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AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

TECHNICAL REPORT 25-20

COORDINATES WHICH UNCOUPLE THE EQUATIONS OF MOTION OF DAMPED LINEAR DYNAMIC SYSTEMS

BY

KENNETH A. FOSS

FOR THE
OFFICE OF NAVAL RESEARCH
CONTRACT N5ori-07833
ONR PROJECT NR 064-259

MARCH 1956

AEROELASTIC AND STRUCTURES RESEARCH LABORATORY

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Reported by: K. A. Foss

Approved by: R. L. Bisplinghoff

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TABLE OF CONTENTS

ABSTRACT

Orthogonality relations between the eigenvectors of damped linear dynamic systems with lumped parameters are derived; and from these relations coordinates are found in terms of which uncoupled equations of motion can be written. Methods are developed for determining transient stresses in terms of these coordinates. The present treatment is extended to systems involving transient damping and to continuous systems.

INTRODUCTION

The equations of motion of a damped linear dynamic system with lumped parameters can be written in matrix notation as

$$[m_{ij}]\{\ddot{q}_j\} + [r_{ij}]\{\dot{q}_j\} + [k_{ij}]\{q_j\} = \{f_i(t)\} \tag{1}$$

The classical method of solving these equations is to find the normal modes of oscillation of the homogeneous equation

$$[m_{ij}]\{\ddot{q}_j\} + [k_{ij}]\{q_j\} = \{0\} \tag{2}$$

and from these modes to determine a set of normal coordinates in terms of which there is no inertial or elastic coupling between the equations of motion. However, unless the matrix [r] happens to be proportional to either [m] or [k], velocity coupling still exists, and the chief object of using normal coordinates is defeated except when the coupling terms are small and can be neglected.

The purpose of the present paper is to derive orthogonality relations between the homogeneous solutions of Eq. (1), and from these relations to develop a set of coordinates in terms of which completely uncoupled equations of motion can be written.

HOMOGENEOUS SOLUTIONS

Equation (1) can be written in reduced[1] form as

$$[R]\{\dot{Z}\} + [K]\{Z\} = \{F(t)\} \tag{3}$$

where

$$[R] = \begin{bmatrix} [0] & [m] \\ [m] & [r] \end{bmatrix}, \qquad [K] = \begin{bmatrix} -[m] & [0] \\ [0] & [k] \end{bmatrix},$$

$$\{Z\} = \begin{bmatrix} \{\dot{q}\} \\ \{q\} \end{bmatrix}, \qquad \{F(t)\} = \begin{bmatrix} \{0\} \\ \{f(t)\} \end{bmatrix}$$

To obtain the homogeneous solution of Eq. (3), let

$$\{Z(t)\} = e^{\alpha t}\{\Phi\} \tag{4}$$

This gives

$$\alpha [R]\{\Phi\} + [K]\{\Phi\} = \{0\} \tag{5}$$

Equation (5) may be manipulated into a form more amenable to matrix iteration, such as

$$[U]\{\Phi\} = \frac{1}{\alpha}\{\Phi\} \tag{6}$$

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1 "Elementary Matrices", R. A. Frazer, W. J. Duncan and A. R. Collar, Cambridge University Press, London, 1946, p. 327.

where

$$[U] = -[K]^{-1}[R] = \begin{bmatrix} [0] & [I] \\ -[c][m] & -[c][r] \end{bmatrix}$$

and where

$$[c] = [k]^{-1}$$

The matrix [c] is a set of flexibility influence coefficients.

The solution of Eqs. (5) or (6) will yield 2N eigenvalues and eigenvectors,

$$\alpha_n \quad and \quad \{\Phi^{(n)}\} = \begin{bmatrix} \alpha_n \{\phi^{(n)}\} \\ \{\phi^{(n)}\} \end{bmatrix}, \qquad n = 1, 2, \cdots, 2N$$

where N is the number of degrees of freedom. For a stable system each a_n is either real and negative or complex with a negative real part. The complex roots occur as complex conjugate pairs with corresponding complex conjugate modal columns. A relatively simple method of iterating Eq. (6) for complex eigenvalues and eigenvectors is described by Frazer, Duncan and Collar [2].

If the matrices [m], [r] and [k] are symmetric, the matrices [R] and [K] are symmetric, and Eq. (5) can be written in either of the two ways

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2 Ibid

$$\alpha_n [R]\{\Phi^{(n)}\} + [K]\{\Phi^{(n)}\} = \{0\} \tag{7}$$

and

$$\alpha_m \{\Phi^{(m)}\}^T [R] + \{\Phi^{(m)}\}^T [K] = \{0\}^T \tag{8}$$

where the superscript T denotes a transposed matrix. The premultiplication of Eq. (7) by $\{\Phi^{(m)}\}^T$ and the postmultiplication of Eq. (8) by $\{\Phi^{(n)}\}$ yields

$$\alpha_n \{\Phi^{(m)}\}^T [R]\{\Phi^{(n)}\} + \{\Phi^{(m)}\}^T [K]\{\Phi^{(n)}\} = 0 \tag{9}$$

and

$$\alpha_m \{\Phi^{(m)}\}^T [R]\{\Phi^{(n)}\} + \{\Phi^{(m)}\}^T [K]\{\Phi^{(n)}\} = 0 \tag{10}$$

When Eq. (10) is subtracted from Eq. (9), one obtains

$$(\alpha_n - \alpha_m)\{\Phi^{(m)}\}^T [R]\{\Phi^{(n)}\} = 0 \tag{11}$$

Thus, unless $\alpha_m = \alpha_n$, the orthogonality relations are

$$\{\Phi^{(m)}\}^T [R]\{\Phi^{(n)}\} = 0, \qquad \text{when} \quad m \neq n \tag{12}$$

and

$$\{\Phi^{(m)}\}^T [K]\{\Phi^{(n)}\} = 0, \qquad \text{when} \quad m \neq n \tag{13}$$

Equation (12) can be used to construct "sweeping matrices" when one iterates Eq. (6). In fact, if there are "rigid body" modes present ($\alpha_n = 0$), these modes must be eliminated from Eq. (5) to make Eq. (6) nonsingular.

NONHOMOGENEOUS SOLUTIONS

Nonhomogeneous solutions of Eqs. (1) or (3) can be obtained by expanding $\{Z\}$ into a modal series; i.e., by letting

$$\{Z\} = \sum_n \{\Phi^{(n)}\} \xi_n(t) \tag{14}$$

Eq. (3) becomes

$$\sum_n [R]\{\Phi^{(n)}\} \dot{\xi}_n + \sum_n [K]\{\Phi^{(n)}\} \xi_n = \{F(t)\} \tag{15}$$

Premultiplication of Eq. (15) by $\{\Phi^{(m)}\}^T$ gives

$$\sum_n \{\Phi^{(m)}\}^T [R]\{\Phi^{(n)}\} \dot{\xi}_n + \sum_n \{\Phi^{(m)}\}^T [K]\{\Phi^{(n)}\} \xi_n = \{\Phi^{(m)}\}^T \{F(t)\} \tag{16}$$

The orthogonality relations, Eqs. (12) and (13), simplify Eq. (16) to

$$R_n \dot{\xi}_n - \alpha_n R_n \xi_n = F_n(t) \tag{17}$$

where

$$R_n = \{\Phi^{(n)}\}^T [R] \{\Phi^{(n)}\} = 2\alpha_n \{\phi^{(n)}\}^T [m] \{\phi^{(n)}\} + \{\phi^{(n)}\}^T [r] \{\phi^{(n)}\}$$

and

$$F_n(t) = \{\Phi^{(n)}\}^T \{F(t)\} = \{\phi^{(n)}\}^T \{f(t)\}$$

From the orthogonality relations it can also be shown that

$$\{F(t)\} = \sum_n \frac{F_n(t)}{R_n} [R] \{\Phi^{(n)}\} \tag{18}$$

and

$$\{f(t)\} = \sum_n \frac{F_n(t)}{R_n} \left[\alpha_n [m] + [r]\right] \{\phi^{(n)}\} \tag{19}$$

also that

$$\sum_n \frac{F_n(t)}{R_n} [m] \{\phi^{(n)}\} = \{0\} \tag{20}$$

and

$$\sum_n \frac{F_n(t)}{R_n} \{\phi^{(n)}\} = \{0\} \tag{21}$$

For zero initial conditions, the solution of Eq. (17) is

$$\xi_n = \frac{1}{R_n} \int_0^t e^{\alpha_n (t-\tau)} F_n(\tau)\, d\tau \tag{22}$$

and

$$\dot{\xi}_n = \frac{F_n(t)}{R_n} + \frac{\alpha_n}{R_n}\int_0^t e^{\alpha_n(t-\tau)} F_n(\tau)\, d\tau \tag{23}$$

Thus, the nonhomogeneous solution of Eq. (1) is

$$\{q_i\} = \sum_n \frac{1}{R_n}\{\phi_i^{(n)}\}\int_0^t e^{\alpha_n(t-\tau)} F_n(\tau)\, d\tau \tag{24}$$

and

$$\{\dot{q}_i\} = \sum_n \frac{\alpha_n}{R_n}\{\phi_i^{(n)}\}\int_0^t e^{\alpha_n(t-\tau)} F_n(\tau)\, d\tau \tag{25}$$

If α_k and α_{k+1} are complex conjugate roots, the corresponding two terms of Eq. (24) are

$$\sum_{n=k}^{k+1} \frac{1}{R_n}\{\phi_i^{(n)}\}\int_0^t e^{\alpha_n(t-\tau)} F_n(\tau)\, d\tau$$

$$= \frac{2}{|R_k|}\int_0^t e^{-\zeta(t-\tau)}|F_k(\tau)|\left\{|\phi_i^{(k)}| \cos\left(\omega(t-\tau) - \theta_R + \theta_F + \theta_i\right)\right\} d\tau \tag{26}$$

where

$$\alpha_k = -\zeta + i\omega$$

$$R_k = |R_k|\, e^{i\theta_R}$$

$$F_k(t) = |F_k(t)|\, e^{i\theta_F}$$

$$\{\phi_i^{(k)}\} = \left\{|\phi_i^{(k)}|\, e^{i\theta_i}\right\}$$

DYNAMIC STRESSES

Two methods for determining the transient stresses in terms of the normal coordinates of a structure have been summarized by Bisplinghoff, Isakson and Pian[3]. These two methods are commonly called the "mode displacement method" and the "mode acceleration method". Two analogous methods can be derived in terms of the coordinates of the present paper.

If Eq. (1) represents the equations of motion of a flexible structure, a stress at point i in the structure is

$$\sigma_i = \lfloor s_i^{(i)} \rfloor [k_{ij}] \{q_j(t)\} \tag{27}$$

where $\lfloor s_i^{(i)} \rfloor$ is a row matrix which sums up the elastic forces to give the total stress σ_i. For the first method, substitute

$$\{q_j(t)\} = \sum_n \{\phi_j^{(n)}\} \xi_n(t) \tag{28}$$

into Eq. (27), which yields

$$\sigma_i = \sum_n^{*} A_n^{(i)} \xi_n(t) \tag{29}$$

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3 "Methods in Transient Stress Analysis", R. L. Bisplinghoff, G. Isakson and T. H. H. Pian, Journal of the Aeronautical Sciences, Vol. 17, No. 5, p. 259, May 1950.

where

$$A_n^{(i)} = \lfloor s^{(i)} \rfloor [k] \{\phi^{(n)}\}$$

$$= -\alpha_n^2 \lfloor s^{(i)} \rfloor [m] \{\phi^{(n)}\} - \alpha_n \lfloor s^{(i)} \rfloor [r] \{\phi^{(n)}\}$$

and where the starred summation symbol indicates that "rigid body" modes are not included. Equation (29) is analogous to the mode displacement method.

For the second method, Eq. (17) can be rearranged as

$$\xi_n = \frac{F_n(t)}{-\alpha_n R_n} + \frac{\dot{\xi}_n}{\alpha_n} \tag{30}$$

The substitution of this into Eq. (29) gives

$$\sigma_i = \sigma_{i(static)} + \sum_n^{*} \frac{A_n^{(i)}}{\alpha_n} \dot{\xi}_n \tag{31}$$

where

$$\sigma_{i(static)} = \sum_n^{*} \frac{A_n^{(i)} F_n(t)}{-\alpha_n R_n}$$

Equation (31) is analogous to the mode acceleration method, and in this case it might be called the "mode velocity method".

From Eq. (19)

$$\{f(t)\} = \sum_n^{**} \frac{F_n(t)}{R_n} [r] \{\phi^{(n)}\} - \sum_n^{*} \frac{F_n(t)}{\alpha_n R_n} [k] \{\phi^{(n)}\} \tag{32}$$

where the double star indicates that only "rigid body" modes are included in the summation. Therefore,

$$\lfloor s^{(i)} \rfloor \{f(t)\} = \sum_{n}^{**} \dot{\xi}_n \lfloor s^{(i)} \rfloor [r] \{\phi^{(n)}\} + \sum_{n}^{*} \frac{A_n^{(i)} F_n(t)}{-\alpha_n R_n} \tag{33}$$

and so,

$$\sigma_{i(static)} = \lfloor s^{(i)} \rfloor \{f(t)\} - \sum_{n}^{**} \dot{\xi}_n \lfloor s^{(i)} \rfloor [r] \{\phi^{(n)}\} \tag{34}$$

Thus, through the use of Eq. (31), the stress due to the applied forces can be accounted for exactly even when a limited number of vibration modes are used.

TRANSIENT DAMPING

Many problems in aeroelasticity require that the velocity in Eq. (1) be replaced by a convolution integral containing the Wagner function; i.e., a single-degree-of-freedom equation of the same type as Eq. (1) would become

$$m\ddot{q} + r\int_0^t \ddot{q}(\tau)\,\dot{\phi}(t-\tau)\,d\tau + kq = f(t) \tag{35}$$

where $\phi(t)$ is the Wagner function which accounts for unsteady flow effects. It can be shown by means of the Laplace transform that, if the usual exponential approximation of the Wagner function is used, Eq. (35) is equivalent to a fourth order differential equation of the type

$$a\ddddot{q} + b\dddot{q} + c\ddot{q} + d\dot{q} + eq = g(t) \tag{36}$$

Thus, Eq. (1) becomes

$$[a_{ij}]\{\ddddot{q}_j\} + [b_{ij}]\{\dddot{q}_j\} + [c_{ij}]\{\ddot{q}_j\} + [d_{ij}]\{\dot{q}_j\} + [e_{ij}]\{q_j\} = \{g_i(t)\} \tag{37}$$

This can also be expressed in reduced form as

$$[R]\{\dot{Z}\} + [K]\{Z\} = \{F(t)\} \tag{38}$$

where now

$$[R] = \begin{bmatrix} [0] & [0] & [0] & [a] \\ [0] & [0] & [a] & [b] \\ [0] & [a] & [b] & [c] \\ [a] & [b] & [c] & [d] \end{bmatrix}, \qquad [K] = \begin{bmatrix} [0] & [0] & -[a] & [0] \\ [0] & -[a] & -[b] & [0] \\ -[a] & -[b] & -[c] & [0] \\ [0] & [0] & [0] & [e] \end{bmatrix},$$

$$\{Z\} = \begin{bmatrix} \{\dddot{q}\} \\ \{\ddot{q}\} \\ \{\dot{q}\} \\ \{q\} \end{bmatrix}, \qquad \{F(t)\} = \begin{bmatrix} \{0\} \\ \{0\} \\ \{0\} \\ \{g(t)\} \end{bmatrix}$$

Orthogonality relations for continuous systems can be obtained in a manner analogous to that for lumped parameter systems. Consider for example the damped motion of a simple beam. The equation of motion is

$$m(y)\,\ddot{z}(y,t) + r(y)\,\dot{z}(y,t) + \int_{y_0}^{y_1} k(y,\eta)\, z(\eta,t)\, d\eta = f(y,t) \tag{39}$$

where $k(y,\eta)$ is a stiffness influence function[4]. This equation can be written in reduced form as

$$\left.\begin{aligned} m(y)\,\dot{z}(y,t) - m(y)\,\zeta(y,t) &= 0 \\ m(y)\,\dot{\zeta}(y,t) + r(y)\,\dot{z}(y,t) + \int_{y_0}^{y_1} k(y,\eta)\, z(\eta,t)\, d\eta &= f(y,t) \end{aligned}\right\} \tag{40}$$

where

$$\zeta(y,t) = \dot{z}(y,t)$$

To obtain the homogeneous solution of Eqs. (39) or (40), let

$$z(y,t) = \phi(y)\, e^{\alpha t} \qquad \text{and} \qquad \zeta(y,t) = \beta(y)\, e^{\alpha t}$$

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4 "Aeroelasticity", R. L. Bisplinghoff, H. Ashley and R. L. Halfman, Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, p. 26.

Equation (40) becomes

$$\left.\begin{aligned} &\alpha\, m(y)\,\phi(y) - m(y)\,\beta(y) = 0 \\ &\alpha\, m(y)\,\beta(y) + \alpha\, r(y)\,\phi(y) + \int_{y_0}^{y_1} k(y,\eta)\,\phi(\eta)\,d\eta = 0 \end{aligned}\right\} \tag{41}$$

The solution of Eqs. (41) would yield an infinite number of eigenvalues and eigenfunctions,

$$\alpha_n \qquad \text{and} \qquad \beta_n(y) = \alpha_n\,\phi_n(y)\,, \qquad n = 1, 2, \cdots, \infty$$

For the n^{th} mode Eqs. (41) become

$$\left.\begin{aligned} &\alpha_n\, m(y)\,\phi_n(y) - m(y)\,\beta_n(y) = 0 \\ &\alpha_n\, m(y)\,\beta_n(y) + \alpha_n\, r(y)\,\phi_n(y) + \int_{y_0}^{y_1} k(y,\eta)\,\phi_n(\eta)\,d\eta = 0 \end{aligned}\right\} \tag{42}$$

If the first of Eqs. (42) is multiplied by $\beta_m(y)$ and the second by $\phi_m(y)$ and if the resulting equations are added together and integrated, the following equation is obtained:

$$\alpha_n \int_{y_0}^{y_1} \Big(m(y)\,\phi_n(y)\,\beta_m(y) + r(y)\,\phi_n(y)\,\phi_m(y) + m(y)\,\beta_n(y)\,\phi_m(y) \Big)\,dy$$

$$+ \int_{y_0}^{y_1} \left(\int_{y_0}^{y_1} k(y,\eta)\,\phi_n(\eta)\,\phi_m(y)\,d\eta - m(y)\,\beta_n(y)\,\beta_m(y) \right) dy = 0 \tag{43}$$

In Eq. (43) the subscripts m and n may be interchanged to give

$$\alpha_m \int_{y_0}^{y_1} \Big(m(y)\, \phi_m(y)\, \beta_n(y) + r(y)\, \phi_m(y)\, \phi_n(y) + m(y)\, \beta_m(y)\, \phi_n(y) \Big)\, dy$$

$$+ \int_{y_0}^{y_1} \Big(\Big(\int_{y_0}^{y_1} k(y,\eta)\, \phi_m(\eta)\, \phi_n(y)\, d\eta - m(y)\, \beta_m(y)\, \beta_n(y) \Big) \Big)\, dy = 0 \tag{44}$$

When Eq. (44) is subtracted from Eq. (43), one obtains

$$(\alpha_n - \alpha_m) \int_{y_0}^{y_1} \Big(m(y)\, \phi_n(y)\, \beta_m(y) + r(y)\, \phi_n(y)\, \phi_m(y) + m(y)\, \beta_n(y)\, \phi_m(y) \Big)\, dy = 0 \tag{45}$$

because $k(y,\eta)$ is a symmetric kernel. Thus, the orthogonality relations are

$$\int_{y_0}^{y_1} \Big(m(y)\, \phi_n(y)\, \beta_m(y) + r(y)\, \phi_n(y)\, \phi_m(y) + m(y)\, \beta_n(y)\, \phi_m(y) \Big)\, dy = 0, \quad \text{when} \quad m \neq n \tag{46}$$

and

$$\int_{y_0}^{y_1} \Big(\Big(\int_{y_0}^{y_1} k(y,\eta)\, \phi_n(\eta)\, \phi_m(y)\, d\eta - m(y)\, \beta_n(y)\, \beta_m(y) \Big) \Big)\, dy = 0, \quad \text{when} \quad m \neq n \tag{47}$$

Through these results uncoupled equations of motion similar to Eq. (17) can be derived for this continuous system.

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