test thrust bearing. This load is then transmitted through the rotor to a set of ball bearings which react it. The thrust bearing clearance is measured by three eddy current proximity probes located 120° from each other, which allows for more detailed information about the clearance profile. Finally, temperature is measured at the inlet and outlet, and flow rate is measured into the bearing.

Wang and Yamaguchi [6] also use a test rig of a similar layout. Their test rig though is vertically aligned and uses a different loading and support mechanism for the test thrust bearing. On the Wang test rig, an oil cylinder loads a mechanism through which a hydrostatic bearing transfers the load to the test thrust bearing. The load is then transmitted as in the Harada test rig. The load-transferring hydrostatic thrust bearing allows the measurement of the torque through a load cell.

Gardner [7] describes a different type of multiple-thrust-bearing test rig. This test rig places two thrust bearings between the thrust disks of the rotor as seen in Figure 3.
This thrust-bearing test-rig style is referred to as a back-to-back configuration. The back-to-back arrangement allows for the use of hydraulic loading pistons between the thrust bearings. The pistons load the test bearing; the load is then reacted with the other bearing after the rotor transmits the load. The load is measured from the supply pressure to the hydraulic loading pistons. The thrust bearing assembly is also supported on a hydrostatic support system, which allows the torque on the thrust bearings to be measured with a load cell. Temperature on the bearing pad, inlet pressures, and flow rates are also measured with this test rig.

Glavatskih [8] presents a very similar back-to-back thrust bearing test rig, and Glavatskih [9] describes the test rig for testing double thrust bearings in Figure 4.
The test rig in Figure 4 is composed of two similar sections each housing a thrust bearing set and a journal bearing. One section is allowed to move freely in the axial direction to apply load to the rotor. The other section is secured and reacts the applied force from the sliding section.

**Previous Thrust Bearing Research**

Thrust bearings are a critical part of any rotating machine to maintain correct axial positioning and counteract axial loads. As such, knowledge of thrust bearing static and dynamic characteristics is crucial for predicting a turbomachine’s position under operating conditions. This review examines the currently available experimental information in the literature on hydrodynamic and hydrostatic bearings.

Hydrostatic bearings rely on external pressurization to support a load. The bearing fluid is supplied into a pocket or the lands through an orifice or capillary feed. The fluid then flows to the sides through the lands and exits to a lower ambient pressure. The resulting high pressure over the bearing surface provides the necessary force to react the applied load. The performance of this type of bearing depends upon the size of the lands, pockets, fluid properties, fluid inlets, clearance, and most importantly the supply pressure. If a hydrostatic bearing experiences rotation it is referred to as a hybrid bearing, and the influence of the rotation must be taken into account as the rotation can have a significant effect on the bearing performance. Hybrid bearings may also
incorporate tilting pads that can significantly alter the behavior of the thrust bearing. Rowe [11] offers additional general information on hydrostatic and hybrid bearings.

San Andrés [12] presents a model for predicting hybrid thrust bearing performance that includes fluid inertia, flow turbulence, and fluid compressibility. This model uses the bulk-flow model that relates shear at the surfaces of the bearing to the average fluid-flow velocities relative to the walls. Separate equations are presented for the lands and recesses (pockets) with the conditions calculated at the edge of the pockets providing boundary conditions for the lands equation. A first-order perturbation is implemented to calculate the bearing force coefficients. The zeroth order equations are solved using a control volume scheme. Numerical results are then provided for a hybrid bearing system. Discussions on this model are expanded in San Andrés [13] where effects of collar misalignment are discussed and predicted to noticeably affect flow rate, stiffness, and damping.

Wang and Yamaguchi [6] present flow rate and clearance results from testing a center pocket hydrostatic thrust bearing with oil. This bearing has a pocket in the center of the thrust bearing and has no inner radius discharge as opposed to the bearing tested here with a ring of fluid-supplied pockets with an inner discharge. Figure 5 shows their relevant results.

![Figure 5: Scaled load versus scaled film thickness and flow rate for the hydrostatic thrust bearing tested in [6].](image)
Measured flow rates in [6] show a decreasing second-order trend with increasing scaled load indicating that resistance to flow increases with the decreasing clearance caused by the load. Clearance shows a higher-order trend, decreasing with increasing scaled load. Similar results are also presented for this geometry in Harada et al [5].

**Test Rig Description**

This section presents the hydrostatic thrust bearing test rig. The two configurations planned and used to date are described, as well as the intended goals of each configuration.

Figure 6 shows a cross section of the hydrostatic test thrust bearing rig in its rotating configuration.

Figure 6 shows the face-to-face thrust-bearing configuration mounted on a rotor that is supported by two hydrostatic journal bearings. This configuration is similar to the back-to-back configuration used by Gardner [7] and Glavatskih [8] but with the thrust bearings facing the outside of the rotor thrust disks. This allows the test bearing to be a single piece instead of split and allows for a lighter loading mechanism. The load is applied to the system with a hydraulic shaker that moves the test thrust bearing axially thus applying load to the rotor. This load is then transmitted by the rotor to the slave thrust bearing where it is reacted.
The test rig uses two bearing-supported shafts. The first is the nonrotating loading shaft supported on two air bearings. These bearings are supplied with filtered 10.34 bar shop air and allow the loading shaft to move axially with very little friction with changing load.

The test rotor is supported by two flexible-pivot tilting-pad hydrostatic journal bearings. The journal bearings are supplied with water from the same source as the test bearings and allow the rotor to move axially with varied loading as well as rotate. An electric motor attaches to the slave bearing side of the rotor through a flexible coupling and drives the rotor.

The test thrust bearing section of the test rig is isolated from the other bearings by an air buffer seal to enable measurement of the test thrust bearing exhaust flow without contamination from the other bearings’ exhaust flow as shown in Figure 7.

Air buffer seals are also used to seal on either side of the test rig, preventing exhaust-flow leakage.

Flow control to the thrust bearings and the journal bearings was established via three pneumatic control valves. Separate control valves were used for the test thrust bearing, the journal bearings, and the slave thrust bearings. Further details can be seen in the flow schematic in Appendix A.
The test rig is intended for rotational testing of the thrust bearing’s characteristics. The rig configuration was also designed with future test programs in mind. For these later test programs, the rig was designed to accept bearings with a range of flange and fluid supply design.

Considerations for testing thrust bearings under dynamic conditions were incorporated. These include the use of air bearings to support the loading shaft to avoid friction losses from the movement of the test thrust bearing and minimizing the mass of the moving assembly that supports the test thrust bearing. Dynamic testing would involve applied measured harmonic excitation using the shaker.

For initial non-rotating testing, the modified configuration shown in Figure 8 was utilized. This configuration allowed for testing without pressurizing both thrust bearings by applying the load at the slave end of the rotor.

![Figure 8: Schematic of hydrostatic thrust bearing test rig non-rotating testing configuration](image)

This configuration simplified the initial testing process by removing the necessity of operating the slave thrust bearing. Thus, alignment issues or other possible problems from the slave thrust bearing did not interfere with the initial tests. The static testing configuration applies load to the rotor directly, which is then reacted by the air bearing pedestal through the test thrust bearing and rotor as shown in Figure 9.
During testing, the test thrust bearing is monitored using eddy current proximity probes to determine the operating clearance, flow meters for recording inlet and exit flow, as well as several pressure and temperature measurements.

The thrust bearing is a hydrostatic bearing with eight pockets. Figure 10 shows a front view.

The bearing is designed to mount in the thrust bearing support of the test rig. Table 1 lists the geometric parameters for the thrust bearing.
Table 1: Important dimensions and parameters for the tested thrust bearing

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bearing inner radius</td>
<td>20.32 mm (0.80 in)</td>
</tr>
<tr>
<td>Bearing outer radius</td>
<td>38.10 mm (1.50 in)</td>
</tr>
<tr>
<td>Number of pockets</td>
<td>8, Equally spaced</td>
</tr>
<tr>
<td>Pocket arc length</td>
<td>0.349 rad (20°)</td>
</tr>
<tr>
<td>Pocket inner radius</td>
<td>23.47 mm (0.924 in)</td>
</tr>
<tr>
<td>Pocket outer radius</td>
<td>31.47 mm (1.239 in)</td>
</tr>
<tr>
<td>Pocket depth</td>
<td>5.08 mm (0.020 in)</td>
</tr>
<tr>
<td>Orifice location</td>
<td>Located at mid-radius and mid-arc of pocket</td>
</tr>
<tr>
<td>Orifice diameter</td>
<td>1.80 mm (0.071 in)</td>
</tr>
</tbody>
</table>

The slave thrust-bearing parameters are identical to the test thrust-bearing parameters listed in Table 1. The slave thrust bearing features a different flange design and is thicker due to a split design.

Instrumentation

Four types of instrumentation were required for the non-rotating tests of the hydrostatic thrust bearing: eddy current proximity probes, thermocouples, pressure sensors, and flow meters. Seven proximity probes were used in the static testing configuration. The proximity probe locations are shown in Figure 11.
These proximity probes are used to monitor the rotor position. Two probes are dedicated to monitoring each of the rotor ends’ radial position to insure rotor lift off. These probes will also be used during tests with rotor rotation to monitor the rotor radial motions. The remaining three probes are used to monitor the axial position of the test thrust bearing relative to the test bearing thrust disk. Three proximity probes in this application define the thrust bearing face’s position relative to the plane of the thrust disk. This capability means that the misalignment of the thrust disk relative to the bearing can be determined. Additional proximity probes will be added to monitor the slave thrust bearing axial position and alignment for rotational testing.

The proximity probe selected was the SKF CMSS-65 5mm system. This system uses a five millimeter diameter probe that attaches to a driver which outputs a voltage to the DAQ. The probe system requires a -24 V power source and outputs a signal from -2 to -20 V [14]. However, the DAQ card requires a signal to be within the ±10 V range [15]. To correct this discrepancy, Encore model 619M signal conditioners were used to offset the signal and to attenuate it if necessary.

Each proximity probe was calibrated in the same configuration used during testing, including extension cable, probe, and driver. Calibration also used the rotor, the intended probe target, to develop the output curve to assure that the inconel composition did not cause measurement error. Measurements were made of the output voltage from the driver with a digital voltmeter and the probe distance from the rotor using a dial gauge. These readings were then compiled to find the linear region of the probe with the rotor target and the equation of the line through the linear region. An example curve is shown in Figure 12. With the calibration curves, the linear region of the probes with inconel was found to be from 0.4 to 1.5 mm approximately, and the best fit line was found for each probe system.
Four thermocouples were also used. Two thermocouples measured the inlet flow to the test thrust bearing and the journal bearings. Plug-style type-K thermocouples were used to prevent failure at maximum operating pressure. The other two thermocouples were mounted to the inner and outer radius of the test thrust bearing as seen in Figure 13.

These two thermocouples are type K and are secured in place with silicone to measure the temperature of the flow from the outer radius and inner radius of the test thrust
bearing. All four thermocouples are wired to Omega DP25B displays and were calibrated with the displays using an ice-water and a boiling-water bath. The displays output a 0 to 5 V signal to the DAQ. An additional plug type thermocouple is also installed in the water supply piping to monitor the inlet supply temperature of the slave thrust bearing for testing with rotor rotation.

Seven pressure sensors were used in the non-rotating testing. Figure 14 shows the three sensors used to monitor the inlet pressures to the test thrust bearing and each of the radial bearings.

![Figure 14: Inlet pressure sensors for journal bearings and test thrust bearing](image)

As shown, the test thrust-bearing supply-pressure sensor and gage were mounted as near as possible to the bearing to provide representative pressure readings. Additionally, the two journal-bearing supply-pressure sensors are used to confirm that both bearings are operating at nearly identical pressures. Two additional pressure transducers will be added to monitor the slave thrust bearing for rotational testing. The remaining four pressure sensors are used to measure the pressures in two of the pockets and at two locations on the lands. Figure 15 shows the pressure tap positions.
Figure 15: Location of pressure taps for pressure measurement on bearing lands and pockets

The taps in the bearing face intersect with a perpendicular hole which leads to the edge of the bearing. From there, the taps are hooked into their respective static pressure transducers by way of flexible tubing. Each pressure transducer was calibrated with its respective DP-25 display using a dead-weight tester over the range of 0 to 20.7 bar. The analog output from each DP-25 display was then routed to the DAQ.

Finally, three flow meters were used during testing. These flow meters were used to monitor the inlet flow to the journal bearings and the test thrust bearing in addition to the exhaust flow from the inner radius of the test thrust bearing. The location of the flow meters can be found in the flow schematic in Appendix A. All of the flow meter outputs were wired to signal linearizers with output displays. Outputs from the linearizers were then wired into the DAQ.

All data from the testing was collected using a single PCI-6225 DAQ board. Table 2 contains a summary of the specification for the DAQ board.

Table 2: Table of DAQ board specifications [15]

<table>
<thead>
<tr>
<th>Board Type</th>
<th>Number of Channels</th>
<th>Sampling Rate (Ksamples/sec)</th>
<th>Resolution (bits)</th>
<th>Signal Range (V)</th>
<th>Signal Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>PCI-6225</td>
<td>80</td>
<td>250</td>
<td>16</td>
<td>±1.0</td>
<td>Analog/Digital Input/Output Analog Input used</td>
</tr>
</tbody>
</table>

For additional details on the transducers and their accessories see Appendix B.
### Non-Rotating Test Procedure

Testing began after the final assembly of the hydrostatic thrust bearing test rig. The testing was divided into three parts based on the supply pressure range of the thrust bearing at 3.45, 10.34, and 17.24 bar. At each of these pressures, ten loads were selected to span the thrust bearing load capacity as determined by initial predictions from XLHydroTHRUST®.

Alignment difficulties between the thrust bearing and thrust disk face were encountered. This problem arose through a combination of the tolerance stack up involved in the air bearing support, and using the air bearing pedestal as a backstop for the test thrust bearing. Initially, the direct stinger attachment to the rotor was thought to cause the misalignment. This possibility was eliminated using a ball to replace the stinger as an interface between the shaker and rotor as shown in Figure 8.

![Figure 16: Tilt adjustment shims for the correction of test thrust bearing tilt in static testing configuration](image)

The problem was solved by shimming the air bearing pedestal. Application of shims at the locations noted in Figure 16 on the air bearing pedestal moved the entire support pedestal to correct the observed misalignment measured during tilt assessment testing. Misalignment was corrected in two directions from the combination of the following two shims: (1) about the horizontal axis of the thrust bearing by “shim 1” located under the pedestal, and (2) about the vertical axis by “shim 2” placed in with the
alignment keyway. With these shims, the misalignment between the thrust bearing face and thrust disk was decreased to 0.013 mm or less for all test cases.

The correction shimming was determined based upon the misalignment slopes of the test bearing in relation to the thrust disk. The formula for the misalignment slope in the horizontal $x$ direction ($M_1$) is shown in Equation (1.1), and the misalignment slope in the vertical $y$ direction ($M_2$) is shown in Equation (1.2).

\[
M_1 = 0.00194(Z_2 - Z_3) \tag{1.1}
\]

\[
M_2 = (-0.00294Z_1 + 0.00277Z_2 + 0.00017Z_3) \tag{1.2}
\]

Where

\[
M_1 = \text{Misalignment in the } x \text{ direction } \left( \frac{\text{mm}}{\text{mm}} \right)
\]

\[
M_2 = \text{Misalignment in the } y \text{ direction } \left( \frac{\text{mm}}{\text{mm}} \right)
\]

$Z_1, Z_2, Z_3 = \text{Gap reading from probe 1, 2, or 3}$

The derivation for the misalignment slopes can be found in Appendix C. These slopes were used with the length of the pivot (the length of the air bearing pedestal) to determine the correction shim dimension.

After several tests, the misalignment was determined to be load dependent, most likely due to the use of the air bearing pedestal as a backstop. Hence, before each test, a tilt assessment test (TAT) was performed over a test load range. The TAT was usually performed with three loads which were then used to calculate the correction shim dimension. For the 3.45 bar tests, one correction was adequate while for the 10.34 and 17.24 bar tests, three and five corrections were necessary, respectively, to cover the entire load range.

Each shim set reduced the misalignment of the thrust bearing face relative to the rotor to approximately 0.013 mm in the $x$ and $y$ directions across the entire thrust bearing face. This misalignment limit was determined by the length of the bearing pedestal where the adjustment was made and the thickness of the available shims. With a longer bearing pedestal or a smaller adjustment method, the misalignment tolerance could be reduced.
When the tilt corrections were complete, data were collected. Fifteen data points were collected for each 3.45 and 10.34 bar supply pressure load. This was increased to twenty data points for each 17.24 bar supply pressure load.

**Non-Rotating Test Results**

After the tests, the collected data needed to be compared to predictions from XLHydroTHRUST®. For more accurate predictions, the orifice discharge coefficient was extracted from the collected data. Equation (1.3) [16] shows the equation used to calculate the discharge coefficient ($C_d$).

\[
C_d = \frac{Q_0}{A_0 \sqrt{\frac{2}{\rho}(P_S - P_R)}}
\]

*(1.3)*

Where

- $A_0$ = Area of the orifice (m$^2$)
- $C_d$ = Orifice discharge coefficient
- $P_R$ = Recess (pocket) pressure (Pa)
- $P_S$ = Supply pressure (Pa)
- $Q_0$ = Flow rate through an orifice (m$^3$/s)
- $\rho$ = Working fluid density (kg/m$^3$)

The equation assumes that the pocket pressures are all equal to the measured average pocket pressure and that the flow through each orifice is equal. This assumption is necessary due to the measurements taken that cannot isolate any specific orifice, forcing the average assumption. Figure 17 shows the derived orifice coefficients for the three test supply pressures.
This figure shows that the orifice discharge coefficient is nearly constant over selected clearances for the 3.45 bar case. However, with the higher bearing supply pressures, decreased clearances are reached where the orifice discharge coefficient drop to lower values. This decrease appears to stabilize for the 17.24 bar thrust bearing supply pressure at a discharge coefficient of approximately 0.15. The 10.34 bar case though does not reach this relatively constant second range, and the 3.45 bar case shows no decrease in the orifice discharge coefficient. To examine this result, the orifice Reynolds number is calculated from

$$Re_{Orifice} = \frac{\rho * V_{Orifice} * D_{Orifice}}{\mu}$$

Where

- $Re_{Orifice}$ = Orifice Reynolds number
- $\rho$ = Working fluid density ($\text{kg/m}^3$)
- $V_{Orifice}$ = Fluid velocity through orifice ($\text{m/s}$)
- $D_{Orifice}$ = Diameter of the inlet orifice (m)
- $\mu$ = Dynamic viscosity ($\text{N*s/m}^2$)

The orifice Reynolds number is shown in Figure 18.
Figure 18: Orifice Reynolds number calculated from test data for 3.45, 10.34, and 17.24 bar test thrust bearing supply pressure tests

Figure 18 indicates that the orifice Reynolds number transitions from laminar to turbulent over the test range. Munson et al. [10] shows that the discharge coefficient can change with the Reynolds number until it is well into the turbulent regime; however, the orifice loss coefficient increases in Munson et al [10], disagreeing with the present calculated coefficients. The Reynolds number at the thrust bearing outer radius is next checked in Equation (1.5).

\[
Re_{\text{Outer Radius}} = \frac{\rho \cdot V_{\text{Outer Radius}} \cdot Z}{\mu}
\]

Where

\[Re_{\text{Outer Radius}} = \text{Outer radius Reynolds number}\]
\[\rho = \text{Working fluid density} \left( \frac{\text{kg}}{\text{m}^3} \right)\]
\[V_{\text{Outer Radius}} = \text{Average fluid velocity through thrust bearing outer radius} \left( \frac{\text{m}}{\text{s}} \right)\]
\[Z = \text{Center clearance of thrust bearing} \ (\text{m})\]
\[\mu = \text{Dynamic viscosity} \left( \frac{\text{N} \cdot \text{s}}{\text{m}^2} \right)\]

Equation (1.5)

The outer radius Reynolds number is shown in Figure 19.
This Reynolds number indicates laminar flow through all tests, which does not explain the change in the orifice loss coefficient. Neither of these Reynolds-number plots explains the observed change in the discharge coefficient. This change may relate to the misalignment across the bearing face, but further data would be necessary to show this.

For XLHydroTHRUST® predictions, a constant orifice discharge coefficient is required. For this purpose an average value was taken at the stable high clearance value. This average is not entirely accurate for the smaller clearances but is an adequate approximation over the entire range for comparison with hypothetical predictions. The averaged values used for the orifice discharge coefficient are shown in Table 3.

**Table 3: Average orifice discharge coefficients used for XLHydroTHRUST® predictions**

<table>
<thead>
<tr>
<th>Thrust Bearing Supply Pressure (bar)</th>
<th>Average orifice discharge coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.45</td>
<td>0.61</td>
</tr>
<tr>
<td>10.34</td>
<td>0.61</td>
</tr>
<tr>
<td>17.24</td>
<td>0.51</td>
</tr>
</tbody>
</table>

Additionally, the static misalignment angles about each axis, in radians, were required for the XLHydroTHRUST® predictions. These misalignments were found from the \( M_1 \) and \( M_2 \) slopes of Equations (1.1) and (1.2). They were converted to misalignment in radians by using the slope in \( \text{mm/mm} \) and taking the inverse tangent function. Because of
the small angle, the resulting angle was approximately the slope in \(\text{mm/mm}\). Figure 20 shows the misalignments for the three supply pressure tests.

![Figure 20: Misalignment over the bearing face about the x and y axis for 3.45, 10.34, and 17.24 bar test thrust bearing supply pressure test. Lines mark desired limits for misalignment ±0.013 mm](image)

This figure shows that tests were performed at reasonably low misalignment values for all cases. For the majority of the points, the misalignment was maintained within the targeted range of ±0.013 mm over the bearing face, indicated by the marker lines on Figure 20, with the few outliers missing the maximum limit by a small amount. For low clearances, smaller misalignments are desirable, but due to correction limitations were not attainable. For more detailed figures of the misalignment for each supply pressure see Appendix D.

Finally, the temperature of the inlet water was determined from an average of the recorded water inlet temperatures. With these parameters, XLHydroTHRUST® was used to predict the thrust bearing behavior. These predictions were then compared to the acquired test data.

Figure 21 shows the measured flow rates into the thrust bearing and out through the inner radius.
Figure 21: Measured inlet flow rates and flow out through inner radius of thrust bearing for 3.45, 10.34, and 17.24 bar thrust bearing supply pressures with uncertainty bars shown.

All uncertainty bars are shown on the plot but are barely noticeable. For details on the uncertainty calculations of the flow rate and the other experimental measurements see Appendix E. The uncertainty of the inlet flow measurement is around 1% or less. The uncertainty of the inner radius out flow is larger but near or less than 5%.

Figure 22 shows both the predictions and the measurements of the inlet flow rate.
This figure shows that the correlation between the predictions and measurements is very good for the 3.45 and 10.34 bar cases but poorer for the 17.24 bar case. The 3.45 and 10.34 bar case predictions generally match the measurements within 20% with the exceptions occurring for the very low flow rates. The 17.24 bar case predictions had differences that were mostly near 20%. The error between the measurements and the 17.24 bar predictions was also significantly over 20% for the data collected at the lowest five clearances. The poorer correlation of the 17.24 bar case is likely due to the more variable orifice loss coefficient which is shown in Figure 17. This causes a less accurate set of predictions from XLHydroTHRUST®, which assumes a constant orifice loss coefficient. All predictions show the general trends observed in the experimental measurements. Figure 23 compares measured flow rates exiting from the inner radius of the bearing plotted with the predictions obtained from XLHydroTHRUST®.
The measurements and predictions in Figure 23 show poorer correlation than those for the inlet flow rate with nearly twice the error between measurements and predictions for the majority of points. The best correlation occurs for the 3.45 and 10.34 bar supply pressure cases where the prediction appear to be offset slightly for the entire range of measured values. Additionally, for the majority of the measured range, the 3.45 bar predictions fall within the error of the measurement of the inner radius flow rate. The 17.24-bar measurements show a significantly higher decrease in flow rate with decreasing clearance than predicted. This is probably related to the large drop off in the discharge coefficient seen in Figure 17.

The calculated differences between the measurements and the XLHydroTHRUST® predictions are calculated using

\[
\% \text{ Difference} = \left| \frac{\text{Measured Value} - \text{Predicted Value}}{\text{Measured Value}} \right| \times 100
\]

The resulting errors are reasonable for all supply pressures with a clearance greater than 0.05 mm. The predicted inlet flow remains within approximately 10% of the measured
value, and the predicted exhaust flow of the inner radius remains within 20% of the measured value in this range. For clearances less than 0.05 mm, the error increases significantly though this may also be due in part to the increased significance of the bearing face on the orifice loss coefficient.

The next parameter compared is the pocket pressure ratio defined in Equation (1.7).

\[
P_{\text{Ratio}} = \frac{P_{\text{Pocket}} - P_{\text{Exhaust}}}{P_{\text{Supply}} - P_{\text{Atmospheric}}}
\]

Where

\[
P_{\text{Ratio}} = \text{Pressure ratio}
\]

\[
P_{\text{Pocket}} = \text{Pocket pressure}
\]

\[
P_{\text{Exhaust}} = \text{Pressure bearing exhausts into}
\]

\[
P_{\text{Supply}} = \text{Supply pressure}
\]

\[
P_{\text{Atmospheric}} = \text{Atmospheric pressure}
\]

Figure 24 shows the pocket pressure ratios over the tested clearance range for the tested supply pressures.

![Figure 24: Pocket pressure ratios over the tested clearance range for 3.45, 10.34, and 17.24 bar thrust bearing supply pressure.](image)

Figure 24 shows that the pocket-pressure ratio for the supply pressures ranges from a low value, ~0.05 for 3.45 bar to ~0.15 for 17.24 bar to approximately 1. These results indicate that most of the useful load range of the thrust bearing was covered in testing.
This figure also shows that the pocket 1 and pocket 4 pressure ratios are nearly identical over the tested range, providing another check that the test thrust bearing and rotor thrust disk misalignment is low. The predictions also match closely for the 3.45 and 10.34 bar supply pressures. The 17.24 bar case predictions are low by approximately 15-25% of the measured value at clearances greater than 0.028 mm. Below this clearance the pressure ratio compares well for this supply pressure. Pressures from the lands were also collected, but the data appeared anomalous at higher pressures. This discrepancy is attributed to a leak in the tube from the pressure taps to the pressure sensor. Thus the pressure data from the lands was considered invalid. For more detailed figures of the pocket pressures for each supply pressure see Appendix D.

Figure 25 depicts the loading behavior of the thrust bearing over the tested clearance range.

All three plots show similar trends to those in San Andres [12] as well as in Wang and Yamaguchi [6]. The figure also shows that the predictions are in reasonable agreement with measurements. Predictions correlate especially well with the 10.34 bar measurements. Experimental errors are also shown in the plot. The load uncertainties shown are small and do not significantly alter the data with their inclusion. The clearance uncertainties are larger due to the inclusion of the tilt in the uncertainty bar to show the
clearance range over the entire bearing face. Predictions for the 3.45 and 17.24 bar supply pressure case show little difference with misalignment. The 10.34 bar supply pressure case does show some change with the addition of the misalignment.

Percentage errors between the experimental data and the predictions are calculated using Equation (1.6). Predictions remain within approximately 20% of the measured values for clearances less than 0.09 mm in all cases. Higher clearances show significantly higher errors. However, these errors confirm that the 10.34 bar predictions correlate best overall with the measurements.

The stiffness predictions from XLHydroTHRUST® cannot be directly compared to results extracted from to date measurements. An estimation of the inverse flexibility can be made though. Equation (1.8) shows the flexibility equation.

\[
\begin{bmatrix}
\Phi \\
\Phi \\
\Phi \\
\Phi \\
\Phi
\end{bmatrix}
= \begin{bmatrix}
f_{FF} & f_{FM\Phi_x} & f_{FM\Phi_y} \\
f_{MFx} & f_{MFx,M\Phi_x} & f_{MFx,M\Phi_y} \\
f_{MFy} & f_{MFy,M\Phi_x} & f_{MFy,M\Phi_y}
\end{bmatrix}
\begin{bmatrix}
F_z \\
M_{\Phi x} \\
M_{\Phi y}
\end{bmatrix}
= \begin{bmatrix}
Z \\
\Phi_x \\
\Phi_y
\end{bmatrix}
\tag{1.8}
\]

where

\( F_z \) = Load on bearing face  \\
\( M_{\Phi x} \) = Moment acting in "a"  \\
\( f_{mn} \) = Flexibility coefficient for direction "m" from variable"n"  \\
\( Z \) = Thrust bearing clearance  \\
\( \Phi_c \) = Misalignment angle about axis"c"

The flexibility matrix can be inverted to determine an expression for the force on the bearing. To estimate the inverse flexibility in respect to the bearing clearance the clearance versus load curve can be differentiated.

To estimate the inverse flexibility of the test thrust bearing over the load range the load-versus-clearance data seen in Figure 25 was curve fitted. A fourth-order polynomial was used to fit the data for each load series with the exception of the 10.34 bar case where a third order curve fit was used for higher clearances. These were the lowest order polynomials that would successfully capture the behavior of both the clearance versus load curve and the inverse flexibility curve. Figure 26 shows the 3.45 bar experimental data with a fourth order curve fit equation displayed.
The plot confirms that the fourth-order curve fits the data well over the data range. With the data in this figure, the thrust bearing axial inverse flexibility can be estimated by taking the curve-fit equation and differentiating. The uncertainty of the inverse flexibility estimation is also a significant parameter. To determine this uncertainty Equation (1.9) was used.

\[
E_{dF/dZ} = \sqrt{E_A^2 + E_B^2 + E_C^2 + E_D^2}
\]  

Where

Load = A \cdot Z_C^4 + B \cdot Z_C^3 + C \cdot Z_C^2 + D \cdot Z_C + E

\[E_N\] = Uncertainty associated with term N

\[Z_C\] = Thrust bearing clearance

The uncertainty from each term of the curve fit was found using the TableCurve2D program. This program provided a means to check the curve fits from Excel and gave an uncertainty associated with each of the terms. Figure 27 shows curve fit differentiation estimation, or overall inverse flexibility \(dF/dZ\).
The uncertainty of the differentiation estimation shown in this figure appears reasonable and for the majority of the estimation the uncertainty is approximately 15% or less.

Figure 28 shows the 10.34 bar experimental data with a fourth order and a cubic curve fit.

Figure 28: Estimated clearance versus axial inverse flexibility for 3.45 bar test thrust bearing supply pressure

Figure 28: Clearance versus load measurements with two fourth order polynomial curve fit for 10.34 bar test thrust bearing supply pressure. Curve fit equations shown.
This clearance versus load series was split into two parts, due to poor curve fitting results when attempting to fit the entire series. Thus, for the 10.34 bar supply pressure range, a high clearance and low clearance fit were established. These curve fits were used to find the axial inverse flexibility estimates with the same methods used for the 3.45 bar case.

Figure 29 shows the resulting estimates with the XLHydroTHRUST® predictions.

![Graph showing clearance versus axial inverse flexibility for 10.34 bar test thrust bearing supply pressure](image)

Figure 29: Estimated clearance versus axial inverse flexibility for 10.34 bar test thrust bearing supply pressure

The uncertainty of the differentiation estimation shown in Figure 29 appears good with the majority of the points having an uncertainty less than 10%.

Figure 30 shows the clearance versus load for the 17.24 bar case with a fourth-order polynomial curve fit.
A single fourth order polynomial was used to curve fit this clearance versus load data. The curve fit and data points were used as previously. Figure 31 shows predicted and estimated inverse flexibility.

The uncertainty of the differentiation estimation is shown in Figure 31: Estimated clearance versus axial inverse flexibility for 17.24 bar test thrust bearing supply pressure
also appears reasonable with the majority of the points having an uncertainty less than 15 %.

**Conclusions**

A face-to-face style test rig was designed and built for testing hydrostatic thrust bearings. Using this test rig a hydrostatic thrust bearing was tested under non-rotating conditions. These test results were compared to predictions for flow rate and load from the XLHydroTHUST® program. The inverse flexibility was also estimated. Predictions correlated reasonably well for flow and load. Stiffness predictions did not compare as well with the measured slopes. The inverse flexibility predictions showed significantly higher maximum inverse flexibilities and much more severe drop offs from this maximum inverse flexibility.

The differences between predictions and measurements may be due to the observed discharge-coefficient variation and thrust bearing face misalignment at low clearance. This discharge coefficient must be given a constant value for XLHydroTHUST®, but significant changes were observed when extracted from the test data. Further investigation may be warranted to account for this discrepancy.
References


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<http://phn.tamu.edu/me626/Notes_pdf/Notes12_Seals_HJBs.pdf>


[18] Coleman, Hugh W. and Steele, W Glenn Jr., 1989, “Experimentation and 
Uncertainty Analysis for Engineers.” John Wiley and Sons, pp 89-100, 172-174
Appendix A –

Flow loop schematic with pertinent instrumentation included
Figure 32: Hybrid thrust bearing test rig flow loop with select instrumentation locations included
Appendix B –

Table of instrumentation and accessories used
<table>
<thead>
<tr>
<th>Item description</th>
<th>Manufacturer's part number</th>
<th>Desired range</th>
<th>Sensor range</th>
<th>Power requirements</th>
<th>Output</th>
<th>Resolution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Vendor: SKF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drivers</td>
<td>CMSS 55 5mm Probe</td>
<td>1-20 mil</td>
<td>10-90 mil</td>
<td>-24V at 15mA</td>
<td>0 to -15V</td>
<td>200 mV/mil</td>
</tr>
<tr>
<td>Proximity Probes</td>
<td>CMSS 65 5mm Probe</td>
<td>6 mil</td>
<td>16-90 mil</td>
<td>-24V at 16mA</td>
<td>0 to -15V</td>
<td>201 mV/mil</td>
</tr>
<tr>
<td>Extension Cables</td>
<td>4 meter extension cable</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Vendor: FTI</strong></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>FT-10</td>
<td>FT-10NEXW-LEG-2</td>
<td>8-15 GPM</td>
<td>0.3-15 GPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FT-12</td>
<td>FT-12NEXW-LEG-2</td>
<td>0.27-13.45 GPM</td>
<td>0.25-20 GPM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Digital Indicator</td>
<td>FTI Linear Link</td>
<td></td>
<td></td>
<td></td>
<td>24V at 300mA</td>
<td>±0.1% of reading</td>
</tr>
</tbody>
</table>
Appendix C –

Test thrust bearing rotor thrust disk plane equation derivation
Determining the position and orientation of the thrust disk relative to the test thrust bearing is critical. The output of three eddy current probes is available for this task. The layout geometry of the probes is known as seen in Figure 33.

![Figure 33: Layout geometry of eddy current sensors on test thrust bearing support](image)

From this diagram, the horizontal and vertical positions of each probe relative to the center of the bearing are calculated. These positions are shown in Table 4.

<table>
<thead>
<tr>
<th>Probe</th>
<th>X (mm)</th>
<th>Y (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probe 1</td>
<td>-33.02</td>
<td>27.69</td>
</tr>
<tr>
<td>Probe 2</td>
<td>-37.34</td>
<td>-21.59</td>
</tr>
<tr>
<td>Probe 3</td>
<td>37.34</td>
<td>-21.59</td>
</tr>
</tbody>
</table>

Table 4: Vertical and horizontal positions of proximity probes on thrust bearing support from bearing center

The equation for the plane of the rotor thrust disk relative to the thrust bearing face can now be found. Both the thrust bearing face and the rotor thrust disk are assumed to be rigid flat planes. The three proximity probe outputs are used to identify three points on the plane of the thrust disk as shown in Figure 34.
These three points on the thrust disk are further defined in Table 5.

Table 5: Coordinates of the three proximity probe points

<table>
<thead>
<tr>
<th></th>
<th>Point Coordinates (x,y,z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point 1</td>
<td>(-33.02, 27.69, Z1) mm</td>
</tr>
<tr>
<td></td>
<td>(-1.30, 1.09, Z1) in</td>
</tr>
<tr>
<td>Point 2</td>
<td>(-37.34, -21.59, Z2) mm</td>
</tr>
<tr>
<td></td>
<td>(-1.47, -0.85, Z2) in</td>
</tr>
<tr>
<td>Point 3</td>
<td>(37.34, -21.59, Z3) mm</td>
</tr>
<tr>
<td></td>
<td>(1.47, -0.85, Z3) in</td>
</tr>
</tbody>
</table>

These three points can next be used to generate two vectors located on the surface of the plane. Equation (2.1) and (2.2) show these two vectors.
\[ \vec{v}_{12} = \text{Point 2} - \text{Point 1} = \langle -4.32, -49.28, Z_2 - Z_1 \rangle \text{ mm} \quad (2.1) \]

\[ \vec{v}_{13} = \text{Point 3} - \text{Point 1} = \langle 70.36, -49.28, Z_3 - Z_1 \rangle \text{ mm} \quad (2.2) \]

**Where**

\[ \vec{v}_{12} = \text{Vector from point 1 to point 2 in the thrust disk plane} \]

\[ \vec{v}_{13} = \text{Vector from point 1 to point 3 in the thrust disk plane} \]

\[ Z_1, Z_2, Z_3 = \text{Z coordinate of probe 1, 2, or 3} \]

The two generated vectors can next be used to create a vector normal to the plane by taking the cross product of the two vectors.

\[ \vec{N} = \vec{v}_{12} \times \vec{v}_{13} \]

\[ \vec{N} = \langle -49.28(Z_2 - Z_3), -74.68Z_1 + 70.36Z_2 + 4.32Z_3, 144.87 \rangle \text{ mm} \quad (2.3) \]

**Where**

\[ \vec{N} = \text{Normal vector to the thrust disk plane} \]

A generic vector must now be generated from the first point and a generic point located on the plane of the thrust disk \((x, y, z)\). Equation (2.4) shows the formula for this point.

\[ \vec{v}_{\text{generic}} = \text{Generic Point} - \text{Point 1} = \langle x+33.02, y-27.69, z-Z_1 \rangle \text{ mm} \quad (2.4) \]

**Where**

\[ \vec{v}_{\text{generic}} = \text{Vector from point 1 to a generic point } (x, y, z) \text{ in the thrust disk plane} \]

Finally, by taking the dot product of the normal vector and the generic vector the equation of the thrust disk plane can be determined.

\[ \vec{N} \cdot \vec{v}_{\text{generic}} = 0 \]

\[ 49.3(Z_2 - Z_3)x + (-74.7Z_1 + 70.4Z_2 + 4.3Z_3)y + 144.8z + (-63.5Z_1 - 12.7Z_2 - 68.8Z_3) = 0 \quad (2.5) \]

This equation can ultimately be reduced to the more useful, final form.

\[ Z(x, y) = M_1x + M_2y + Z_c \]

\[ Z(x, y) = 8.6(Z_3 - Z_2)x + (13.1Z_1 - 12.3Z_2 - 0.76Z_3)y + (11.1Z_1 + 2.2Z_2 + 12.1Z_3) \]

**Where**

\[ M_1 = \text{Slope of the thrust disk plane in the X-axis} \]

\[ M_2 = \text{Slope of the thrust disk plane in the Y-axis} \]

\[ Z_c = \text{Clearance at the center of the test thrust bearing} \]
Appendix D –

Detailed figures of misalignment across bearing face and pocket pressures
Figure 35: Misalignment over the bearing face about the x and y axis for 3.45 bar test thrust bearing supply pressure.

Figure 36: Misalignment over the bearing face about the x and y axis for 10.34 bar test thrust bearing supply pressure.
Figure 37: Misalignment over the bearing face about the x and y axis for 17.24 bar test thrust bearing supply pressure.

Figure 38: Pocket pressure ratios over the tested clearance range for 3.45 bar thrust bearing supply pressure.
Figure 39: Pocket pressure ratios over the tested clearance range for 10.34 bar thrust bearing supply pressure.

Figure 40: Pocket pressure ratios over the tested clearance range for 17.24 bar thrust bearing supply pressure.
Appendix E –

Uncertainty analysis of experimental measurements
To determine the validity of any experimentally obtained data sets, the error must be estimated. Errors can come from two sources (bias and precision). In this analysis, bias error is considered negligible and precision error is assumed to be the major error source. The precision error can come from several sources and must be propagated appropriately. To combine these errors Equation (3.1) [1] is used.

\[
E_N = \left[ \sum \left( \frac{dF(X_1, X_2, \ldots, X_N) \cdot E_{X_N}}{dX_N} \right) \right]^{1/2}
\]

(3.1)

With this equation, so long as the function and individual variable errors are known, the overall error can be estimated. The measurement uncertainty (\(\sigma\)) can also be included into Equation (3.1) as one half of a summed squared term. Most of these individual errors are known from the error of the instrumentation; however, to determine the error from the calibration of the instrumentation Equation (3.2) [1] must be introduced,

\[
SEE = \left[ \frac{\sum (Y_i - (aX_i - b))^2}{M - 2} \right]^{1/2}
\]

(3.2)

to determine the standard error of estimation (SEE). The SEE can then be used similarly to determine standard deviation of the error associated with a given curve fit parameter. The error derived from the SEE that is associated with a given variable is 2*SEE. This error band to either side of the calculated value will include approximately 95% of the possible points. With these two equations the errors of the experimental measurements are defined.

Equation (3.3) was used to define the flow rate.

\[
FlowRate = A_{FlowMeter} \cdot V - B_{FlowMeter}
\]

(3.3)

Where

\[
\begin{align*}
A_{FlowMeter} & = \text{Linear calibration constant of the flow meter} \\
B_{FlowMeter} & = \text{Flow meter calibration offset} \\
V & = \text{Flow meter output voltage}
\end{align*}
\]

This equation can then be used with Equation (3.1) to find the expression for overall error.
\[ E_{\text{Flowrate}} = \sqrt{(V \cdot E_{\text{FlowMeter}})^2 + (A_{\text{FlowMeter}} \cdot E_Y)^2 + (2 \cdot \sigma_{\text{Flow}})^2} \]  \hspace{1cm} (3.4)

Where
- \( E_{\text{FlowMeter}} \) = Flow meter error
- \( E_{\text{FlowMeter}} \) = Error of \( A_{\text{FlowMeter}} \) term
- \( E_Y \) = Error of flow meter voltage measurment
- \( \sigma_{\text{Flow}} \) = Standard deviation of flow meter reading

This process can be repeated to find the error associated with the load measurement from

\[ \text{Load} = A_{\text{LoadCell}} \cdot V - B_{\text{LoadCell}} \]  \hspace{1cm} (3.5)

Where
- \( A_{\text{LoadCell}} \) = Linear calibration constant of the load cell
- \( B_{\text{LoadCell}} \) = Load cell calibration offset
- \( V \) = Load cell output voltage

Applying Equation (3.1) to Equation (3.5) the error expression is

\[ E_{\text{Load}} = \sqrt{(A_{\text{Load}} \cdot E_Y)^2 + (2 \cdot \sigma_{\text{Load}})^2} \]  \hspace{1cm} (3.6)

Where
- \( E_{\text{Load}} \) = Load cell error
- \( E_Y \) = Error of load cell voltage measurement
- \( \sigma_{\text{Load}} \) = Standard deviation of flow meter reading

Finally the error associated with the differentiation estimation of the stiffness can be determined using

\[ E_{\frac{dF}{dZ}} = \sqrt{E_A^2 + E_B^2 + E_C^2 + E_D^2} \]  \hspace{1cm} (3.7)

Where
- \( \text{Load} = A \cdot Z_C^4 + B \cdot Z_C^3 + C \cdot Z_C^2 + D \cdot Z_C + E \)
- \( E_N \) = Error associated with term N
- \( Z_C \) = Thrust bearing clearance

This equation requires that the error from each term is known; hence, the TableCurve2D software was utilized. This software calculated the error from each of the terms in the polynomial curve fit load expression. These errors were then substituted into Equation (3.7).