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TI-WADC TECHNICAL REPORT 54-28

A METHOD FOR PREDICTING DYNAMIC LANDING LOADS

(This report supersedes Memorandum Report
MCREXA-5-4595-8-2, 20 February 1948)

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SEPTEMBER 1954

WRIGHT AIR DEVELOPMENT CENTER

NOTICE

WADC TECHNICAL REPORT 54-28

A METHOD FOR PREDICTING DYNAMIC LANDING LOADS

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September 1954

Project 1367

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

Carpenter Litho & Prtg. Co., Springfield, O.
100 - 7 December 1954

FOREWORD

This report supersedes Air Materiel Command Memorandum Report MCREXA5-4595-8-2, "A Method for Predicting Dynamic Landing Loads", prepared by Lee S. Wasserman under date of 20 February 1948. The purpose of re-writing this report is to expand and revise details, to present an additional derivation of the theory and to present a new computation form.

This report was prepared in the Dynamic Loads Section, Dynamics Branch, Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center under Research and Development Project 1367, Structural Design Criteria.

ABSTRACT

This report supersedes Memorandum Report MCREXA5-4595-8-2, "A Method for Predicting Dynamic Landing Loads", 20 February 1948. The purpose of rewriting is to correct minor errors and make several refinements.

Dynamic responses may be computed as the sum of the rigid body response and the vibratory responses in each normal mode. The rigid body response is determined first from basic airplane parameters and in this report is assumed trapezoidal in shape. This trapezoid is then applied to the equation of motion of the elastic system to determine the vibratory response.

The vibratory response of an elastic system to a trapezoidal forcing function can be computed algebraically or graphically. The algebraic computation method is motivated by two distinct principles; discontinuity and superposition; and the graphical computation method is motivated by the superposition principle. New computation forms are provided for both the algebraic and the graphical methods.

Three particular problems are solved to compare theoretical and measured results, to serve as a computation guide and to illustrate the flexibility of the approach. In the first problem the effect of varying basic parameters is discussed with a flow chart.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

E H Schwartz
for DANIEL D. McKEE
Colonel, USAF
Chief, Aircraft Laboratory
Directorate of Laboratories

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INTRODUCTION

The evolution of vastly stepped up performance of airplanes has placed increased importance on designing structures to close tolerances to minimize weight penalties. The problem of predicting and analyzing dynamic landing loads provides a fertile field for replacing empirical criteria by rational computations.

This report presents an acceptable method for the computation of dynamic landing loads. The basic assumption of a trapezoidal shape for the rigid body loads was suggested by Mr. Lee Wasserman. This assumption is in agreement with test results. Fortunately it is also simple to handle as a forcing function for the differential equation of motion of the vibratory system.

The present paper supersedes MCREXA5-4595-8-2, same title, dated 20 February 1948. The object of this revision is to:

1. Correct the following errors in the examples:

Page in MCREXA5-4595-8-2	Reads	Should Read
13, 26 and 33, column 10	$\frac{(9) - (4)}{\omega_n}$	$\frac{(9) + (4)}{\omega_n}$
33, column 12, row "A" to "B"	∞	$-\infty$

These errors had little effect on the final results.

2. Expand and revise the details of the original theory for easier reading.

3. Provide an additional derivation of the acceleration response to a trapezoidal forcing function.

4. Present a new self-contained computation form which eliminates a significant amount of superfluous arithmetic.

5. Present a flow-chart analysis of the basic parameters in the first example. The effect of changing basic parameters is shown to be intricate but predictable.

In the examples of Sections III through V experimental data were used to determine fuselage and landing gear frequencies and the relevant modes of vibration. For other aircraft experimental values of frequencies may not be available. The computation of fuselage frequencies presents no problem (see Section IV); but the computation of landing gear frequencies requires further investigation.

If it is not known which natural vibration modes are relevant, calculations may be necessary beginning with the mode of lowest frequency and continuing until the computed responses are no longer significant. Reference 9 discusses the difficult question of selecting relevant modes.

SECTION I

BASIC THEORY

1. It is assumed that at the moment of landing wing lift exactly counterbalances the weight of the airplane, so that the vertical velocity is constant. Consequently the response is due entirely to dissipation of kinetic energy at impact and may be considered in two stages:

a. Rigid body response of the whole structure.

b. Vibratory response within the structure in each normal mode.

2. Energy equilibrium conditions must be satisfied:

a. For the structure as a whole (determining rigid body response) i.e., Kinetic Energy = Potential Energy.

b. For each particle of mass (determining vibratory response) i.e., Inertial Work - Elastic Work = External Work.

3. a. The rigid body response is determined from the equilibrium conditions for the structure as a whole. Briefly, the kinetic energy at impact is determined from the rate of descent and gross weight. But this is equal to the potential energy of the tire and strut work. The tire deflection vs load curves determine the tire work, and the strut work is determined assuming isothermal expansion and quasi-adiabatic compression.

Assuming a trapezoidal shape for rigid body load, the time history is now easy to evaluate. For further detail of this method see Reference 3.

b. The vibratory response in each mode is determined by the local equilibrium conditions:

$$\sum \text{Inertia work} - \sum \text{elastic work} = \sum \text{external work} \qquad (1)$$

In particular, let the entire mass be represented by a finite number of elements m_i (e.g. gear, body, tail, wing, etc.) each located at a point in space. Then the vertical displacement and acceleration of the element m_i in each mode depend on the element's location and can be written $c_i x$ and $c_i \ddot{x}$ respectively, where c_i is a constant determined by the position and mode.

Now consider incremental displacements $c_i\,dx$ of the elements m_i caused by external forces $f_i F(t)$ where $F(t)$ is the time history of the external forcing function with unit amplitude. Temporarily neglecting damping:

	Force	x	Distance	=	Work
Inertia:	$m_i c_i \ddot{x}$	x	$c_i\, dx$	=	$(m_i c_i\ \ddot{x})(c_i\, dx)$
Elastic:	$K_i^* c_i x$	x	$c_i\, dx$	=	$(K_i c_i\ x)(c_i\, dx)$
External:	$f_i F(t)$	x	$c_i\, dx$	=	$(f_i \cdot F(t))(c_i\, dx)$

And so equation (1) can be written:

$$\sum_i (m_i c_i^2 \ddot{x})(dx) - \sum_i (K_i c_i^2 x)(dx) = \sum_i (f_i c_i)\, dx \cdot F(t) \qquad (2\text{-a})$$

or

$$(\Sigma m_i c_i^2)\ddot{x} - (\Sigma K_i c_i^2)x = (\Sigma f_i c_i) \cdot F(t) \qquad (2\text{-b})$$

It will now be shown that $\Sigma K_i c_i^2 = -\omega^2 \Sigma m_i c_i^2$. For in the particular case $F(t)=0$: $(\Sigma m_i c_i^2)\ddot{x} = (\Sigma K_i c_i^2)x$ and there is simple harmonic motion so that: $\ddot{x} = -\omega^2 x$ where ω is the mode frequency. Thus $-\omega^2(\Sigma m_i c_i^2) = \Sigma K_i c_i^2$
Substituting

$$(\Sigma m_i c_i^2)\ddot{x} + (\Sigma m_i c_i^2)\omega^2 x = (\Sigma f_i c_i) \cdot F(t) \qquad (2\text{-c})$$

or

$$\ddot{x} + \omega^2 x = \frac{\Sigma f_i c_i}{\Sigma m_i c_i^2} F(t) \qquad (2\text{-d})$$

4. The effect of structural damping can be approximated using a dimensionless "damping coefficient" $\bar{g}$ acting on the displacement so that:

$$\ddot{x} + \omega^2(1 + \bar{g} j)\, x = \frac{\Sigma f_i c_i}{\Sigma m_i c_i^2} F(t) \qquad (3)$$

It is this equation (3) which gives the vibratory acceleration response $\ddot{x}$. In this report $\bar{g}$ is rather arbitrarily assumed to be .10.

5. It is sometimes convenient to by-pass the (constant) coefficient of $F(t)$ by defining:

$$\text{GAF} = \text{Generalized Acceleration factor} = \frac{\text{Generalized force}}{\text{Generalized mass}} \qquad (4)$$

(In "g" units.)

$$= \frac{\Sigma f_i c_i}{\Sigma m_i c_i^2}$$

---

* K_i = spring constant

6. This equation (3) can be solved separately for each normal mode because of the original assumption that normal modes do not feed energy to each other or to the rigid body modes. The assumption is validated by the excitation of reasonably pure natural modes during ground vibration tests.

7. The next problem is to determine which normal modes are important. (See Reference 9 for further discussion.) The experience of AMC dynamic tests shows that only the first few modes are important unless there is appreciable coupling between landing gear fore and aft vibrations and higher structural modes.

8. An outline of the computational procedure:

Step I: Compute rigid body vertical load time history from basic airplane parameters.

Step II: Compute rigid body drag load time history if appropriate. Assume (empirically) the coefficient of friction is .55 until the wheel gets up to speed, after which the rigid body drag load falls to zero in one-quarter the spinup time.

Step III: Determine which modes of vibration are important. Experimental data of frequencies will be used if available. If data is not available equations of motion of the structure can be used, but care is required in selecting the appropriate degrees of freedom.

Step IV: Compute generalized acceleration factor if appropriate.

Step V: Compute vibratory response in each mode. The theory for this computation is discussed in Section II.

Step VI: Obtain the time history of total acceleration or structural force by appropriate combination of rigid and vibratory components. Notice that the trapezoid considered as rigid body component may have a different ordinate from the trapezoid considered as forcing function for vibrations.

SECTION II

DERIVATION OF THE ACCELERATION RESPONSE TO A TRAPEZOIDAL FORCING FUNCTION (Solution of Equation (3))

1. Given forcing function *

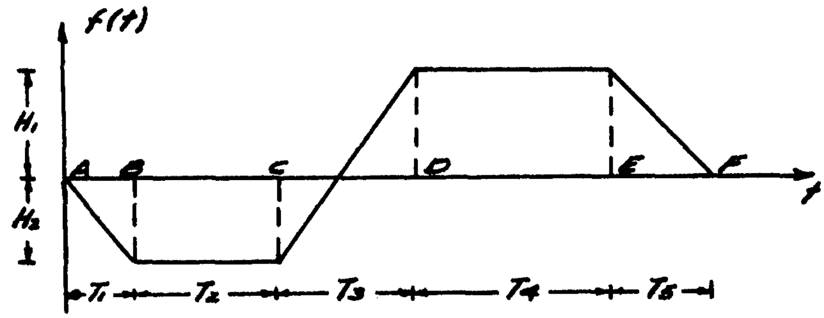


Fig. 1. Generalized Trapezoidal Forcing Function

2. It is required to determine explicitly the vibratory acceleration response for each interval AB, BC....In symbols, find $\ddot{x}$ explicitly where $\ddot{x}$ for each interval is given implicitly by:

$$\ddot{x} + \omega^2(1+\bar{g}j)x = f(T) \qquad (5)$$

with T defined as follows:

AB:	$T = t$	and	$0 \leq t \leq T_1$	
BC:	$T = t - T_1 = t'$		$0 \leq t' \leq T_2$	
CD:	$T = t-(T_1+T_2) = t''$		$0 \leq t'' \leq T_3$	(6)
DE:	$T = t-(T_1+T_2+T_3) = t'''$		$0 \leq t''' \leq T_4$	
EF:	$T = t-(T_1+T_2+T_3+T_4) = t''''$		$0 \leq t'''' \leq T_5$	
→F:	$T = t-(T_1+T_2+T_3+T_4+T_5) = t'''''$		$0 \leq t'''''$	

---

* The function has been sketched in sufficiently general terms to satisfy all common problems. In some cases, e.g., $H_1 = 0$ and $H_2 = GHF$ (pos. or neg.).

3. It is apparent * that (5) has an explicit solution of the form:

$$\ddot{x} = e^{-\bar{g}\omega T/2}\left[a \sin\omega T + b \cos\omega T\right] \qquad (7)$$

For example in AB the initial conditions at $T=0$ are $\ddot{x}=0$ and $\dddot{x}=f'(t)$

Then:

$$\ddot{x} = \frac{f'(t)}{\omega} e^{-\bar{g}\omega t/2} \sin \omega t \qquad (8)$$

The problem is thus solved in the interval AB.

4. To get the solution in each successive interval two approaches may be taken:

a. Discontinuity Derivation: Evaluate the constants a and b in (7) by applying the initial conditions.

b. Superposition Derivation: Resolve trapezoid into the sum of straight lines all of which emanate from points on the t-axis.

Each of these approaches will now be demonstrated. Of course they must lead to identical results.

Discontinuity Method
(Paragraphs 5-9)

5. It is required to determine the constants a and b in (7) in each interval from the initial conditions $\ddot{x}_0$ and $\dddot{x}_0$. (The values of $\ddot{x}$ and $\dddot{x}$ when $T=0$.)

6. The solution in AB, according to paragraph 3, is (8). Thus the value of $\ddot{x}$ and $\dddot{x}$ will be known at the end of AB. Similarily once the interval BC is solved, $\ddot{x}$ and $\dddot{x}$ will be known at the end of BC. So if each interval is solved in turn it is fair to assume in all cases that $\ddot{x}$ and $\dddot{x}$ are known at the end of the previous interval.

---

* For since $\ddot{x} + \omega^2(1+\bar{g}j)x = f(T)$ is to be solved for $\ddot{x}$ (not x), in present form it is an integral equation (involving x which is $\iint \ddot{x}$). To get a differential equation in $\ddot{x}$ differentiate (5) twice with respect to T to get

$$\frac{d^2}{dT^2}\ddot{x} + \omega^2(1+\bar{g}j)\ddot{x} = 0$$

since $f(T)$ is always linear in T so that $\frac{d^2}{dT^2} f(T) = 0$

This is a standard homogeneous equation in $\ddot{X}$ with the general solution: *(approximate)*

$$\ddot{x} = e^{-\bar{g}\omega T/2}\left[a \sin\omega T + b \cos\omega T\right]$$

7. According to paragraphs 5 and 6, it will be sufficient to evolve $\ddot{x}_o$ and $\dddot{x}_o$ from $\ddot{x}_e$ and $\dddot{x}_e$ (where "e" denotes evaluation at the end of the previous interval). Since $\ddot{x}$ is continuous from one interval to the next,

$$\ddot{x}_o = \ddot{x}_e \tag{9}$$

But $\dddot{x}$ is <u>not</u> continuous, so that $\dddot{x}_o \neq \dddot{x}_e$

So a gimmick will be introduced to evaluate $\dddot{x}_o$. Basically, we transport the <u>discontinuous</u> variable $\dddot{x}$ through the medium of the <u>continuous</u> variable $\dot{x}$.

First consider equation (5) when differentiated once:

$$\dddot{x} + \omega^2(1+\bar{g}j)\dot{x} = f'(T) \tag{10}$$

Evaluating (10) at $T=0$

$$\dddot{x}_o = f_R' - \omega^2(1+\bar{g}j)\dot{x}_o \tag{11}$$

From continuity, $\dot{x}_o = \dot{x}_e$ (12)

So evaluating (10) at the end of the previous interval:

$$\omega^2(1+\bar{g}j)\dot{x}_e = f_e' - \dddot{x}_e \tag{13}$$

or, from (12): $\omega^2(1+\bar{g}j)\dot{x}_o = f_e' - \dddot{x}_e$ (14)

Finally, from (11) and (14),

$$\dddot{x}_o = \dddot{x}_e + [f_R' - f_e'] \tag{15}$$

8. Solving (7) and its derivative at $T=0$, and applying (9) and (15), the constants a and b can be determined:

$$\ddot{x}_o = b \qquad\qquad a = \frac{\dddot{x}_e + f_R' - f_e'}{\omega} + \bar{g}\frac{\ddot{x}_e}{2}$$

$$\dddot{x}_o = a\omega - \bar{g}\frac{\omega b}{2} \qquad \text{or} \qquad b = \ddot{x}_e \tag{17}$$

9. In particular, the solution in each interval is:

a. In AB:

$$\ddot{X} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t \qquad (18)$$

at B:

$$\ddot{X}_B = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \qquad (19)$$

$$\dddot{X}_B = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{T_1}\left[\cos \omega T_1 - \frac{\bar{g}}{2} \sin \omega T_1\right] \qquad (20)$$

b. In BC:

$$\ddot{X}_0 = \ddot{X}_B \qquad \dddot{X}_0 = \dddot{X}_B - \frac{H_2}{T_1} \qquad (21)$$

So from (17)

$$q = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1}\left[\cos \omega T_1 - \frac{\bar{g}}{2} \sin \omega T_1\right] - \frac{H_2}{\omega T_1} + \frac{\bar{g} H_2 e^{-\bar{g}\omega T_1/2}}{2\omega T_1} \sin \omega T_1$$

$$q = \frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1} \cos \omega T_1 - \frac{H_2}{\omega T_1} \qquad (22)$$

So that (7) becomes:

$$\ddot{X} = e^{-\bar{g}\omega t'/2}\left[\left(\frac{H_2 e^{-\bar{g}\omega T_1/2}}{\omega T_1} \cos \omega T_1 - \frac{H_2}{\omega T_1}\right) \sin \omega t' + \ddot{X}_B \cos \omega t'\right] \qquad (23)$$

This may be rewritten:

$$\ddot{x} = e^{-\bar{\zeta}\omega t'/2}\sqrt{\ddot{x}_B^2 + R_1^2}\,\sin(\omega t' + \phi_1) \tag{24}$$

where

$$\begin{cases} R_1 = \dfrac{H_2 e^{-\bar{\zeta}\omega T_1/2}}{\omega T_1}\cos\omega T_1 - \dfrac{H_2}{\omega T_1} & (25) \\ \phi_1 = \arc\tan\dfrac{\ddot{x}_B}{R_1} & (26) \end{cases}$$

Note: The choice of the correct quadrant for ϕ is crucial. The quadrant must be selected so that:

$$\begin{Bmatrix} \sin\phi_1 \\ \cos\phi_1 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{x}_B \\ R_1 \end{Bmatrix} \tag{27}$$

i.e.: $\sin\phi_1$ has the algebraic sign of $\ddot{x}_B$
and $\cos\phi_1$ has the algebraic sign of R_1

Finally at C:

$$\ddot{x}_C = e^{-\bar{\zeta}\omega T_2/2}\sqrt{\ddot{x}_B^2 + R_1^2}\,\sin(\omega T_2 + \phi_1)$$

$$\dddot{x}_C = e^{-\bar{\zeta}\omega T_2/2}\sqrt{\ddot{x}_B^2 + R_1^2}\left[-\frac{\bar{\zeta}\omega}{2}\sin(\omega T_2 + \phi_1) + \omega\cos(\omega T_2 + \phi_1)\right] \tag{28}$$

c. In CD: $\ddot{x}_D = \ddot{x}_C$

$$\dddot{x}_D = \dddot{x}_C + \frac{H_1 - H_2}{T_3} \tag{29}$$

from (15).

So from (17)

$$a = \frac{H_1 - H_2}{\omega T_3} + e^{-\bar{\zeta}\omega T_2/2}\sqrt{\ddot{x}_B^2 + R_1^2}\,\cos(\omega T_2 + \phi_1) \tag{30}$$

And (after simplification):

$$\ddot{x} = e^{-\bar{\zeta}\omega t''/2}\sqrt{\ddot{x}_C^2 + R_2^2}\,\sin(\omega t'' + \phi_2) \tag{31}$$

where
$$\begin{cases} R_2 = \dfrac{H_1 - H_2}{\omega T_3} + e^{-\bar{\zeta}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\cos(\omega T_2 + \phi_1) \\ \phi_2 = \arctan \dfrac{\ddot{X}_C}{R_2} \end{cases} \tag{32}$$

Note: ϕ_2 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin\phi_2 \\ \cos\phi_2 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_C \\ R_2 \end{Bmatrix} \tag{33}$$

(b) Angle: $\tan\phi_2 = \dfrac{\ddot{X}_C}{R_2}$

Finally

$$\ddot{X}_D = e^{-\bar{\zeta}\omega T_3/2}\sqrt{\ddot{X}_C^2 + R_2^2}\,\sin(\omega T_3 + \phi_2) \tag{34}$$

d. In DE the process is similar, resulting in:

$$\ddot{X} = e^{-\bar{\zeta}\omega t'''/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\sin(\omega t''' + \phi_3) \tag{35}$$

where:
$$\begin{cases} R_3 = e^{-\bar{\zeta}\omega T_3/2}\sqrt{\ddot{X}_C^2 + R_2^2}\cos(\omega T_3 + \phi_2) - \dfrac{H_1 - H_2}{\omega T_3} \\ \phi_3 = \arctan \dfrac{\ddot{X}_D}{R_3} \end{cases} \tag{36}$$

Note: ϕ_3 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin\phi_3 \\ \cos\phi_3 \end{Bmatrix} \text{ Has the algebraic sign of } \begin{Bmatrix} \ddot{X}_D \\ R_3 \end{Bmatrix} \tag{37}$$

(b) Angle: $\tan\phi_3 = \dfrac{\ddot{X}_D}{R_3}$

and

$$\ddot{X}_E = e^{-\bar{\zeta}\omega T_4/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\sin(\omega T_4 + \phi_3) \tag{38}$$

e. And in EF:

$$\ddot{X} = e^{-\bar{\zeta}\omega t''''/2}\sqrt{\ddot{X}_E^2 + R_4^2}\,\sin(\omega t'''' + \phi_4) \tag{39}$$

where
$$\begin{cases} R_4 = \dfrac{-M_1}{\omega T_5} + e^{-\zeta\omega T_4/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\cos(\omega T_4 + \phi_3) \\ \phi_4 = \arctan \dfrac{\ddot{X}_E}{R_4} \end{cases} \qquad (40)$$

Note: ϕ_4 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin\phi_4 \\ \cos\phi_4 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_E \\ R_4 \end{Bmatrix} \qquad (41)$$

(b) Angle: $\tan\phi_4 = \dfrac{\ddot{X}_E}{R_4}$

and
$$\ddot{X}_F = e^{-\zeta\omega T_5/2}\sqrt{\ddot{X}_E^2 + R_4^2}\,\sin(\omega T_5 + \phi_4) \qquad (42)$$

As stated in paragraph 4, the derivation of paragraphs 5-9 could be replaced by paragraphs 10-13.

Superposition Method
(Paragraphs 10-13)

10. According to paragraph 3, it is easy to find the response to any straight line emanating from the time axis. The forcing function of figure 1 can be resolved into such lines:

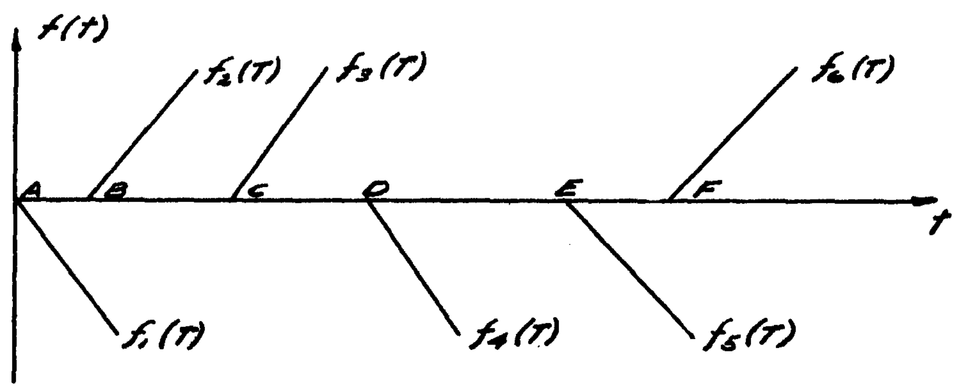


Fig. 2. Linear Decomposition of the Trapezoidal Forcing Function of Figure 1

where the slope $f_1'(T)$ is $\frac{H_2}{T_1}$

$$f_2'(T) \quad \frac{-H_2}{T_1}$$

$$f_3'(T) \quad \frac{H_1-H_2}{T_3} \tag{43}$$

$$f_4'(T) \quad -\frac{H_1-H_2}{T_3}$$

$$f_5'(T) \quad -\frac{H_1}{T_5}$$

$$f_6'(T) \quad \frac{H_1}{T_6}$$

11. Then, according to paragraph 3, the response to

$f_1(T)$ is $\ddot{X}_1 = \frac{H_2}{\omega T_1} e^{-\bar{\zeta}\omega t/2} \sin \omega t \qquad t \geq 0$

$$f_2(T) \qquad \ddot{X}_2 = -\frac{H_2}{\omega T_1} e^{-\bar{\zeta}\omega t'/2} \sin \omega t' \qquad t' \geq 0$$

$$f_3(T) \qquad \ddot{X}_3 = \frac{H_1-H_2}{\omega T_3} e^{-\bar{\zeta}\omega t''/2} \sin \omega t'' \qquad t'' \geq 0 \tag{44}$$

$$f_4(T) \qquad \ddot{X}_4 = -\frac{H_1-H_2}{\omega T_3} e^{-\bar{\zeta}\omega t'''/2} \sin \omega t''' \qquad t''' \geq 0$$

$$f_5(T) \qquad \ddot{X}_5 = -\frac{H_1}{T_5} e^{-\bar{\zeta}\omega t''''/2} \sin \omega t'''' \qquad t'''' \geq 0$$

12. But since

$$f = f_1 + f_2 + f_3 + f_4 + f_5 + f_6 \tag{45}$$

The response to $f(t)$ must be the sum of the responses to f_1, f_2 etc.

13. That is:

a. In AB:

$$\ddot{X} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t \qquad (46)$$

and

$$\ddot{X}_B = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \qquad (47)$$

b. In BC: $\ddot{X} = \ddot{X}_1 + \ddot{X}_2$

$$\ddot{X} = \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t/2} \sin \omega t - \frac{H_2}{\omega T_1} e^{-\bar{g}\omega t'/2} \sin \omega t' \qquad (48)$$

But since $t = t' + T_1$

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \left[\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega (t' + T_1) - \frac{H_2}{\omega T_1} \sin \omega t' \right] \qquad (49)$$

Expanding and collecting:

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \left[\left(\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \cos \omega T_1 - \frac{H_2}{\omega T_1} \right) \sin \omega t' + \left(\frac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \sin \omega T_1 \right) \cos \omega t' \right] \qquad (50)$$

and finally:

$$\ddot{X} = e^{-\bar{g}\omega t'/2} \sqrt{\ddot{X}_B^2 + R_1^2} \sin (\omega t' + \phi_1) \qquad (51)$$

where

$$\begin{cases} R_1 = \dfrac{H_2}{\omega T_1} e^{-\bar{g}\omega T_1/2} \cos \omega T_1 - \dfrac{H_2}{\omega T_1} \\ \phi_1 = \arctan \dfrac{\ddot{X}_B}{R_1} \end{cases} \qquad (52)$$

Note: ϕ_1 must be determined as follows:

(a) Quadrant

$\begin{Bmatrix} \sin \phi_1 \\ \cos \phi_1 \end{Bmatrix}$ has the algebraic sign of $\begin{Bmatrix} \ddot{X}_B \\ R_1 \end{Bmatrix}$ (53)

(b) Angle: $\tan \phi_1 = \dfrac{\ddot{X}_B}{R_1}$

And

$$\ddot{X}_C = e^{-\bar{g}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\sin(\omega T_2 + \phi_1) \qquad (54)$$

c. In CD

$$\ddot{X} = \ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 = (\ddot{X}_1 + \ddot{X}_2) + \ddot{X}_3 \qquad (55)$$

$$\ddot{X} = e^{-\bar{g}\omega t'/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\sin(\omega t' + \phi_1) + \frac{H_1 - H_2}{\omega T_3} e^{-\bar{g}\omega t''/2} \sin\omega t'' \qquad (56)$$

But since $t' = t'' + T_2$

$$\ddot{X} = e^{-\bar{g}\omega t''/2}\left[e^{-\bar{g}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\sin[\omega(t'' + T_2) + \phi_1] + \frac{H_1 - H_2}{\omega T_3}\sin\omega t''\right] \qquad (57)$$

Expanding and collecting:

$$\ddot{X} = e^{-\bar{g}\omega t''/2}\left[\left(e^{-\bar{g}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\cos[\omega T_2 + \phi_1] + \frac{H_1 - H_2}{\omega T_3}\right)\sin\omega t'' \right. \qquad (58)$$

$$\left. + \left(e^{-\bar{g}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\sin[\omega T_2 + \phi_1]\right)\cos\omega t''\right]$$

and finally:

$$\ddot{X} = e^{-\bar{g}\omega t''/2}\sqrt{\ddot{X}_C^2 + R_2^2}\,\sin(\omega t'' + \phi_2) \qquad (59)$$

where

$$\begin{cases} R_2 = e^{-\bar{g}\omega T_2/2}\sqrt{\ddot{X}_B^2 + R_1^2}\,\cos(\omega T_2 + \phi_1) + \dfrac{H_1 - H_2}{\omega T_3} \\ \phi_2 = \arctan \dfrac{\ddot{X}_C}{R_2} \end{cases} \qquad (60)$$

Note: ϕ_2 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin\phi_2 \\ \cos\phi_2 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_C \\ R_2 \end{Bmatrix} \qquad (61)$$

(b) Angle: $\tan \phi_2 = \dfrac{\ddot{X}_C}{R_2}$

and

$$\ddot{X}_D = e^{-\bar{g}\omega T_3/2}\sqrt{\ddot{X}_C^2 + R_2^2}\,\sin(\omega T_3 + \phi_2) \tag{62}$$

d. In DE: $\ddot{X} = (\ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3) + \ddot{X}_4$

$$\ddot{X} = e^{-\bar{g}\omega t''/2}\sqrt{\ddot{X}_C^2 + R_2^2}\,\sin(\omega t'' + \phi_2) - \frac{H_1 - H_2}{\omega T_3}\, e^{-\bar{g}\omega T_3/2} \sin \omega t''' \tag{63}$$

But since $t'' = t''' + T_3$

by the same process as before:

$$\ddot{X} = e^{-\bar{g}\omega t'''/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\sin(\omega t''' + \phi_3) \tag{65}$$

where

$$\begin{cases} R_3 = e^{-\bar{g}\omega T_3/2}\sqrt{\ddot{X}_C^2 + R_2^2}\,\cos(\omega T_3 + \phi_2) - \dfrac{H_1 - H_2}{\omega T_3} \\ \phi_3 = \arctan \dfrac{\ddot{X}_D}{R_3} \end{cases} \tag{66}$$

Note: ϕ_3 must be determined as follows:

(a) Quadrant:

$\begin{Bmatrix} \sin \phi_3 \\ \cos \phi_3 \end{Bmatrix}$ has the algebraic sign of $\begin{Bmatrix} \ddot{X}_D \\ R_3 \end{Bmatrix}$ (67)

(b) Angle: $\tan \phi_3 = \dfrac{\ddot{X}_D}{R_3}$

and

$$\ddot{X}_E = e^{-\bar{g}\omega T_4/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\sin(\omega T_4 + \phi_3) \tag{68}$$

e. In EF,

$$\ddot{X} = (\ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 + \ddot{X}_4) + \ddot{X}_5 \tag{69}$$

$$\ddot{X} = e^{-\bar{g}\omega t''''/2}\sqrt{\ddot{X}_D^2 + R_3^2}\,\sin(\omega t'''' + \phi_3) - \frac{H_1}{\omega T_5}\, e^{-\bar{g}\omega t''''/2} \sin \omega t'''' \tag{70}$$

Again since $t''' = t'''' + T_4$

$$\ddot{X} = e^{-\bar{\zeta}\omega t''''/2}\sqrt{\ddot{X}_E^2 + R_4^2}\,\sin(\omega t'''' + \phi_4) \tag{71}$$

Where

$$\begin{cases} R_4 = e^{-\bar{\zeta}\omega T_4/2}\sqrt{\ddot{X}_0^2 + R_3^2}\,\cos(\omega T_4 + \phi_3) - \dfrac{H_1}{\omega T_5} \\ \phi_4 = \arctan \dfrac{\ddot{X}_E}{R_4} \end{cases} \tag{72}$$

Note: ϕ_4 must be determined as follows:

(a) Quadrant:

$$\begin{Bmatrix} \sin \phi_4 \\ \cos \phi_4 \end{Bmatrix} \text{ has the algebraic sign of } \begin{Bmatrix} \ddot{X}_E \\ R_4 \end{Bmatrix} \tag{73}$$

(b) Angle: $\tan \phi_4 = \dfrac{\ddot{X}_E}{R_4}$

The results of paragraph 13 (that is, $\ddot{X}$ in each interval and at the end of each interval) are identical with the results of paragraph 9. From these the zeros, peaks and discontinuities of the acceleration response are easily found and plotted. A self-explanatory form for the cumbersome computations will be used in Sections III - V.

SECTION III

VERTICAL INCREMENTAL ACCELERATION OF THE WING TIP OF AN F-80A AIRPLANE WITH FULL WING TIP TANKS

This problem uses the parameters of landing 2, flight 37, of the AMC F-80A tests reported in reference 4.

Comparisons (but not computations) are also shown for the cases of half-empty (landing 30-1) and empty (landing 28-2) tanks in Figure 3.

These particular landings were selected because their values of the rigid body incremental acceleration correlated better than any others with the theoretical

$$\frac{P_{MAX}}{\frac{1}{2}\text{ Airplane Weight}}$$

<u>Step I</u>. Vertical Load Time History:

1. Basic airplane data:

   Gross weight = 14,000 lbs

   V_0 = Rate of descent = 6 ft/sec (assumed)

   M = Mass per main gear = 217 slugs (2 wheel landing)

   W = Static load per main gear = 6250 lbs (3 point position)

   KE = Kinetic energy per main gear = 3906 ft-lbs (2 wheel landing)

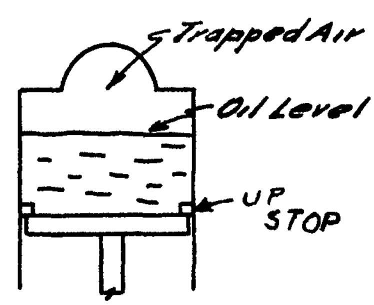
   ω = Natural frequency of wing = 16.75 rad/sec

2. Basic oleo data:

   E_T = Total extension plus latent air column* = (7.95 + 2.00) in. = .8292 ft

---

* Latent air column = $\frac{\text{trapped air volume}}{\text{piston cross section area}}$



$\mathcal{E}_s$ = Static extension plus latent air column = (3.00 + 2.00) in. = .4167 ft.

Assuming isothermal expansion from static to fully extended position, the "load factor" at total extension is:

$$n = \frac{\mathcal{E}_s}{\mathcal{E}_T} = 0.5025 \qquad (74)$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor n is:

$$\mathcal{E}_n = \frac{\mathcal{E}_T}{\left(\frac{n}{n_T}\right)^{\frac{1}{\gamma}}} = \mathcal{E}_T\left(\frac{n_T}{n}\right)^{\frac{1}{\gamma}} \qquad (75)$$

where n = load/6250 lbs and γ = 1.3 (from reference 8).

3. Basic tire data: (Manufacturer's data)

(1) Load (Lbs)	(2) Tire Deflection (Ft)	(3) Incremental Tire Work * (Ft-Lbs)	(4) Tire Work = Σ (3) (Ft-Lbs)
2500	.058	73	73
6500	.125	302	375
9000	.166	318	693

4. Total work:

(5) $n = \frac{(1)}{6250}$	(6) $\frac{n_T}{n} = \frac{0.5025}{(5)}$	(7) $(6)^{1/1.3}$	(8) $1 - (7)$	(9) $\mathcal{E}_T - \mathcal{E}_n = 0.8292\ (8)$	(10) OLEO WK. (1) × (9)	(11) TOTAL WORK (4) + (10)
.40	1.3565				0**	73
1.04	.4832	.5715	.4285	.3553	2309	2684
1.44	.3490	.4455	.5545	.4598	4138	4831

*(3) = (average value of load during increment) (deflection in increment); Approximating the area under the tire curve by trapezoids.

** The load is not yet sufficient to compress the strut.

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflections (Ft)	Oleo Deflection (Ft)
2684	6500	.125	.355
KE = 3906	P_{MAX} = 7923	X_T = .148	X_o = .415
4831	9000	.166	.460

6. The time for tire compression T_T oleo compression T_o and tire-oleo expansion T_{oT} are determined from the formulas of reference 3, assuming a trapezoidal time history for the strut axial load:

$$T_T = 3\left(\frac{MV_o}{P_{MAX}}\right) - \frac{1}{2}\sqrt{\left(\frac{6MV_o}{P_{MAX}}\right)^2 - \frac{24MX_T}{P_{MAX}}} = 0.025\,sec. \quad (76)$$

$$T_o = \sqrt{\frac{2MX_o}{P_{MAX}}} = 0.151\,sec. \quad (77)$$

$$T_{oT} = \sqrt{\frac{3M(X_o + X_T)}{P_{MAX}}} = 0.215\,sec. \quad (78)$$

Step II. Drag Load Time History:

The drag load is not used since fore and aft forces do not put appreciable energy into the first uncoupled bending mode.

Step III: Important Modes:

Since this example involves the vertical acceleration at the elastic axis, the torsional mode of the wing is expected to have little effect. But if the dynamic torque were to be predicted the torsional mode and the torque caused by the drag load would be considered.

Step IV: Generalized Acceleration Factor for Vibratory Response:

Reference 4 gives the computed frequencies and mode shapes for the first bending mode for full tanks, half full tanks, and empty tanks. GAF (per "g" load at gear) = $\dfrac{W h_{LG}}{\Sigma dm_i h_i^2}$ (g units)

Where:

W = static load per wheel, assuming all loads taken by the main gear

h_{LG} = $\dfrac{\text{vertical deflection of wing at gear}}{\text{vertical deflection of wing tip}}$ (first bending mode)

dm_i = mass of wing element at station "i" (slugs)

h_i = $\dfrac{\text{vertical deflection of wing at station "i"}}{\text{vertical deflection of wing tip}}$ (first bending mode)

The results per "g" landing load: GAF = -1.03 (full)

= -1.06 (half-full)

= -1.15 (empty)

Step V. Vibratory Acceleration Response:

1. Table 1-a computes the vibratory response by the desk-calculator method. The form is designed to make the cumbersome computations as mechanical and well-grouped as possible. It is meant to be self-explanatory after following the theory in Section II.

2. Table 1-b computes the vibratory response by the graphical method.

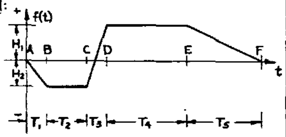
Step VI. Total Acceleration Time History:

Figure 3 shows the total tip acceleration as the sum of the rigid body and vibratory accelerations. The results for half-full and empty wing tip tanks are also shown.

The measured and computed results agree well for full tanks, but for empty tanks additional modes should probably be included in the computations.

I. THE PROBLEM:

GIVEN FORCING FUNCTION $f(t)$
FIND THE RESPONSE $\ddot{x}$ FROM:
$\ddot{x}+\omega^2(1+\bar{g}j)x = f(t)$



II. BASIC DATA

TITLE: Vib. Acc. F80A Wing Tip (Full Tanks)
COMPUTER: [illegible]

1 T_1	2 T_2	3 T_3	4 T_4	5 T_5
.025	.151	.215		

6 H_1	7 H_2	8 ω	9 $\bar{g}$
0	-1.133	16.75	.10

(ω in) RADIANS/SEC.

III. INITIALLY COMPUTABLE CONSTANTS

10 ωT_1 = 8X1	11 ωT_2 = 8X2	12 ωT_3 = 8X3	13 ωT_4 = 8X4	14 ωT_5 = 8X5
.4188	2.5293	3.6013		
15 $\frac{\bar{g}\omega}{2} = \frac{1}{2}(8X9)$	16 $\frac{H_2}{\omega T_1} = \frac{7}{10}$	17 $H_1 - H_2$ = 6-7	18 $\frac{H_1-H_2}{\omega T_3} = \frac{17}{12}$	19 $\frac{H_1}{\omega T_5} = \frac{6}{14}$
.8375	-2.7053	1.133	.3146	
20 $\frac{\bar{g}\omega T_1}{2}$ = 15X1	21 $\frac{\bar{g}\omega T_2}{2}$ = 15X2	22 $\frac{\bar{g}\omega T_3}{2}$ = 15X3	23 $\frac{\bar{g}\omega T_4}{2}$ = 15X4	24 $\frac{\bar{g}\omega T_5}{2}$ = 15X5
.0209	.1265	.1801		
25 e^{-20}	26 e^{-21}	27 e^{-22}	28 e^{-23}	29 e^{-24}
.9793	.8812	.8353		

IV. IN AB,

$\ddot{x} = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t$

AT B. $\ddot{x}_B = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \sin \omega T_1$

30 16X25	31 sin 10	32 30X31
-2.6493	+.4067	-1.0775 → $\ddot{x}_B$
	33 cos 10	34 30X33
	+.9136	-2.4204

V. IN BC,

$\ddot{x} = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega t'}{2}} \sin(\omega t'+\phi_1)$ $t' = t - T_1$

AT C, $\ddot{x}_C = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \sin(\omega T_2+\phi_1)$

WHERE $R_1 = -\frac{H_2}{\omega T_1} + \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \cos \omega T_1$

$\phi_1 = \arctan \frac{\ddot{x}_B}{R_1}$ (SEE BELOW)

35 34-16	36 32^2+35^2	37 $\sqrt{36}$	38 26X37	
.2849	1.242174	1.115	+.9825	
39 32/35	40 ϕ_1 [radians]	41 11+40	42 sin 41	43 38X42
-3.7820	-1.312	1.217	+.9381	+.9217 → $\ddot{x}_C$
			44 cos 41	45 38X44
			+.3465	+.3404

TO DETERMINE
(a) QUADRANT: III
$\begin{Bmatrix}\sin\phi_1\\ \cos\phi_1\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}32\\35\end{Bmatrix}$
(b) ANGLE: $\tan \phi_1$ = 39

VI. IN CD,

$\ddot{x} = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega t''}{2}} \sin(\omega t''+\phi_2)$ $t'' = t' - T_2$

AT D, $\ddot{x}_D = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \sin(\omega T_3+\phi_2)$

WHERE $R_2 = \frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \cos(\omega T_2+\phi_1)$

$\phi_2 = \arctan \frac{\ddot{x}_C}{R_2}$ (SEE BELOW)

46 45+18	47 43^2+46^2	48 $\sqrt{47}$	49 27X48	
+.6550	1.27855387	1.1307	+.9445	
50 43/46	51 ϕ_2 [RADIANS]	52 12+51	53 sin 52	54 49X53
+1.4072	.953	4.5543	-.9887	-.9338 → $\ddot{x}_D$
			55 cos 52	56 49X55
			-.1574	-.1487

TO DETERMINE
(a) QUADRANT: I
$\begin{Bmatrix}\sin\phi_2\\ \cos\phi_2\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}43\\46\end{Bmatrix}$
(b) ANGLE: $\tan \phi_2$ = 50

VII. IN DE,

$\ddot{x} = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega t'''}{2}} \sin(\omega t'''+\phi_3)$ $t''' = t'' - T_3$

AT E, $\ddot{x}_E = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega T_4}{2}} \sin(\omega T_4+\phi_3)$

WHERE $R_3 = -\frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \cos(\omega T_3+\phi_2)$

$\phi_3 = \arctan \frac{\ddot{x}_D}{R_3}$ (SEE BELOW)

57 56-18	58 54^2+57^2	59 $\sqrt{58}$	60 28X59	
-.4633	1.08662933	1.0424		
61 54/57	62 ϕ_3 [RADIANS]	63 13+62	64 sin 63	65 60X64
+2.0153	4.252			→ $\ddot{x}_E$
			66 cos 63	67 60X66

TO DETERMINE
(a) QUADRANT: III
$\begin{Bmatrix}\sin\phi_3\\ \cos\phi_3\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}54\\57\end{Bmatrix}$
(b) ANGLE: $\tan \phi_3$ = 61

VIII. IN EF,

$\ddot{x} = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega t''''}{2}} \sin(\omega t''''+\phi_4)$ $t'''' = t''' - T_4$

AT F, $\ddot{x}_F = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega T_5}{2}} \sin(\omega T_5+\phi_4)$

WHERE $R_4 = -\frac{H_1}{\omega T_5} + e^{-\frac{\bar{g}\omega T_4}{2}} \sqrt{\ddot{x}_D^2+R_3^2} \cos(\omega T_4+\phi_3)$

$\phi_4 = \arctan \frac{\ddot{x}_E}{R_4}$ (SEE BELOW)

68 67-19	69 65^2+68^2	70 $\sqrt{69}$	71 29X70	
72 65/68	73 ϕ_4 [RADIANS]	74 14+73	75 sin 74	76 71X75
				→ $\ddot{x}_F$

TO DETERMINE
(a) QUADRANT:
$\begin{Bmatrix}\sin\phi_4\\ \cos\phi_4\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}65\\68\end{Bmatrix}$
(b) ANGLE: $\tan \phi_4$ = 72

IX. INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $\ddot{x} = Ke^{-\frac{\bar{g}\omega T}{2}} \sin(\omega T+\phi)$. THE VALUES AT DISCONTINUITIES HAVE BEEN FOUND IN IV - VIII. FOR SKETCHING, THE ZEROS [ie., $\sin(\omega T+\phi)=0$] AND PEAKS [ie., $\sin(\omega T+\phi) = \pm 1$, APP.] ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega T+\phi$ FOR WHICH $\sin(\omega T+\phi) = 0, \pm 1$ ARE IN XI. BUT SINCE $0 \le T \le T_i$ IN EACH INTERVAL, ONLY THE VALUES OF $\omega T+\phi$ BETWEEN ϕ_i AND $\omega T_i+\phi_i$ CAN BE USED. THESE LIMITS OF $\omega T+\phi$ ARE LISTED IN X. PARTS XII - XVI THEN COMPUTE PLOTTING POINTS AT PEAKS AND ZEROS.

X. LIMITS OF $(\omega T+\phi)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	ϕ_1 = 40	ϕ_2 = 51	ϕ_3 = 62	ϕ_4 = 73
		-1.312	.953	4.252	
TO	ωT_1 = 10	$\omega T_2+\phi_1$ = 41	$\omega T_3+\phi_2$ = 52	$\omega T_4+\phi_3$ = 63	$\omega T_5+\phi_4$ = 74
	.419	1.217	4.559	∞	

XI. VALUES OF $\sin(\omega T+\phi)$ FOR COMPUTING ZEROS & PEAKS

$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$
-10.996	+1	-3.142	0	+4.712	-1
-9.425	0	-1.571	-1	+6.283	0
-7.854	-1	0	0	+7.854	+1
-6.283	0	+1.571	+1	+9.425	0
-4.712	+1	+3.142	0	+10.996	-1

XII. ZEROS AND PEAKS BETWEEN A & B

NOTE: $\phi = 0$ IN A-B

	77 ωt [FROM X & XI]	78 $\sin \omega t$ [FROM XI]	79 $\frac{\bar{g}}{2} \times \omega t$ = ___ x ωt	80 e^{-79}	81 $t = \frac{77}{8}$ (PLOT)	82 $\ddot{x}$ = 78X80X16 (PLOT)
a						
b			None			
c						
d						
e						

XIII. ZEROS & PEAKS BETWEEN B & C.

	83 $\omega T+\phi$ [FROM X & XI]	84 $\sin(\omega T+\phi)$ [FROM XI]	85 ωT = 83-40	86 $\frac{\bar{g}}{2} \times \omega T$ = .05 X85	87 e^{-86}	88 $T = \frac{85}{8}$	89 t = 1+88 (PLOT)	90 $\ddot{x}$ = 84X87X37 (PLOT)
a.	0	0	1.312	———	———	.078	.103	0
b.								
c.								
d.								
e								

XIV. ZEROS & PEAKS BETWEEN C & D

	91 $\omega T+\phi$ [FROM X & XI]	92 $\sin(\omega T+\phi)$ [FROM XI]	93 ωT = 91-51	94 $\frac{\bar{g}}{2} \times \omega T$ = .05 X93	95 e^{-94}	96 $T = \frac{93}{8}$	97 t = 1+2+96 (PLOT)	98 $\ddot{x}$ = 92X95X48 (PLOT)
a.	1.571	+1	.618	.031	.969	.037	.213	+1.096
b	3.142	0	2.189	———	———	.131	.307	0
c.								
d.								
e.								

XV. ZEROS & PEAKS BETWEEN D & E

	99 $\omega T+\phi$ [FROM X & XI]	100 $\sin(\omega T+\phi)$ [FROM XI]	101 ωT = 99-62	102 $\frac{\bar{g}}{2} \times \omega T$ = .05 X101	103 e^{-102}	104 $T = \frac{101}{8}$	105 t = 1+2+3+104 (PLOT)	106 $\ddot{x}$ = 100X103X59 (PLOT)
a	4.712	-1	.460	.023	.977	.027	.418	-1.018
b.	6.283	0	2.031	———	———	.121	.512	0
c								
d								
e.								

XVI. ZEROS & PEAKS BETWEEN E & F.

	107 $\omega T+\phi$ [FROM X & XI]	108 $\sin(\omega T+\phi)$ [FROM XI]	109 ωT = 107-73	110 $\frac{\bar{g}}{2} \times \omega T$ = ___ X 109	111 e^{-110}	112 $T = \frac{109}{8}$	113 t = 1+2+3+4+112 (PLOT)	114 $\ddot{x}$ = 108X111X70 (PLOT)
a								
b.								
c.								
d.								
e.								

TABLE I-a

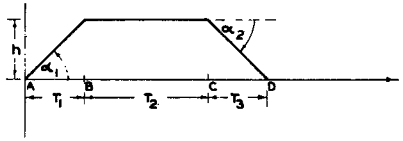
TABLE I-b

GRAPHICAL SOLUTION OF VIBRATORY ACCELERATION RESPONSE TO A TRAPEZOIDAL FORCING FUNCTION

I. THE PROBLEM:

GIVEN FORCING FUNCTION $f(t)$.
FIND THE RESPONSE $\ddot{X}$ FROM:

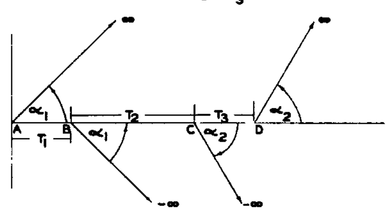
$$\ddot{X} + \omega^2(1+\bar{g}j)X = f(t)$$



II. DECOMPOSITION:

$$f(t) = f_1(t) + f_2(t') + f_3(t'') + f_4(t''')$$

WHERE $t' = t - T_1$, $t'' = t - T_2$, $t''' = t - T_3$



III. APPLYING SUPERPOSITION PRINCIPLE:

$$\ddot{X} = \ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 + \ddot{X}_4$$
$$\ddot{X}_1 + \omega^2(1+\bar{g}j)X_1 = f_1(t)$$
$$\ddot{X}_2 + \omega^2(1+\bar{g}j)X_2 = f_2(t')$$
$$\ddot{X}_3 + \omega^2(1+\bar{g}j)X_3 = f_3(t'')$$
$$\ddot{X}_4 + \omega^2(1+\bar{g}j)X_4 = f_4(t''')$$

WHERE: $t \geq 0$, $t' \geq 0$, $t'' \geq 0$, $t''' \geq 0$

IV. SOLUTIONS TO III:

$$\ddot{X}_1 = \frac{-h}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t \qquad (t \geq 0)$$
$$\ddot{X}_2 = \frac{h}{\omega T_2} e^{-\frac{\bar{g}\omega t'}{2}} \sin \omega t' \qquad (t' \geq 0)$$
$$\ddot{X}_3 = \frac{h}{\omega T_3} e^{-\frac{\bar{g}\omega t''}{2}} \sin \omega t'' \qquad (t'' \geq 0)$$
$$\ddot{X}_4 = \frac{-h}{\omega T_4} e^{-\frac{\bar{g}\omega t'''}{2}} \sin \omega t''' \qquad (t''' \geq 0)$$

V. BASIC DATA & BASIC COMPUTATIONS:

TITLE: *Full Tanks – F80A – Vib. Acc. of Wing Tips*
COMPUTER: *A KG*

1. T_1	2. T_2	3. T_3	4. $\bar{g}/2$	5. $\omega \frac{RAD}{SEC}$
.025	.151	.215	.05	16.75
6. h	7. $\omega T_1 = 5 \times 1$	8. $\omega T_3 = 5 \times 3$	9. $\frac{h}{\omega T_1} = 6/7$	10. $\frac{h}{\omega T_3} = 6/8$
-1.133	.4188	3.6013	-2.7057	-.3146

VI. COMPUTATIONS OF IV AT PEAKS:

	A.	B.	C.	D.	E.	F.	G.
11. ωT*	0	1.5708	4.7124	7.8540	10.9956	14.1372	17.2788
12. $\sin \omega T$	0	+1	-1	+1	-1	+1	-1
13. $\frac{\bar{g}\omega t}{2} = 4 \times 11$	—	.0785	.2356	.3927	.5498		
14. $e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t = 12 \times e^{-13}$	0	+ .9245	− .7901	+ .6752	− .5771		
15. $\frac{h}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t = 9 \times 14$	0	− 2.5014	+ 2.1378	− 1.8270	+ 1.5614		
16. $\frac{h}{\omega T_3} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t = 10 \times 14$	0	− .2909	+ .2486	− .2124	+ .1816		
17. $t = 11/5$	0	.0938	.2813	.4689	.6565		
18. $t + T_1 = 17 + 1$	.025	.1188	.3063	.4939	.6815		
19. $t + T_2 = 18 + 2$	.176	.2698	.4573	.6449			
20. $t + T_3 = 19 + 3$	.391	.4848	.6723				

$T = t, t', t''$, OR t''' DEPENDING ON INTERVAL CONSIDERED

VII. INSTRUCTIONS FOR SKETCHING:

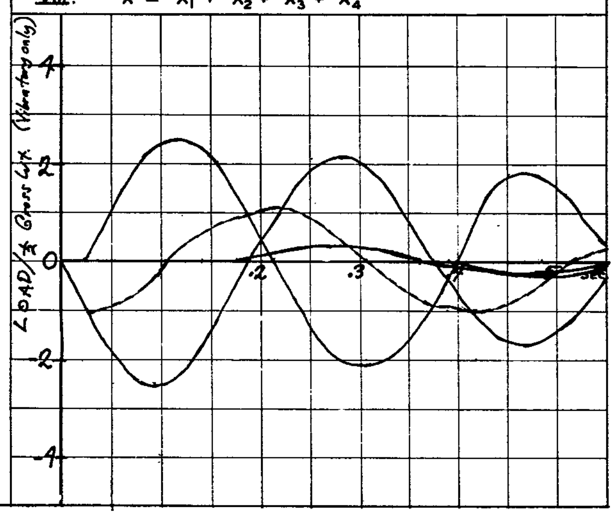
IN VIII PLOT AND CONNECT BY EYE AS DAMPED SINE WAVES:

	ORDINATE	ABSCISSA
A. $\ddot{X}_1 =$	15	17
B. $\ddot{X}_2 =$	-15	18
C. $\ddot{X}_3 =$	-16	19
D. $\ddot{X}_4 =$	16	20

NOW ADD ALL ORDINATES TO GET:

$$\ddot{X} = \ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 + \ddot{X}_4$$

VIII. $\ddot{X} = \ddot{X}_1 + \ddot{X}_2 + \ddot{X}_3 + \ddot{X}_4$



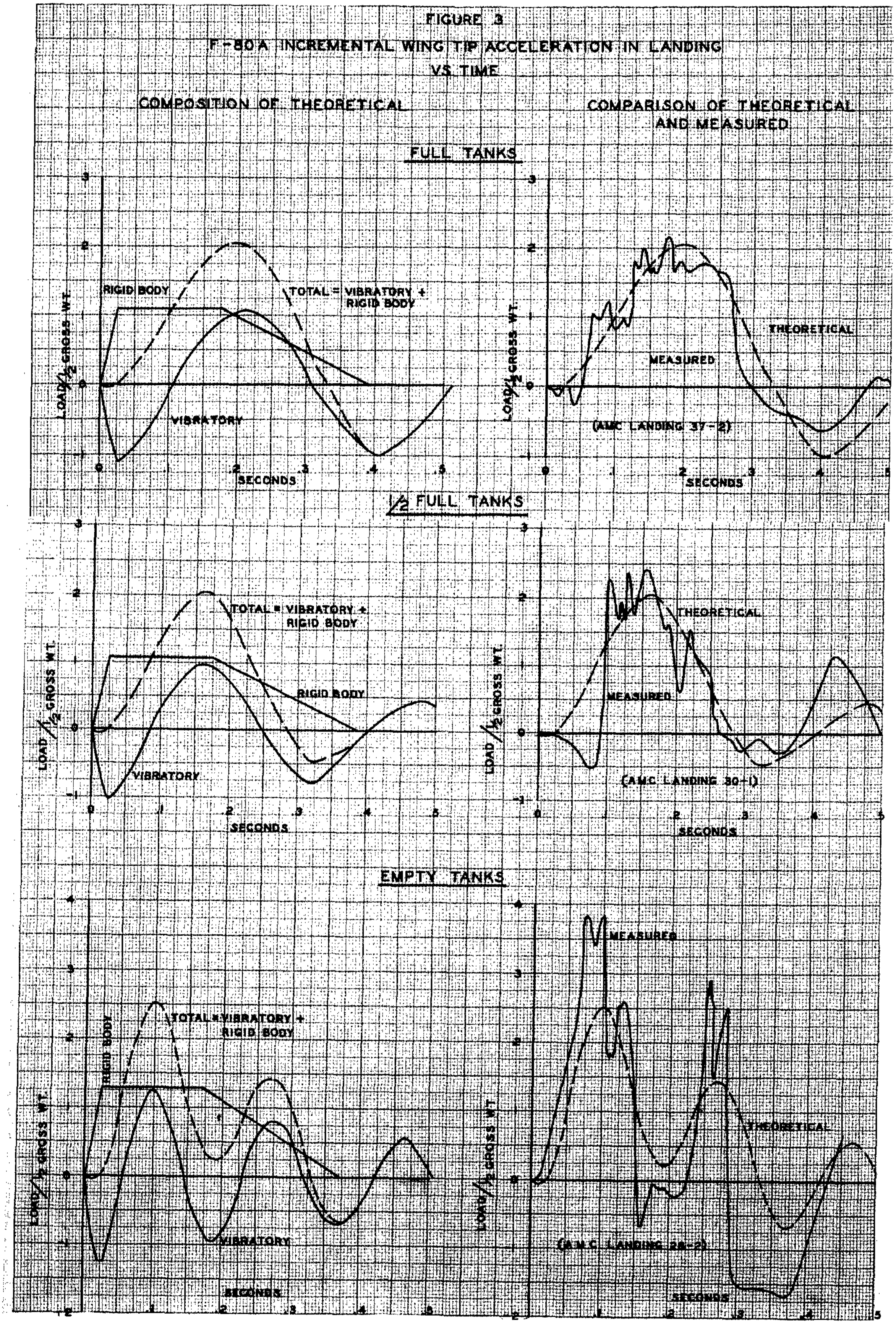


FIGURE 3
F-80A INCREMENTAL WING TIP ACCELERATION IN LANDING VS TIME
COMPOSITION OF THEORETICAL
COMPARISON OF THEORETICAL AND MEASURED
FULL TANKS
RIGID BODY
TOTAL = VIBRATORY + RIGID BODY
VIBRATORY
LOAD/½ GROSS WT.
SECONDS
THEORETICAL
MEASURED
(AMC LANDING 37-2)
½ FULL TANKS
(AMC LANDING 30-1)
EMPTY TANKS
(A.M.C. LANDING 28-2)

Step VII. Variation of Basic Parameters:

1. In this example many parameters were involved - about twelve either previously measured or assumed and the others computed. Will it be possible to predict (without completely new computations) the effect on the final results caused by changing any of these parameters?

2. To facilitate this discussion, a flow chart (Table 1-c) of the computations is shown. Each parameter is oriented vertically by its order of convenient computation and horizontally according to the number of previous parameters which are used to compute this one. Lines emanating upward from any block lead to all of the other parameters which effect this one. The chart gives little if any quantitative help in particular cases.

3. But the following general conclusions seem worthwhile:

a. Change in a basic parameter has an intricate effect, and in many cases the best procedure is to recompute from scratch.

b. There are a few parameters (ω, $\bar{g}$, GAF) which effect only the vibratory response. The effect of altering any of these is rather easily predicted.

c. The problem is conveniently considered in three stages (separated by broken horizontal lines in Table 1-c).

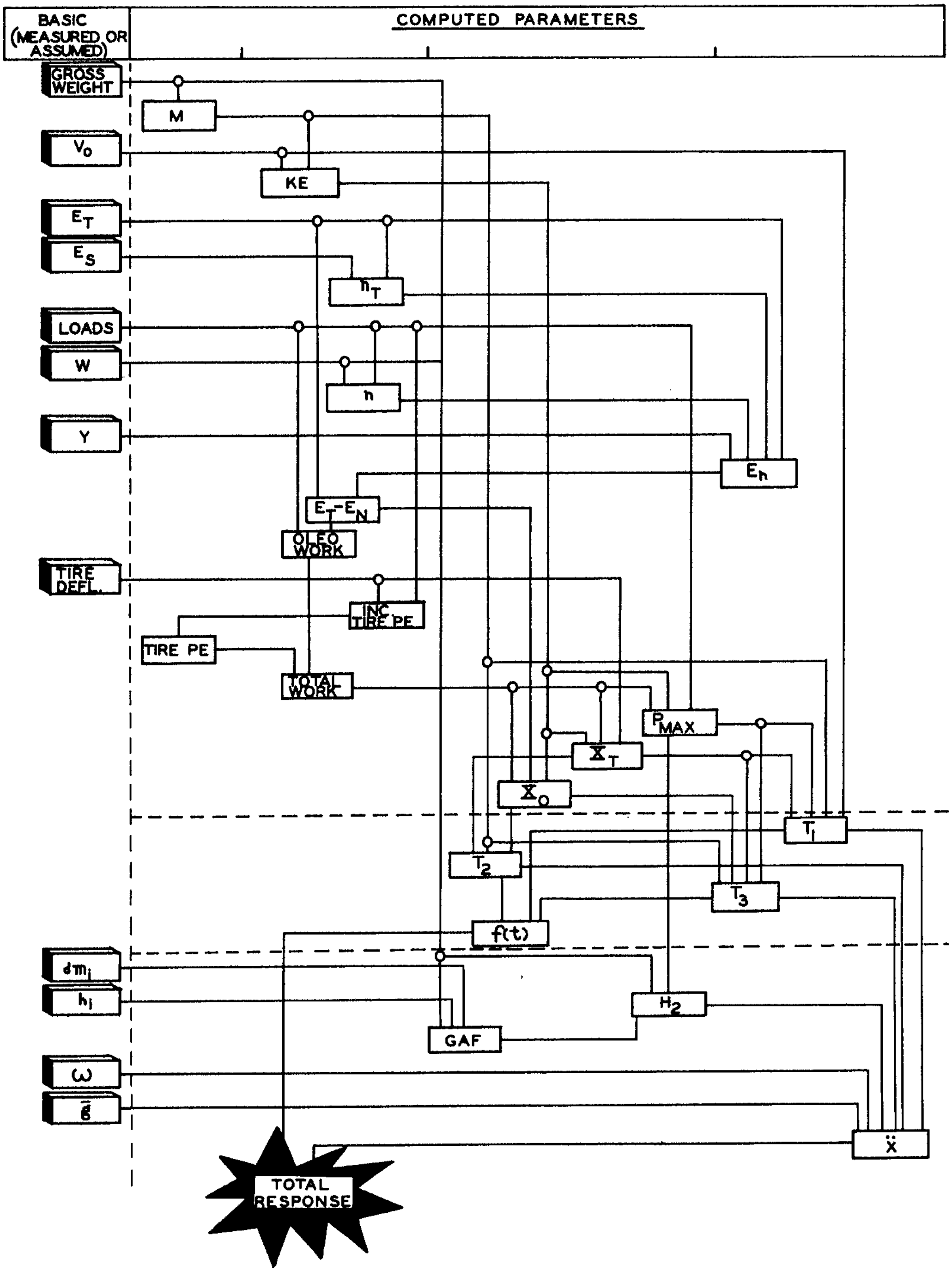
(1) Equating energies.

(2) The rigid-body trapezoid.

(3) Vibratory response and summation.

4. If many changes were contemplated in the first section, it may be profitable to solve the second and third sections only once in general terms, then plug in the particular numbers computed from Part I. See reference 10 for some computations on altering basic parameters.

TABLE 1-C: PARAMETER FLOW - CHART (EXAMPLE 1)



SECTION IV

TAIL BOOM INCREMENTAL ACCELERATIONS OF AN F-61 AIRPLANE

This problem uses the parameters of landing eighteen of the AMC tests reported in reference 5. This particular landing is selected because the maximum rigid body vertical load is closest to the computed value.

Step 1. Vertical Load Time History:

1. Basic airplane data:

Gross weight = 25,000 pounds (app.)

M = Mass per main gear = 388 slugs (2 wheel landing)

V_o = Rate of descent = 8 ft/sec (assumed)

W = Kinetic energy per gear = 12,400 ft-lbs

2. Basic oleo data:

E_T = Total extension * = 10 in. = .833 ft

E_S = Static extension * = 2.87 in. = .239 ft.

Assuming isothermal expansion from static to fully extended position, the "load factor" at total extension =

$$n_T = \frac{E_S}{E_T} = 0.287 \tag{79}$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor "n" during impact is:

$$E_n = \frac{E_T}{\left(\frac{n}{n_T}\right)^{1/\gamma}} = E_T\left(\frac{n_T}{n}\right)^{1/\gamma} \tag{80}$$

where n = load/12500 lbs and γ = 1.3.

* Assuming latent air column = 0

3. Basic tire data:

① Load (Lbs)	② Tire Deflection (Ft)	③ Incremental Tire Work * (Ft-lbs)	④ Tire Work = Σ ③ (Ft-Lbs)
6000	.125	375	375
13000	.250	1188	1563
22000	.333	1453	3016

4. Total work:

⑤ $n = \frac{①}{12500}$	⑥ $\frac{n}{n_r} = \frac{0.287}{⑤}$	⑦ ⑥$^{1/1.3}$	⑧ 1-⑦	⑨ $E_T - E_n$ .833 x ⑧	⑩ OLEO WK. ①x⑨	⑪ TOTAL WORK ④+⑩
.48	.5979	.6733	.3267	.272	1644	2019
1.04	.2760	.3715	.6285	.523	6199	8362
1.76	.1631	.2478	.7522	.627	13794	16810

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflection (Ft)	Oleo Deflection (Ft)
8362	13000	.250	.523
KE = 12400	P_{MAX} = 17302	X_T = .290	X_O = .573
16810	22000	.333	.627

6. The times for tire compression T_O , oleo compression T_T and tire-oleo expansion T_{OT} are determined from the formulas of reference 3.

$$T_T = \frac{3MV_0}{P_{MAX}} - \frac{1}{2}\sqrt{\left(\frac{6MV_0}{P_{MAX}}\right)^2 - \frac{24MX_T}{P_{MAX}}} = 0.0376 \text{ sec} \quad (81)$$

$$T_O = \sqrt{\frac{2MX_O}{P_{MAX}}} = 0.1603 \text{ sec.} \quad (82)$$

$$T_{OT} = \sqrt{\frac{3M(X_O + X_T)}{P_{MAX}}} = 0.2409 \text{ sec.} \quad (83)$$

* ③ = (Average load during increment) (deflection in increment) Approximating the area under the tire curve by trapezoids

Step II. Drag Load Time History

1. Basic parameters:

I_A = moment of inertia of each landing gear rolling assembly = 12.3 slug ft^2

R_R = rolling radius of wheel = 1.65 ft.

V_L = landing speed = 147 ft/sec.

Tire coefficient of friction = 0.55 (assumed)

2. Computation:

θ = angular velocity of wheel after spin up = $\dfrac{V_L}{R_R} = \dfrac{147}{1.05}$ rad/sec = 89.1 rad/sec (84)

θ_T = angular velocity after tire compression = $\dfrac{0.55\, P_{MAX}\, R_R\, T_T}{2 I_A}$ = 24.0 rad/sec. (85)

Assuming (empirically) that peak drag load = .55 P_{max}

θ_0 = angular velocity required for spin up during oleo compression = $\theta - \theta_T$ = 65.1 rad/sec (86)

T_S = duration of skid during oleo compression = $\dfrac{\theta_0\, I_A}{0.55\, P_{MAX}\, R_R}$ = .0510 sec. (87)

$T_T + T_S$ = total spin up time = .0886 sec.

T_Z = time to drop to zero drag load = $\dfrac{T_T + T_S}{4}$ = .0221 sec. (Empirical formula) (88)

Step III. Important Modes

1. The vibratory acceleration of the tail boom is caused partially by the vertical load and partially by the drag load. It is assumed here that the vertical load acts directly to set up a vertical vibration in the tail at the natural frequency of the tail.

2. In order to determine the important vibratory modes set up by the drag load, the airplane is assumed to have the configuration shown in figures 4 and 5 with the three degrees of freedom illustrated.

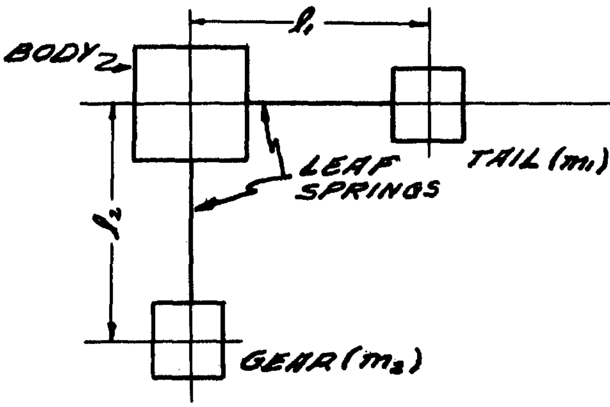


Fig. 4. Geometric Representation

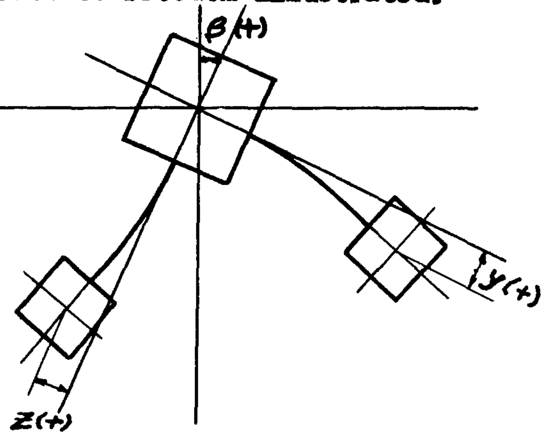


Fig. 5. Positive Direction of Generalized Coordinates

Then (Lagrange's equation for zero external torque)

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right) + \frac{\partial P}{\partial q_i} = 0 \qquad (89)$$

$$q_i = \beta,\ y,\ \text{or}\ z$$

With:

$$K = \frac{1}{2} I \dot{\beta}^2 + \frac{1}{2} m_1 (l_1 \dot{\beta} + \dot{y})^2 + \frac{1}{2} m_2 (l_2 \dot{\beta} + \dot{z})^2 \qquad (90)$$

$$P = \frac{1}{2} m_1 \omega_1^2 y^2 + \frac{1}{2} m_2 \omega_2^2 z^2$$

for which I is the moment of inertia of the body and ω_1 and ω_2 are natural frequencies of tail and gear respectively.

Directly from (89) and (90)

$$\overbrace{(I + m_1 l_1^2 + m_2 l_2^2)}^{I_P}\ddot{\beta} + m_1 l_1 \ddot{y} + m_2 l_2 \ddot{z} = 0$$

$$m_1 \ddot{y} + m_1 \omega_1^2 y + m_1 l_1 \ddot{\beta} = 0 \qquad (91)$$

$$m_2 \ddot{z} + m_2 \omega_2^2 z + m_2 l_2 \ddot{\beta} = 0$$

These equations are unwieldy because they contain both the parameters β, y, z and their second derivatives $\ddot{\beta}$, $\ddot{y}$, $\ddot{z}$.

But the spring action is assumed simple harmonic motion. Then:

$$\beta = a_1 e^{i\omega(t+b_1)} \qquad y = a_2 e^{i\omega(t+b_2)} \qquad z = a_3 e^{i\omega(t+b_3)} \tag{92}$$

where a_i and b_i are constants and ω is the response frequency of the system. The object is to determine the value of ω for each of the possible modes of action.

Directly from (92):

$$\ddot{\beta} = -\omega^2 \beta \qquad \ddot{y} = -\omega^2 y \qquad \ddot{z} = -\omega^2 z \tag{93}$$

Rewriting (91) by substituting (93):

$$\begin{aligned} I_p \beta + m_1 l_1 y + m_2 l_2 z &= 0 \\ l_1 \beta + \left[1 - \left(\frac{\omega_1}{\omega}\right)^2\right] y &= 0 \\ l_2 \beta \qquad\qquad + \left[1 - \left(\frac{\omega_2}{\omega}\right)^2\right] z &= 0 \end{aligned} \tag{94}$$

This system of three equations in the coordinates β, y, z has all constant terms zero. Therefore it has only the trivial solution $\beta = y = z = 0$ unless the determinant of the coefficients of β, y, z is zero. That is:

$$\begin{vmatrix} I_p & m_1 l_1 & m_2 l_2 \\ l_1 & \left[1 - \left(\frac{\omega_1}{\omega}\right)^2\right] & 0 \\ l_2 & 0 & \left[1 - \left(\frac{\omega_2}{\omega}\right)\right] \end{vmatrix} = 0 \tag{95}$$

The only parameter unknown in (95) is ω. For the F-61 airplane, the other parameters have the values (for the complete airplane)

$$I_p = \frac{1.11 \times 10^6}{g} \text{ ft}^2 \text{ slugs}$$

$$m_1 = \frac{650}{g} \text{ slugs} \qquad m_2 = \frac{700}{g} \text{ slugs}$$

$$\omega_1 = \omega_2 = 47.1 \text{ rad/sec}$$

$$l_1 = 24 \text{ ft} \qquad l_2 = 7.7 \text{ ft}$$

Expanding (95) and solving for ω gives two solutions, and corresponding values of the ratios $\frac{\beta}{y}$, $\frac{z}{y}$. These are:

1st Coupled Mode

$$\omega = \omega_1 = 47.1 \text{ rad/sec.}$$

$$\beta / y = 0$$

$$z/y = -\frac{m_1 l_1}{m_2 l_2} = -2.895$$

2nd Coupled Mode

$$\omega = \omega_1 \sqrt{\frac{I_p}{I_p - m_1 l_1^2 - m_2 l_2^2}} = 59.56 \text{ rad./sec} \tag{96}$$

$$\frac{\beta}{y} = \left[\left(\frac{\omega_1}{\omega}\right)^2 - 1\right] / l_1 = -0.03993 \text{ rad/ft.}$$

$$\frac{z}{y} = \frac{l_1}{l_2} = 3.117$$

Step IV. Generalized Acceleration Factors.

1. For the vertical load:

$$GAF = \frac{(17302)\left(-\frac{650}{24350}\right)}{\frac{650}{29}(1)^2 + \frac{24350}{29}\left(-\frac{650}{24350}\right)^2} = -1.384g \qquad (97\text{-a})$$

per foot of tail deflection.

2. For the drag load:

a. In the first mode:

$$GAF = \frac{(0.55)(17302)(-2.895)}{\frac{650}{29}(1)^2 + \frac{700}{29}(-2.895)^2} = -8.444g \qquad (97\text{-b})$$

per foot of tail deflection.

b. In the second mode:

Tail deflection = 1 ft = $y + l_1 \beta$

Thus gear deflection = $z + l_2 \beta$ = .3208 ft.

And β = -.02497 rad.

So that:

$$GAF = \frac{(0.55)(17302)(0.3208)}{\left(\frac{694097}{29}\right)(0.02497)^2 + \frac{650}{29}(1)^2 + \frac{700}{29}(0.3208)^2} = 5.287g \qquad (97\text{-c})$$

per foot of tail deflection

Step V. Vibratory Acceleration Response.

1. Since the mode for the vertical load has the same frequency as the first mode for drag load, the two rigid body forcing functions can be combined into one trapezoidal forcing function as shown in Figure 6.

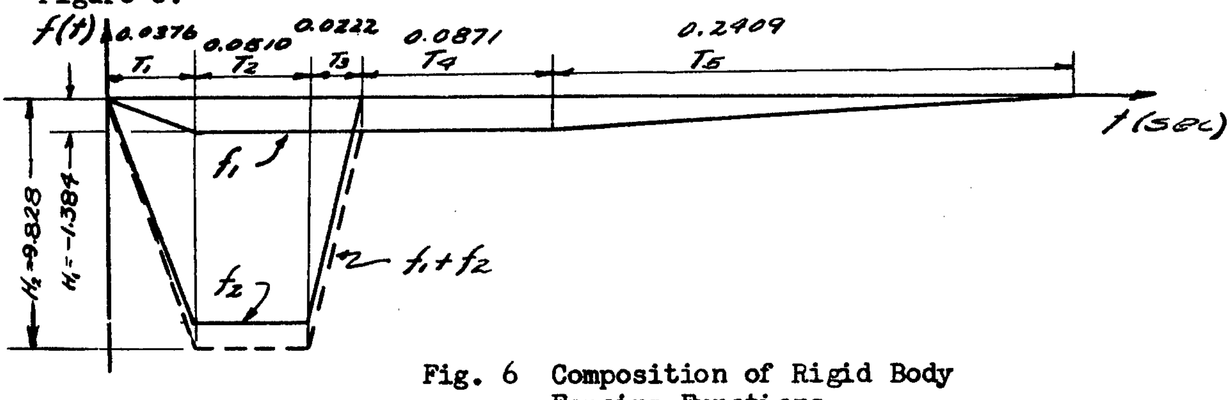


Fig. 6 Composition of Rigid Body Forcing Functions

The response to this function is computed in table 2-a.

2. The response in the second mode due to drag load is computed in table 2-b.

Step VI. Total Acceleration Time History.

1. The total acceleration is the sum of:

a. Rigid body response to vertical load. This and all other components of the final response are nondimensionalized in units of 1/2 airplane weight. Thus the maximum value of the rigid body response will be scaled as $\frac{17302}{12500} = 1.384$

b. Rigid body response to drag load. The maximum value is determined from the definition of torque:

$$\text{Torque} = (.55)\ (17302)\ (7.7)\ \text{ft-lbs}$$

But: $\text{Torque} = I\alpha = \frac{(1.11 \times 10^6)}{2}\left[\text{Max. vertical acceleration}/24\right]\text{ft lbs}$

So:

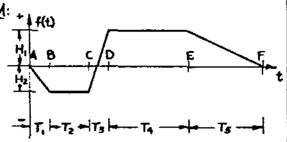
$$\text{Max vertical acceleration} = \frac{(.55)(17302)(7.7)(24)}{(1.11)\ (10^6)/2}\ g = 3.169\ g \qquad (98)$$

c. Vibratory responses to vertical and drag loads.

2. All the above components are summed and compared with experimental results in figure 7.

I. THE PROBLEM:

GIVEN FORCING FUNCTION f(t)
FIND THE RESPONSE $\ddot{x}$ FROM:
$\ddot{x}+\omega^2(1+\bar{g}j)x = f(t)$



II. BASIC DATA

TITLE: *Vib Acc F61 Tail Boom (1st Mode)*
COMPUTER: *KLE*

1 T_1	2 T_2	3 T_3	4 T_4	5 T_5
.0376	.0510	.0222	.0871	.2409
6 H_1	7 H_2	8 ω	9 $\bar{g}$	
-1.384	-9.828	47.1	.10	

↳ RADIANS/SEC.

III. INITIALLY COMPUTABLE CONSTANTS

10 $\omega T_1 = 8\times1$	11 $\omega T_2 = 8\times2$	12 $\omega T_3 = 8\times3$	13 $\omega T_4 = 8\times4$	14 $\omega T_5 = 8\times5$
1.7710	2.4021	1.0456	4.1024	11.3464
15 $\frac{\bar{g}\omega}{2} = \frac{1}{2}(8\times9)$	16 $\frac{H_2}{\omega T_1} = \frac{7}{10}$	17 $H_1 - H_2 = 6-7$	18 $\frac{H_1-H_2}{\omega T_3} = \frac{17}{12}$	19 $\frac{H_1}{\omega T_5} = \frac{6}{14}$
2.355	-5.5494	+8.444	+8.0757	-.1220
20 $\frac{\bar{g}\omega T_1}{2} = 15\times1$	21 $\frac{\bar{g}\omega T_2}{2} = 15\times2$	22 $\frac{\bar{g}\omega T_3}{2} = 15\times3$	23 $\frac{\bar{g}\omega T_4}{2} = 15\times4$	24 $\frac{\bar{g}\omega T_5}{2} = 15\times5$
.0885	.1201	.0523	.2051	.5673
25 e^{-20}	26 e^{-21}	27 e^{-22}	28 e^{-23}	29 e^{-24}
.9153	.8868	.9491	.8146	.5671

IV. IN AB, $\ddot{x} = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t$

AT B. $\ddot{x}_B = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \sin \omega T_1$

30 16×25	31 sin 10	32 30×31
-5.0794	+.9800	-4.9778 → $\ddot{x}_B$
	33 cos 10	34 30×33
	-.1989	+1.0103

V. IN BC, $\ddot{x} = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega t'}{2}} \sin(\omega t'+\phi_1)$ $t' = t - T_1$

AT C, $\ddot{x}_C = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \sin(\omega T_2+\phi_1)$

WHERE $R_1 = -\frac{H_2}{\omega T_1} + \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \cos \omega T_1$

$\phi_1 = \arctan \frac{\ddot{x}_B}{R_1}$ (SEE BELOW)

35 34-16	36 32^2+35^2	37 $\sqrt{36}$	38 26×37	
+6.5597	67.80815693	8.2346	7.3024	
39 32/35	40 ϕ_1 [radians]	41 11+40	42 sin 41	43 38×42
-.7588	-.649	+1.7531	+.9834	+7.1812 → $\ddot{x}_C$
			44 cos 41	45 38×44
			-.1813	-1.3239

TO DETERMINE
(a) QUADRANT: *II*
$\begin{Bmatrix}\sin\phi_1\\ \cos\phi_1\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}32\\35\end{Bmatrix}$
(b) ANGLE: $\tan \phi_1 = 39$

VI. IN CD, $\ddot{x} = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega t''}{2}} \sin(\omega t''+\phi_2)$ $t'' = t' - T_2$

AT D, $\ddot{x}_D = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \sin(\omega T_3+\phi_2)$

WHERE $R_2 = \frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \cos(\omega T_2+\phi_1)$

$\phi_2 = \arctan \frac{\ddot{x}_C}{R_2}$ (SEE BELOW)

46 45+18	47 43^2+46^2	48 $\sqrt{47}$	49 27×48	
+6.7518	97.15693468	9.8568	9.3551	
50 43/46	51 ϕ_2 [RADIANS]	52 12+51	53 sin 52	54 49×53
+1.0636	+.816	+1.8616	+.9580	+8.9622 → $\ddot{x}_D$
			55 cos 52	56 49×55
			-.2867	-2.6821

TO DETERMINE
(a) QUADRANT: *I*
$\begin{Bmatrix}\sin\phi_2\\ \cos\phi_2\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}43\\46\end{Bmatrix}$
(b) ANGLE: $\tan \phi_2 = 50$

VII IN DE, $\ddot{x} = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega t'''}{2}} \sin(\omega t'''+\phi_3)$ $t''' = t'' - T_3$

AT E, $\ddot{x}_E = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega T_4}{2}} \sin(\omega T_4+\phi_3)$

WHERE $R_3 = -\frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \cos(\omega T_3+\phi_2)$

$\phi_3 = \arctan \frac{\ddot{x}_D}{R_3}$ (SEE BELOW)

57 56-18	58 54^2+57^2	59 $\sqrt{58}$	60 28×59	
-10.7578	196.0515[illegible]	14.0018	11.4059	
61 54/57	62 ϕ_3 [RADIANS]	63 13+62	64 sin 63	65 60×64
-.8331	2.4470	6.5494	+.2631	+3.0009 → $\ddot{x}_E$
			66 cos 63	67 60×66
			+.9648	+11.0044

TO DETERMINE
(a) QUADRANT: *II*
$\begin{Bmatrix}\sin\phi_3\\ \cos\phi_3\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}54\\57\end{Bmatrix}$
(b) ANGLE: $\tan \phi_3 = 61$

VIII IN EF, $\ddot{x} = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega t''''}{2}} \sin(\omega t''''+\phi_4)$ $t'''' = t''' - T_4$

AT F, $\ddot{x}_F = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega T_5}{2}} \sin(\omega T_5+\phi_4)$

WHERE $R_4 = -\frac{H_1}{\omega T_5} + e^{-\frac{\bar{g}\omega T_4}{2}} \sqrt{\ddot{x}_D^2+R_3^2} \cos(\omega T_4+\phi_3)$

$\phi_4 = \arctan \frac{\ddot{x}_E}{R_4}$ (SEE BELOW)

68 67-19	69 65^2+68^2	70 $\sqrt{69}$	71 29×70	
+11.1264	132.8021977	11.5240	6.5353	
72 65/68	73 ϕ_4 [RADIANS]	74 14+73	75 sin 74	76 71×75
+.2697	+.2634	+11.6098	-.8172	-5.3406 → $\ddot{x}_F$

TO DETERMINE
(a) QUADRANT: *I*
$\begin{Bmatrix}\sin\phi_4\\ \cos\phi_4\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}65\\68\end{Bmatrix}$
(b) ANGLE: $\tan \phi_4 = 72$

IX INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $\ddot{x} = Ke^{-\frac{\bar{g}\omega T}{2}} \sin(\omega T+\phi)$. THE VALUES AT DISCONTINUITIES HAVE BEEN FOUND IN IV-VIII. FOR SKETCHING, THE ZEROS [ie., $\sin(\omega T+\phi)=0$] AND PEAKS [ie., $\sin(\omega T+\phi)=\pm1$, APP.] ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega T+\phi$ FOR WHICH $\sin(\omega T+\phi)=0,\pm1$ ARE IN XI. BUT SINCE $0 \le T \le T_i$ IN EACH INTERVAL, ONLY THE VALUES OF $\omega T+\phi$ BETWEEN ϕ_i AND $\omega T_i+\phi_i$ CAN BE USED. THESE LIMITS OF $\omega T+\phi$ ARE LISTED IN X. PARTS XII-XVI THEN COMPUTE PLOTTING POINTS AT PEAKS AND ZEROS.

X. LIMITS OF $(\omega T+\phi)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	ϕ_1 = 40 -.649	ϕ_2 = 51 +.816	ϕ_3 = 62 +2.447	ϕ_4 = 73 +.263
TO	ωT_1 = 10 +1.7710	$\omega T_2+\phi_1$ = 41 +1.753	$\omega T_3+\phi_2$ = 52 +1.862	$\omega T_4+\phi_3$ = 63 +6.549	$\omega T_5+\phi_4$ = 74 +11.610

XI VALUES OF $\sin(\omega T+\phi)$ FOR COMPUTING ZEROS & PEAKS

$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$
-10.996	+1	-3.142	0	+4.712	-1
-9.425	0	-1.571	-1	+6.283	0
-7.854	-1	0	0	+7.854	+1
-6.283	0	+1.571	+1	+9.425	0
-4.712	+1	+3.142	0	+10.996	-1

XII ZEROS AND PEAKS BETWEEN A & B

NOTE: $\phi = 0$ → IN A-B

	77 ωt (FROM XI)	78 $\sin\omega t$ (FROM XI)	79 $\frac{\bar{g}}{2}\times\omega t$ = .05×ωt	80 e^{-79}	81 $t = \frac{77}{8}$	82 PLOT $\ddot{x}$ = 78×80×16
a	1.571	+1	.0786	.9244	.0334	-5.130
b						
c						
d						
e						

XIII ZEROS & PEAKS BETWEEN B & C

	83 $\omega T+\phi$ (FROM XI)	84 $\sin(\omega T+\phi)$ (FROM XI)	85 ωT = 83-40	86 $\frac{\bar{g}}{2}\times\omega T$ = .05×85	87 e^{-86}	88 $T = \frac{85}{8}$	89 PLOT t = 1+88	90 $\ddot{x}$ = 84×87×37
a	0	0	.649	—	—	.0138	.0514	0
b	1.571	+1	2.220	.1110	.8949	.0471	.0847	+7.369
c								
d								
e								

XIV ZEROS & PEAKS BETWEEN C & D

	91 $\omega T+\phi$ (FROM XI)	92 $\sin(\omega T+\phi)$ (FROM XI)	93 ωT = 91-51	94 $\frac{\bar{g}}{2}\times\omega T$ = .05×93	95 e^{-94}	96 $T = \frac{93}{8}$	97 PLOT t = 1+2+96	98 $\ddot{x}$ = 92×95×48
a	1.571	+1	.755	.0378	.9629	.0160	.1046	+9.491
b								
c								
d								
e								

XV ZEROS & PEAKS BETWEEN D & E

	99 $\omega T+\phi$ (FROM XI)	100 $\sin(\omega T+\phi)$ (FROM XI)	101 ωT = 99-62	102 $\frac{\bar{g}}{2}\times\omega T$ = .05×101	103 e^{-102}	104 $T = \frac{101}{8}$	105 PLOT t = 1+2+3+104	106 $\ddot{x}$ = 100×103×59
a	3.142	0	.695	—	—	.0148	.1256	0
b	4.712	-1	2.265	.1133	.8929	.0481	.1589	-12.502
c	6.283	0	3.836	—	—	.0814	.1922	0
d								
e								

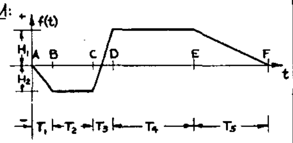
XVI ZEROS & PEAKS BETWEEN E & F

	107 $\omega T+\phi$ (FROM XI)	108 $\sin(\omega T+\phi)$ (FROM XI)	109 ωT = 107-73	110 $\frac{\bar{g}}{2}\times\omega T$ = .05×109	111 e^{-110}	112 $T = \frac{109}{8}$	113 PLOT t = 1+2+3+4+112	114 $\ddot{x}$ = 108×111×70
a	1.571	+1	1.308	.0654	.9367	.0278	.2257	10.795
b	3.142	0	2.879	—	—	.0611	.2590	0
c	4.712	-1	4.449	.2225	.8005	.0945	.2924	-9.245
d	6.283	0	6.020	—	—	.1278	.3257	0
e	7.854	+1	7.591	.3796	.6841	.1612	.3591	+7.859

TABLE 2-a

I. THE PROBLEM:

GIVEN FORCING FUNCTION $f(t)$
FIND THE RESPONSE $\ddot{x}$ FROM:
$\ddot{x}+\omega^2(1+\bar{g}j)x = f(t)$



II. BASIC DATA

TITLE: *Vib Acc F61 Tail Boom (2nd Mode)*
COMPUTER: *[illegible]*

1 T_1	2 T_2	3 T_3	4 T_4	5 T_5
.0376	.0510	.0222		
6 H_1	7 H_2	8 ω	9 $\bar{g}$	
0	5.287	59.56	.10	

(ω in RADIANS/SEC.)

III. INITIALLY COMPUTABLE CONSTANTS

10 ωT_1 = 8X1	11 ωT_2 = 8X2	12 ωT_3 = 8X3	13 ωT_4 = 8X4	14 ωT_5 = 8X5
2.2395	3.0376	1.3222		
15 $\frac{\bar{g}\omega}{2}=\frac{1}{2}(8X9)$	16 $\frac{H_2}{\omega T_1}=\frac{7}{10}$	17 H_1-H_2 = 6-7	18 $\frac{H_1-H_2}{\omega T_3}=\frac{17}{12}$	19 $\frac{H_1}{\omega T_5}=\frac{6}{14}$
2.9780	2.3608	-5.287	-3.9986	
20 $\frac{\bar{g}\omega T_1}{2}$ = 15X1	21 $\frac{\bar{g}\omega T_2}{2}$ = 15X2	22 $\frac{\bar{g}\omega T_3}{2}$ = 15X3	23 $\frac{\bar{g}\omega T_4}{2}$ = 15X4	24 $\frac{\bar{g}\omega T_5}{2}$ = 15X5
.1120	.1519	.0661		
25 e^{-20}	26 e^{-21}	27 e^{-22}	28 e^{-23}	29 e^{-24}
.8940	.8591	.9360		

IV. IN AB, $\ddot{x} = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t$

AT B. $\ddot{x}_B = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \sin \omega T_1$

30 16X25	31 sin 10	32 30X31
2.1106	.7846	1.6560 → $\ddot{x}_B$
	33 cos 10	34 30X33
	-.6200	-1.3086

V. IN BC, $\ddot{x} = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega t'}{2}} \sin(\omega t'+\phi_1)$ $t' = t - T_1$

AT C, $\ddot{x}_C = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \sin(\omega T_2+\phi_1)$

WHERE
$R_1 = -\frac{H_2}{\omega T_1} + \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \cos \omega T_1$
$\phi_1 = \arctan \frac{\ddot{x}_B}{R_1}$ (SEE BELOW)

35 34-16	36 32^2+35^2	37 $\sqrt{36}$	38 26X37	
-3.6694	16.2068[illegible]	4.0258	3.4586	
39 32/35	40 ϕ_1 [radians]	41 11+40	42 sin 41	43 38X42
-.4513	2.7177	5.7553	-.5037	-1.7421 → $\ddot{x}_C$
			44 cos 41	45 38X44
			.8639	2.9879

TO DETERMINE
(a) QUADRANT: *II*
$\begin{Bmatrix}\sin\phi_1\\ \cos\phi_1\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}32\\35\end{Bmatrix}$
(b) ANGLE: $\tan\phi_1$ = 39

VI. IN CD, $\ddot{x} = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega t''}{2}} \sin(\omega t''+\phi_2)$ $t'' = t' - T_2$

AT D. $\ddot{x}_D = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \sin(\omega T_3+\phi_2)$

WHERE
$R_2 = \frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \cos(\omega T_2+\phi_1)$
$\phi_2 = \arctan \frac{\ddot{x}_C}{R_2}$ (SEE BELOW)

46 45+18	47 43^2+46^2	48 $\sqrt{47}$	49 27X48	
-1.0107	4.0564269	2.0141	1.8852	
50 43/46	51 ϕ_2 [RADIANS]	52 12+51	53 sin 52	54 49X53
1.7237	4.1866	5.5088	-.6993	-1.3183 → $\ddot{x}_D$
			55 cos 52	56 49X55
			.7149	1.3477

TO DETERMINE
(a) QUADRANT: *III*
$\begin{Bmatrix}\sin\phi_2\\ \cos\phi_2\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}43\\46\end{Bmatrix}$
(b) ANGLE: $\tan\phi_2$ = 50

VII IN DE. $\ddot{x} = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega t'''}{2}} \sin(\omega t'''+\phi_3)$ $t''' = t'' - T_3$

AT E. $\ddot{x}_E = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega T_4}{2}} \sin(\omega T_4+\phi_3)$

WHERE
$R_3 = -\frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \cos(\omega T_3+\phi_2)$
$\phi_3 = \arctan \frac{\ddot{x}_D}{R_3}$ (SEE BELOW)

57 56-18	58 54^2+57^2	59 $\sqrt{58}$	60 28X59	
5.3463	30.3208[illegible]	5.5069		
61 54/57	62 ϕ_3 [RADIANS]	63 13+62	64 sin 63	65 60X64
-.2466	-.2418			→ $\ddot{x}_E$
			66 cos 63	67 60X66

TO DETERMINE
(a) QUADRANT: *IV*
$\begin{Bmatrix}\sin\phi_3\\ \cos\phi_3\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}54\\57\end{Bmatrix}$
(b) ANGLE: $\tan\phi_3$ = 61

VIII IN EF, $\ddot{x} = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega t''''}{2}} \sin(\omega t''''+\phi_4)$ $t'''' = t''' - T_4$

AT F. $\ddot{x}_F = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega T_5}{2}} \sin(\omega T_5+\phi_4)$

WHERE
$R_4 = -\frac{H_1}{\omega T_5} + e^{-\frac{\bar{g}\omega T_4}{2}} \sqrt{\ddot{x}_D^2+R_3^2} \cos(\omega T_4+\phi_3)$
$\phi_4 = \arctan \frac{\ddot{x}_E}{R_4}$ (SEE BELOW)

68 67-19	69 65^2+68^2	70 $\sqrt{69}$	71 29X70	
72 65/68	73 ϕ_4 [RADIANS]	74 14+73	75 sin 74	76 71X75
				→ $\ddot{x}_F$

TO DETERMINE
(a) QUADRANT: ——
$\begin{Bmatrix}\sin\phi_4\\ \cos\phi_4\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}65\\68\end{Bmatrix}$
(b) ANGLE: $\tan\phi_4$ = 72

IX. INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $\ddot{x} = Ke^{-\frac{\bar{g}\omega T}{2}} \sin(\omega T+\phi)$. THE VALUES AT DISCONTINUITIES HAVE BEEN FOUND IN IV - VIII. FOR SKETCHING, THE ZEROS [ie., $\sin(\omega T+\phi) = 0$] AND PEAKS [ie, $\sin(\omega T+\phi) = \pm 1$, APP.] ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega T+\phi$ FOR WHICH $\sin(\omega T+\phi) = 0, \pm 1$ ARE IN XI. BUT SINCE $0 \le T \le T_i$ IN EACH INTERVAL, ONLY THE VALUES OF $\omega T+\phi$ BETWEEN ϕ_i AND $\omega T_i+\phi_i$ CAN BE USED. THESE LIMITS OF $\omega T+\phi$ ARE LISTED IN X. PARTS XII - XVI THEN COMPUTE PLOTTING POINTS AT PEAKS AND ZEROS.

X. LIMITS OF $(\omega T+\phi)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	ϕ_1 = 40	ϕ_2 = 51	ϕ_3 = 62	ϕ_4 = 73
		2.7177	4.1866	-.2418	
TO	ωT_1 = 10	$\omega T_2+\phi_1$ = 41	$\omega T_3+\phi_2$ = 52	$\omega T_4+\phi_3$ = 63	$\omega T_5+\phi_4$ = 74
	2.2395	5.7553	5.5088	∞	

XI VALUES OF $\sin(\omega T+\phi)$ FOR COMPUTING ZEROS & PEAKS

$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$
-10.996	+1	-3.142	0	+4.712	-1
-9.425	0	-1.571	-1	+6.283	0
-7.854	-1	0	0	+7.854	+1
-6.283	0	+1.571	+1	+9.425	0
-4.712	+1	+3.142	0	+10.996	-1

XII ZEROS AND PEAKS BETWEEN A & B

NOTE: $\phi = 0$ IN A-B

	77 ωt [FROM X & XI]	78 $\sin \omega t$ [FROM XI]	79 $\frac{\bar{g}}{2} \times \omega t$ = .05 × ωt	80 e^{-79}	81 $t = \frac{77}{8}$ (PLOT)	82 $\ddot{x}$ = 78X80X16 (PLOT)
a	1.571	+1	.0786	.9244	.0264	2.1823
b						
c						
d						
e						

XIII. ZEROS & PEAKS BETWEEN B & C.

	83 $\omega T+\phi$ [FROM X & XI]	84 $\sin(\omega T+\phi)$ [FROM XI]	85 ωT = 83-40	86 $\frac{\bar{g}}{2}\times\omega T$ = .05X85	87 e^{-86}	88 $T = \frac{85}{8}$	89 t = 1+88 (PLOT)	90 $\ddot{x}$ = 84X87X37 (PLOT)
a	3.142	0	.4243	——	——	.0071	.0447	0
b	4.712	-1	1.9943	.0997	.9051	.0335	.0711	-3.6[illegible]
c								
d								
e								

XIV. ZEROS & PEAKS BETWEEN C & D

	91 $\omega T+\phi$ [FROM X & XI]	92 $\sin(\omega T+\phi)$ [FROM XI]	93 ωT = 91-51	94 $\frac{\bar{g}}{2}\times\omega T$ = .05X93	95 e^{-94}	96 $T = \frac{93}{8}$	97 t = 1+2+96 (PLOT)	98 $\ddot{x}$ = 92X95X48 (PLOT)
a	4.712	-1	.5254	.0263	.9740	.0088	.0974	-1.9617
b								
c								
d								
e								

XV. ZEROS & PEAKS BETWEEN D & E

	99 $\omega T+\phi$ [FROM X & XI]	100 $\sin(\omega T+\phi)$ [FROM XI]	101 ωT = 99-62	102 $\frac{\bar{g}}{2}\times\omega T$ = .05X101	103 e^{-102}	104 $T = \frac{101}{8}$	105 t = 1+2+3+104 (PLOT)	106 $\ddot{x}$ = 100X103X59 (PLOT)
a	0	0	.2418	——	——	.0041	.1149	0
b	1.571	+1	1.8128	.0906	.9134	.0304	.1412	5.0295
c	3.142	0	3.3838	——	——	.0568	.1676	0
d	4.712	-1	4.9538	.2477	.7806	.0832	.1940	-4.2983
e	6.283	0	6.5248	——	——	.1096	.2204	0
	7.854	+1	8.0958	.4048	.6671	.1359	.2467	3.6733

XVI. ZEROS & PEAKS BETWEEN ~~E & F~~ D/E (Cont'd)

	107 $\omega T+\phi$ [FROM X & XI]	108 $\sin(\omega T+\phi)$ [FROM XI]	109 ωT = 107-[illegible]	110 $\frac{\bar{g}}{2}\times\omega T$ = .05X109	111 e^{-110}	112 $T = \frac{109}{8}$	113 t = 1+2+3+[illegible]+112 (PLOT)	114 $\ddot{x}$ = 108X111X[illegible] (PLOT)
a	9.425	0	9.667	——	——	.1623	.2731	0
b	10.996	-1	11.238	.5619	.5701	.1887	.2995	-3.1392
c	12.566	0	12.808	——	——	.2150	.3258	0
d	14.137	+1	14.379	.7190	.4872	.2414	.3522	+2.6827
e	15.708	0	15.950	——	——	.2678	.3786	0

TABLE 2-b

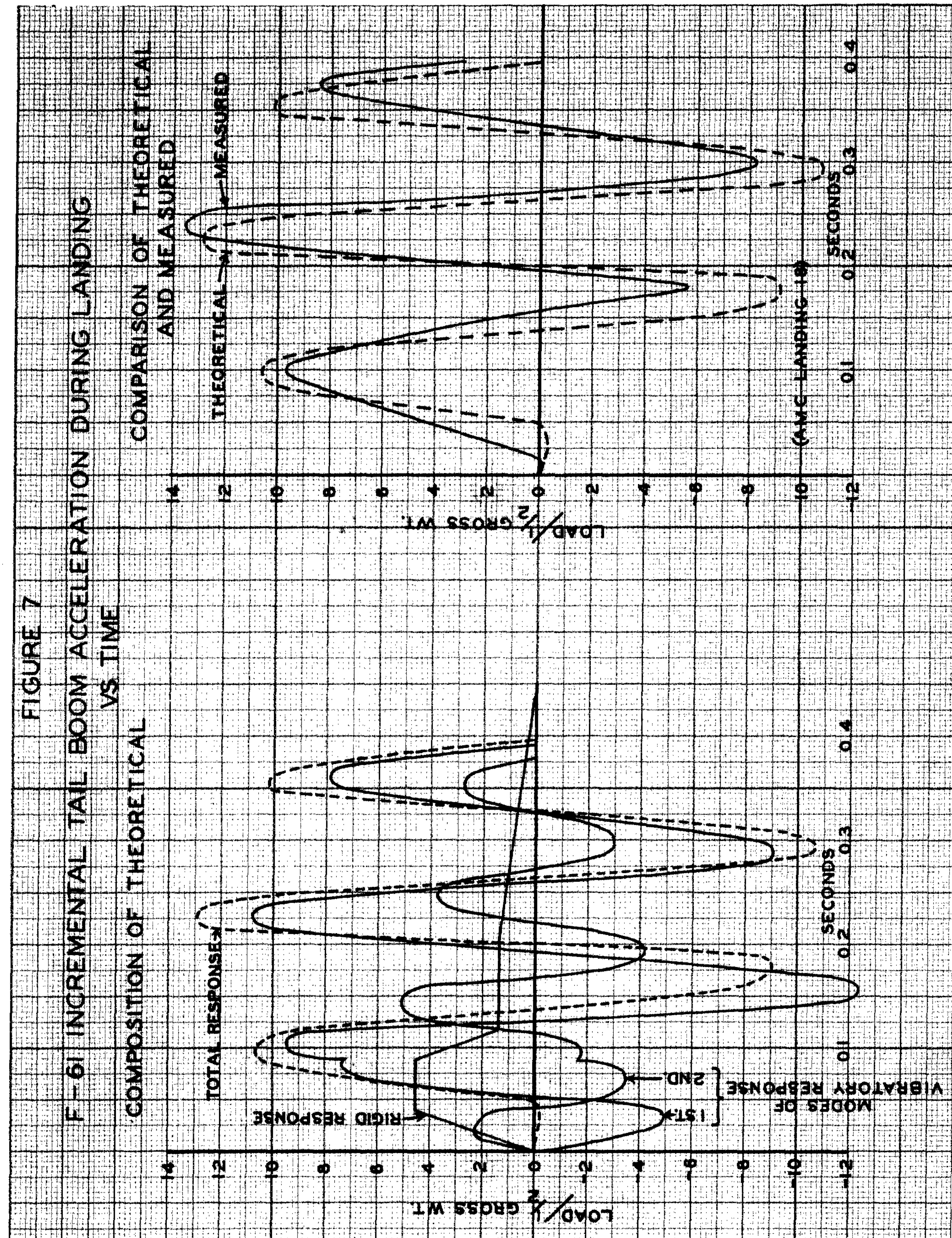


FIGURE 7
F-61 INCREMENTAL TAIL BOOM ACCELERATION DURING LANDING VS TIME

SECTION V

B-17G LANDING GEAR DRAG LOAD (NORMAL TO STRUT)

Parameters are taken from landing number 1 of flight 3 of the B-17G landing load tests of AMC, reported in reference 6.

This example is a particularly good criteria of the accuracy of the predictions based on the trapezoidal theory, because V_0 was actually measured.

Step I. Vertical Load Time History:

1. Basic airplane data:

Gross weight = 48,137 pounds

M = Mass per main gear = 750 slugs (two wheel landing)

W = Static load per gear = 21,775 lbs (3 point position)

V_0 = Rate of descent = 7 ft/sec (measured)

KE = Kinetic energy per gear = 18,380 ft-lbs

ω = Natural frequency of gear fore and aft = 73.2 rad/sec (measured from landing records)

2. Basic oleo data:

E_T = Total extension + latent air column = (9.6 + 1.56) in. = .930 ft.

E_s = Static extension + latent air column = (1.44 + 1.56) in. = .250 ft.

Assuming isothermal expansion from static to fully loaded position, the "load factor" at total extension =

$$n_T = \frac{E_s}{E_T} = 0.2688 \qquad (99)$$

Then assuming quasi-adiabatic compression from the fully extended position during impact, the extension at any load factor "n" during impact is:

$$E_n = \frac{E_T}{\left(\frac{n}{n_T}\right)^{1/r}} = E_T\left(\frac{n_T}{n}\right)^{1/r} \qquad (100)$$

where n = load/21,775 lbs and γ = 1.3 (assumed).

3. Basic tire data:

(1) Load (Lbs)	(2) Tire Deflection (Ft)	(3) Incremental Tire Work * (Ft-lbs)	(4) Tire Work = Σ (3) (Ft-Lbs)
10000	.160	800	800
15000	.223	792	1592
20000	.283	1050	2642
25000	.337	1219	3861

4. Total Work:

(5) $n = \frac{(1)}{21775}$	(6) $\frac{n_T}{n} = \frac{0.2688}{(5)}$	(7) $\left(\frac{n_T}{n}\right)^{1/\gamma} = (6)^{1/1.3}$	(8) $1-(7)$	(9) $\frac{E_T - E_n}{0.930 \cdot (8)}$	(10) OLEO WK. $(1)\cdot(9)$	(11) TOTAL WORK $(4)+(10)$
.4592	.5854	.8339	.1661	.1545	1545	2345
.6889	.3902	.4849	.5151	.4790	7185	8777
.9185	.2927	.3886	.6114	.5686	11372	14014
1.1481	.2341	.3273	.6727	.6256	15640	19501

5. Tire deflection, oleo deflection and load when kinetic energy per gear = total work:

Total Work (Ft-Lbs)	Load (Lbs)	Tire Deflection (Ft)	Oleo Deflection (Ft)
14014	20000	.283	.569
KE = 18380	P_{MAX} = 23978	X_T = .326	X_o = .614
19501	25000	.337	.626

6. The times for tire compression T_T oleo compression T_o and tire-oleo expansion T_{oT} are determined from the formulas of reference 3.

$$T_T = \frac{3MV_o}{P_{MAX}} - \frac{1}{2}\sqrt{\left(\frac{6MV_o}{P_{MAX}}\right)^2 - \frac{24\,M\,X_T}{P_{MAX}}} = .0484 \text{ sec} \quad (101)$$

* (3) = (average load during increment) (deflection in increment) (Approximating the area under the tire curve by trapezoids)

$$T_0 = \sqrt{\frac{2MX_0}{P_{MAX}}} = 0.1968 \text{ sec.} \qquad (102)$$

$$T_{0T} = \sqrt{\frac{3M(X_0 + X_T)}{P_{MAX}}} = 0.2970 \text{ sec.} \qquad (103)$$

Step II. Drag Load Time History:

1. Basic parameters:

I_A = moment of inertia of each landing gear rolling assembly = 31.23 slug ft^2

R_R = rolling radius of wheel = 1.942 ft.

V_L = landing speed = 147 ft/sec.

Tire coefficient of friction = .55 (assumed)

2. Computation:

θ = angular velocity of wheel after spin up = $\frac{V_L}{R_R}$ = $\frac{147}{1.942}$ rad/sec = 75.70 rad/sec. (104)

θ_T = angular velocity after tire compression = $\frac{R_R\, 0.55\, P_{MAX}\, T_T}{2 I_A}$ = 19.68 rad/sec (peak drag load = .55 P_{max}). (105)

θ_0 = angular velocity for spin up during oleo compression = $\theta - \theta_T$ = 56.02 rad/sec (106)

T_S = duration of skid during oleo compression = $\frac{\theta_0 I_A}{0.55\, P_{MAX}\, R_R}$ = .0683 sec. (107)

$T_T + T_S$ = total spin up time = .1167 sec.

T_Z = time to drop to zero drag load = $\frac{T_T + T_S}{4}$ = .0292 sec. (empirical formula). (108)

Step III. Important Modes:

The first mode only, (frequency = natural frequency of gear fore and aft = 73.2 rad/sec) is assumed to be excited by the vertical load component normal to the strut and by the drag load component normal to the strut.

Step IV. Generalized Acceleration Factor of Vibratory Response:

1. It will first be shown that it is not necessary to compute the vibratory response to the normal component of vertical load and the vibrating response to the normal component of drag load separately. Rather the entire vibratory response can be computed from a "double" trapezoid combination of the two forcing functions. Consider:

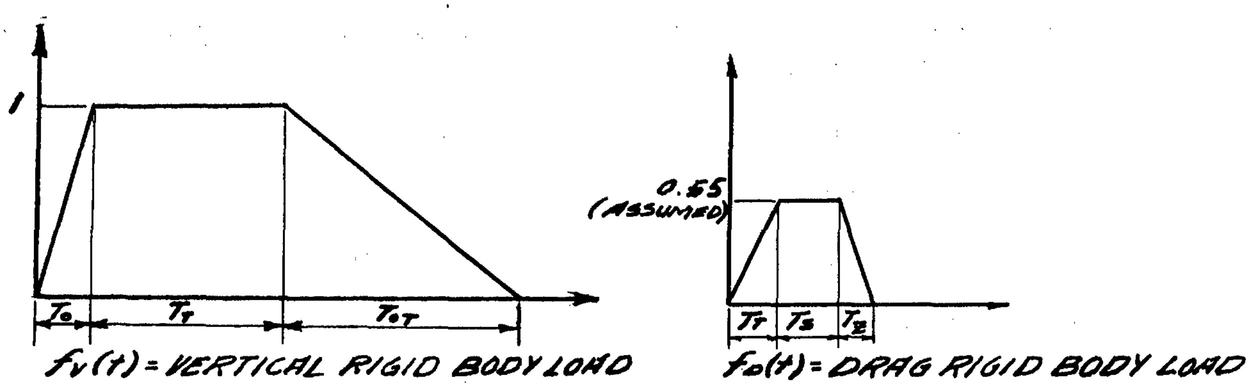


Fig. 8. Time Histories of Vertical Load and of Drag Load in Units of P_{MAX}

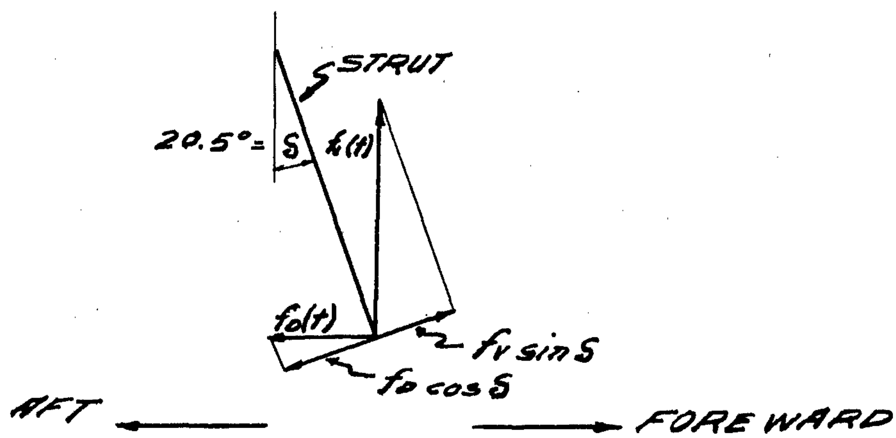


Fig. 9. Load Normal to Strut at Time t Due to Vertical Load and Drag Load

2. Thus the load normal to the strut due to rigid body motion is:

$$a_R(t) = \sin\delta \cdot f_V(t) - \cos\delta \cdot f_\theta(t) \qquad (109)$$

3. It is therefore possible to consider a combined function to represent rigid body acceleration and as forcing function for the vibratory acceleration:

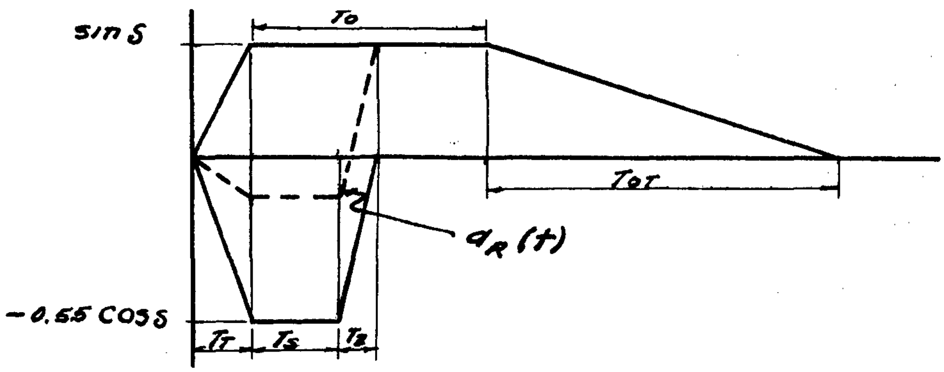


Fig. 10. The Composition of the Trapezoidal Forcing Function

4. In Examples I and II the vibratory acceleration $\ddot{X}$ and rigid body acceleration $F(t)$ are found and added to find the total acceleration response.

5. But in this example the structural force rather than the total acceleration is to be computed. That is, in this example find structural force normal to the strut at the axle due to a one-foot vertical deflection of the axle. Writing equation (1) with $dx = 1$ ft:

$$(\Sigma c_i^2 \, dm_i)\ddot{X} + (\Sigma c_i^2 m_i)\omega^2(1+\bar{g}j)X = (\Sigma c_i f_i)\cdot F(t) \qquad (110)$$

where: $(\Sigma c_i^2 m_i)\omega^2(1+\bar{g}j)X$ is the required structural force

$(\Sigma c_i^2 m_i)\ddot{X}$ is the inertial force

$(\Sigma c_i f_i)\cdot F(t)$ is the external force

Then *: Structural force = $(\Sigma c_i f_i)\cdot F(t) - (\Sigma c_i^2 m_i)\ddot{x}$ (111)

But $\Sigma c_i f_i = P_{MAX}$. and $\ddot{x}$ is obtained in terms of $\frac{\Sigma c_i f_i}{(\Sigma c_i^2 m_i)} f(t)$
Therefore $\Sigma c_i^2 m_i$ can be eliminated from equation (111).

6. It is clear that the generalized acceleration factor does not appear as such.

7. Using the "double" trapezoid derived above, the right side of equation (110) is already scaled relative to P_{max}. In the computation of Table 3, the ordinate of the trapezoid will be multiplied by $\frac{P_{max}}{\text{1/2 Airplane Weight}}$ so that the structural force is finally non-dimensionalized in units of 1/2 airplane weight.

Step V. Vibratory Acceleration Response:

Table 3 computes the vibratory response by the desk-calculator method.

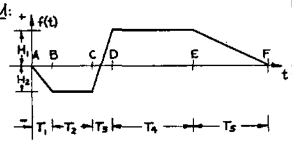
Step VI. Total Acceleration Time History:

The final dimensionless result of $\frac{\text{Force normal to strut at axle}}{\text{One-half weight of airplane}}$ is composed and compared with experimental results in Figure 11. The experimental force normal to the strut at the axle was computed from the measured drag link load by taking a summation of moments about the top of the strut assuming the oleo to be fully extended.

* Of course the differential equation (5) could be solved this time for x rather than $\ddot{x}$, but it is convenient to utilize the methods of Section II.

I. THE PROBLEM:

GIVEN FORCING FUNCTION $f(t)$
FIND THE RESPONSE $\ddot{x}$ FROM:
$\ddot{x}+\omega^2(1+\bar{g}j)x = f(t)$



II. BASIC DATA (Low)

TITLE: B17G - Vib. Acc. Normal to Strut at Axle
COMPUTER: KLE

1 T_1	2 T_2	3 T_3	4 T_4	5 T_5
.0484	.0683	.0292	.0993	.2970
6 H_1	7 H_2	8 ω	9 $\bar{g}$	
-.3489	+.1644	73.2	.10	

(ω in) RADIANS/SEC.

III. INITIALLY COMPUTABLE CONSTANTS

10 ωT_1 = 8X1	11 ωT_2 = 8X2	12 ωT_3 = 8X3	13 ωT_4 = 8X4	14 ωT_5 = 8X5
3.5429	4.9996	2.1374	7.2688	21.7404
15 $\frac{\bar{g}\omega}{2} = \frac{1}{2}(8X9)$	16 $\frac{H_2}{\omega T_1} = \frac{7}{10}$	17 $H_1 - H_2$ = 6-7	18 $\frac{H_1-H_2}{\omega T_3} = \frac{17}{12}$	19 $\frac{H_1}{\omega T_5} = \frac{6}{14}$
3.66	+.0464	-.5133	-.2402	-.0160
20 $\frac{\bar{g}\omega T_1}{2}$ = 15X1	21 $\frac{\bar{g}\omega T_2}{2}$ = 15X2	22 $\frac{\bar{g}\omega T_3}{2}$ = 15X3	23 $\frac{\bar{g}\omega T_4}{2}$ = 15X4	24 $\frac{\bar{g}\omega T_5}{2}$ = 15X5
.1771	.2500	.1069	.3634	1.0870
25 e^{-20}	26 e^{-21}	27 e^{-22}	28 e^{-23}	29 e^{-24}
.8377	.7788	.8986	.6953	.3372

IV. IN AB, $\ddot{x} = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega t}{2}} \sin \omega t$

AT B. $\ddot{x}_B = \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \sin \omega T_1$

30 16X25	31 sin 10	32 30X31
.0389	-.3906	-.0152 ← $\ddot{x}_B$
	33 cos 10	34 30X33
	-.9206	-.0358

V. IN BC, $\ddot{x} = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega t'}{2}} \sin(\omega t'+\phi_1)$ $t' = t - T_1$

AT C, $\ddot{x}_C = \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \sin(\omega T_2+\phi_1)$

WHERE $\begin{cases} R_1 = -\frac{H_2}{\omega T_1} + \frac{H_2}{\omega T_1} e^{-\frac{\bar{g}\omega T_1}{2}} \cos \omega T_1 \\ \phi_1 = \arctan \frac{\ddot{x}_B}{R_1} \text{ (SEE BELOW)} \end{cases}$

35 34-16	36 32^2+35^2	37 $\sqrt{36}$	38 26X37	
-.0822	.00698788	.0836	.0651	
39 32/35	40 ϕ_1 [radians]	41 11+40	42 sin 41	43 38X42
.18491	3.3244	8.3240	+.8916	+.0580 ← $\ddot{x}_C$
			44 cos 41	45 38X44
			-.4529	-.0295

TO DETERMINE
(a) QUADRANT: III
$\begin{Bmatrix}\sin\phi_1 \\ \cos\phi_1\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}32 \\ 35\end{Bmatrix}$
(b) ANGLE: $\tan \phi_1 = 39$

VI. IN CD, $\ddot{x} = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega t''}{2}} \sin(\omega t''+\phi_2)$ $t'' = t' - T_2$

AT D, $\ddot{x}_D = \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \sin(\omega T_3+\phi_2)$

WHERE $\begin{cases} R_2 = \frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_B^2+R_1^2}\, e^{-\frac{\bar{g}\omega T_2}{2}} \cos(\omega T_2+\phi_1) \\ \phi_2 = \arctan \frac{\ddot{x}_C}{R_2} \text{ (SEE BELOW)} \end{cases}$

46 45+18	47 43^2+46^2	48 $\sqrt{47}$	49 27X48	
-.2697	.07610209	.2759	.2479	
50 43/46	51 ϕ_2 [RADIANS]	52 12+51	53 sin 52	54 49X53
-.21505	2.9298	5.0672	-.9377	-.2325 ← $\ddot{x}_D$
			55 cos 52	56 49X55
			+.3474	+.0861

TO DETERMINE
(a) QUADRANT: II
$\begin{Bmatrix}\sin\phi_2 \\ \cos\phi_2\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}43 \\ 46\end{Bmatrix}$
(b) ANGLE: $\tan \phi_2 = 50$

VII. IN DE, $\ddot{x} = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega t'''}{2}} \sin(\omega t'''+\phi_3)$ $t''' = t'' - T_3$

AT E, $\ddot{x}_E = \sqrt{\ddot{x}_D^2+R_3^2}\, e^{-\frac{\bar{g}\omega T_4}{2}} \sin(\omega T_4+\phi_3)$

WHERE $\begin{cases} R_3 = -\frac{H_1-H_2}{\omega T_3} + \sqrt{\ddot{x}_C^2+R_2^2}\, e^{-\frac{\bar{g}\omega T_3}{2}} \cos(\omega T_3+\phi_2) \\ \phi_3 = \arctan \frac{\ddot{x}_D}{R_3} \text{ (SEE BELOW)} \end{cases}$

57 56-18	58 54^2+57^2	59 $\sqrt{58}$	60 28X59	
+.3263	.16052794	.4006	.2785	
61 54/57	62 ϕ_3 [RADIANS]	63 13+62	64 sin 63	65 60X64
-.71253	-.6191	6.6497	+.3584	+.0998 ← $\ddot{x}_E$
			66 cos 63	67 60X66
			+.9336	+.2600

TO DETERMINE
(a) QUADRANT: IV
$\begin{Bmatrix}\sin\phi_3 \\ \cos\phi_3\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}54 \\ 57\end{Bmatrix}$
(b) ANGLE: $\tan \phi_3 = 61$

VIII. IN EF, $\ddot{x} = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega t''''}{2}} \sin(\omega t''''+\phi_4)$ $t'''' = t''' - T_4$

AT F, $\ddot{x}_F = \sqrt{\ddot{x}_E^2+R_4^2}\, e^{-\frac{\bar{g}\omega T_5}{2}} \sin(\omega T_5+\phi_4)$

WHERE $\begin{cases} R_4 = -\frac{H_1}{\omega T_5} + e^{-\frac{\bar{g}\omega T_4}{2}} \sqrt{\ddot{x}_D^2+R_3^2} \cos(\omega T_4+\phi_3) \\ \phi_4 = \arctan \frac{\ddot{x}_E}{R_4} \text{ (SEE BELOW)} \end{cases}$

68 67-19	69 65^2+68^2	70 $\sqrt{69}$	71 29X70	
+.2760	.08613604	.2935	.0990	
72 65/68	73 ϕ_4 [RADIANS]	74 14+73	75 sin 74	76 71X75
+.36159	.3470	22.0874	.0962	.0095 ← $\ddot{x}_F$

TO DETERMINE
(a) QUADRANT: I
$\begin{Bmatrix}\sin\phi_4 \\ \cos\phi_4\end{Bmatrix}$ HAS THE ALGEBRAIC SIGN OF $\begin{Bmatrix}65 \\ 68\end{Bmatrix}$
(b) ANGLE: $\tan \phi_4 = 72$

IX. INSTRUCTIONS FOR SKETCHING

IN EACH INTERVAL THE RESPONSE IS OF THE FORM $\ddot{x} = Ke^{-\frac{\bar{g}\omega T}{2}} \sin(\omega T+\phi)$. THE VALUES AT DISCONTINUITIES HAVE BEEN FOUND IN IV - VIII. FOR SKETCHING, THE ZEROS [ie., $\sin(\omega T+\phi)=0$] AND PEAKS [ie., $\sin(\omega T+\phi)=\pm 1$, APP.] ARE SUFFICIENT ADDITIONAL POINTS. SOME VALUES OF $\omega T+\phi$ FOR WHICH $\sin(\omega T+\phi)=0,\pm 1$ ARE IN XI. BUT SINCE $0 \le T \le T_i$ IN EACH INTERVAL, ONLY THE VALUES OF $\omega T+\phi$ BETWEEN ϕ_i AND $\omega T_i+\phi_i$ CAN BE USED. THESE LIMITS OF $\omega T+\phi$ ARE LISTED IN X. PARTS XII - XVI THEN COMPUTE PLOTTING POINTS AT PEAKS AND ZEROS.

X. LIMITS OF $(\omega T+\phi)$

INTERVAL	AB	BC	CD	DE	EF
FROM	0	ϕ_1 = 40	ϕ_2 = 51	ϕ_3 = 62	ϕ_4 = 73
		3.3244	2.9298	-.6191	.3470
TO	ωT_1 = 10	$\omega T_2+\phi_1$ = 41	$\omega T_3+\phi_2$ = 52	$\omega T_4+\phi_3$ = 63	$\omega T_5+\phi_4$ = 74
	3.5429	8.3240	5.0672	6.6497	22.0874

XI. VALUES OF $\sin(\omega T+\phi)$ FOR COMPUTING ZEROS & PEAKS

$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$	$\omega T+\phi$	$\sin(\omega T+\phi)$
-10.996	+1	-3.142	0	+4.712	-1
-9.425	0	-1.571	-1	+6.283	0
-7.854	-1	0	0	+7.854	+1
-6.283	0	+1.571	+1	+9.425	0
-4.712	+1	+3.142	0	+10.996	-1

XII. ZEROS AND PEAKS BETWEEN A & B

NOTE: $\phi = 0$ IN A-B

	77 ωt [FROM X & XI]	78 $\sin \omega t$ [FROM XI]	79 $\frac{\bar{g}}{2}\times\omega t$ = .05 x ωt	80 e^{-79}	81 $t = \frac{77}{8}$ (PLOT)	82 $\ddot{x}$ = 78X80X16 (PLOT)
a	1.571	+1	.0786	.9244	.0215	+.0429
b	3.142	0	—	—	.0429	0
c						
d						
e						

XIII. ZEROS & PEAKS BETWEEN B & C

	83 $\omega T+\phi$ [FROM X & XI]	84 $\sin(\omega T+\phi)$ [FROM XI]	85 ωT = 83-40	86 $\frac{\bar{g}}{2}\times\omega T$ = .05X85	87 e^{-86}	88 $T = \frac{85}{8}$	89 t = 1+88 (PLOT)	90 $\ddot{x}$ = 84X87X37 (PLOT)
a	4.712	-1	1.388	.0694	.9330	.0190	.0674	-.0780
b	6.283	0	2.959	—	—	.0404	.0888	0
c	7.854	+1	4.530	.2265	.7973	.0619	.1103	+.6667
d								
e								

XIV. ZEROS & PEAKS BETWEEN C & D

	91 $\omega T+\phi$ [FROM X & XI]	92 $\sin(\omega T+\phi)$ [FROM XI]	93 ωT = 91-51	94 $\frac{\bar{g}}{2}\times\omega T$ = .05X93	95 e^{-94}	96 $T = \frac{93}{8}$	97 t = 1+2+96 (PLOT)	98 $\ddot{x}$ = 92X95X48 (PLOT)
a	3.142	0	.212	—	—	.0029	.1196	0
b	4.712	-1	1.783	.0892	.9147	.0244	.1411	-.2524
c								
d								
e								

XV. ZEROS & PEAKS BETWEEN D & E

	99 $\omega T+\phi$ [FROM X & XI]	100 $\sin(\omega T+\phi)$ [FROM XI]	101 ωT = 99-62	102 $\frac{\bar{g}}{2}\times\omega T$ = .05X101	103 e^{-102}	104 $T = \frac{101}{8}$	105 t = 1+2+3+104 (PLOT)	106 $\ddot{x}$ = 100X103X59 (PLOT)
a	0	0	.619	—	—	.0085	.1544	0
b	1.571	+1	2.190	.1095	.8963	.0299	.1758	+.3591
c	3.142	0	3.761	—	—	.0514	.1973	0
d	4.712	-1	5.331	.2666	.7660	.0728	.2187	-.3069
e	6.283	0	6.902	—	—	.0943	.2402	0

XVI. ZEROS & PEAKS BETWEEN E & F

	107 $\omega T+\phi$ [FROM X & XI]	108 $\sin(\omega T+\phi)$ [FROM XI]	109 ωT = 107-73	110 $\frac{\bar{g}}{2}\times\omega T$ = .05 X 109	111 e^{-110}	112 $T = \frac{109}{8}$	113 t = 1+2+3+4+112 (PLOT)	114 $\ddot{x}$ = 108X111X70 (PLOT)
a	1.571	+1	1.224	.0612	.9406	.0167	.2619	+.2761
b	3.142	0	2.795	—	—	.0382	.2834	0
c	4.712	-1	4.365	.2183	.8039	.0596	.3048	-.2359
d	6.283	0	5.936	—	—	.0811	.3263	0
e	7.854	+1	7.507	.3754	.6870	.1026	.3478	+.2016
	10.996	-1	10.649	.5325	.5871	.1455	.3907	-.1723
	14.137	+1	13.790	.6895	.5018	.1884	.4336	+.1473
	17.279	-1	16.932	.8466	.4289	.2313	.4765	-.1259

TABLE 3

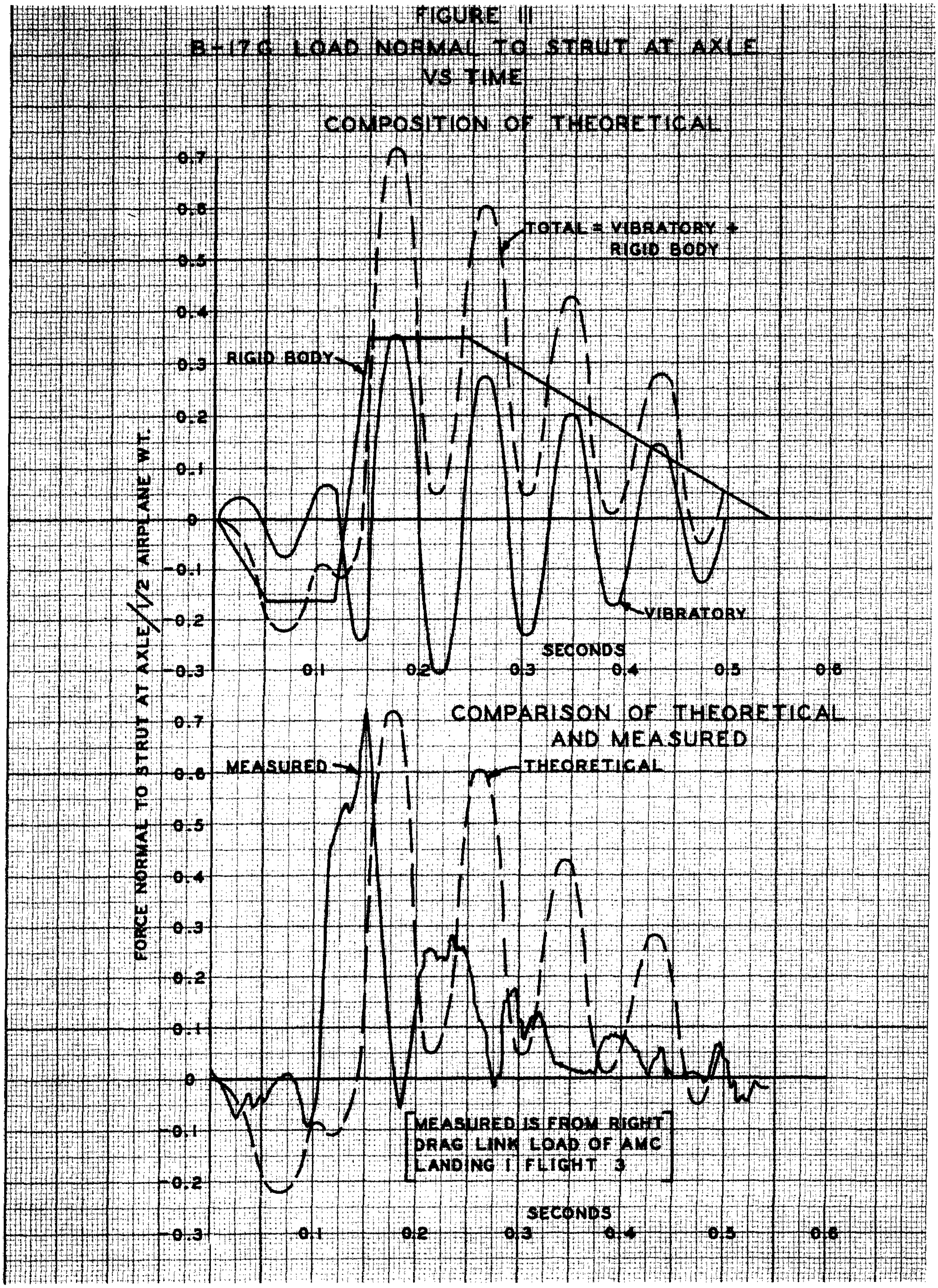


FIGURE 11
B-17G LOAD NORMAL TO STRUT AT AXLE
VS TIME
COMPOSITION OF THEORETICAL
TOTAL = VIBRATORY + RIGID BODY
RIGID BODY
VIBRATORY
FORCE NORMAL TO STRUT AT AXLE/1/2 AIRPLANE WT.
SECONDS
COMPARISON OF THEORETICAL AND MEASURED
MEASURED
THEORETICAL
MEASURED IS FROM RIGHT DRAG LINK LOAD OF AMC LANDING 1 FLIGHT 3
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0.1
0.2
0.3
0.4
0.5
0.6

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