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ANALYSIS OF THE ROTARY REGENERATOR FOR GAS TURBINE APPLICATIONS AND INVESTIGATION OF REGENERATOR SEAL LEAKAGE

BY

D. B. HARPER

FOR

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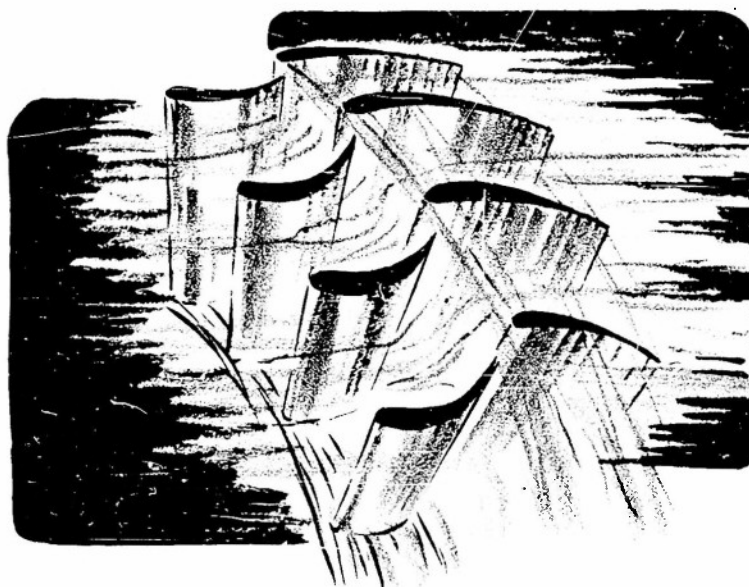
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ANALYSIS OF THE ROTARY REGENERATOR
FOR GAS-TURBINE-CYCLE APPLICATIONS;
AND AN ANALYSIS AND INVESTIGATION OF
ROTARY REGENERATOR SEAL LEAKAGE

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FOR

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TABLE OF SYMBOLS

A — leakage area constant, $= \alpha \delta \delta L_s \left(\frac{2g}{RT_{oh}} \right)^{1/2}$

A' — matrix area/unit volume

a — area

B — leakage displacement area constant $= \frac{V_c}{RT_o}L$

c — specific heat of matrix material

C_p — specific heat of gas, constant pressure

D_e — equivalent diameter

D_R — rotor diameter

G — mass velocity of flow

G' — mass velocity (matrix frontal area)

g — acceleration of gravity

h — coefficient of heat transfer

k — ratio of specific heats

L — length of seal shoe in direction of leakage

L_s — length of seal shoe perpendicular to leakage path

L_R — length of matrix flow path

m' — matrix mass/unit volume

N — number, number of screens

n — number, number of passages under seal

P — pressure

ΔP — pressure difference

R — gas constant

R_p — pressure loss ratio $= \left(\frac{\Delta P}{P} \right)_G / \left(\frac{\Delta P}{P} \right)_A$

r — pressure ratio

r_h — hydraulic radius

r_c — compression ratio

S	– matrix velocity at seal shoe
S_A	– air side matrix frontal area
S_g	– gas side matrix frontal area
T	= temperature
V	= velocity
v^2	– rotor, matrix, free volume under seal shoe
W, W_T	– flow rate, total leakage
W_L	– flow rate, clearance leakage
W_D	– flow rate, displacement leakage
W_{NRT}	– flow rate, net regenerator seal leakage
W_x	– net cycle work output Btu/lb
W_{xc}	– compressor work Btu/lb
W_{xt}	– turbine work Btu/lb
$\Delta W/W$	– leakage/compressor flow ratio
x	– length dimension
α	– flow coefficient
γ	– carry over factor for labyrinth
δ	– seal shoe clearance
ρ	– density
ξ	– matrix voids fraction
θ	– time period
η	– efficiency, cycle
η_R	– regenerator effectiveness
η_{RI}	– regenerator effectiveness, internal
η_{RE}	– regenerator effectiveness, external
Λ	– reduced length, $A'hL_a/G'c_p$
Λ_o	$= \Lambda_c \left[\dfrac{1}{1 + \dfrac{\Lambda_c}{\Lambda_H} \cdot \dfrac{(Wc_p)_c}{(Wc_p)_H}} \right]$

w - reduced time period $= \dfrac{A'h\theta}{m'c}$

N_{st} - Stanton number $= \dfrac{h}{GC_p}$

N_{Pr} - Prandtl number $= C_p \mu/\underline{c}$

N_{Re} - Reynolds number $= \dfrac{GD_e}{\mu} = \dfrac{G4r_h}{\mu}$

BS/A   - dimensionless leakage parameter

Subscripts

A   - air side

G   - gas side

H   - hot side

C   - cold side

1   - inlet

2   - outlet

ABSTRACT

This report covers those aspects of the rotary-regenerator gas-turbine-cycle combination that have not yet received treatment.

The space requirements of heat-exchangers for gas-turbine-cycles are discussed and an analysis is made to illustrate the size advantage of the rotary-regenerator.

An analysis for rotary-regenerator leakage is presented and the verifying results of experimental tests of a simulated sealing situation are included. The effect of leakage on the calculated regenerator effectiveness is analyzed.

The optimum efficiency conditions of the gas-turbine cycle including leakage and pressure loss are studied; and a sample regenerator design for a specific gas turbine plant is carried as far as the presentation of design point curves for sizing the rotary-regenerator.

A method of reducing leakage is presented and the results of experimental verification are shown.

Conclusions are drawn regarding the best general configuration of the rotary regenerator with respect to the leakage problem.

I. INTRODUCTION

Heat exchangers that store energy in a solid medium, by virtue of a temperature rise, during the passage of a hot gas and reject it during the passage of a colder gas are usually only found in regenerative appli-cations in which what would otherwise be waste heat is recovered and used to the benefit of the process. Hence, storage heat exchangers are commonly called regenerators. They have been in use for many years in connection with open hearth furnaces, glass ovens, and blast furnaces. Such regenerators have large brickwork matrices for a storage medium, and the alternate flows of air and hot gas are controlled by valves. A duplicate regenerator is usually employed to insure continuity of operation as the reversal period may be long as an hour. The classical regenerator analyses of Hausen (13) and Nusselt (24) were concerned with this type of regenerator.

In the rotary-regenerator, Fig. 1-1, the matrix (or storage medium) is mounted on a disk or drum which, as it slowly rotates, is passed through the hot and cold streams. Heat is transferred to the matrix while in the hot stream, and from the matrix while in the cold stream. The net result being a heat transfer from the hot stream to the cold stream. A sealing system separates the flow streams and serves as a valving system. The rotary-regenerator is known as a cross-flow regenerator, as relative to the gas streams, the solid is passing in cross flow. A longitudinal flow regenerator is possible, and would utilize a chain or thin disk type matrix rotating in the plane of the flow streams. Cox and Stevens (8) show that a longitudinal flow regenerator is very sensitive to the relative velocity between matrix and fluid. This thesis is only concerned with the cross-flow type rotary regenerator.

The rotary-regenerator made its appearance in the Ljungström air-preheater which is used in many steam power plants to preheat the combustion air by heat transfer from the flue gases. The conception of a rotary-regenerator for use as a gas-turbine plant heat exchanger is generally attributed to Ritz (25). Rotary-regenerators for this service have been the subject of a number of theoretical and experimental investigations in recent years. No rotary-regenerator suitable for industrial type gas turbine plants is known to have been built at this time.

This thesis analyses the effect of the rotary-regenerator on the gas turbine cycle, and includes the results of an experimental investigation of the seal leakage flow. The object is to enable unified design of the gas-turbine cycle, rotary-regenerator combination as their performance parameters are closely related.

II. THE SPACE REQUIREMENTS OF REGENERATIVE HEAT-EXCHANGERS IN A GAS TURBINE CYCLE

The simple CBT[1] gas-turbine cycle has a relatively low thermal efficiency compared to that of conventional prime-movers. Other than increasing the compression ratio to 12 or 16, and with it the upper temperature level, an improvement can only be attained by addition of a regenerative heat-exchanger. Intercooled compression and re-heat expansion, while increasing the specific work output, reduce the thermal efficiency unless in a regenerative cycle.

If conventional shell and tube type heat-exchangers with normal tube diameters are used in a gas-turbine cycle a very large unit results if a high thermal recovery is desired. The thermal recovery system at the Beznau 40,000 kw plant in Switzerland occupies a space 216 ft. by 29 ft. (2) and illustrates this point. While stationary power plants have almost unlimited space available, the equipment and building costs might well make a more compact recovery system desirable.

The use of a CBTX locomotive cycle probably represents the extreme of restricted space in non-aircraft applications. The shell and tube heat-exchangers in Brown Boveri built locomotives (3), while using considerable space are only about 40% effective giving a thermal efficiency for the cycle scarcely different from that obtained with CBT cycle locomotives in the United States and England. A compact heat-exchanger of high effectiveness would be most desirable in this application as the gas-turbine locomotive will have to compete with the highly efficient diesel type.

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1. The conventional notation: C-compressor, B-burner or combustion chamber, T-turbine, X-heat exchanger.

The space requirements for ship propulsion gas turbine plant heat-exchangers would depend on the type of vessel and would be of importance.

In aircraft applications the regenerative cycle could only be used, on turbo-prop engines. While Ritz's original rotary regenerator studies were for this application, the weight and the ducting complications rule out all recovery systems with the possible exception of the liquid coupled system.

The preceding discussion indicates that a gas-turbine plant heat-exchanger should occupy the smallest possible volume. A brief study of the flow friction and heat transfer relationships indicate that small heat-exchanger volume can be attained with passages of small hydraulic diameter (neglecting fouling problems for the present). As passage size is reduced the number of passages increase while the passage length must decrease if the pressure losses are to be maintained constant. The conventional counter-flow shell and tube type heat-exchanger cannot exploit this characteristic very far as a large number of small tubes introduce construction difficulties and short tube lengths prevent the use of counterflow.

THE CROSS-FLOW PLATE-FIN HEAT EXCHANGER

The cross-flow plate fin heat exchanger, Fig. 2-1(a) can utilize small size passages without difficulty. This type of exchanger is analyzed in Appendix A for the conditions of a gas turbine cycle having a cold stream at 60 psia and 400° F, a hot stream at 15 psia and 1140° F, and a flow rate of 45 lb/sec. (approximately equivalent to the conditions of a 4000 H.P. plant). The heat exchanger is assumed to have a constant temperature effectiveness η_R of 0.80 and a constant pressure loss

$$\left(\frac{\Delta P}{P}\right)_G + \left(\frac{\Delta P}{P}\right)_A = 0.04$$

The solution for minimum exchanger volume as a function of plate spacing X is shown by Fig. 2-1(b).

Fig. 2-1(b) shows the decrease of exchanger volume as passage size is decreased and also the reduction of passage length (L_A and L_G). As the air and gas passages are perpendicular to each other, the height of the plate pile L_T increases rapidly as L_A and L_G decrease. For 25 plates per inch (X = 0.04 in.) the dimensions are 0.16 ft. by 0.7 ft. by 130 ft. Thus, while a plate-fin heat exchanger can be of small volume its dimensions become unwieldy. The total length L_T would have to be divided into a number of interconnected units.

THE ROTARY REGENERATOR

In Appendix A a rotary regenerator is analyzed under the same assumptions as the preceding cross-flow exchanger, using the same plate-fin type of surface, and solving for minimum exchanger volume as a function of plate spacing X. (The rotary regenerator heat transfer relations used are discussed in detail in Section III).

The results of this calculation are shown by dotted lines on Fig. 2-1(b). Here the volume can be seen as essentially the same as for the cross-flow type, but exchanger dimensions as given by length L_R diameter D_R remain reasonable even for the smallest plate spacings. Thus, a rotary regenerator can be made small in volume and still be built in a single unit up to quite large flow capacities. In addition to this advantage, the adjacent passages do not have to withstand the full pressure difference as a large number of them will be under the seal shoe at one time and the pressure difference between adjacent passages will be small.

Another possibility is to completely separate the pressure retaining and the heat transfer components of the rotary regenerator. This can be accomplished by constructing a compartmented rotor. The compartment walls together with the seal shoes will prevent leakage and retain the pressures. The matrix will be individually mounted in each compartment and may consist of layers of wire screen or any other finely divided material which gives the desired heat transfer and flow friction characteristics.

A fouling problem will be present in any heat exchanger with small flow passages but to some extent the rotary regenerator will be self-clearing as the air and gas flows take place in opposite directions in the same passages. Additionally there will be fairly sudden pressure changes when a passage or compartment passes under the seal shoes resulting in rapid changes in local velocity within the matrix.

The preceeding discussion has pointed out the characteristics of the rotary regenerator which lead to considering it for application as a gas-turbine cycle heat exchanger. Together with these, the inherent disadvantages of the rotary regenerator; namely loss of compressed air due to the positive displacement occasioned by the rotation of the matrix and due to leakage through the clearances of the sealing arrangements. These problems are discussed in more detail in the following sections.

III. HEAT TRANSFER RELATIONSHIPS IN THE ROTARY REGENERATOR

Rotary Regenerator Theory:

Fig. 3-1(a) represents a passage from the regenerator matrix. During the hot flow period, hot gas at a constant initial temperature T_{H1} flows through the passage at a constant flow rate W_H for the duration of the period θ_H. Then air enters the same passage from the opposite end at a constant initial temperature T_{C1} and constant flow rate W_C for a period of length θ_C.

Fig. 3-1(b) is a passage element during the gas flow period.
The principal assumptions made are:

1. infinite thermal conductivity in the passage
   wall perpendicular to the gas flow.

2. zero thermal conductivity in the passage
   wall parallel to the gas flow.

These assumptions become exact as the thickness of the passage
wall approaches zero. Assumption (1) has been analyzed by Saunders and
Smolenieo (30) and they show that for an error of 0.5% ceramic walls must
be less than 0.05 inches thick, and for steel less than 0.2 inches.
Assumption (2) is negligible for ceramic matrices or for packed matrices
(such as screens). For continuous metallic walls the effect is slight
and has been evaluated by Schultz (36) and Hahnemann (10).

Under these assumptions analysis of Fig. 3-1(b) will lead to
the differential equations

$$\frac{\partial T_H}{\partial Y_H} = T_S - T_H$$

$$\frac{\partial T_S}{\partial Z_H} = T_H - T_S$$

<div align="right">3.1</div>

where

$$0 \leq Y_H \leq \Lambda_H = \frac{A' h_H}{G'_H} \frac{L_R}{C_{PH}}$$

$$0 \leq Z_H \leq \pi_H = \frac{A' h_H}{m' c} \Theta_H$$

<div align="right">3.2</div>

Equations (3.1) together with an identical set relating the
variables during the air blow period must be solved under boundary condi-
tions which fix the entry condition of gas and air temperature, and the
temperature gradient in the passage walls at the ends (assumed equal to
zero); together with a reversal condition which fixes the temperature
gradient in the passage walls at the end of one period equal to that at
the beginning of the next.

Solutions have been obtained by Hausen (13), Iliffe (16), Saun-
ders and Smolenieo (30), Boestad (4) and Johnson (17); and their scope and
limitations are summarized by Coppage and London (6). These results
involve finite difference methods at some stage of solution. Fig. 3-2
shows the solution of Saunders and Smolenieo for the symmetrical regenera-
tor $\Lambda_H = \Lambda_C$; $\pi_H = \pi_C$.

Approximate solutions are obtained by Tipler (20), Coppage and London (6) and Schack (32). Coppage and London have extrapolated the more exact solutions by use of their approximate results to allow for variations in the flow heat capacity rates and for non-symmetrical regenerators. Harper, in the discussion to reference (6), presents a partially emperical closed form equation for rotary regenerator internal effectiveness as

$$\eta_{RI} = \left[\frac{\Lambda_o}{\frac{1}{2}\left(1 + \frac{(\overline{W C_P})_C}{(\overline{W C_P})_H}\right)\Lambda_o + 1} \right] \cdot \left[1 - \frac{1}{9}\left(\frac{\pi}{\Lambda}\right)_C^2 \right] \qquad 3.3$$

where

$$\Lambda_o = \Lambda_C \left[\frac{1}{1 + \frac{\Lambda_C}{\Lambda_H} \cdot \frac{(\overline{W C_P})_C}{(\overline{W C_P})_H}} \right] \qquad 3.4$$

which is accurate within about two percent over the range

$$1 \leq \Lambda_o \leq 10$$

$$0 \leq \left(\frac{\pi}{\Lambda}\right)_C \leq 1$$

$$0.9 \leq \frac{(\overline{W C_P})_C}{(\overline{W C_P})_H} \leq 1$$

Equation 3.3 enables a close value of the effectiveness to be obtained without requiring interpolation of values from a series of graphs.

HEAT TRANSFER AND FLOW FRICTION DATA FOR MATRIX MATERIALS

Heat transfer and flow friction data for materials suitable for use in rotary regenerators has till recently been scarce. The most complete work dealing with rotary regenerator matrices is that of Coppage (7) which contains the results of extensive experimentation with wire screens and spheres, and includes a large bibliography of related topics. Kays, London and Johnson (15) give data for plate fin type surfaces; and Saunders and Ford (29) and Rose (26) give data for beds of granular particles.

IV. ROTARY REGENERATOR LEAKAGE

GENERAL DISCUSSION OF LEAKAGE

The prevention of leakage flow between the hot and cold streams in a rotary regenerator presents a design problem, the success of which will determine the success of this form of heat exchanger.

The sealing problem will be severe in applications such as the gas turbine where a high pressure difference exists between the two streams. The sealing arrangements must adequately prevent leakage flow, and at the same time not introduce high resistance to rotation. Neglecting small aerodynamic effects[2] the power required to rotate the rotor is entirely absorbed in friction. Since this power input is charged directly against the heat exchanger, the friction loss must be small.

Leakage in a rotary regenerator in a gas turbine cycle involves the loss of compressed air to the low pressure exhaust side. This loss is occasioned by flow through finite clearances and by the positive displacement of air trapped in the matrix voids as the rotor rotates.

The sealing arrangements are of two general types; continuous cylindrical seals similar to a shaft seal, and the main seals which divide the high and low pressure sides in the plane of fluid streams. The seals must accommodate deflections of the rotor due to thermal distortion and the main seals must be loaded to balance the pressure existing beneath their sealing faces. The continuous cylindrical seals can be handled by careful mechanical design. The main seals present a more difficult problem because of the effect of the displacement or carry over loss and the pressure forces on the sealing face which must be balanced.

The main seals are analyzed in this report to study their probable leakage characteristics and the results of experimental tests on this type of seal is recorded.

ANALYSIS OF THE MAIN SEAL SHOE LEAKAGE

The two possible classes of sealing arrangement in a rotary regenerator are shown in Fig. 4-1. Fig. 4-1(a) illustrates the case where the seal bears directly upon the regenerator matrix. This type assumes the use of a matrix of passages, the walls of which prevent the direct flow of air from the high to the low pressure side. Such a matrix could be made up of a close packed bundle of fine tubes, or from various arrangements of flat and corrugated plate. As shown in the illustration, a number of these passages would be between the seal shoes at any instant

---

2. Kryniszak (14, 14a) proposes to drive a rotary regenerator by installing guide vanes and rotor (matrix) blades.

so that the pressure difference to be retained by a single passage wall is a small part of the total pressure difference.

Fig. 5-1(b) illustrates the case where the seal bears against a system of rotor compartments. This type of rotor must be used where packed screens or other porous matrix types are to be used. Here too, a number of compartments are underneath the seal at any instant.

A cross section through the seals of Fig. 4-1 is illustrated schematically in Fig. 4-2(a) where the important relationships are reproduced.

Here the clearance δ through which leakage will take place is shown. Movement of the matrix will cause a positive displacement of air from one side to the other. The probable shape of the pressure gradient under the seal shoe is also indicated.

The leakage at the seal may be analyzed if the following assumptions are made:

a) low air and gas velocities approaching the matrix.

b) low velocity of matrix movement compared with the air velocity under the sealing restrictions.

c) that the flow under the seal shoe past a passage end is similar to flow through an orifice.

d) neglect the resistance to flow of air from within the passage, i.e. the air pressure within the passage is equal to that at the seal face.[3]

e) sufficiently large number of passages, N, under the seal shoe for Δp between adjacent passages to be small.

f) that a mean temperature can be used for the air trapped within half a passage.

g) that the temperature during the expansion of the air trapped in a compartment is constant, (isothermal expansion).

Assumptions (a), (d) and (f) enable the matrix and rotor to be split on the center plane and the passages considered to be closed off at that plane. Thus, Fig. 4-2(b) will be analyzed and considered equivalent, with respect to the leakage, as Fig. 4-2(a).

---

3. Appendix C - "The Effect of Passage Resistance on Leakage Flow."

Since the velocity of matrix movement, S, is much less than the gas velocity under the restrictions (assumption b), we may assume that viscous effects due to matrix movement does not change the form of the flow. Hence, the flow rate through a given restriction will be assumed to be a function of the pressure, and the pressure difference, (assumptions (c) and (e)) similar to flow through an orifice where the critical pressure ratio has not been reached. Thus, from equation B.9 of Appendix B

$$W_{L_x} = \propto \gamma \delta L_s \left(- \frac{2g}{RT_x} P_x \Delta P_x \right)^{\frac{1}{2}} \qquad 4.1$$

In order to simplify the analysis of the combined effects of the flow due to clearance and the flow due to matrix movement the pressure under the seal shoe will be assumed a continuous function of X. This assumption becomes more correct for a large number of passages. In addition the large heat capacity of the matrix material in each compartment compared to that of the trapped air will be assumed to cause heat transfer sufficient to maintain T_x constant in successive compartments equal to T_o (assumption g).

Since
$$\Delta P = \frac{dP}{dx} \Delta x$$

$$\Delta x = \frac{L}{n}$$

$$\Delta P = \frac{L}{n} \frac{dP}{dx} \qquad 4.2$$

Rearranging equation 4.1 and introducing 4.2

$$W_{L_x} = A \left(- LP \frac{dP}{dx} \right)^{\frac{1}{2}} \qquad 4.3$$

where
$$A = \propto \gamma \delta L_s \left(\frac{2g}{RT_o n} \right)^{\frac{1}{2}} \qquad 4.3a$$

At any point X under the seal shoe the displacement flow due to matrix movement will be proportional to the density and the free volume flow rate

$$W_D = \rho \, (\text{vol. flow rate})$$

$$W_{D_x} = \rho_x \frac{v^e}{L} S \qquad 4.4$$

Where $\mathcal{V}^e$ is the free volume of rotor and matrix beneath the seal shoe at any one time. For disk type regenerators where angular measure is more representative of sealing length equation 4.4 would become

$$W_{D_\theta} = \rho_\theta \frac{\mathcal{V}^e}{\theta_T} \cdot \omega \qquad\qquad 4.4a$$

As a function of pressure equation 4.4 can be written

$$W_{D_x} = \frac{P_x}{RT_o} \cdot \frac{\mathcal{V}^e}{L} S$$

or $\qquad\qquad W_D = BPS \qquad\qquad 4.5$

where $\qquad\qquad B = \dfrac{\mathcal{V}^e}{RT_\eta L} \qquad\qquad 4.5a$

Since the temperature is constant, by assumption (g), and since P is a function of X, and since the total leakage W_T must be constant from continuity considerations we have

$$W_T = W_L (P) + W_D (P)$$

$$W_T = W_L (x) + W_D (x)$$

but $\qquad\qquad \dfrac{dW_T}{dx} = 0 \qquad\qquad 4.6$

therefore $\qquad\qquad \dfrac{dW_L}{dx} = - \dfrac{dW_D}{dx} \qquad\qquad 4.7$

Using 4.3 and 4.5 in 4.7 will give

$$\left(\frac{dP}{dx}\right)^2 + P \frac{d^2P}{dx^2} = 2 \underline{K} P^{\frac{1}{2}} \left(- \frac{dP}{dx}\right)^{\frac{3}{2}} \qquad\qquad 4.8$$

$$\underline{K} = \frac{BS}{AL^{1/2}} \qquad\qquad 4.8a$$

the differential equation defining the relationship which must exist between the pressure and the distance along the sealed area.[4]

With the substitution $\qquad \dfrac{dP}{dx} = - y$

$$\left(\frac{dP}{dx}\right)^2 = y^2$$

$$\frac{d^2P}{dx^2} = y\frac{dy}{dP} \qquad\qquad 4.8c$$

equation 4.8 may be rearranged to give

$$\frac{y}{P} + \frac{dy}{dP} = 2\underline{k}\left(\frac{y}{P}\right)^{\frac{1}{2}} \qquad\qquad 4.9$$

with the substitution $\quad \dfrac{y}{P} = z$

$$\frac{dy}{dP} = z + P\frac{dz}{dP} \qquad\qquad 4.9a$$

equation 4.9 becomes

$$\frac{dz}{2z\left(\underline{k}z^{-1/2} - 1\right)} = \frac{dP}{P} \quad 5. \qquad\qquad 4.10$$

Integration of 4.10 gives

$$\ln\frac{z^{-\frac{1}{2}}}{\underline{k}z^{-1/2} - 1} = \ln P + c' = \ln Po$$

which, on clearing of logs and rearranging gives

$$z = \left(\frac{1}{Po} - \underline{k}\right)^2 \qquad\qquad 4.11$$

Using equations 4.8c and 4.10 in 4.11 gives

$$\frac{dP}{dx} = -P\left(\frac{1}{Po} - \underline{k}\right)^2 \qquad\qquad 4.12$$

---

4. If 4.1 had been obtained from B.8 instead of B.9 the differential equation would be

$$\left(-P\frac{dP}{dx}\right)\left(1 - \frac{2L}{kPN}\right)^{-1}\left(\frac{2L}{kPN}\right)\left(\frac{1}{P}\frac{dP}{dx} - \frac{d^2P}{dx^2}\right) + \left(\frac{dP}{dx}\right)^2 + P\frac{d^2P}{dx^2} = 2\underline{k}P^{\frac{1}{2}}\left(-\frac{dP}{dx}\right)^{\frac{3}{2}}\left(1 - \frac{2L}{kPN}\frac{dP}{dx}\right)^{\frac{1}{2}} \quad 4.8b$$

5. If 4.8b had been solved this result would give

$$\frac{dz}{2z\left(1 + \frac{2L}{kN}z\right)\left(\underline{k}z^{-1/2}\left\{1 + \frac{2L}{kN}z\right\} - 1\right)} = \frac{dP}{P} \qquad\qquad 4.10a$$

Rearranging 4.12 and integrating between limits

$$\int_{P_1}^{P} \frac{P\,dP}{\left[\frac{1}{c} - \underline{k}P\right]^2} = -\int_{0}^{x} dx$$

giving

$$\frac{1}{1 - \underline{k}\,cP_1} - \frac{1}{1 - \underline{k}\,cP} - \ell n \frac{1 - \underline{k}\,cP}{1 - \underline{k}cP_1} = \underline{k}^2\,x \qquad 4.13$$

From 4.3, 4.5 and 4.6

$$W_T = A\left(- LP \frac{dP}{dx}\right)^{\frac{1}{2}} + BSP \qquad 4.14$$

Substitute for dP/dx from 4.12 giving

$$W_T = AL^{\frac{1}{2}} \left(\frac{1}{c} - \underline{k}\,P\right) + BSP$$

but

$$\underline{k} = \frac{BS}{AL^{1/2}}$$

therefore

$$W_T = \frac{AL^{1/2}}{c}$$

or

$$c = \frac{AL^{1/2}}{W_T} \qquad 4.15$$

Substituting 4.8a and 4.15 into 4.13 gives

$$\frac{1}{1 - \frac{BS}{W_T} P_1} - \frac{1}{1 - \frac{BS}{W_T} P} - \ell n \frac{1 - \frac{BS}{W_T} P}{1 - \frac{BS}{W_T} P_1} = \left(\frac{BS}{A}\right)^2 \frac{x}{L} \qquad 4.16$$

the solution for rotary regenerator seal leakage. As 4.16 gives neither the leakage quantity W_T or the pressure gradient P vs. X in closed form it is tedius to solve.

Equation 4.16 may be written in the form

$$\frac{1}{1 - \frac{W_{D_1}}{W_T}} - \frac{1}{1 - \frac{W_{D_1}}{W_T} \cdot \frac{r}{r_T}} - \ell n \frac{1 - \frac{W_{D_1}}{W_T} \cdot \frac{r}{r_T}}{1 - \frac{W_{D_1}}{W_T}} = \left(\frac{BS}{A}\right)^2 \frac{x}{L} \qquad 4.17$$

where $r = P/P_2$ and $W_{D_1} = BS\ P_1$, the displacement flow at the high pressure side. This equation has been used to construct the solutions graphs of Figs. 4-3 and 4-4 which give the leakage and pressure gradient readily. To use these plots enter with the known value of

$$\left(\frac{BS}{A}\right)^2 \frac{x}{L}, \quad \frac{x}{L} = 1, \quad \text{and} \quad \frac{r_T}{r_2} = r_T = \frac{P_1}{P_2}.$$

Read the quantity W_T/W_{D_1}, which, since $W_{D_1} = BS\ P_1$ is known, gives W_T. Along the same horizontal line of constant W_T/W_{D_1}, the r_T/r vs. x/L

quantities can be picked off and solved for r vs. x/L. The critical nature of matching the boundary condition at $x/L = 1$ is such that high accuracy in obtaining the pressure gradient will not be realized at large values of BS/A.

Equation 4.16 may be written in the form

$$\frac{1}{1 - \left(\frac{BS}{A}\right)\left(\frac{AP_2}{W_T}\right)r_T} - \frac{1}{1 - \left(\frac{BS}{A}\right)\left(\frac{AP_2}{W_T}\right)r} - \ell n \frac{1 - \left(\frac{BS}{A}\right)\left(\frac{AP_2}{W_T}\right)r}{1 - \left(\frac{BS}{A}\right)\left(\frac{AP_2}{W_T}\right)r_T} = \left(\frac{BS}{A}\right)^2 \frac{x}{L}$$

which is perhaps most convenient for direct numerical solution. Values of Values of $\left(\frac{BS}{A}\right)\left(\frac{AP_2}{W_T}\right)r$ are assumed, and with $\frac{x}{L} = 1$, r must be unity, and $\frac{BS}{A}$ may be determined. With $\left(\frac{BS}{A}\right)$ known $\frac{AP_2}{W_T}$ is obtained by substitution in the assumed quantity. Values of x/L vs. r may now be determined.

Figs. 4-5 and 4-6 show the form of the solution for $r_T = 4$. In Fig. 4-5 the leakage quantity for negative and positive rotation are shown. Since a regenerator must have the seals in pairs, one rotating with the pressure gradient and one against it, the net leakage will be the sum of the leakage from each seal. Fig. 4-5 shows net leakage plotted assuming $-\left(\frac{BS}{A}\right)^+ = \left(\frac{BS}{A}\right)^-$. Fig. 4-6 shows the variation of pressure gradient with rotation. Fig. 4-7 shows a range of values of net leakage if $-\left(\frac{BS}{A}\right)^+ = \left(\frac{BS}{A}\right)^-$ for various pressure ratios.

The effect of choked flow under the seals is analyzed in Appendix D.

Choking will not occur till high values of $\left|\left(\frac{BS}{A}\right)\right|$ are reached. With large $\frac{BS}{A}$ the quantity $\frac{W_L}{W_T} = \frac{W_T - W_D}{W_T} \longrightarrow 0$ as $W_T \longrightarrow W_D$, that is W_L

becomes small and W_T is the term affected by choking. Thus choking under the seal makes no practical change in the value of W_T under the conditions which choking will occur.

EXPERIMENTAL INVESTIGATION

The experimental apparatus described in Appendix E was used to check the leakage theory described in the preceding section. The calibration of the apparatus is described in Appendix E. The apparatus allowed for variation of S, H, r_T, and S, all of which appear in the dimensionless leakage parameter (BS/A) and tests were conducted to verify that this quantity does determine the leakage flow and the form of the pressure gradient.

Figs. 4-8, 4-9 and 4-10 show the measured points and the theoretical curve for $r = 2$, 3, and 4. The correlation is fairly good except for at negative values of BS/A. This may be due partly to small extraneous leakage passages in the apparatus distorting the flow under the steep pressure gradients occuring during negative rotation, and partly due to the steep gradients no longer making the assumption of small pressure differences between adjacent compartments valid. Fig. 4-11 shows the leakage due to clearance vs. BS/A. This quantity is seen to decrease with rotation in either direction.

Figs. 4-12 through 4-17 show the measured pressures and the theoretical pressure gradients. The agreement is seen to be good for moderate BS/A, but the measured points fall short of the theoretical curves for large BS/A. The correlation of different conditions with BS/A is seen to be good even when the theoretical curve is not reached.

The theoretical analysis seems to be verified by the experimental tests with sufficient accuracy to predict seal leakage and the pressure gradient under the seal shoes in design problems.

THE EFFECT OF LEAKAGE ON REGENERATOR EFFECTIVENESS

The regenerator effectiveness calculated by equation 3.3 or any of the references provided may be described as internal regenerator effectiveness, η_{RI}, as it takes no account of the effect of the leakage flow. An analysis by Harper and Rohsenow (12) under the assumptions of Fig. 4-18(a) gives the correction to calculated effectiveness as

$$\eta_{RI} - \eta_{RE} = \frac{\eta_{RI}(\frac{\Delta W}{W})(1 - \eta_{RI})}{2 - (\frac{\Delta W}{W})(1 + \eta_{RI})} \qquad 4.19$$

This quantity is shown plotted on Fig. 4-18(b). The correction is seen to be slight and would not justify a more rigorous analysis.

V. EFFECT OF ROTARY REGENERATOR PERFORMANCE
ON GAS-TURBINE CYCLE PERFORMANCE

In order to evaluate the effect of a rotary-regenerator in a gas-turbine cycle it is necessary to consider not only the regenerator effectiveness but also the pressure and leakage losses occuring due to its use. Considering the gas-turbine cycle of Fig. 5-1 together with the T-S diagram of Fig. 5-2 the following cycle analysis has been carried out.

Assuming constant specific heat and carrying the analysis out on the basis of one pound of air compressed per second, the compressor work is given by

$$\bar{w}_{x_c} = \frac{c_p T_1}{\eta_c} (r_c^{\frac{k-1}{k}} - 1) \qquad 5.1$$

Allowing for the leakage quantity $\Delta W/W$, the turbine work is given by

$$W_{x_T} = (1 - \frac{\Delta W}{W}) \eta_T c_p T_3 (1 - \frac{1}{r_T^{\frac{k-1}{k}}}) \qquad 5.2$$

But $r_T \neq r_c$ due to the pressure losses in the system. These pressure losses occur on the air and gas sides of the regenerator and in the ducting. Assuming small $\Delta P/P$ in the various components, the effect of pressure loss may be introduced as a correction to turbine efficiency rather than expansion ratio (Hawthorne Ref. 13a). Thus

$$\frac{\Delta \eta_T}{T} = \frac{k-1}{k} \frac{\sum \frac{\Delta P}{P}}{(r_c^{\frac{k-1}{k}} - 1)} \qquad 5.3$$

where

$$\sum \frac{\Delta P}{P} = \frac{\Delta P}{P}\Big)_{duct} + \frac{\Delta P}{P}\Big)_H + \frac{\Delta P}{P}\Big)_e \qquad 5.3$$

Introducing 5.3 in 5.2 and rearranging gives

$$W_{x_T} = (1 - \frac{\Delta W}{W}) c_p T_3 \eta_T \left[1 - \frac{(1 + \frac{k-1}{k} \sum \frac{\Delta P}{P})}{r_c^{\frac{k-1}{k}}} \right] \qquad 5.4$$

The heat transferred to the cycle by combustion is given by

$$q = (1 - \frac{\Delta W}{W})(c_p)(T_5 - T_x)$$

or

$$q = (1 - \frac{\Delta W}{W})\left[c_p(T_5 - T_1) - c_p(T_x - T_2) - c_p(T_2 - T_1)\right] \qquad 5.5$$

From the definition of external regenerator effectiveness

$$\eta_{RE} = \frac{T_x - T_2}{T_4 - T_2}$$

or

$$T_x - T_2 = \eta_{RE}\left[(T_5 - T_1) - (T_2 - T_1) - (T_5 - T_4)\right] \qquad 5.6$$

The turbine and compressor work may be written

$$\frac{W_{xT}}{(1 - \frac{\Delta W}{W})c_p} = T_5 - T_4 \qquad 5.7$$

$$W_{xc} = c_p(T_2 - T_1) \qquad 5.8$$

Using 5.7 in 5.6; 5.8 in 5.5 and 5.6; and 5.6 in 5.5 gives

$$q = (1 - \frac{\Delta W}{W})\left\{(1 - \eta_{RE})\left[c_p(T_5 - T_1) - W_{xc}\right] + \eta_{RE}\frac{W_{xT}}{(1 - \frac{\Delta W}{W})}\right\} \qquad 5.9$$

For a regenerator to be considered for a gas turbine cycle, thermal efficiency must be one of the most important factors. The cycle should therefore be designed to operate at or near its maximum thermal efficiency point. The cycle efficiency may be written

$$\eta = \frac{W_x}{q_{in}} = \frac{W_{xT} - W_{xc}}{q} \qquad 5.10$$

If equations 5.1, 5.4 and 5.9 are introduced into 5.10, it can be seen that for fixed temperature limits, leakage, pressure loss, and component efficiencies the efficiency of the cycle is a function of the compression ratio r_c only. For maximum cycle efficiency under given conditions $d\eta/dr_c$ must equal zero. This differentiation has been carried out and gives as its result

$$r_c^{\frac{k-1}{k}} = \frac{\eta_T \frac{T_3}{T_1}(1+\psi)}{\phi - (\phi - \eta_T)\frac{T_3}{T_1}} \cdot \left\{1 \pm \sqrt{\frac{\frac{T_3}{T_1} - 1}{\eta_T \frac{T_3}{T_1}} \cdot \frac{\phi}{1+\psi}\left(\frac{\eta_T \frac{T_3}{T_1}}{\frac{T_3}{T_1} - 1}\right) \cdot \frac{\psi}{\phi} + 1 + \varphi \eta_o\left[(\frac{T_3}{T_1} - 1)\phi - \eta_T \frac{T_3}{T_1}\right]}\right\}$$

$$5.11$$

where

$$\phi = \frac{1 - \eta_{RE}}{\varphi - (1+\varphi)\eta_{RE}}$$

$$\varphi = 1 - \frac{\Delta W}{W}$$

$$\psi = \frac{k-1}{k}\sum \frac{\Delta P}{P}$$

Equation 5.11 has been solved for a cycle operating between 60°F and 1500°F with compressor and turbine efficiencies of 0.85 and 0.88 respectively. A range of regenerator effectiveness, pressure loss and leakage loss have been used. The results of these calculations are given on Figs. 5-3 through 5-7. Each plot is for a given value of $\sum \frac{\Delta P}{P}$ and is entered with the desired value of η_{RE} and $\frac{\Delta W}{W}$. The maximum cycle efficiency, the compression ratio for maximum efficiency, and the net work output may be read off. These results show clearly the loss in cycle efficiency and work output due to regenerator leakage. For regenerators of high effectiveness the point of maximum efficiency occurs at a low compression ratio which is advantageous for reducing leakage loss in the regenerator. However, the specific work output of the cycle is seen to be low at this point. The necessity of keeping regenerator leakage at a minimum and of predicting the leakage quantity as closely as possible is made apparent by this analysis.

VI. ROTARY REGENERATOR DESIGN

In designing the rotary-regenerator, it will be necessary to fix the overall matrix dimensions along with the dimensions of any rotating ducts such that when installed in the gas-turbine plant the desired plant performance will be realized.

A preliminary analysis of the effect of the principle matrix dimensions was made by Harper and Rohsenow (12). This showed that, for optimum plant performance there exists a best rotational speed, matrix flow length, and frontal area. Therefore, for a fixed gas-turbine plant performance there must exist an optimum set of regenerator dimensions. The analysis here will be for a gas turbine cycle of 4000 horsepower output and maximum cycle efficiency of 34 percent. Figures 5-3 through 5-5 of Section V, define a plane in W_x, η_{RE}, $\Delta P/P$ coordinate system on which cycles with this maximum efficiency will be found. Any point on this plane can be defined by the quantities $\Delta P/P$ and $\Delta W/W$ which will be determined by the proportions of the rotary-regenerator (the required effectiveness being attained for each configuration). The correction to the regenerator effectiveness for leakage can be obtained from Fig. 4-12b.

MATRIX CHARACTERISTICS

The matrix type chosen has the geometry found by Coppage (7) to have the most favorable heat transfer-friction power characteristics. This matrix consists of layers of wire screen arranged perpendicularly to the flow direction. The mesh chosen (20 x 20) will not give as compact a unit as closer meshes, but it probably is about the closest mesh for an open cycle plant where combustion products must be passed.

Matrix data:

mesh, 20 x 20 wires per inch x 0.01 inch diameter
18/8 stainless steel
specific heat, $c = 0.12$ Btu/lb F

heat transfer area, $A' = 784$ ft^2/ft^3

mass, $m' = 82.5$ lb/ft^3

hydraulic radius, $r_h = 1.0625 \times 10^{-3}$ ft

screen thickness = 0.02 in

porosity $\xi = 0.852$

From Fig. 6 of reference 7 the heat transfer correlation is given as

$$N_{st} \circ N_{Pr} = 1.50 \, (N'_{Re})^{-0.50} \qquad 6.1$$

and the friction factor curve has been approximated by

$$f = 36 \, (N'_{Re})^{-0.715} \qquad 6.2$$

ROTOR TYPE

The rotor type assumed is shown on Fig. 6-1a. Here, the matrix is mounted in drum form and a section of inlet and outlet ducting rotates with the matrix. This arrangement allows for small main seal shoes placed at the ends of the drum which simplifies the equalizing of seal shoe thrust and the allowance for thermal expansion of the rotor. It does, however, increase the displacement loss. For analysis the voids volume of matrix plus rotor is assumed to be five times the total matrix volume. The drum diameter to length ratio is assumed to be 1.5, to allow space within the drum for the internal ducting. This rotor type also allows for the turbine-compressor shaft to pass through the rotary regenerator shaft, giving a compact plant layout. This arrangement would be particularly desirable for application in a locomotive.

Five rotor partitions are assumed to be under the seal shoe at one time. Each seal shoe is assumed to cover 20 degrees of angle. That is, the useful matrix volume is $320/360$ of the total matrix volume.

ROTARY REGENERATOR HEAT TRANSFER RELATIONSHIPS

From Section III, with the assumption of equal air and gas heat capacity rates, the equation for regenerator effectiveness is given by

$$\eta_{RI} = \left[\frac{\Lambda_o}{\Lambda_o + 1}\right]\left[, -\frac{1}{9}\left(\frac{\pi}{\Lambda}\right)_A^2\right] \qquad 6.3$$

where $\Lambda_o = \Lambda_A\left[\dfrac{1}{1 - \dfrac{\Lambda_A}{\Lambda_G}}\right]$

Here

$$\left(\frac{\pi}{\Lambda}\right)_A = \frac{G'_A C_p \Theta_A}{m' c L_R} = \frac{W_A C_p \Theta_A}{S_A m' c L_R} \qquad 6.4$$

but $\dfrac{S_A L_R}{\Theta_A}$ equals the matrix volume flow rate, and hence, the total displacement leakage may be given by

$$W_D = 5\frac{S_A L_R}{\Theta_A}(\rho_A - \rho_G)$$

or

$$\left(\frac{\pi}{\Lambda}\right)_A = 5\frac{W_A}{W_D} \circ \frac{(r-1)P_G C_p}{RT_{AV} m' c} \qquad 6.5$$

where T_{AV} is the average of the air and gas temperatures occurring under the seal shoes; a good approximation for regenerators of high effectiveness. Thus, for a fixed value of T_{AV} and the gas side pressure P_G, $\left(\frac{\pi}{\Lambda}\right)_A$ is a function of $\dfrac{W_A}{W_D}$ and r only.

Since $\Lambda = \dfrac{A'hL_R}{G'C_p}$ from the heat transfer correlation, Equation 6.1, we can derive

$$\frac{\Lambda_A}{\Lambda_G} = \left(\frac{G_G'}{G_A'r}\right)^{0.50} \qquad \text{6.6}$$

and

$$\Lambda_A = k_1 \frac{L_R}{(G_A')^{0.50}} \qquad \text{6.7}$$

where

$$k_1 = \frac{1.50 \, A'}{N_{Pr}} \cdot \left(\frac{\mu}{4 r_n \xi}\right)^{0.50}$$

Equations 6.5, 6.6 and 6.7 have defined all the quantities appearing in Equation 6.3.

ROTARY REGENERATOR PRESSURE DROP RELATIONSHIPS

The total regenerator pressure drop parameter, $\left(\frac{\Delta P}{P}\right)_R$ is defined as

$$\left(\frac{\Delta P}{P}\right)_R = \left(\frac{\Delta P}{P}\right)_A + \left(\frac{\Delta P}{P}\right)_G \qquad \text{6.8}$$

Let R_p equal the pressure loss ratio

$$R_p = \left(\frac{\Delta P}{P}\right)_G \Big/ \left(\frac{\Delta P}{P}\right)_A \qquad \text{6.9}$$

From 6.8 and 6.9, with $r = P_A/P_G$ we have

$$\frac{\Delta P_G}{\Delta P_A} = \frac{R_p}{r} \qquad \text{6.10}$$

and

$$\Delta P_A = \frac{r P_G \left(\frac{\Delta P}{P}\right)_R}{1 + R_p} \qquad \text{6.11}$$

From the pressure drop formula

$$\Delta P = f \frac{L_R}{r_h} \cdot \frac{G^2}{\rho \cdot 2g} \qquad \text{6.12}$$

and the friction factor correlation, Equation 6.2, and Equation 6.11 we derive from the air side pressure drop equations,

$$L_R (G_A')^{1.285} = k_2 F_1 \left(\frac{\Delta P}{P}\right)_R \qquad \text{6.13}$$

where

$$F_1 = \frac{r^2}{1 + R_p}$$

$$k_2 = \frac{2g \, P_G^2 \, \xi^{1.285} \, r_n^{1.715}}{36 \, R T_A \, (\mu/4)^{0.715}}$$

Using Equations 6.12 and 6.2 for the air and gas sides we obtain

$$\frac{G_G'}{G_A'} = \left(\frac{T_A}{T_G} \cdot \frac{R_p}{r^2}\right)^{0.78} = F_2^{0.78} \tag{6.14}$$

and hence

$$\frac{S_A}{S_G} = F_2^{0.78} \tag{6.15}$$

METHOD OF ANALYSIS

The matrix volume, allowing for the 40° of "dead" matrix is

$$Vol = \frac{360}{320} L_R (S_A + S_G) = \frac{360}{320} \frac{L_R W_A}{G_A'} \left(1 + \frac{1}{F_2^{.78}}\right) \tag{6.16}$$

and on using 6.4, 6.6, 6.7 and 6.15 gives

$$Vol = \frac{360}{320} \left[\frac{W_A}{k_2 \left(\frac{\Delta P}{P}\right)_R}\right]^{0.28} \left[\frac{\Lambda_o}{k_1}\right]^{1.28} \cdot F_3 \tag{6.17}$$

where

$$F_3 = \left[\frac{1}{F_1}\right]^{0.28} \left[1 + F_2^{0.39}\right]^{1.28} \left[1 + \left(\frac{1}{F_2}\right)^{.78}\right] \tag{6.18}$$

and F_3 = function of r and R_p only if T_A/T_G is assumed nearly constant.

Since the volume, Equation 6.17 is defined in terms of Λ_o, W_A, $\left(\frac{\Delta P}{P}\right)_R$ and F_3 and since Λ_o, W_A, $\left(\frac{\Delta P}{P}\right)_R$ and r are defined for a given regenerator design, the minimum volume will occur where

$$\frac{dF_3}{dR_p} = 0, \quad r = \text{constant} \tag{6.19}$$

Equation 6.19 was solved for R_p over a range of r. Thus, for a given value of compression ratio, r, the functions F_1, F_2 and F_3 have a given constant value.

The matrix passage length can be derived as

$$L_R = \left\{k_2 \left(\frac{\Delta P}{P}\right)_R F_1 \left[\frac{\Lambda_o}{k_1} \left(1 + F_2^{0.39}\right)\right]^{2.75}\right\}^{0.28}$$

The total frontal area of the matrix is given by Vol/L_R and the air and gas side areas can be obtained from Equations 6.15 and 6.16.

RESULTS OF ANALYSIS

The results of the preceding analysis are shown on Fig. 6-1 plotted against $\Delta W/W$ as a parameter. All points on these design curves define a regenerator which will give the gas-turbine plant analysed in Section V the desired output and efficiency. In addition, the gas side to air side pressure loss parameter as defined by R, has taken on values, throughout the calculation of these curves, that give minimum matrix volume as determined by Equation 6.19. While the results are plotted against $\Delta W/W$ which is a convenient parameter, and one of the principal quantities investigated in this report, they could have been transferred and plotted against one of the dimensional quantities: rotor diameter, rotor RPM, or number of layers (passage length).

Fig. 6-1b gives the matrix volume for various pressure drops through the regenerator. A constant $\Delta P/P = 0.03$ for cycle ducting has been assumed. A minimum matrix volume is seen to occur at $\Delta W/W = 0.035$ and $(\Delta P/P)_R = 0.03$.

Fig. 6-1c shows the regenerator diameter which is perhaps a more significant measure than matrix volume. This minimum occurs at $\Delta W/W = 0.03$ and $(\Delta P/P)_R = 0.09$, however, there is no significant difference in diameter between the minimum for $(\Delta P/P)_R = 0.03$ and 0.09.

Fig. 6-1d gives rotor RPM which has a value of about 30 at minimum diameter.

Fig. 6-1e shows the number of screen layers required in the matrix, and is a measure of the matrix length in the flow direction, (the radial direction).

With the rotor type here analyzed, the main seal shoe length is short compared to the rotor voids volume, hence, even for clearances of 0.003 inches under the seals the leakage parameter BS/A is large. This indicates that the displacement leakage constitutes almost the entire regenerator leakage.

In conclusion, this design study of a 4000 horsepower gas-turbine plant incorporating a rotary-regenerator shows that this type of heat-exchanger can attain the required effectiveness within a small space. Careful design of plant layout and ducting should enable this type of plant to be incorporated in a locomotive, or in small ships.

VII. REDUCTION OF LEAKAGE IN THE ROTARY-REGENERATOR

The amount of leakage in the rotary regenerator when applied to a gas turbine cycle has been shown to be intimately connected with the design of the cycle components. That is, an increase of leakage at the optimum design point decreases the work output of the cycle per pound of air compressed at a greater rate than the leakage increase. Thus, all the components of the cycle must accomodate an increased flow rate. To attain a given cycle efficiency, the regenerator leakage also requires an increase in regenerator effectiveness, and hence its size.

METHODS OF REDUCING LEAKAGE

1. Clearances -

If the internal structure of the rotor-matrix combination is such that the volume flow rate during rotation is small (as with seals bearing directly on the matrix passage ends), i.e. small BS/A, reducing seal shoe clearance will give good results. However, as this type of regenerator will have very large seal shoes small clearances may be voided by thermal distortion. In addition the forces required to support large sealing areas against pressure may introduce distortion on long seal shoes, and excess seal loading will cause high frictional forces resisting rotation.

If the structure of the rotor matrix combination creates large volumetric flow rates, hence, large displacement losses, i.e., large BS/A; clearance, within limits, has little effect on leakage flow. This is the type of rotary regenerator where there is rotating ducting and very small seal shoes as in Fig. 6-1.

2. Seal Length -

If the seal length is reduced in order to reduce the flow passage under the clearances the configuration of the rotor is forced to a type with large rotational losses. Thus, as the seal length is reduced, the leakage due to clearance becomes negligible and the displacement losses increase.

3. Rotational Speed -

Reducing rotational speed below its optimum value for a given effectiveness, increases rotor size and the sealing clearance length at a greater rate than the decrease of displacement leakage and results in increased leakage. Increasing the rotational speed above its optimum will cause large and rapid increase of leakage due to displacement effects.

4. Recovery of the Displacement Loss

With small seal shoes the problems of seal clearance, support, and friction can be conveniently handled. However, as pointed out already, small seal shoes will dictate the use of rotating internal ducting and large displacement losses. Any method to reduce the displacement loss will enable the use of small seal shoes and higher rotational speeds without increase of the total leakage flow.

CROSS CONNECTED SEAL SHOES

Rotary-regenerators with small clearance areas (small seal shoes) and large rotating volumes will establish large values of the leakage parameter BS/A. Large BS/A cause large differing pressure gradients under the seal shoe pairs. The pressure gradient where the rotor rotates into the high pressure region is low, and where it rotates out of the high pressure region it is high. Thus, if a pipe or series of pipes were connected between ported seal shoes, the pressure gradient would be equalized between shoes at each port. It can readily be shown that as BS/A → ∞ the pressure gradients become stepped at each port, and that for this case the leakage is given by

$$\frac{W_T}{BP_2} = \frac{r-1}{N_p+1} \qquad\qquad 7.1$$

where N_p is the number of ports. This will give adequate results for BS/A > 10 providing the cross connecting pipes are sized to give negligible pressure drop.

Fig. 7-1 shows the results of an experimental test of a ported seal shoe set in the range 0 = BS/A = 6. The percent leakage curve shows that no effect is obtained below a value of BS/A = 1, and then, with increased BS/A the leakage is approaching the asymptotic value given by Equation 7.1.

The measured pressure gradients graphically illustrate the equalization of the pressure between shoes at each port.

From Equation 7.1, or from Fig. 7-1, it is clear that the successive reduction of leakage becomes less with each added connection. Thus, more than three or four cross connections between seals would not be warranted.

VIII. CONCLUSIONS AND RECOMMENDATIONS

In the preceding Sections the rotary regenerator has been analyzed in detail with respect to its application in a gas-turbine cycle. This has shown that, even though a definite small leakage between high and low pressure sides must exist it will not detract from the size advantages of this type of heat exchanger.

An analysis and experimental investigation of seal leakage in the rotary regenerator has been described. The theoretical results are quite adequate for seal design purposes. The method of reducing the displacement loss by cross-connecting ported seal shoes is shown and gives considerable promise.

A rotary-regenerator has been sized for a specific gas-turbine plant requirement. The effect of choice of pressure drop and leakage rate are shown by design point curves. A suggested plant layout is presented to enable visualization of the space requirements of the rotary-regenerator.

A general conclusion may be drawn that the best overall rotary regenerator arrangement will have a rotor containing the matrix and a syst m of rotating compartmented ducts leading out to small seal shoes. The seal shoes can be pressure balanced by a diaphram or piston which can be pressurized by a port in the seal shoe itself. The location of this port along the pressure gradient will depend on the seal area and piston area. This loading force should be just sufficient to keep the seal in close contact with the rotor compartments, but not to cause excess friction. If the seals act on opposite faces of a drum type rotor, thermal expansion of the rotor will not result in great difficulties. As such a rotor will contain large rotating volumes a system of seal porting and cross-connection should be used to minimize leakage. This general arrangement with the matrix mounted in a number of separate compartments would facilitate provision for separate removal and replacement of matrix elements.

RECOMMENDATIONS

The work of Coppage (7) has given adequate heat transfer and flow friction data for matrix forms. This report has covered the problems of seal leakage adequately; providing the number of compartments under the seal at one time is greater than about six or eight. While the results can probably be applied with fair accuracy to leakage for fewer compartments under the seal, the assumptions of the analysis are no longer valid and the pressure gradients will no longer apply. Finite difference methods should be carried out for this case. The effects of cross-connected seal shoes have only been analyzed for large values of the leakage parameter, BS/A, where it is most useful. Additional analysis could be carried out to obtain a result for BS/A between 0 and 10.

The preceding recommendations for further work are somewhat detailed. It would appear that the most fruitful next step would be to design an experimental rotary regenerator of convenient size and construction. Among the desirable properties would be to arrange for interchanging matrix elements. This would allow actual regenerator tests with varying matrix configurations, along with the other variables of rotational speed, flow rate, and temperature levels. The sealing design would also reveal any problems that have not been apparent in an idealized study of sealing alone.

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APPENDIX A

EFFECT OF PASSAGE DIAMETER ON HEAT EXCHANGER VOLUME

For the triangular passage of Fig. 2-1a a flow friction and heat transfer correlation was obtained from Fig. 16 of Kays, London and Johnson (15), for the laminar flow range, as follows.

$$(N_{st})(N_{pr})^{2/3} = 0.34 \ (N_{Re})^{-0.58} \qquad \text{A.1}$$

$$f = 4.60 \ (N_{Re})^{-0.77} \qquad \text{A.2}$$

Assuming perfect geometry of the triangular passage, the equivalent diameter is βX, where X is the center to center plate spacing and β has a value of 0.555. The fraction of frontal area available for flow is ξ where ξ has a value of 0.80.

Assume the following conditions for the heat exchangers, which are typical of those found in a gas turbine cycle.

$$(\frac{\Delta P}{P})_A + (\frac{\Delta P}{P})_G = 0.04$$

$$T_{A_1} = 860^{\circ} R \quad T_{G_1} = 1600^{\circ} R$$

$$\eta_R = 0.80 \qquad W_A = W_G = 45 \ \#/sec \qquad \text{A.3}$$

From the definition of effectiveness, is derived

$$T_{A_2} = 1452^{\circ}R, \quad T_{G_2} = 1008^{\circ}R$$

$$\Delta T_M = 148^{\circ} \qquad \text{A.4}$$

The Plate-Fin Cross-Flow Heat Exchanger

Using the dimensional notation of Fig. 2-1a, the Fanning equation for the air side pressure drop may be written

$$\Delta P_A = 4 \ f_A \frac{L_A}{D_e} \cdot \frac{G_A^2}{\rho_A \cdot 2g} \qquad \text{A.5}$$

and using

$$G_A = \frac{W_A}{\xi \ NL_G x} \qquad \text{A.6}$$

and defining

$$R_p = (\frac{\Delta P}{P})_G / (\frac{\Delta P}{P})_A \qquad \text{A.7}$$

together with equation A.2 gives

$$\frac{L_A}{L_G^{1.25} N^{1.25} x^5} = \frac{k_1}{1 + R_p} \qquad \text{A.8}$$

where the quantities in k_1 are constant and have been evaluated at mean temperatures where applicable. Proceeding similarly we have

$$\frac{L_G}{L_A} = k_2 R_p^{0.45} \qquad \text{A.9}$$

$$h_A = \frac{k_3}{x N^{0.42} L_G^{0.42}} \qquad \text{A.10}$$

$$\frac{h_A}{h_G} = \frac{k_4}{R_p^{0.19}} \qquad \text{A.11}$$

From the Newton relation

$$q = UAY\Delta T_M$$

where Y = cross flow factor

$$U = h_A / (1 + \frac{h_A}{h_G})$$

$$A = 6 L_A L_G N \mathcal{E}$$

where $\mathcal{E}$ is fin effectiveness, and assumed constant and equal to 0.90; together with A.10 and A.11 we derive

$$\frac{L_A L_G^{0.58} N^{0.58}}{x} = k_5 (1 + \frac{1.41}{R_p^{0.19}}) \qquad \text{A.12}$$

Using A.8, A.9 and A.12

$$N = \frac{F_1(R_p)}{x^{2.75}} \qquad \text{A.13}$$

$$L_G = \frac{F_2(R_p)}{N^{0.60}} \qquad \text{A.14}$$

and from A.9

$$L_A = F_3(R_p) L_G \qquad \text{A.15}$$

The volume is given by $L_A L_G \cdot 2\pi x$

$$\text{Vol} = \left[\frac{F_2^2 (R_p) F_3 (R_p)}{F_1^{0.2} (R_p)} \right] x^{1.55} = F(R_p) x^{1.55} \qquad A.16$$

For minimum volume for a fixed plate spacing, the value of R_p for $F(R_p)$ a minimum, was determined and found to be about unity.

That is $(\frac{\Delta P}{P})_G = (\frac{\Delta P}{P})_A$ for minimum volume.

With R_p = unity equations A.13 through A.16 were used to determine the solid curves of Fig. 2-1.

Rotary Regenerator with Plate-Fin Type Surface

Proceeding in a similar manner as above, but using the notation S for matrix frontal area, hence

$$G_A = \frac{W_A}{S_A \xi} \; , \quad G'_A = \frac{W_A}{S_A}$$

we can derive

$$\frac{c_A}{S_G} = k_6 \; R_p^{0.813}$$

$$h_A = \frac{k_7}{x^{0.58} \; S_A^{0.42}} \qquad A.17$$

$$\frac{h_A}{h_G} = \frac{k_8}{R_p^{0.34}}$$

From Section III we have

$$\eta_R = (\frac{\Lambda_o}{\Lambda_o + 1})(1 - \frac{1}{9}(\frac{\pi}{\Lambda})^2_A)$$

$$\Lambda_o = \Lambda_A/(1 + \frac{\Lambda_A}{\Lambda_G}) ; \quad \Lambda = \frac{hA'L_R}{G'C_p} ; \quad \pi = \frac{hA'G}{m'c} \qquad A.18$$

where $A' = \frac{6}{x}$ ft^{-1}, $m' = 98$ #/ft^3

and $\frac{W_A}{\Lambda_A} = \frac{W_A}{W_D} \cdot \frac{C_p}{c} \cdot \frac{\xi}{m'} (\rho_A - \rho_G)$

where W_D is the displacement leakage and $\frac{W_D}{W_A}$ is taken as 0.02.

From the relationships A.17 and A.18 are found

$$S_A = F_4(R_p) x^{-0.105}$$

$$L_R = F_5(R_p) x^{1.64}$$

and from A.17 $S_G = F_6(R_p) S_A$ A.19

Allowing 40 degrees of dead rotor for sealing

$$Vol = \frac{360}{320} L_R(S_A + S_G)$$

$$= \frac{360}{320} F_5(R_p)\left[1 + F_6(R_p)\right] F_4(R_p) x^{1.535}$$

$$= F(R_p) x^{1.535}$$ A.20

Here the value of R_p for minimum $F(R_p)$ is 20, that is

$$\left(\frac{\Delta P}{P}\right)_G = 20 \left(\frac{\Delta P}{P}\right)_A$$

With $R_p = 20$ equations A.19 and A.20 were solved for the dotted curves of Fig. P2-1. For drum diameter it was taken that the drum length and diameter were equal. The value of $R_p = 20$ gives $S_G/S_A = 1.22$, the ratio of the gas to air sides of the regenerator.

APPENDIX B

THE STRAIGHT THROUGH LABYRINTH

The leakage flow between the seal shoe and the partition or passage wall of the rotor is assumed to act as an orifice. See Fig. A-1.

The flow equation through the clearance δ may be written

$$\frac{V^2_{(n+1)'} - V^2_n}{2g} = C_p (T_n - T_{(n+1)'})$$ B.1

where $V_{(n+1)'}$ is at a section a'; where the pressure equals P_{n+1}. Generally the kinetic energy of approach to the nth clearance will be a function of $\frac{\delta}{\Delta}$, $\left(\frac{V^2}{2g}\right)_n$, and n. Assuming that $V_{n'}$ and $V_{(n+1)'}$ are of the same order of magnitude $\left(\frac{V^2}{2g}\right)_n = \left(\frac{V^2}{2g}\right)_{n'} \cdot f(\frac{\delta}{\Delta}, n)$

$$= \left(\frac{V^2}{2g}\right)_{(n+1)'} \cdot f'(\frac{\delta}{\Delta}, n)$$

or
$$\left(\frac{V^2}{2g}\right)_n = F\left(\frac{V^2}{2g}\right)_{(n+1)'}$$
<div align="right">B.2</div>

where
$$F = f'\left(\frac{\delta}{\Delta}, n\right)$$

Introducing B.2 into B.1

$$\left(\frac{V^2}{2g}\right)_{(n+1)'}(1-F) = C_p\left(T_n - T_{(n+1)'}\right)$$

$$\left(\frac{V^2}{2g}\right)_{(n+1)'} = F_1 C_p\left(T_n - T_{(n+1)'}\right)$$

where
$$F_1 = 1/(1-F)$$

For an adiabatic expression $\quad P/\rho^k = C$

thus

$$\left(\frac{V^2}{2g}\right)_{(n+1)'} = F_1 C_p T_n\left[1 - \left\{\frac{P_{(n+1)'}}{P_n}\right\}^{\frac{k-1}{k}}\right]$$
<div align="right">B.3</div>

But
$$P_{(n+1)'} = P_{n+1}$$

Expanding $\quad \left(\frac{P_{n+1}}{P_n}\right)^{\frac{k-1}{k}} = \left(1 + \frac{\Delta P}{P}\right)^{\frac{k-1}{k}}$, where $\Delta P = P_{n+1} - P_n$

by use of the binomial expansion gives

$$\left(\frac{P_{n+1}}{P_n}\right)^{\frac{k-1}{k}} = 1 - \frac{k-1}{k}\left(\frac{\Delta P}{P_n}\right) - \frac{k-1}{k}\cdot\frac{1}{2k}\left(\frac{\Delta P}{P_n}\right)^2 + \ldots$$

Which when introduced into equation B.3 gives

$$\left(\frac{V^2}{2g}\right)_{(n+1)'} = -F_1 C_p T_n \frac{k-1}{k}\left[\frac{\Delta P}{P_n} - \frac{1}{2k}\left(\frac{\Delta P}{P_n}\right)^2 + \ldots\right]$$

Since this formula is valid only for $\frac{P_{n+1}}{P} > 0.53$ that is $\frac{\Delta P}{P_n} < 0.47$,
$\frac{\Delta P}{P_n} > \frac{1}{2k}\left(\frac{\Delta P}{P_n}\right)^2$, and the squared term will be neglected.

So that $\quad \left(\frac{V^2}{2g}\right)_{(n+1)'} = -F_1 RT_n \frac{\Delta P}{P_n}$

The equation of continuity for the jet gives

$$W_n = (\rho AV)_{(n+1)'}$$
<div align="right">B.4</div>

The density ρ_{n+1} is obtained from

$$\frac{\rho_n}{\rho_{n+1}} = \left(\frac{P_n}{P_{n+1}}\right)^{-\frac{1}{k}} = \left(1 + \frac{\Delta P}{P_n}\right)^{-\frac{1}{k}}$$

which when expanded gives

$$\frac{\rho_n}{\rho_{n+1}} = 1 - \frac{1}{k}\frac{\Delta P}{P_n} + \frac{k+1}{2k^2}\left(\frac{\Delta P}{P_n}\right)^2 - \ldots$$

Again neglecting the squared term, and rearranging gives

$$\rho_{n+1} = \frac{\rho_n}{1 - \frac{1}{k}\frac{\Delta P}{P_n}}$$

or

$$\rho^2_{n+1} = \frac{\rho_n^2}{1 - \frac{2}{k}\frac{\Delta P}{P_n}} \qquad\qquad B.5$$

Introducing B.3 and B.5 into B.4

$$W_n = a \left[\frac{2g\rho_n^2}{1 - \frac{2}{k}\frac{\Delta P}{P_n}} \cdot \frac{- F_1 RT_n \Delta P}{P_n} \right]^{\frac{1}{2}}$$

$$= a' \left[\frac{- F_1 \, 2g \, P_n \, \Delta P}{RT_n \left(1 - \frac{2}{k}\frac{\Delta P}{P_n}\right)} \right]^{\frac{1}{2}}$$

Introduce a coefficient α such that $a\alpha = a'$ where $a = L_s \cdot \delta$, the dimensions of the clearance.

$$\frac{W_n}{a\alpha} = \left[\frac{- F_1 2g}{RT_n} \cdot \frac{P_n \, \Delta P}{1 - \frac{2}{k}\frac{\Delta P}{P_n}} \right]^{\frac{1}{2}} \qquad\qquad B.6$$

Equation B.6 has been solved by Egli (ref. 9a) to give the flow through a labyrinth seal.

$$\frac{W}{a\alpha} = \left[\frac{F_1 g}{RT_o} \cdot \frac{P_1^2 - P_2^2}{n + \ln \frac{P_1}{P_2}} \right]^{\frac{1}{2}} \qquad\qquad B.7$$

In this solution Egli uses $\gamma = f(\frac{\delta}{\Delta}, n) = \sqrt{F_1}$, the carry-over coefficient for the straight through labyrinth and has obtained experimental values of for a range of δ/Δ and n under the assumption of constant flow coefficient α for all the orifices.

Using $a = \delta L$ the clearance area and the carry-over coefficient as determined by Egli's (Fig. B-2) equation B.6 may be written

$$W_n = \alpha \gamma \delta L_s \left[\frac{-2g}{RT_n} \cdot \frac{P_n \Delta P}{1 - \frac{2}{k} \frac{\Delta P}{P_n}} \right]^{\frac{1}{2}}$$ B.8

If the term $\frac{2}{k} \frac{\Delta P}{P_n}$ is considered small compared to unity

$$\overline{W}_n = \alpha \gamma \delta L_s \left[\frac{-2g}{RT_n} \cdot P_n \Delta P \right]^{\frac{1}{2}}$$ B.9

which gives for a solution of the labyrinth problem

$$\overline{W} = \alpha \gamma \delta L_s \left[\frac{g}{RT_o} \cdot \frac{\overline{P}_1^2 - P_2^2}{n} \right]^{\frac{1}{2}}$$ B.10

Comparing the approximate solution of equation B.10 with B.7 gives

$$\frac{\overline{W}}{W} = \left[\frac{n + \ln P_1/P_2}{n} \right]^{\frac{1}{2}}$$

$$= \left[1 + \frac{\ln P_1/P_2}{n} \right]^{\frac{1}{2}} = 1 + \frac{1}{2n} \ln P_1/P_2 - \ldots$$

Thus for the approximate solution to be valid the natural log of the pressure ratio P_1/P_2, divided by twice the number of throttlings n must be small compared to unity.

APPENDIX C

EFFECT OF PASSAGE RESISTANCE

In the analysis of rotary regenerator seal leakage given in Section IV, the assumption was made that the resistance to outflow from a matrix passage or compartment, during its traverse of the seal shoe, could be neglected. That is, the pressure at any point within a matrix passage was taken as equal to the pressure in the passage end at the seal face.

To verify this assumption, a simple conservative, analysis was made. Assuming the passage resistance to be concentrated at the exit to a chamber of volume equal to the passage volume, the equation for outflow under a sudden pressure release gives

$$\frac{P - P_B}{P_1 - P_B} = e^{-t/k}$$

<div align="right">C.1</div>

where

$$k = \frac{64\mu}{2g \, P_B} \left(\frac{L}{D}\right)^2 \quad \left(f = \frac{16}{w_{Re}}\right)$$

and L = one half the total passage length.

The maximum passage $\frac{L_T}{D}$ in a gas turbine cycle rotary regenerator is likely to be about 500, i.e. L/D here equals 250. For this case the pressure in the passage (chamber) will fall 90 percent of the total pressure drop in about .002 seconds. This is much a shorter time than the minimum of about 0.02 seconds to traverse a seal shoe. Thus, we may consider the assumption of no internal passage resistance as valid.

If in some particular application the passage resistances were higher a sealing analysis could be carried out using finite difference methods. However, the work required might not be justified as the introduction of high passage resistance will bring the total leakage closer to the sum of stationary leakage plus displacement loss.

APPENDIX D

CHOKED FLOW UNDER THE SEALING SHOE

The flow rate through a simple orifice will reach a maximum, or choked value, when the sonic velocity is reached in the contracted jet. The critical pressure ratio at which choking occurs is given by

$$\frac{P_1}{P_B} = \left(\frac{k+1}{2}\right)^{\frac{k}{k-1}}$$

<div align="right">D.1</div>

The value of P_1/P_B for air, $k = 1.4$, is 1.89. Since dp/dx and d^2p/dx^2 are negative for positive rotation, choking can only occur in the last restriction. For negative rotation, since dp/dx is negative and d^2p/dx^2 is positive, choking will occur in the first restriction. This condition will be reached in the seal leakage passages for $|BS/A|$ large and n small. For a given case, for $|S|$ large and n small. To determine if, for a given BS/A, choked flow has been reached, the leakage equation is solved for $W_T/\Delta P_2$. For positive rotation $(BS/A = +)$ it is next solved for x/L with $P/P_2 = 1.89$ and if $x/L > n-1/n$ choking has occurred. For negative rotation $(BS/A = -)$, x/L is solved for with $\frac{P}{P_2} = \frac{P_1}{P_2} \cdot \frac{P}{P_1} = \frac{P_1/P_2}{1.89}$ and if $x/L < \frac{1}{n}$ choking has occurred.

Solutions for Leakage if Choking has Occurred

The equation for flow rate through a choked restriction may be derived as

$$W_L = \alpha \breve{\delta} \delta L_s \, P_o \left[\frac{2g}{RT_o} \cdot \frac{k}{k+1} \left(\frac{2}{k+1}\right)^{\frac{2}{k-1}} \right]^{\frac{1}{2}} \qquad \text{D.2}$$

which, for air $k = 1.4$, reduces to

$$W_L = 0.483 \, \alpha \breve{\delta} \delta L_s P_o \left(\frac{2g}{RT_o}\right)^{\frac{1}{2}} \qquad \text{D.3}$$

Positive Rotation: -

Since the choked flow occurs at $\frac{x}{L} = \frac{n-1}{n}$ we can write

$$\overline{W}_T = W_{D_{(n-1)}} + \overline{W}_{L_{(n-1)}}$$

where

$$W_{D_{(n-1)}} = BSP_{(n-1)} \qquad \text{D.4}$$

and $W_{L_{(n-1)}}$ is given by D.3

thus

$$\frac{W_{D_{(n-1)}}}{W_T} = \frac{BSP_{(n-1)}}{BSP_{(n-1)} + 0.483 \alpha \breve{\delta} \delta L_s P_{(n-1)} \left(\frac{2g}{RT}\right)^{1/2}}$$

$$= \frac{1}{1 + \dfrac{0.483 \alpha \breve{\delta} \delta L_s}{BS} \left(\frac{2g}{RT}\right)^{1/2}} \qquad \text{D.5}$$

Since $A = \alpha \delta \delta L_s \left(\frac{2g}{RTn}\right)^{1/2}$

$$\frac{W_{D_{(n-1)}}}{W_T} = \frac{1}{1 + 0.483\sqrt{n} \, \dfrac{A}{BS}}$$

or

$$= \frac{1}{1 + 0.483\sqrt{n}/\left(\frac{BS}{A}\right)} \qquad \text{D.6}$$

From the leakage solution

$$\frac{1}{1-\frac{W_{D_1}}{W_T}} - \frac{1}{1-\frac{W_{D_{(n-1)}}}{W_T}} - \ln\frac{1-\frac{W_{D_{(n-1)}}}{W_T}}{1-\frac{W_{D_1}}{W_T}} = \left(\frac{BS}{A}\right)^2\frac{n-1}{n}$$

using D.6 to give

$$\frac{1}{1-\left(\frac{BS}{A}\right)\left(\frac{AP_1}{W_T}\right)} - \frac{\frac{BS}{A}}{0.485\sqrt{n}} - 1 - \ln\frac{1}{\left(\frac{\frac{BS}{A}}{0.485\sqrt{n}}+1\right)\left(1-\frac{BS}{A}\cdot\frac{AP_2}{W_T}\right)}$$

$$= \left(\frac{BS}{A}\right)^2\frac{n-1}{n} \qquad\qquad \text{D.7}$$

where W_T is the only unknown for a given case. With W_T determined equation D.7 may be solved for the pressure gradient for $P_1 \geq P \geq P_{(n-1)}$ giving values of $0 \leq \frac{x}{L} \leq \frac{n-1}{n}$.

Negative Rotation

Here the choking will occur at the high pressure end of the shoe. From equation D.6

$$\frac{W_{D_1}}{W_T} = \frac{1}{1+\frac{0.485\sqrt{n}}{BS/A}}$$

For this case the leakage solution becomes

$$\frac{\frac{BS}{A}}{0.485\sqrt{n}} + 1 - \frac{1}{1-\frac{BS}{A}\cdot\frac{AP_2}{W_T}} - \ln\left(\frac{\frac{BS}{A}}{0.485\sqrt{n}}+1\right)\left(1-\frac{BS}{A}\cdot\frac{AP_2}{W_T}\right) = \left(\frac{BS}{A}\right)^2$$

$$\text{D.8}$$

which may be solved for W_T .

Now, with W_T determined

$$\frac{1}{1-\frac{BS}{A}\frac{AP_2}{W_T}\frac{P}{P_2}} - \frac{1}{1-\frac{BS}{A}\frac{AP_2}{W_T}} - \ln\frac{1-\frac{BS}{A}\cdot\frac{AP_2}{W_T}}{1-\frac{BS}{A}\cdot\frac{AP_2}{W_T}\cdot\frac{P}{P_2}} = \left(\frac{BS}{A}\right)^2\left(1-\frac{x}{L}\right) \quad \text{D.9}$$

May be solved for the pressure gradient in the range.

$$\frac{1}{n} = \frac{x}{t} = 1$$

$$P_{n_1} = P = P_2$$

APPENDIX E

DESCRIPTION OF APPARATUS

The diagram of Fig. E-1 shows the method used to carry out the regenerator seal tests and to record data.

The rotor passages were laid out on a circle eight inches in diameter, the section of the compartments being one inch wide at the seal face and two inches deep. The compartments were spaced at three degree intervals around the rotor. Figure E-3 shows a general view of the rotor and the underside of the upper assembly. Loss of air to the outside was prevented by two inch wide mating faces on either side of the compartments. The upper assembly was spring loaded against the rotor. The spring force being adjusted to balance the air pressure underneath the upper assembly.

Figure E-2 is a general view of the apparatus layout. The electric motor, with variable speed pulley, drives the rotor through a four step pulley set, a worm reducer and chain drive. The drive gives a continuous variation of rotor speed of ten to one. Since there is only one test seal shoe, the electric motor is reversed to simulate positive or negative rotation seals.

Figure E-4 shows the three test seals used. These shoes, being of different angular lengths, enable tests to be run with the number of sealed passages (n) equal to 8, 12 or 16. The clearance between the seal shoe face and rotor was varied over a range from 0.001 to 0.009 inches. Figure E-5 shows the static test piece used to calibrate the apparatus and to determine the flow coefficient under the individual restrictions.

Method of Obtaining Data

Preliminary data was obtained by connecting the static test piece (Fig. E-5) to the air supply, manometer connections, and metering system of Fig. E-2. A given pressure ratio was applied across the test section and air flow readings taken. This was done for a range of clearances by changing the gaskets and shims between the static test piece halves. The static test piece originally had 17 partitions. These were removed in steps of four and the above procedure repeated.

The main test section (Figs. E-1 and E-3) was now connected up and flow readings taken, with the rotor stationary, over a range of seal shoe clearances in order to calibrate the unwanted leakage past corners and small cracks inherent with the use of an adjustable interchangeable seal shoe.

With the calibration complete, tests were made for various rotor speeds, shoe lengths, clearances, and pressure ratios as recorded in Section IV.

During tests the pressure was equalized between P_2 and P_B of Figure E-1. Thus, the total leakage measurement was the sum of the flow meter reading plus (or minus) the volumetric flow rate at constant pressure between P_2 and P_B. The pressure P_B was adjusted by controlling the flow out of that section or if necessary by introducing air from the air supply tank.

The supply air pressure was held constant by an adjustable pressure reducing valve. The metered air was measured in either of two flow meters giving a flow measurement range of 100 to 1. The pressure P_1 and the seal shoe pressures were measured on 106 inch mercury U-tubes. The pressures P_2, P_B and the flow meter pressure were obtained on 40 inch water U-tubes.

The rotor speed was obtained from revolution counter readings and a time period reading.

LIST OF ILLUSTRATIONS

SEALS

MATRIX

SEAL SHOE

FIG. I-I --ROTARY REGENERATOR

(a) PLATE-FIN SURFACE

X, PLATE SPACING, INCHES

——————— CROSS FLOW
– – – – – ROTARY REGENERATOR

(b) EFFECT OF HYDRAULIC DIAM. ON H.E. SIZE

FIG. 2-1

THERMAL RECOVERY FOR REGENERATOR
OF GIVEN REDUCED TIMES, REDUCED
LENGTHS, AND UTILIZATION FACTORS

FIG. 3-2
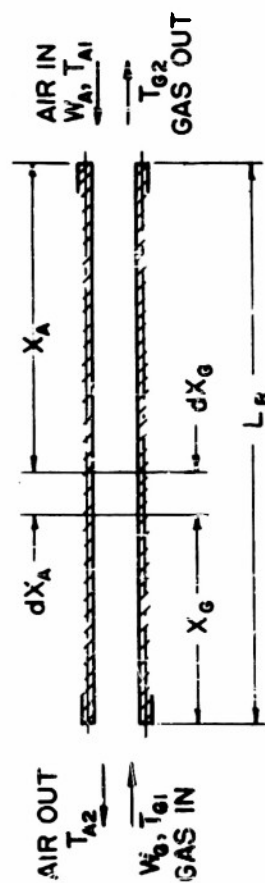
(a) REGENERATOR PASSAGE

(b) HEAT FLOW, GAS PERIOD

FIG. 3-1

(a) SEALING SYSTEM

(b) EQUIVALENT SYSTEM
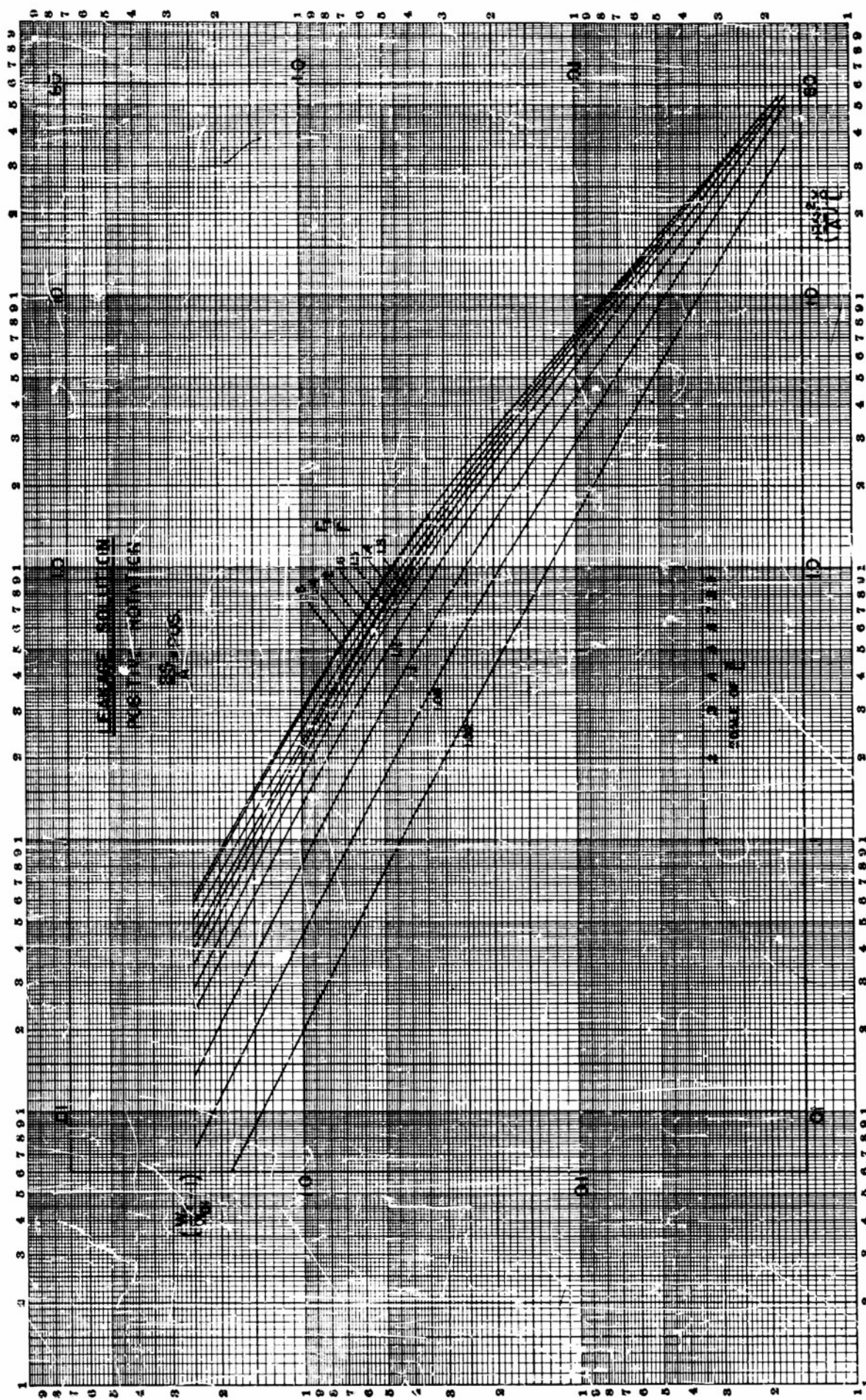
FIG. 4-2

(a) CONTINUOUS PASSAGE ROTOR

(b) COMPARTMENT ROTOR

FIG. 4-1

FIG. 4-5

FIG. 4-4

SEAL LEAKAGE
r = 4

FIG. 4 - 5

PRESSURE GRADIENTS
UNDER SEAL SHOE
r = 4

FIG. 4-6

NET ROTARY-REGENERATOR LEAKAGE

$$\left(\frac{BS}{A}\right)^+ = \left(\frac{BS}{A}\right)^-$$

FIG. 4-7

SEAL SHOE LEAKAGE
Γ = 2

FIG. 4 - 8

SEAL SHOE LEAKAGE
Γ = 3

FIG. 4 - 9

FIG. 4 — 10

FIG. 4 —11

SEAL SHOE
PRESSURE GRADIENTS

$\Gamma = 4$

FIG. 4-14

SEAL SHOE
PRESSURE GRADIENTS

$\Gamma = 4$

FIG. 4-13

SEAL SHOE
PRESSURE GRADIENTS

$\Gamma = 4$

FIG. 4-12

SEAL SHOE
PRESSURE GRADIENTS
$\Gamma = 2$

FIG. 4-17

SEAL SHOE
PRESSURE GRADIENTS
$\Gamma = 3$

FIG. 4-16

SEAL SHOE
PRESSURE GRADIENTS
$\Gamma = 4$

FIG. 4-15

$\Delta \omega / \omega$
0.10
0.08
0.05
0.04
0.02

CALCULATED EFFECTIVENESS η_{RI} %

CORRECTION $\eta_{RI} - \eta^*_{RE}$; %

1.2
0.8
0.4
0

0   20   40   60   80   100

FIGURE 4-18 b

EFFECT OF LEAKAGE ON REGENERATOR
EFFECTIVENESS

(a) ASSUMED LEAKAGE PATHS

MATRIX

$\omega - \frac{\Delta \omega}{2}$

t_{a_2}

$\omega - \Delta \omega$

$\frac{\Delta \omega}{2}, t_{a_2}$

$t^*_{g_1}$

$\omega - \Delta \omega$

$\frac{\Delta \omega}{2}, t_{a_1}$

$\omega - \frac{\Delta \omega}{2}$

t_{g_2}

t_{g_1}

HOT SEAL

COLD SEAL

t_{a_1}

ω

AIR

$t_{g_2}^*$

ω

GAS

(b) GAS SIDE OF HOT SEAL

$\omega - \Delta \omega$

$t^*_{g_1}$

$\frac{\Delta \omega}{2}, t_{a_2}$

$\omega - \frac{\Delta \omega}{2}$

t_{g_1}

FIGURE 4-18 a

EXHAUST

REGENERATOR

FUEL

COMBUSTION
CHAMBER

COMPRESSOR

TURBINE

ELECTRIC
GENERATOR

AIR
INLET
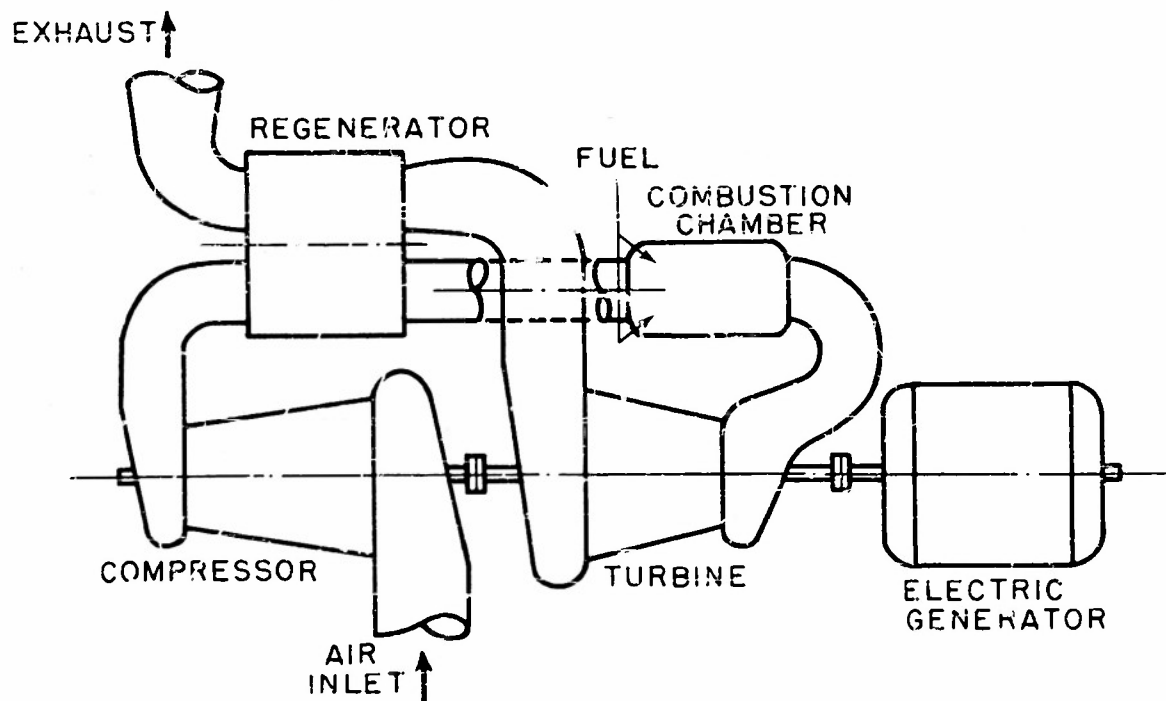
REGENERATIVE G.T. CYCLE

FIG. 5-1

T

1500°F

P_2

P_3

3

X

P_4

P_1

2

60°F

S
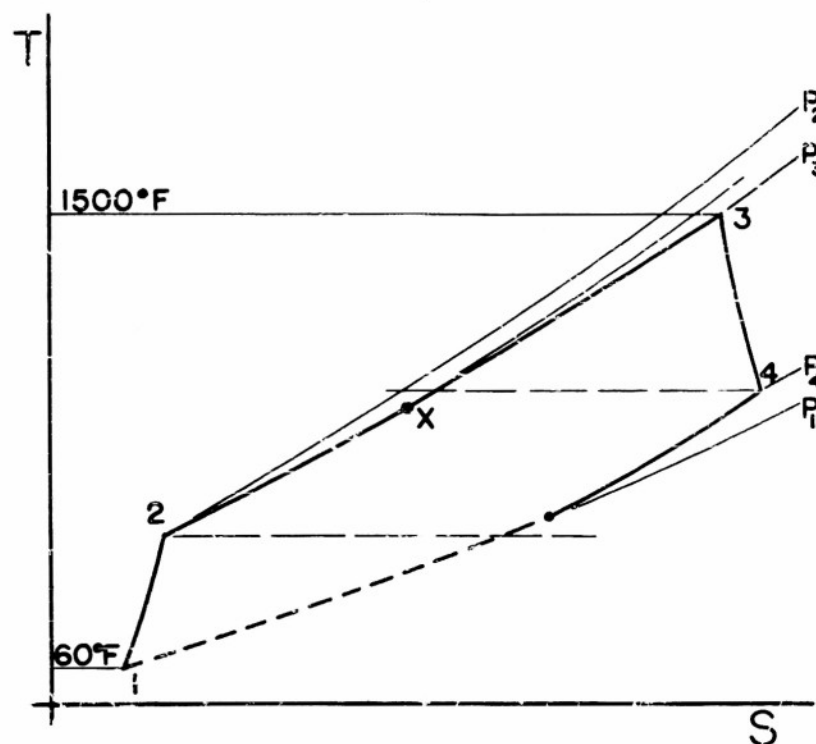
T-S DIAGRAM OF CBTX CYCLE

FIG. 5-2

CBTX CYCLE DESIGN POINTS

CBTX CYCLE DESIGN POINTS

CBTX CYCLE DESIGN POINTS

FIG 5-3

CRTX CYCLE DESIGN POINTS

FIG 5-7

GRYX CYCLE DESIGN POINTS

FIG. 5-6

(a) PLANT LAYOUT
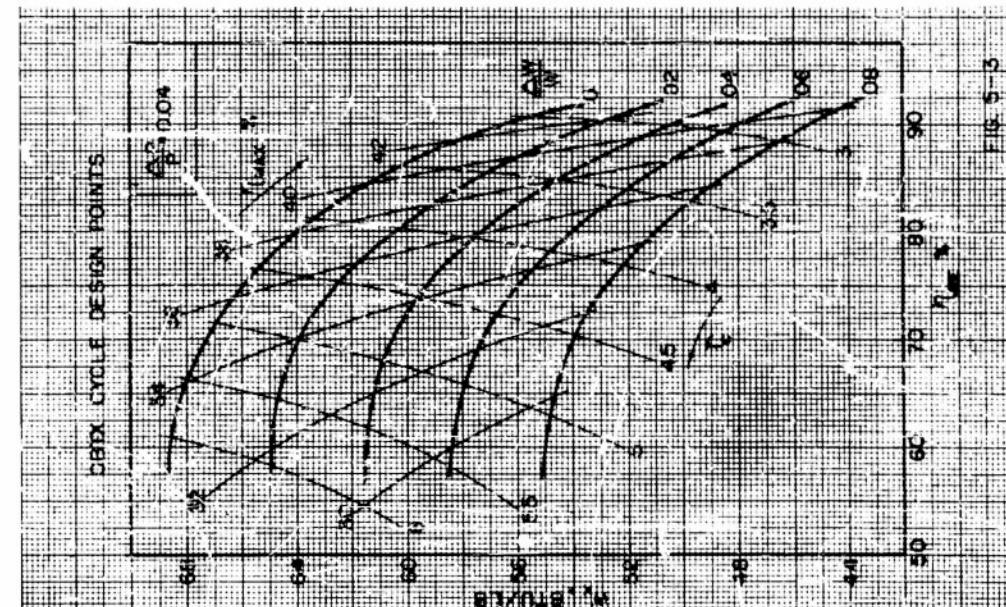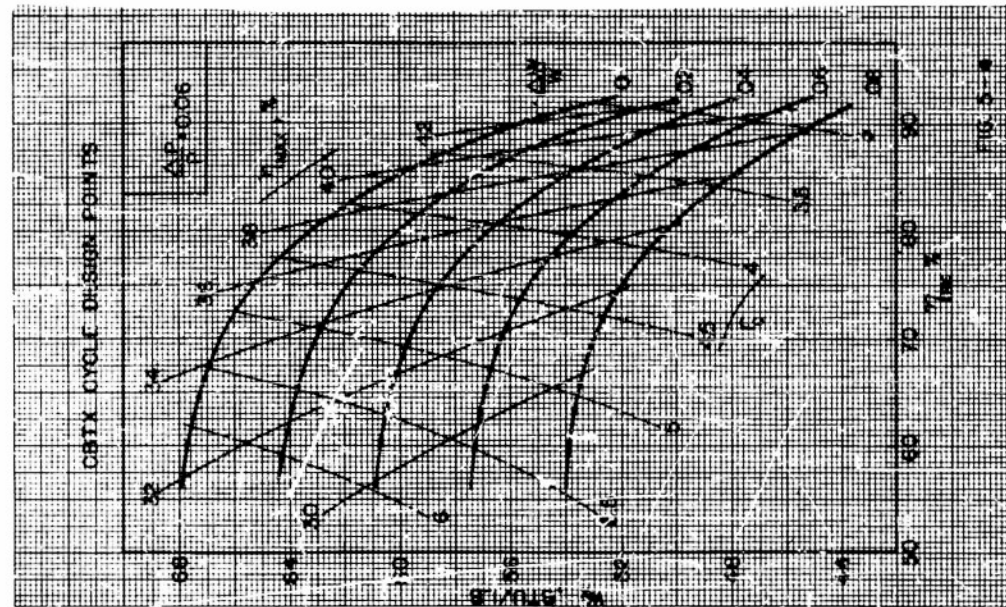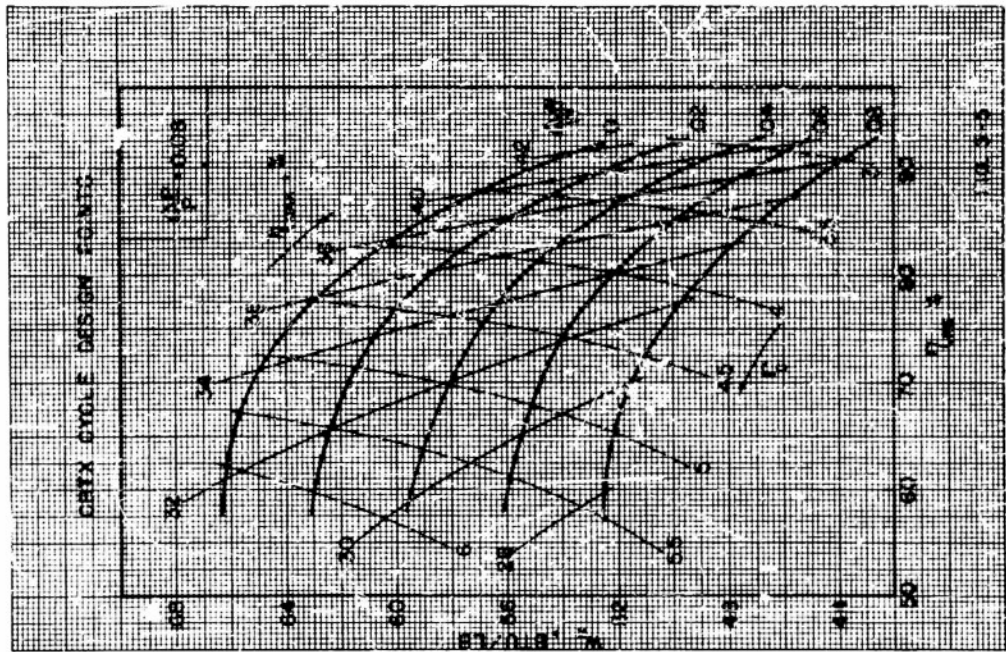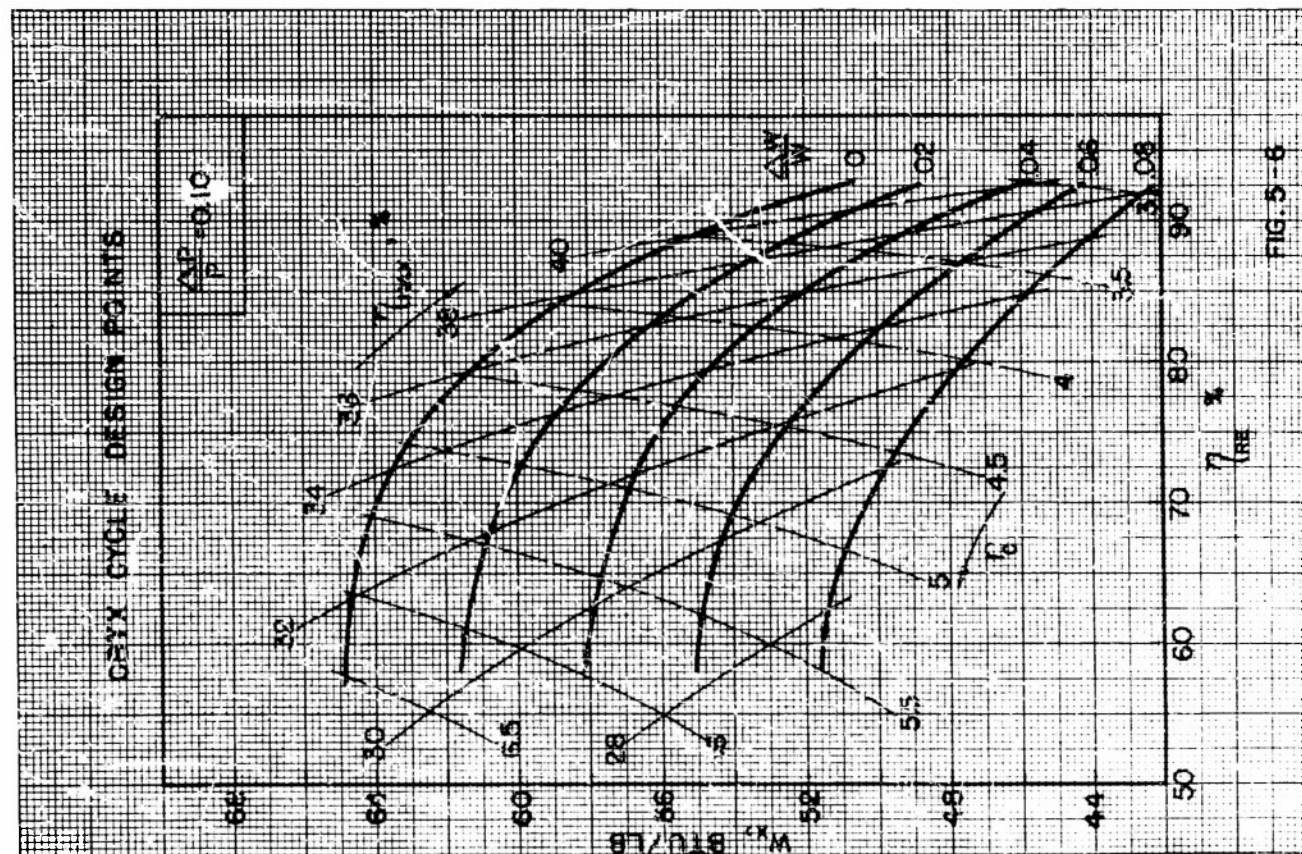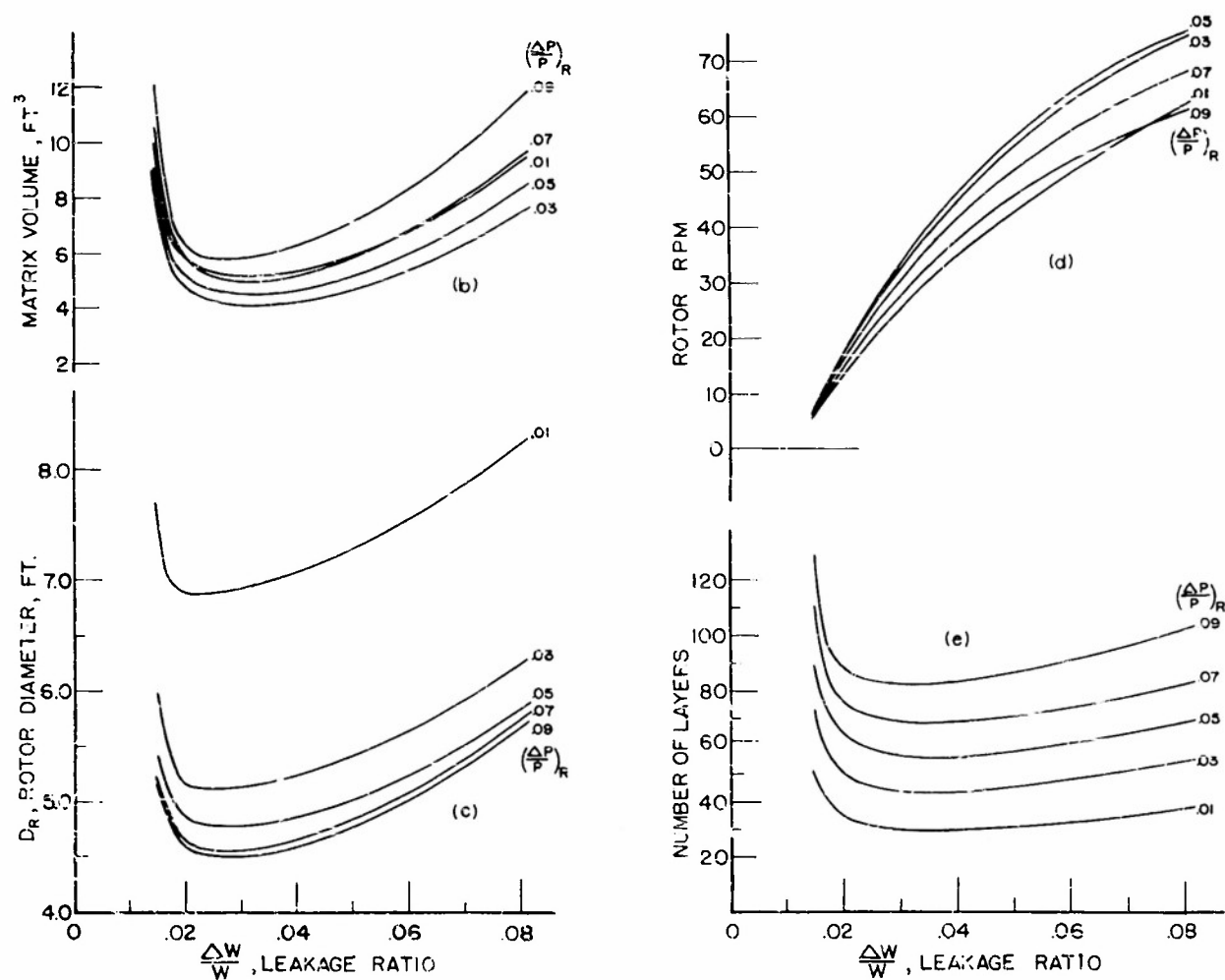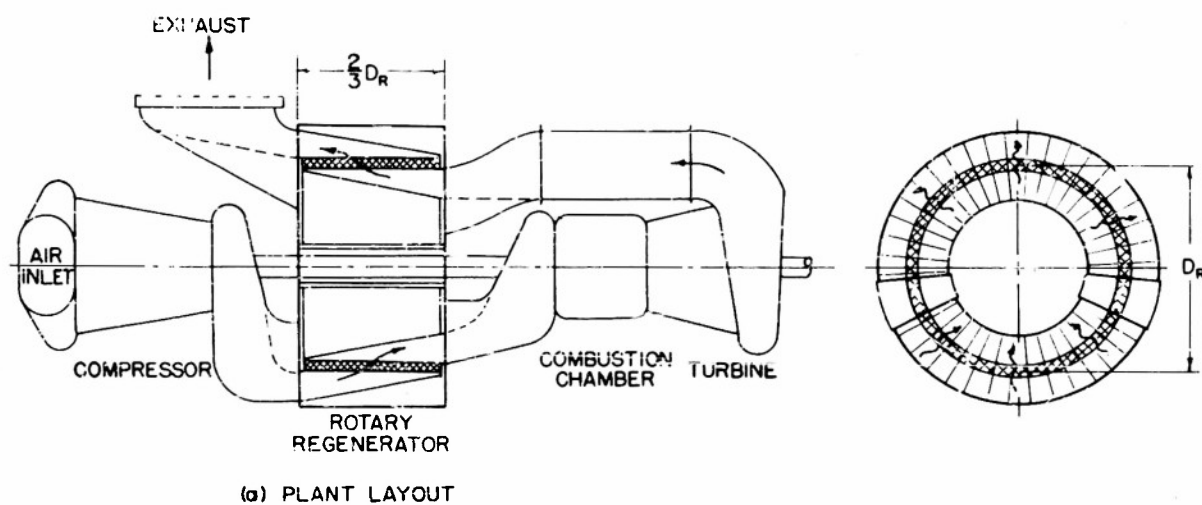
ROTARY REGENERATOR DESIGN POINTS

4000 HP, 34% EFF, 60°-1500°F, G.T. CYCLE

$$\left(\frac{\Delta P}{P}\right)_R = \left(\frac{\Delta P}{P}\right)_T - 0.03$$

FIG. 6-1

DETAIL OF CONNECTIONS

PORT LOCATION

$\frac{BS}{A} = \pm 5.75$

$\frac{X}{L}$

$\frac{P_1 - P}{P_1 - P_2}$

PERCENT LEAKAGE

PLAIN SEAL ○
1 PORT □
2 PORTS △
3 PORTS +

$\frac{BS}{A}$
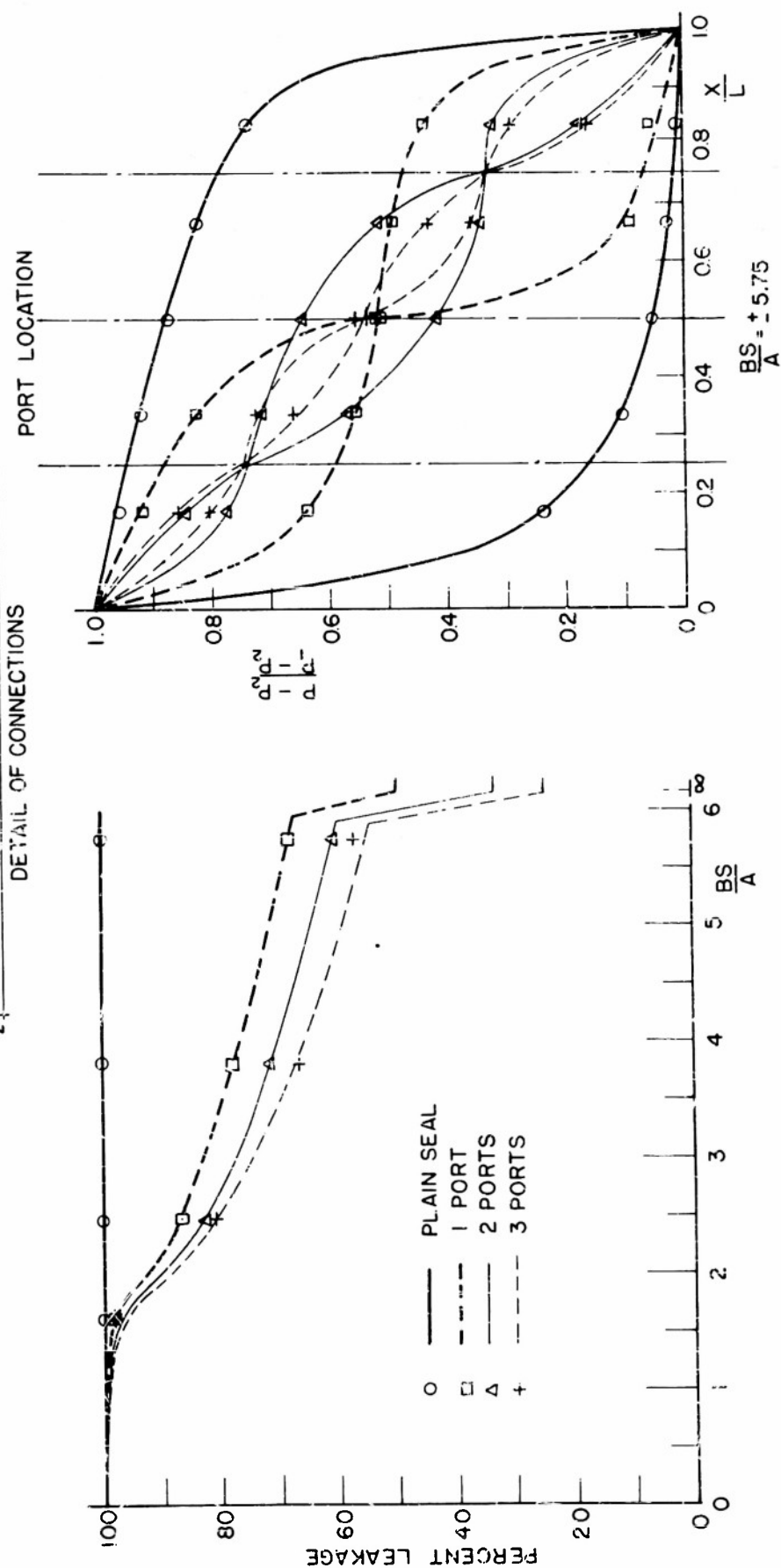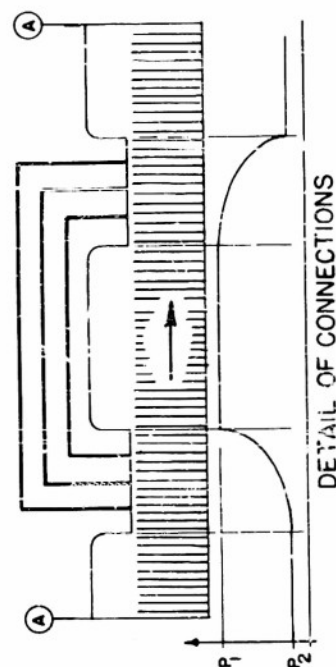
TEST OF PORTED SEAL SHOES

FIG. 7 — 1
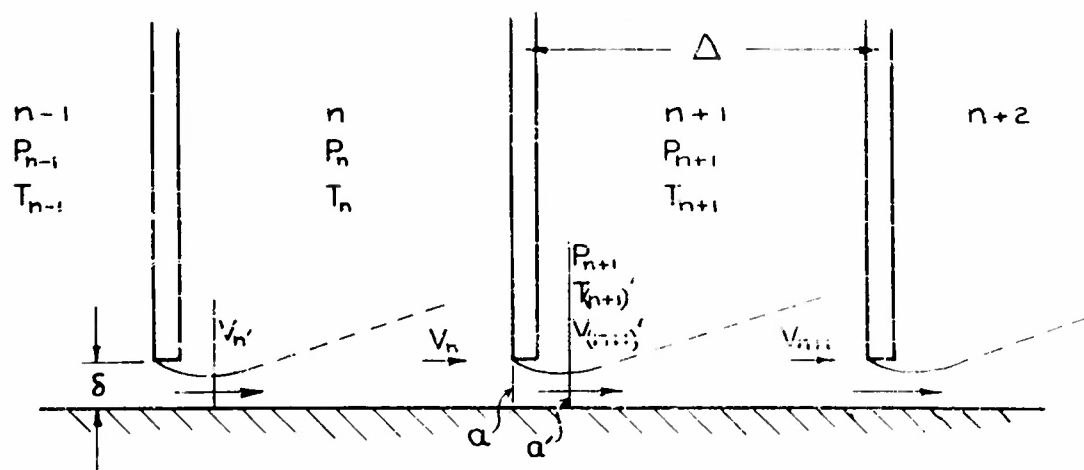
FIG. B-1, LABYRINTH FLOW
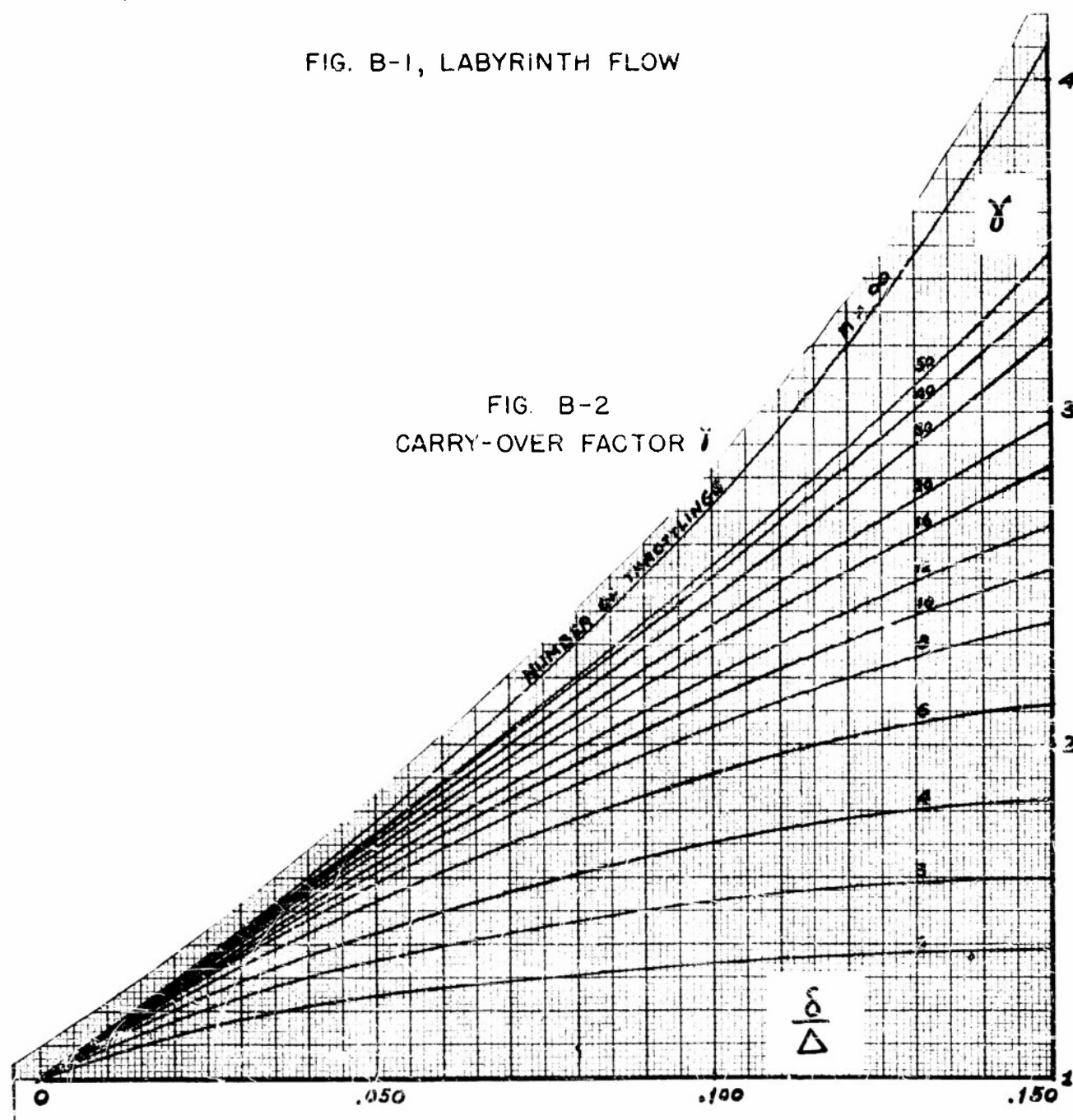
FIG. B-2
CARRY-OVER FACTOR ĭ
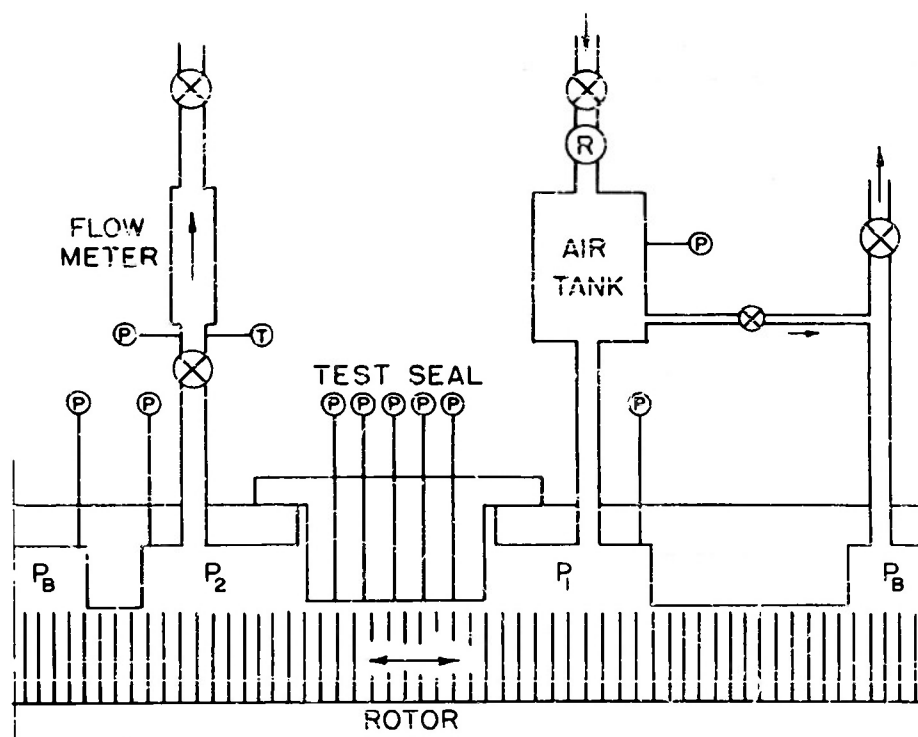
DIAGRAM OF TEST APPARATUS

FIG. E - 1
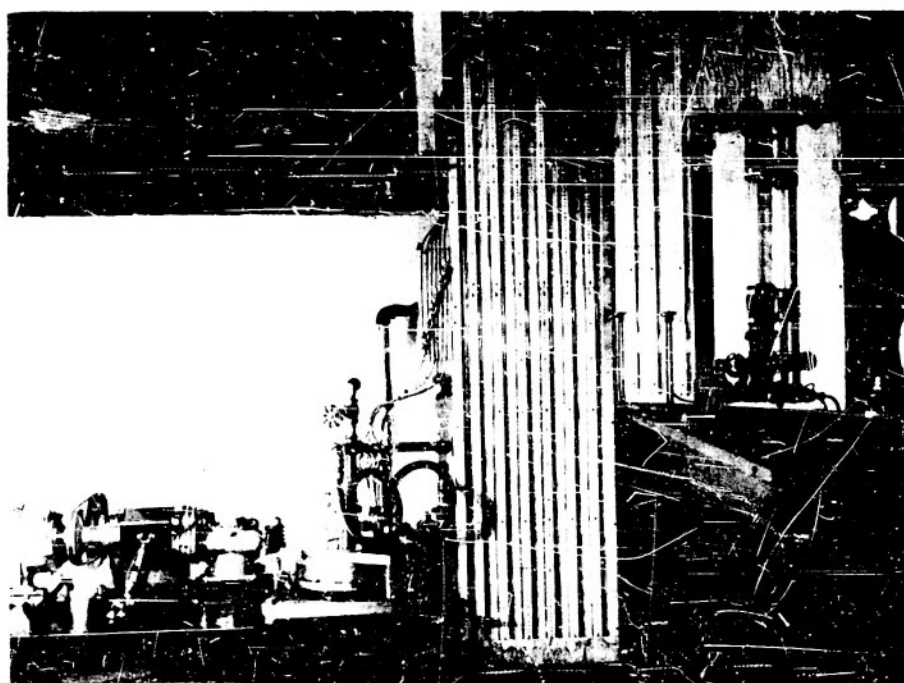
FIG. E - 2 , VIEW OF APPARATUS
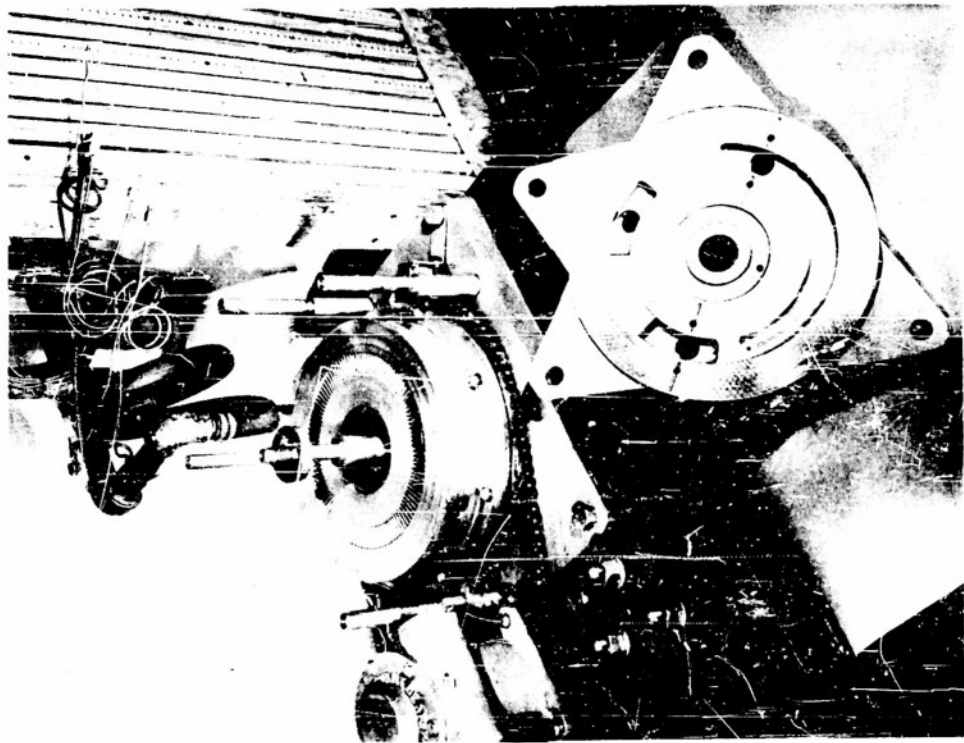
FIG. E-4, SEAL SHOES

FIG. E-5, STATIC TEST PIECE

FIG. E-3, ROTOR, UPPER PLATE

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