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WALL CORRECTIONS FOR A WIND TUNNEL WITH  
LONGITUDINAL SLOTS AT SUBSONIC VELOCITIES

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GOTTFRIED GUDERLEY

AIRCRAFT LABORATORY

JANUARY 1954

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**WALL CORRECTIONS FOR A WIND TUNNEL WITH  
LONGITUDINAL SLOTS AT SUBSONIC VELOCITIES**

*Gottfried Guderley*

*Aircraft Laboratory*

*January 1954*

*RDO No. 458-429*

Wright Air Development Center  
Air Research and Development Command  
United States Air Force  
Wright-Patterson Air Force Base, Ohio

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## FOREWORD

This report was prepared by the Wind Tunnel Branch, Aircraft Laboratory, Directorate of Laboratories, Wright Air Development Center, Wright-Patterson Air Force Base, Ohio. The project was administered under Research and Development Order Number 458-429, (UNCLASSIFIED), "Two-Dimensional and Axially Symmetric Transonic Flow," with Gottfried Guderley acting as project engineer.

This entire report is classified CONFIDENTIAL because it contains informative and descriptive materials relating to the design of a new type of test-section wall for circular wind tunnels. The security classification for such material has been established by the National Advisory Committee for Aeronautics.

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ABSTRACT

On the basis of a simplified boundary condition which has been derived in a previous report, the wind-tunnel corrections for walls with slots of different widths have been determined by a method analogous to that of I. Lotz. Formulae have been given for the source, the doublet and the infinitesimally small horseshoe vortex. The correction velocities at the wind-tunnel axis, the average velocities at the wind-tunnel walls and the height up to which the jet will penetrate into the slots have been computed and are represented in curves.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

  
DANIEL D. MCKEE  
Colonel, USAF  
Chief, Aircraft Laboratory  
Directorate of Laboratories

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NOTE: Wherever the number of slots in the wind tunnel is used (Figures 1 - 14, inclusive), it is assumed to be eight.

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LIST OF SYMBOLS

$x, r, \omega$	system of cylindrical coordinates
$x, y, z$	system of Cartesian coordinates, the $z$ axis lies in the plane $\omega = 0$
$n$	number of slots in the tunnel
$c$	ratio of open to the entire jet surface (opening ratio)
$C^*$	$= -\frac{2}{n} \log \sin \frac{c\pi}{2}$
$r_0$	radius of the tunnel
$r_1$	radius of half body due to a source
$F$	wing area
$c_L$	lift coefficient
$U$	free stream velocity
$V$	Volume of slender body
$\rho$	density of the undisturbed flow
$\Gamma$	circulation
$s$	span of horseshoe vortex
$\varphi_1$	potential which gives the deviation of the flow over the body in free air from a parallel flow
$\varphi_2$	potential which describes the wall influence
$\varphi = \varphi_1 + \varphi_2$	the potential which would describe the flow over the body in the tunnel, if the simplified boundary condition (1) were valid

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LIST OF SYMBOLS (Continued)

- $\lambda$  variable of integration which occurs in a Fourier integral
- $a_S, a_D, a_L$  functions which occur in Fourier integrals and which are related to the source, the doublet or the horseshoe vortex
- $F_S, F_D, F_L$  functions which express the wall influence along the wind-tunnel axis for the source, the doublet and the horseshoe vortex
- $G_S, G_D, G_L$  functions which express the average wall velocities for the source, the doublet and the horseshoe vortex
- $H_S, H_D, H_L$  the height up to which the jet penetrates into the slots for the source, the doublet and the horseshoe vortex
- $K_S, K_D, K_L$  functions by which the H's are expressed
- R the ratio of the maximum of the additional wall velocity to the average wall velocity at  $x = 0$ .

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## INTRODUCTION

The present report gives subsonic wall corrections for circular wind tunnels with longitudinal slots. The most important point is the introduction of the boundary conditions. The simplifications which one can make are studied in a separate report (Reference 1). There the conclusion is reached that for slots with a constant width which extend from negative to positive infinity, the wall acts in the average as if everywhere the condition

$$\varphi + C^* r_0 \varphi_r = 0 \quad (1)$$

with

$$C^* = -\frac{2}{n} \log \sin \frac{c\alpha}{2}$$

is imposed. Here  $\varphi$  is the potential which would describe the deviation of the flow from a parallel flow, if Equation (1) were the actual boundary condition at the wall (not only an average expression),  $n$  is the number of slots,  $r_0$  is the wind-tunnel radius,  $c$  is the ratio of open area to the entire surface of the jet (opening ratio). The above boundary condition as an average condition does not show a difference between the solid part of the wall and the slot. So one cannot expect that one obtains  $\varphi_r = 0$  along the solid part of the wall and  $\varphi = 0$  in the slots as the actual boundary conditions would require. Incidentally the quantity  $C^*$  used here is denoted in Reference 1 by  $K$ .

With this boundary condition the wind-tunnel corrections can be determined by a procedure which is quite analogous to that of I. Lotz (Reference 2).

As usual the body is replaced by singularities at the tunnel axis. The following singularities will be treated:

1. the doublet
2. the source
3. the infinitesimally small horseshoe vortex.

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The opening ratio  $c$  will be varied from zero (closed tunnel) to one (open tunnel). For these singularities the following data will be given:

1. the additional velocities at the tunnel axis due to the wall influence, or the downwash at the axis
2. the average values of  $\phi_x$  at the wind-tunnel wall
3. the height up to which, under the present assumptions, the slots will be filled with air from the tunnel.

The data will be described in a separate section.

For the computation the potential  $\phi$  is split up into two parts-- the first one,  $\phi_1$ , describes the flow in free air; the second one,  $\phi_2$ , gives the influence of the walls.

Attention is drawn to a report by Don D. Davis, Jr. and Dewey Moore, Reference 6, which appeared after the present report was finished and treats the same subject in a very similar manner.

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## SECTION I

### EXPRESSIONS FOR THE SINGULARITIES IN FREE AIR

In the following computations only an incompressible flow need be considered since the results can readily be transformed to subsonic flow fields by means of a Prandtl-Glauert correction.

In this paragraph representations of the various singularities which are assumed at the axis will be derived in the form of Fourier integrals. Once these expressions are known the correction potential can be found nearly immediately.

Let  $x, r, \omega$  be a system of cylindrical coordinates whose axis coincides with the tunnel axis

$U$  the velocity of the undisturbed flow

$\rho$  the density of the undisturbed flow.

Occasionally a system of Cartesian coordinates  $x, y, z$  will be used. The  $z$  axis will lie in the plane  $\omega = 0$ .

For an axisymmetric flow the potential equation  $\Delta \varphi = 0$  has the form

$$\varphi_{rr} + \frac{1}{r} \varphi_r + \varphi_{xx} = 0 \quad (2)$$

We shall use particular solution of this differential equation in the form

$$\varphi = Z_0(i\lambda r) \cdot \cos \lambda x \quad (3)$$

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Here  $Z_0$  denotes a linear combination of Bessel functions of zero order,  $\lambda$  is a real quantity. Among these solutions the following ones are of particular importance

$$i H_0^{(1)}(i\lambda r) \cos \lambda x \quad (4a)$$

and

$$J_0(i\lambda r) \cos \lambda x \quad (4b)$$

The first functions vanish exponentially as  $r$  goes to infinity, functions of this type will be used for the representation of the singularities at the axis. The other particular solution is the only one which is regular for  $r = 0$ , it will be used for the representation of the wall influence.

To discuss the expression (4a), we determine the behavior of the Hankel functions for small values of  $r$ . With the known formulae for Bessel functions (cf., e.g., Reference 4) one obtains-

$$i H_0^{(1)}(i\lambda r) \sim -N_0(i\lambda r) \sim -\frac{2}{\pi} \ln r$$

Close to the axis one hence obtains

$$\frac{d}{dr} (i H_0^{(1)}(i\lambda r)) = -\frac{2}{\pi} \cdot \frac{1}{r}$$

Thus it follows that the expression (4a) gives a flow field with a source distribution along the  $x$  axis, for which the source strength per unit length is given by

$$-4\rho \cos \lambda x$$

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A suitable superposition of the expression (4a) will give a source of unit strength at the origin. The Fourier integral for a  $\delta$ -function of weight one at the origin is given by

$$\frac{1}{\pi} \int_0^{\infty} \cos \lambda x \, d\lambda$$

Thus one obtains as the desired expression for the potential of a source with unit strength at the origin

$$\varphi_{1S} = -\frac{1}{4\rho r_0 \pi} \int_0^{\infty} i H_0^{(1)}(i\lambda \frac{r}{r_0}) \cos(\lambda \frac{x}{r_0}) \, d\lambda \quad (5)$$

In the notation of  $\varphi$  the subscript S indicates that the expression refers to a source; later the subscripts D for the doublet and L for the lifting element will be introduced. The subscript one with  $\varphi$  indicates that the expression refers to free-stream conditions. The last expression converges everywhere except at the axis; actually, it will be used only at the wind-tunnel wall. The corresponding closed form for the potential would be

$$\varphi_{1S} = -\frac{1}{4\rho r_0 \pi} \frac{1}{\sqrt{(\frac{x}{r_0})^2 + (\frac{r}{r_0})^2}}$$

The potential for a doublet is obtained by a differentiation with respect to  $x$  and a change of sign. Then the moment of this doublet has the value one. Thus one obtains

$$\varphi_{1D} = -\frac{1}{4\rho r_0^2 \pi} \int_0^{\infty} i H_0^{(1)}(i\lambda \frac{r}{r_0}) \cdot \lambda \cdot \sin(\lambda \frac{x}{r_0}) \, d\lambda \quad (6)$$

According to a well-known result of slender-body theory the doublet strength equivalent to a body of volume  $V$  is given by  $-UV\rho$ .

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For the sphere one obtains a very similar result, namely,  $-3/2 \cdot UV \varphi$ . For intermediate bodies one will use a correction factor with the expression  $-UV \varphi$  which depends upon the body shape (see e.g., Reference 5, Figure 2).

To arrive at the potential of an infinitesimally small horseshoe vortex with a distance of the two trailing vortices  $s$  and a circulation  $\Gamma$ , we investigate the behavior of its potential close to the axis. The horseshoe vortex may lie in the plane  $z = 0$ . If  $s$  is small, then at a station  $x$  sufficiently away from the airfoil one will have essentially the two dimensional potential, i.e.,

$$\frac{\Gamma}{2\pi} \left\{ \operatorname{arctg} \left( \frac{z}{y-\frac{s}{2}} \right) - \operatorname{arctg} \left( \frac{z}{y+\frac{s}{2}} \right) \right\}$$

For a small value of  $s$  this gives

$$-\frac{s\Gamma}{2\pi} \cdot \frac{d}{dy} \left( \operatorname{arctg} \frac{z}{y} \right) = \frac{s\Gamma}{2\pi} \cdot \frac{z}{r^2}$$

The result indicates that along the positive  $x$  axis the potential of a horseshoe vortex behaves like

$$\frac{s\Gamma}{2\pi} \cdot \frac{1}{r} \cos \omega.$$

Along the negative  $x$  axis it is obviously regular. To obtain an expression for such a potential we first decompose it into a symmetrical and into an antisymmetrical part, with respect to the plane  $x = 0$ , i.e., into the potential of two vortices of opposite sign with strength

$\frac{\Gamma}{2}$  which extend from negative infinity, and into the potential of a

vortex system which agrees with the symmetrical part for positive values of  $x$  and has the opposite sign for negative values of  $x$ . The potential of the first part is given by

$$\frac{s\Gamma}{4\pi} \cdot \frac{1}{r} \cos \omega$$

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The potential of the second part will be obtained by a super-position of expressions of the form

$$- H_1^{(1)}(i\lambda \frac{r}{r_0}) \cos \omega \cdot \sin(\lambda \frac{x}{r_0})$$

They are solutions of the potential equation. Along the x axis one has

$$- H_1^{(1)}(i\lambda \frac{r}{r_0}) \sim -i N_1(i\lambda \frac{r}{r_0}) \sim \frac{i}{\pi} \left( \frac{2}{i\lambda \frac{r}{r_0}} \right) = \frac{2}{\pi \lambda \frac{r}{r_0}} \quad (7)$$

The Fourier integral for the unit step function is

$$\frac{i}{\pi} \int_0^{\infty} \frac{1}{\lambda} \cdot \sin(\lambda \frac{x}{r_0}) d\lambda$$

Thus one obtains for the potential of the antisymmetric vortex distribution

$$\frac{s \cdot \Gamma \cos \omega}{4\pi r_0} \int_0^{\infty} (-H_1^{(1)}(i\lambda \frac{r}{r_0}) \cdot \sin(\lambda \frac{x}{r_0}) d\lambda$$

The lift produced by the bound vortex is

$$s \cdot \Gamma \cdot \rho \cdot u$$

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Thus the potential for an infinitesimally small horseshoe vortex with unit lift is given by

$$\psi_{1L} = \frac{\cos \omega}{\rho U 4\pi r_0} \left( \frac{r_0}{r} + \int_0^{\infty} (-H_1''(i\lambda \frac{r}{r_0}) \cdot \sin(\lambda \frac{x}{r_0}) d\lambda \right) \quad (9)$$

An alternative procedure to derive this expression is the following: one starts with an even source distribution in that region of the plane  $z = 0$  which is enclosed by the horseshoe vortex. The source strength per unit of area may be one. The potential of this flow is continuous everywhere, its derivative in the  $z$  direction makes a unit jump along this sheet. Since the derivative of the potential with respect to  $z$  fulfills also the potential equation, this derivative represents a new potential function which is continuous everywhere except in the portion of the plane  $z = 0$  which is enclosed by the horseshoe vortex. This expression then represents the desired potential of the horseshoe vortex. Naturally, this procedure can be used for an infinitesimally narrow horseshoe vortex. The potential of the source distribution is at some distance from the vortex, that of a line source whose discharge of mass per unit length is given by  $s$  ( $s$  was introduced as the span of the horseshoe vortex). This expression is given by

$$\frac{s}{4\pi\rho} \ln r - \frac{s}{4\pi\rho} \int_0^{\infty} \frac{i H_0''(i\lambda \frac{r}{r_0})}{\lambda} \cdot \sin(\lambda \frac{x}{r_0}) d\lambda \quad (10)$$

The derivative with respect to  $z$  gives then the potential of the horseshoe vortex. Thus one obtains again the previous formula.

## SECTION II

### THE WALL INFLUENCE

One must now determine a solution  $\psi_2$  of the potential equation which is regular inside of the tunnel and which, if added to the

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expression for the potential of a source, a doublet and a horseshoe vortex, fulfills the boundary condition (1).

In the following equations  $a_S(\lambda)$ ,  $a_D(\lambda)$  and  $a_L(\lambda)$  will be suitable functions of  $\lambda$ , used to express the potential by means of a Fourier integral.

The correction potential for a source may be expressed by

$$\varphi_{2S} = \frac{1}{4\rho r_0 \pi} \int_0^{\infty} a_S(\lambda) J_0(i\lambda \frac{r}{r_0}) \cos(\lambda \frac{x}{r_0}) d\lambda \quad (11)$$

The function  $a_S(\lambda)$  is determined by inserting the expression

$\varphi_{1S} + \varphi_{2S}$  into the boundary condition (1).  $\varphi_{1S}$  is given by Equation (5). At the tunnel boundary one has  $r = r_0$ . One thus obtains

$$-\frac{1}{4\rho r_0 \pi} \int_0^{\infty} (iH_0''(i\lambda) + r_0 \cdot C \cdot \frac{d}{dr} (iH_0''(i\lambda \frac{r}{r_0})) \Big|_{r=r_0} - a_S(\lambda) (J_0(i\lambda) + r_0 \cdot C \cdot \frac{d}{dr} (J_0(i\lambda \frac{r}{r_0})) \Big|_{r=r_0}) \cos(\lambda \frac{x}{r_0}) d\lambda = 0$$

Hence one finds

$$a_S(\lambda) = \frac{iH_0''(i\lambda) + C \cdot r_0 \frac{d}{dr} (iH_0''(i\lambda \frac{r}{r_0})) \Big|_{r=r_0}}{J_0(i\lambda) + C \cdot r_0 \frac{d}{dr} (J_0(i\lambda \frac{r}{r_0})) \Big|_{r=r_0}}$$

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Using the formula for the derivative of a Bessel function and inserting the resulting expression for  $a_S(\lambda)$  into Equation (11) one obtains

$$\varphi_{2s} = \frac{1}{4\rho r_0 \pi} \int_0^{\infty} \frac{i H_0''(i\lambda) - C^* \lambda (-H_1''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} J_0(i\lambda \frac{r}{r_0}) \cos(\lambda \frac{x}{r_0}) d\lambda \quad (12)$$

The corresponding expression for the doublet can be found in an analogous manner, but it can be found more quickly, if we remember that the potential  $\varphi_{1d}$  is the negative derivative of  $\varphi_{1s}$  with respect to  $x$ . The boundary condition is prescribed along a surface  $r = r_0 =$  constant; therefore, it can be differentiated with respect to  $x$ . Accordingly, if one differentiates the entire expression for the source

$\varphi_{1s} + \varphi_{2s}$  with respect to  $x$ , the result will fulfill the boundary condition (1). Thus

$$\varphi_{2d} = -\frac{\partial}{\partial x} (\varphi_{2s})$$

or explicitly

$$\varphi_{2d} = \frac{1}{4\rho r_0^2 \pi} \int_0^{\infty} \frac{i H_0''(i\lambda) - C^* \lambda (-H_1''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \cdot J_0(i\lambda \frac{r}{r_0}) \lambda \cdot \sin(\lambda \frac{x}{r_0}) d\lambda \quad (13)$$

For the lift case the investigation is quite similar. We set

$$\varphi_{2L} = \frac{\cos \omega}{\rho U 4\pi r_0} \left( \delta \frac{r}{r_0} + \int_0^{\infty} a_L(\lambda) (-i J_1(i\lambda \frac{r}{r_0})) \sin(\lambda \frac{x}{r_0}) d\lambda \right)$$

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where  $\gamma$  is a constant which must be determined. Inserting the expression  $\varphi_{L1} + \varphi_{L2}$  into the boundary conditions one obtains finally

$$\varphi_{L2} = \frac{\cos \omega}{\rho u 4\pi \rho_0} \left( \frac{C^* - 1}{C^* + 1} \frac{r}{r_0} - \int_0^{\infty} \frac{(1-C^*)(-H_1''(i\lambda) - C^* \lambda i H_0(i\lambda))}{(1-C^*)(-J_1(i\lambda) + C^* \lambda J_0(i\lambda))} \cdot (-i J_1(i\lambda \frac{r}{r_0}) \cdot \sin(\lambda \frac{x}{r_0})) d\lambda \right) \quad (14)$$

### SECTION III

#### SPECIALIZED FORMULAE

From the expressions (5), (6), (9), (12), (13) and (14) one obtains without difficulty the quantities which are of practical interest.

Since  $J_0(0) = 1$

one finds for the wind-tunnel axis  $r = 0$

$$\frac{\partial \varphi_{2s}}{\partial x} = - \frac{1}{4\rho r_0^2 \pi} \int_0^{\infty} \frac{i H_0'''(i\lambda) - C^* \lambda (-H_1''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda \cdot \sin(\lambda \frac{x}{r_0}) d\lambda \quad (15)$$

and

$$\frac{\partial \varphi_{20}}{\partial x} = \frac{1}{4\rho r_0^3 \pi} \int_0^{\infty} \frac{i H_0'''(i\lambda) - C^* \lambda (-H_1''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda^2 \cos(\lambda \frac{x}{r_0}) d\lambda \quad (16)$$

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For reasons of symmetry the horseshoe vortex does not produce additional values of  $\phi_x$  along the wind-tunnel axis, but it will give a downwash. The downwash is determined as

$$\frac{d\phi_{2L}}{dz}$$

Now one has

$$\frac{d\phi_{2L}}{dz} = \frac{\partial\phi_{2L}}{\partial r} \cos\omega - \frac{1}{r} \frac{\partial\phi_{2L}}{\partial\omega} \sin\omega$$

The series development of Bessel functions yields for small values of  $\lambda \frac{r}{r_0}$

$$(-iJ_1(i\lambda \frac{r}{r_0})) \sim \frac{1}{2} \lambda \frac{r}{r_0}$$

Thus one obtains from Equation (14)

$$\frac{\partial\phi_{2L}}{\partial z} = \frac{1}{\rho U 4\pi r_0^2} \left( \frac{C^* - 1}{C^* + 1} - \frac{1}{2} \int_0^\infty \frac{(1-C^*)[-H_1''(i\lambda) - C^* \lambda (iH_0''(i\lambda))]}{(1-C^*)(-iJ_1(i\lambda) + C^* \lambda J_0(i\lambda))} \lambda \sin(\lambda \frac{x}{r_0}) d\lambda \right) \quad (17)$$

Equations (15), (16) and (17) may be written in a form which puts the geometrical quantities into better evidence. As is known a source in a parallel flow corresponds to the flow over a half body. If the radius of the half body is  $r_1$  then the source strength is  $\pi r_1^2 \rho U$ .

Thus the expression (15) can be rewritten as

$$\frac{\partial\phi_{2S}}{\partial x} = U \frac{r_1^2}{r_0^2} \cdot F_S(x) \quad \text{along the axis} \quad (18)$$

with

$$F_S(x) = -\frac{1}{4} \int_0^\infty \frac{iH_0''(i\lambda) - C^* \lambda (-H_1''(i\lambda))}{J_0(i\lambda + C^* \lambda (-iJ_1(i\lambda)))} \lambda \sin(\lambda \frac{x}{r_0}) d\lambda \quad (18a)$$

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It was mentioned previously that a slender body of volume  $V$  is equivalent to a doublet of strength  $-UV\varrho$ . Hence one obtains from Equation (16)

$$\frac{\partial \varphi_{2D}}{\partial x} = U \frac{V}{r_0^3} F_D(x) \quad \text{along the axis} \quad (19)$$

with

$$F_D(x) = -\frac{1}{4\pi} \int_0^\infty \frac{iH_0''(i\lambda) - C^*\lambda(-H_1''(i\lambda))}{J_0(i\lambda) + C^*\lambda(-iJ_1(i\lambda))} \lambda^2 \cos(\lambda \frac{x}{r_0}) d\lambda \quad (19a)$$

Finally one can express the lift  $L$  by means of the lift coefficient  $c_L$  and the area of the model  $F$

$$L = F \cdot c_L \cdot 1/2 \varrho U^2$$

Then one obtains from Equation (17) for the change of the angle of attack

$$\Delta \alpha = \frac{F c_L}{8\pi r_0^2} \left( \frac{C^* - 1}{C^* + 1} - F_L(x) \right) \quad \text{along the axis} \quad (20)$$

with

$$F_L(x) = \frac{1}{2} \int_0^\infty \frac{(1-C^*)(-H_1''(i\lambda) - C^*\lambda(iH_0''(i\lambda)))}{(1-C^*)(-iJ_1(i\lambda) + C^*\lambda J_0(i\lambda))} \lambda \sin(\lambda \frac{x}{r_0}) d\lambda \quad (20a)$$

The average velocities at the walls are obtained by putting  $r = r_0$  in the expression

$$\varphi_x = \varphi_{1x} + \varphi_{2x}$$

Thus one finds

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$$\varphi_{xS} = u \frac{r_1^2}{r_0^2} \cdot \frac{1}{4} \int_0^{\infty} \left( i H_0'''(i\lambda) - \frac{i H_0'''(i\lambda) - C^* \lambda (-H_1'''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} J_0(i\lambda) \right) \lambda \sin(\lambda \frac{x}{r_0}) d\lambda$$

By the formulae on page 144 of Reference 4 the last expression can be simplified and one obtains

$$\varphi_{xS} = u \cdot \frac{r_1^2}{r_0^2} \cdot G_S(x) \quad \text{along the wall} \quad (21)$$

where

$$G_S(x) = \frac{C^*}{2\pi} \int_0^{\infty} \frac{1}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda \cdot \sin(\lambda \frac{x}{r_0}) d\lambda \quad (21a)$$

similarly

$$\varphi_{xD} = u \cdot \frac{V}{r^3} G_D(x) \quad \text{along the wall} \quad (22)$$

where

$$G_D = \frac{C^*}{2\pi^2} \int_0^{\infty} \frac{1}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda^2 \cdot \cos(\lambda \frac{x}{r_0}) d\lambda \quad (22a)$$

and

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$$\varphi_{xL} = \frac{F \cdot c_L \cdot u}{8\pi r_0^2} \cos \omega G_L(x) \quad \text{along the wall} \quad (23)$$

where

$$G_L = \frac{2c^*}{\pi} \int_0^{\infty} \frac{\lambda \cdot \cos(\lambda \frac{x}{r_0})}{(1-c^*)(-J_1(i\lambda)) + c^* \lambda \cdot J_0(i\lambda)} d\lambda \quad (23a)$$

Furthermore, the height up to which the wind-tunnel air enters the slots will be determined. It is found by an integration of the streamline slopes in the slots. The procedure is the following equations. From Equations (21) and (21a) one obtains by an integration

$$\varphi_S = \frac{4r_1^2}{r_0^2} \int_{-\infty}^x G_S\left(\frac{x}{r_0}\right) d\left(\frac{x}{r_0}\right)$$

This is the average potential along the wall. Hence one obtains by means of the boundary condition (1)

$$\varphi_{Sr} = - \frac{4r_1^2}{r_0^2} \cdot \frac{1}{c^*} \int_{-\infty}^x G_S\left(\frac{x}{r_0}\right) d\left(\frac{x}{r_0}\right)$$

This is the average value of  $\varphi_r$ . For reasons of continuity the value of  $\varphi_r$  in the slot is this average value multiplied with the reciprocal of the opening ratio  $c$ . Thus

$$\varphi_{r \text{ slot}} = - \frac{1}{c} \frac{4r_1^2}{r_0^2} \cdot \frac{1}{c^*} \int_{-\infty}^x G_S\left(\frac{x}{r_0}\right) d\left(\frac{x}{r_0}\right)$$

Let us denote the heights up to which the slots are filled by  $H_S$ ,  $H_D$  and  $H_L$ . One then finds

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$$\frac{H_S}{r_0} = \frac{1}{c} \cdot \frac{r_0^2}{r_0^2} \cdot K_S(x) \quad (24)$$

where

$$K_S(x) = -\frac{1}{c} \int_{-\infty}^x d\eta \int_{-\infty}^{\eta} G_S(\xi) d\xi \quad (24a)$$

Here  $\xi$  and  $\eta$  are variables of integration.

Similarly

$$\frac{H_D}{r_0} = \frac{1}{c} \left( \frac{V}{r_0^3} \right) \cdot K_D(x) \quad (25)$$

where

$$K_D(x) = -\frac{1}{c} \int_{-\infty}^x d\eta \int_{-\infty}^{\eta} G_D(\xi) d\xi \quad (25a)$$

and

$$\frac{H_L}{r_0} = \frac{1}{c} \cdot \frac{F \cdot c_L}{8\pi r_0^2} K_L(x) \cdot \cos \omega \quad (26)$$

where

$$K_L(x) = -\frac{1}{c} \int_{-\infty}^x d\eta \int_{-\infty}^{\eta} G_L(\xi) d\xi \quad (26a)$$

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## SECTION IV

### SURVEY OF THE FIGURES

The functions necessary to determine the wall influence along the wind-tunnel axis are shown in Figures 1, 2 and 3 for different values of  $C^*$  as a function of  $x/r_0$ . Besides the values of  $C^*$  the values of  $c$  for a wind tunnel with eight slots have been given. According to Equations (18) and (19) Figures 1 and 2 give the velocities along the wind-tunnel axis caused by the wall influence for a source and a doublet.

Figure 1 shows that a slotted tunnel does not require a Mach number correction due to the wake of the body. Such a Mach number correction will occur if the Mach number change due to the wall far upstream of the model is different from the Mach number change at the location of the model. However, this happens only for a closed tunnel and not for slotted tunnels, since the presence of the slots enforces far upstream and far downstream a Mach number which is determined by the pressure outside of the slots.

In a closed tunnel a correction of the drag due to the pressure gradient caused by the wake may have some importance. Figure 1 shows that in slotted tunnels this correction would be only a fraction of the correction in a closed tunnel.

Figure 2 gives the influence of the thickness of a body. One is primarily interested in the Mach number correction at the location of the model. Therefore, Figure 10 shows the function  $F_D$  versus  $1/C^*$ . The scattered points are the points which can be found in the corresponding figure of Reference 3. In Figure 11 the same quantity has been plotted over the opening ratio  $c$  for a wind tunnel with eight slots. This graph shows very distinctly that even an extremely small opening ratio gives a very appreciable deviation from the values of a closed tunnel. The behavior of the curve 11 at  $c = 0$  is studied in an appendix. Practically, non-linear effects would give a more gradual transition for small values of the opening ratio.

Figure 3 gives essentially the downwash along the axis except for the quantity  $(C^* - 1)/(C^* + 1)$  in Equation (20). At the location of the model the downwash is determined by this quantity only. Therefore,

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Figure 3 illustrates only how the downwash changes. Eventually the results could be expressed as a camber correction. For this purpose Figure 14 shows for  $x = 0$

$$\frac{d F_L}{d \left( \frac{x}{r_0} \right)} \quad \text{versus } 1/C^*.$$

It is remarkable that the value of  $C^*$  for zero Mach number correction (Figure 10) and for zero camber correction (Figure 14) are not too different; thus camber correction and Mach number correction can practically be eliminated simultaneously.

The average wall velocities can be obtained from the graphs 4, 5 and 6 by Equations (21), (22) and (23). The order of magnitude of these perturbations is quite appreciable and suggests caution in the application of the present results in the transonic range. To show the relative orders of magnitudes, the ratio of the Mach number correction along the tunnel axis at the location of the model to the Mach number deviation at the wall from the free-stream Mach number (also for  $x = 0$ ) has been plotted for a doublet in Figure 12 versus  $C^*$ . In the small range of values  $C^*$  shown there, this ratio is quite small.

The average wall velocity  $\varphi_x$  which is obtained from Equations (21), (22) and (23) cannot be measured directly. However, one can measure the maximum wall velocity by pressure holes in the middle of the solid part of the wall. The ratio between the average and the maximum velocity deviation, may be denoted by  $R$ , depends upon the opening ratio  $c$  and can be found from the consideration of Reference 1. This ratio is represented in Figure 13.

The penetration of the wind-tunnel air into the slots can be found from the curves 7, 8 and 9 by means of the formulae (24), (25) and (26). Reference 1 showed that the penetration of the tunnel air into the slot may have a very essential influence with respect to the boundary condition. These graphs can be used in order to obtain an insight into possible errors for a given case.

In general, it is remarkable that a relatively small opening ratio suffices to make the behavior of the slotted tunnel quite similar to that of an open one. For values of  $C^*$  for which the Mach number corrections at the axis are small, the results are not too sensitive against a change of the constant  $C^*$ . This is quite encouraging from the practical point of view.

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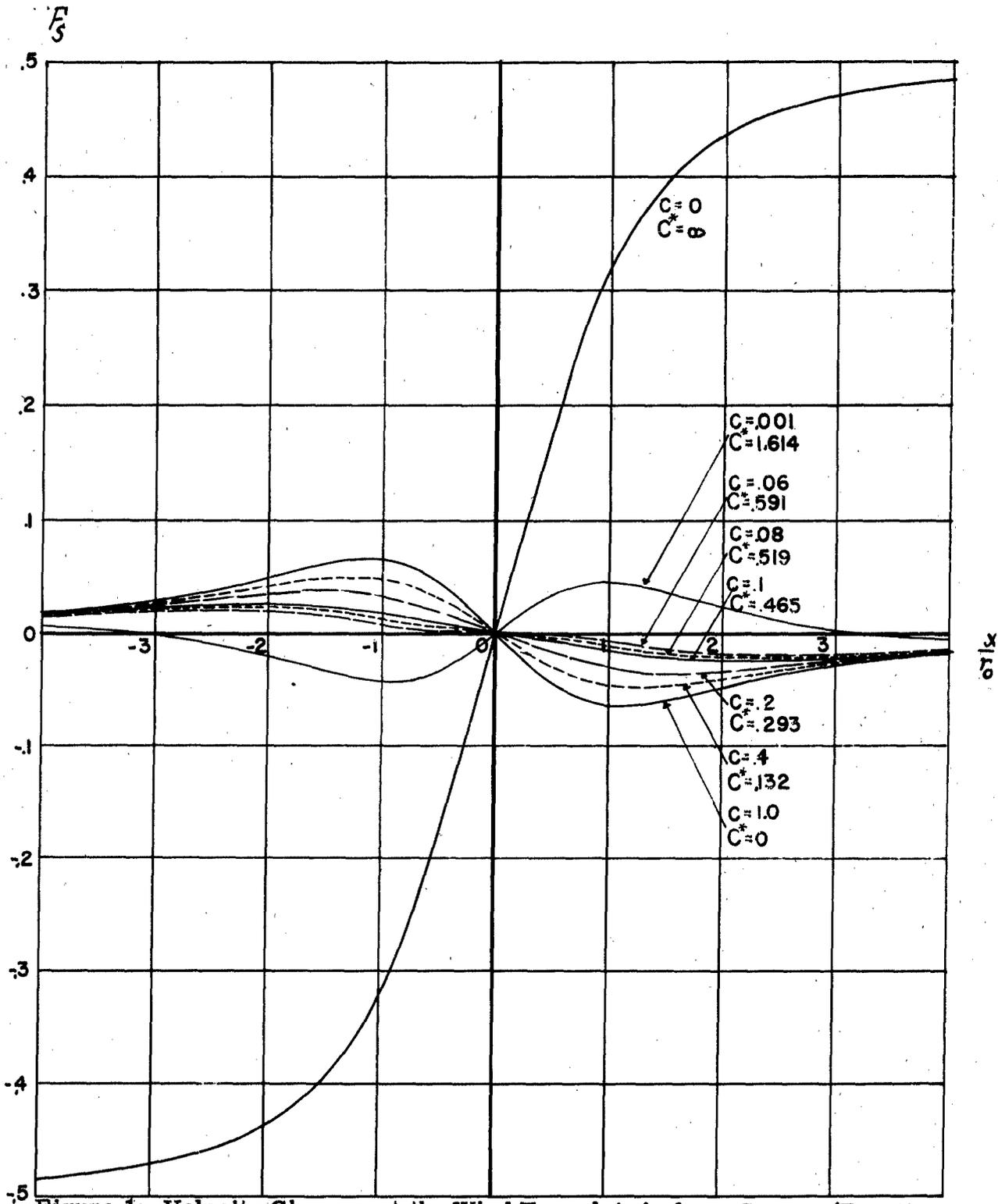


Figure 1: Velocity Changes at the Wind Tunnel Axis for a Source (Functions  $F_S(x)$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

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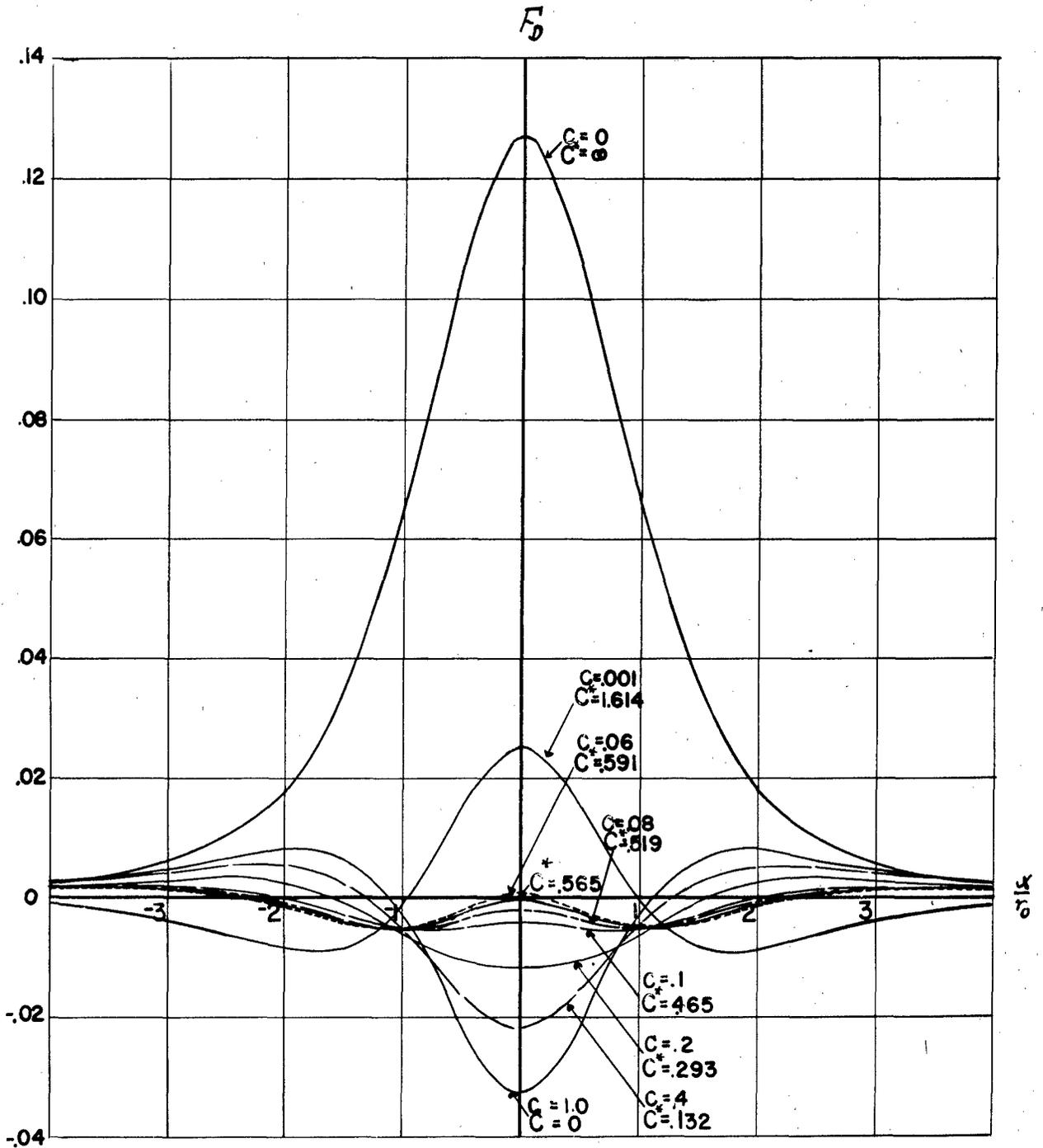


Figure 2: Velocity Changes at the Wind Tunnel Axis for a Doublet  
(Function  $F_D$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

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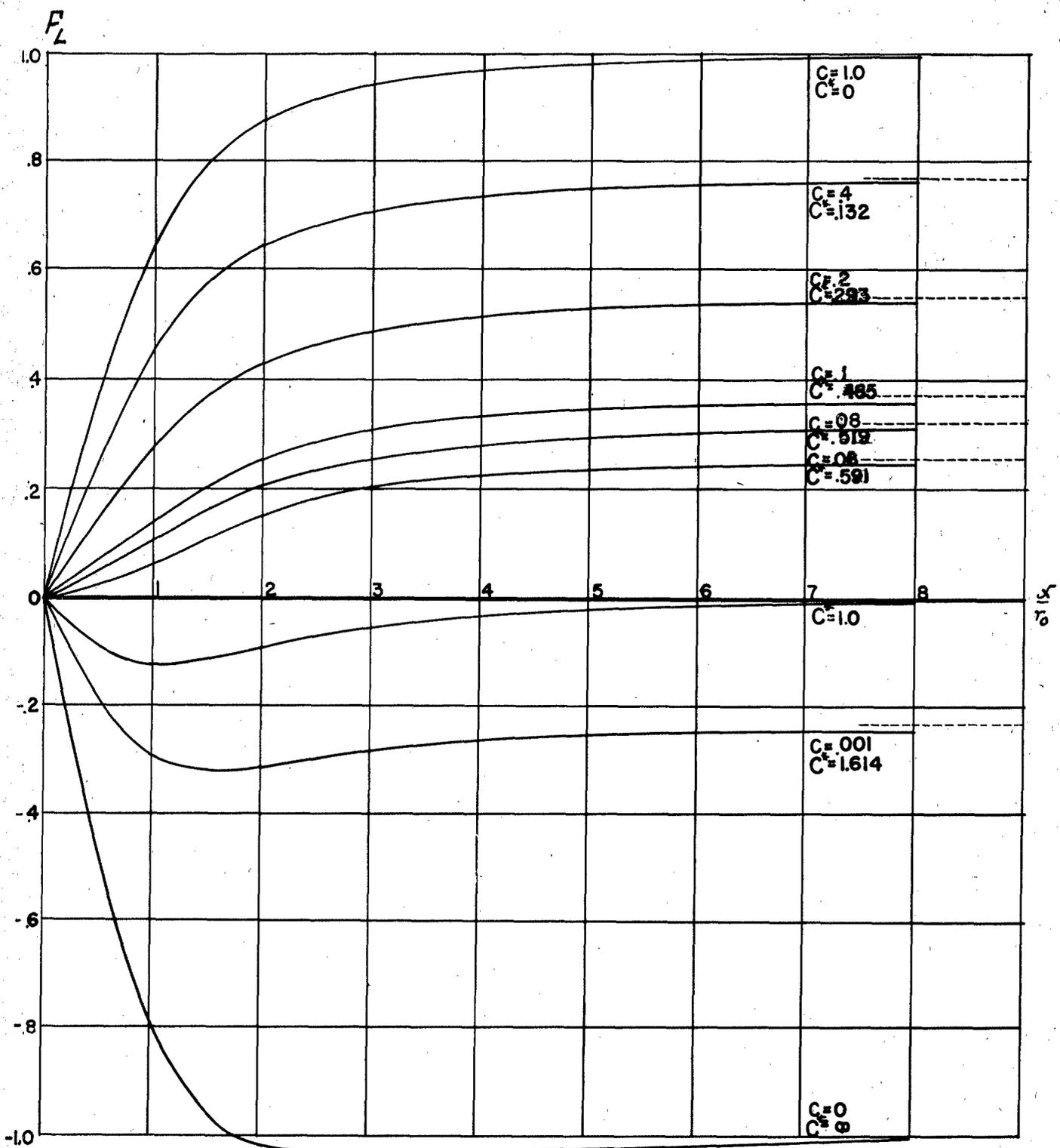


Figure 3: Downwash Due to the Wall Influence for a Horseshoe Vortex (Function  $F_L$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

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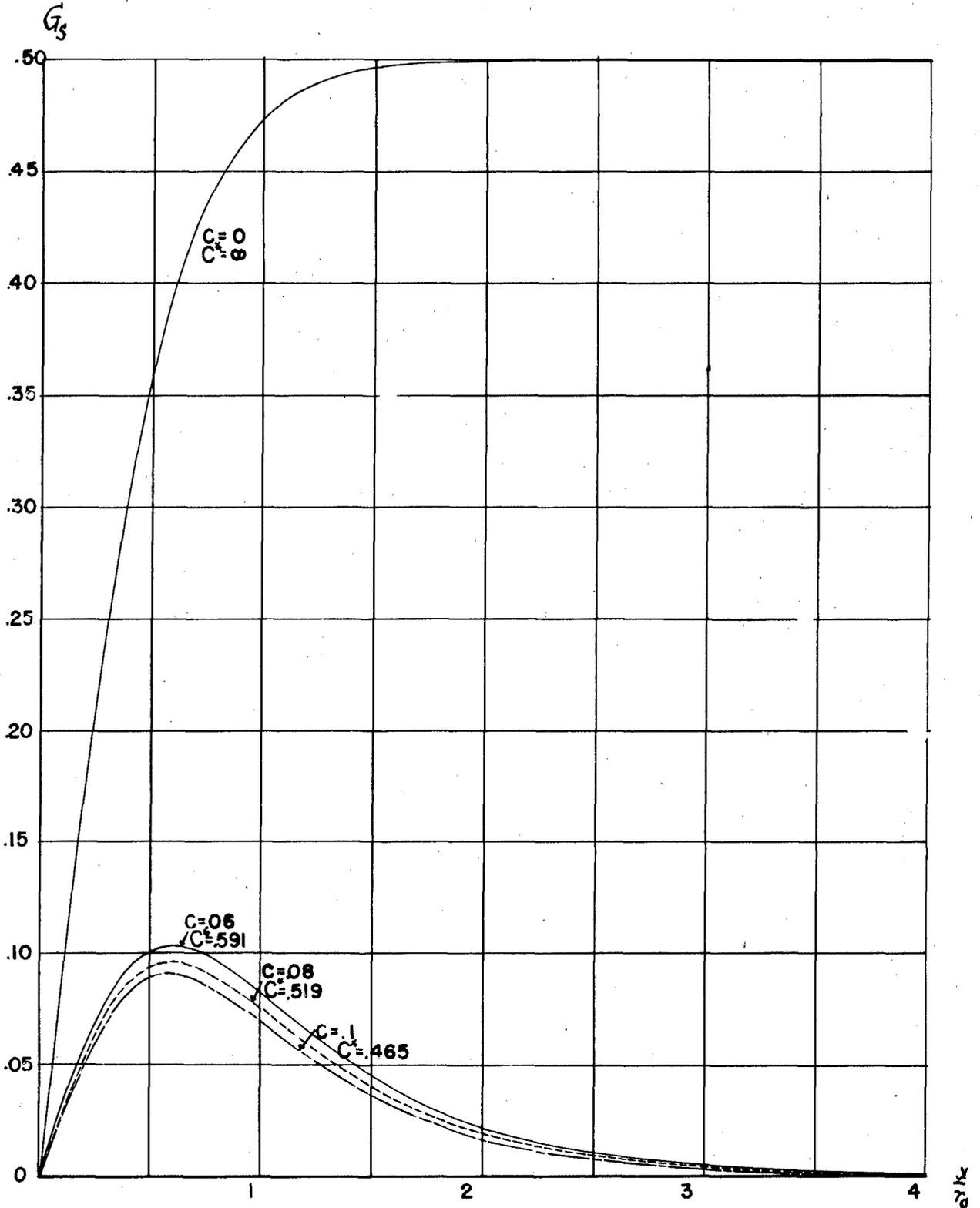


Figure 4: Average Wall Velocity for a Source (Function  $G_S$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

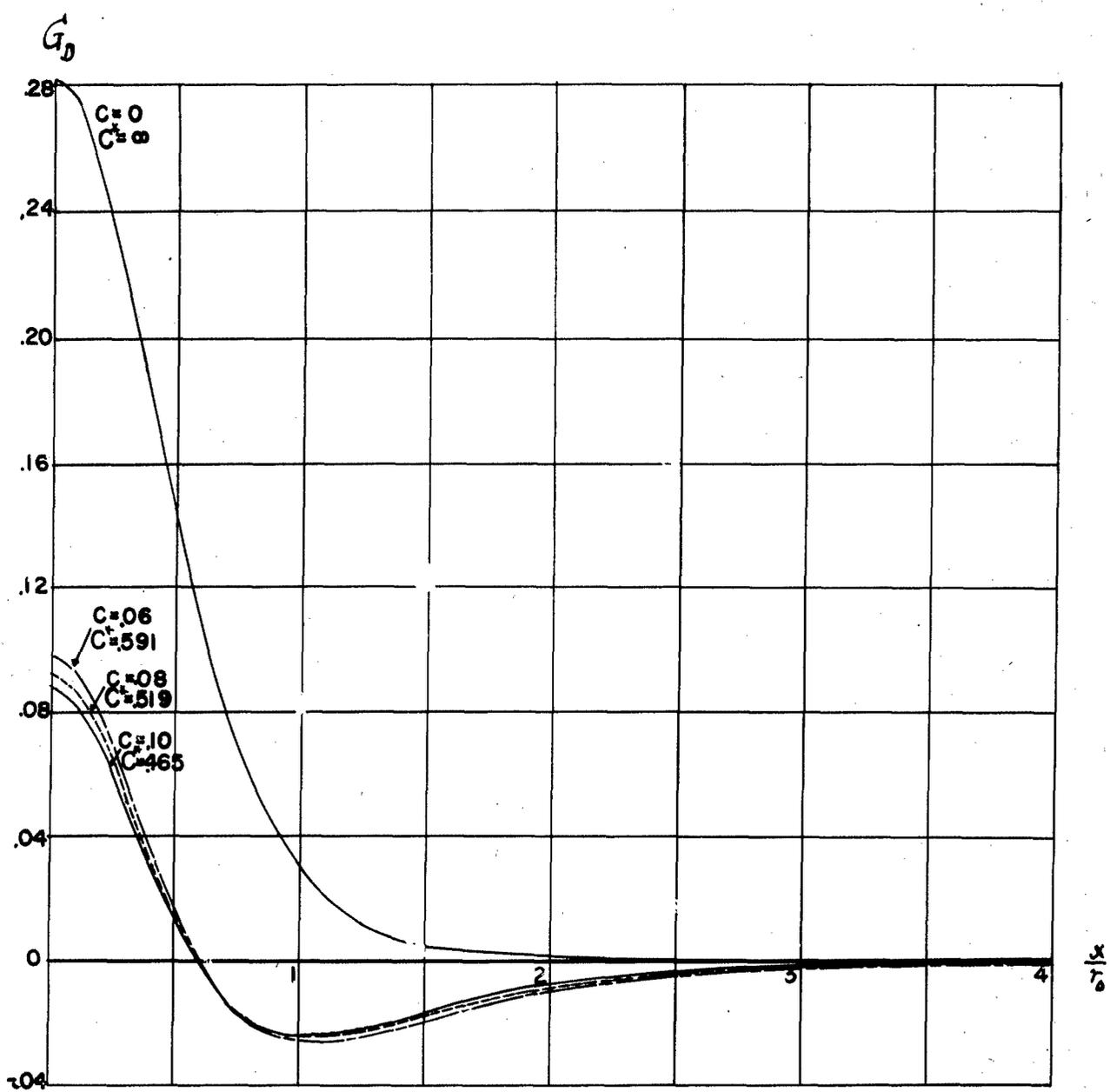


Figure 5: Average Wall Velocity for a Doublet (Function  $G_D$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

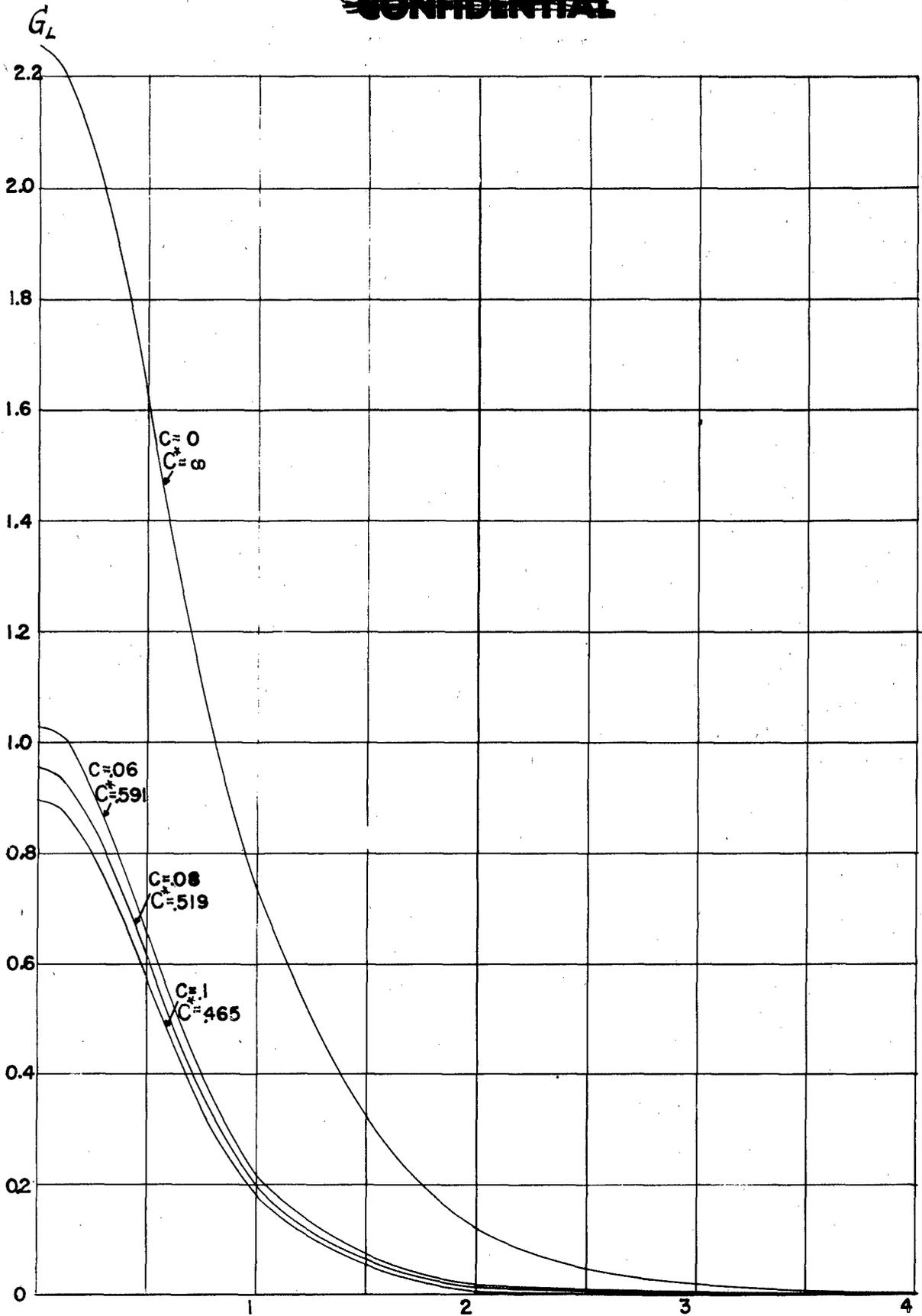


Figure 6: Average Wall Velocity for a Horseshoe Vortex (Function  $G_L$  Versus  $x/r_0$  for Various Values of  $c$  or of  $C^*$ )

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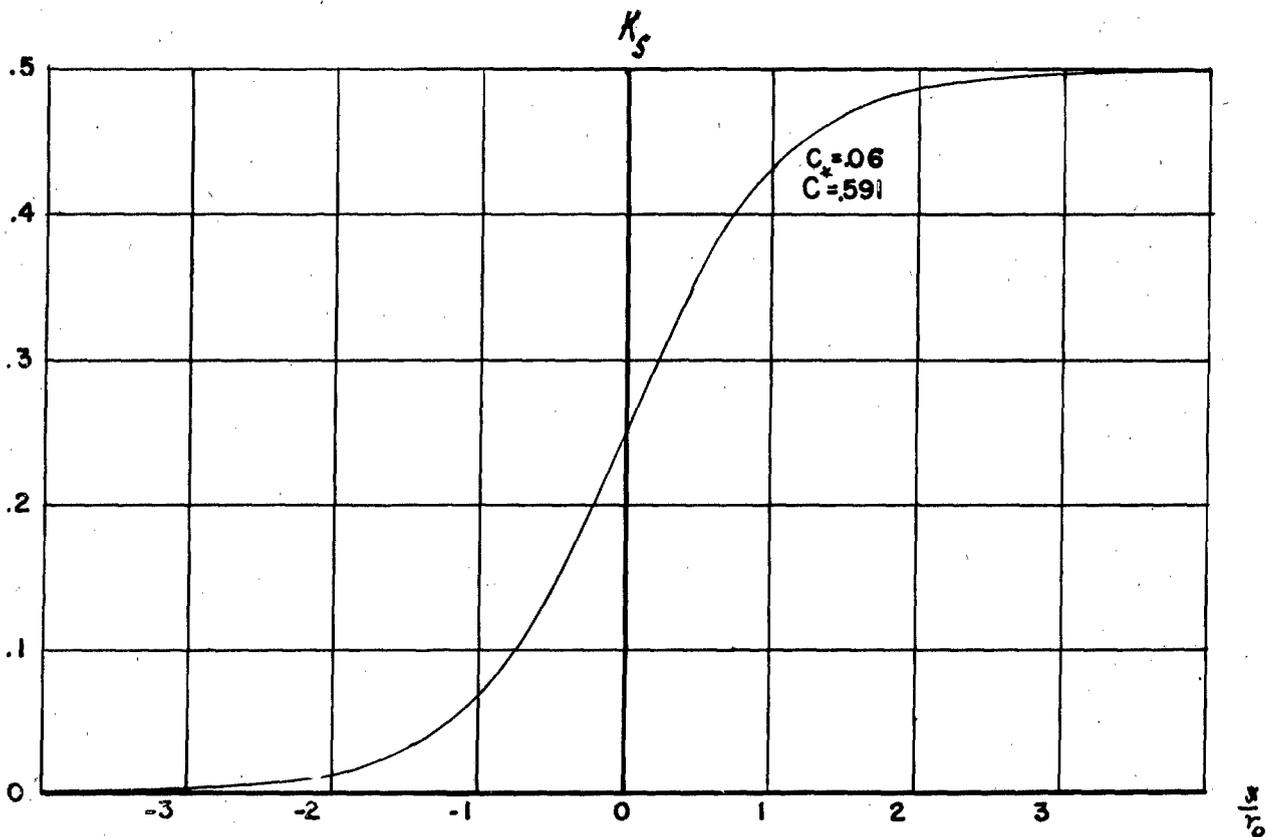


Figure 7: Penetration of the Jet into the Slots for Source (Function  $K_s$  Versus  $x/r_0$  for One Value of  $c$  and the Corresponding Value of  $C^*$ ). (The value  $C^*$  is chosen in such a manner that the correction velocity for the doublet is close to zero.)

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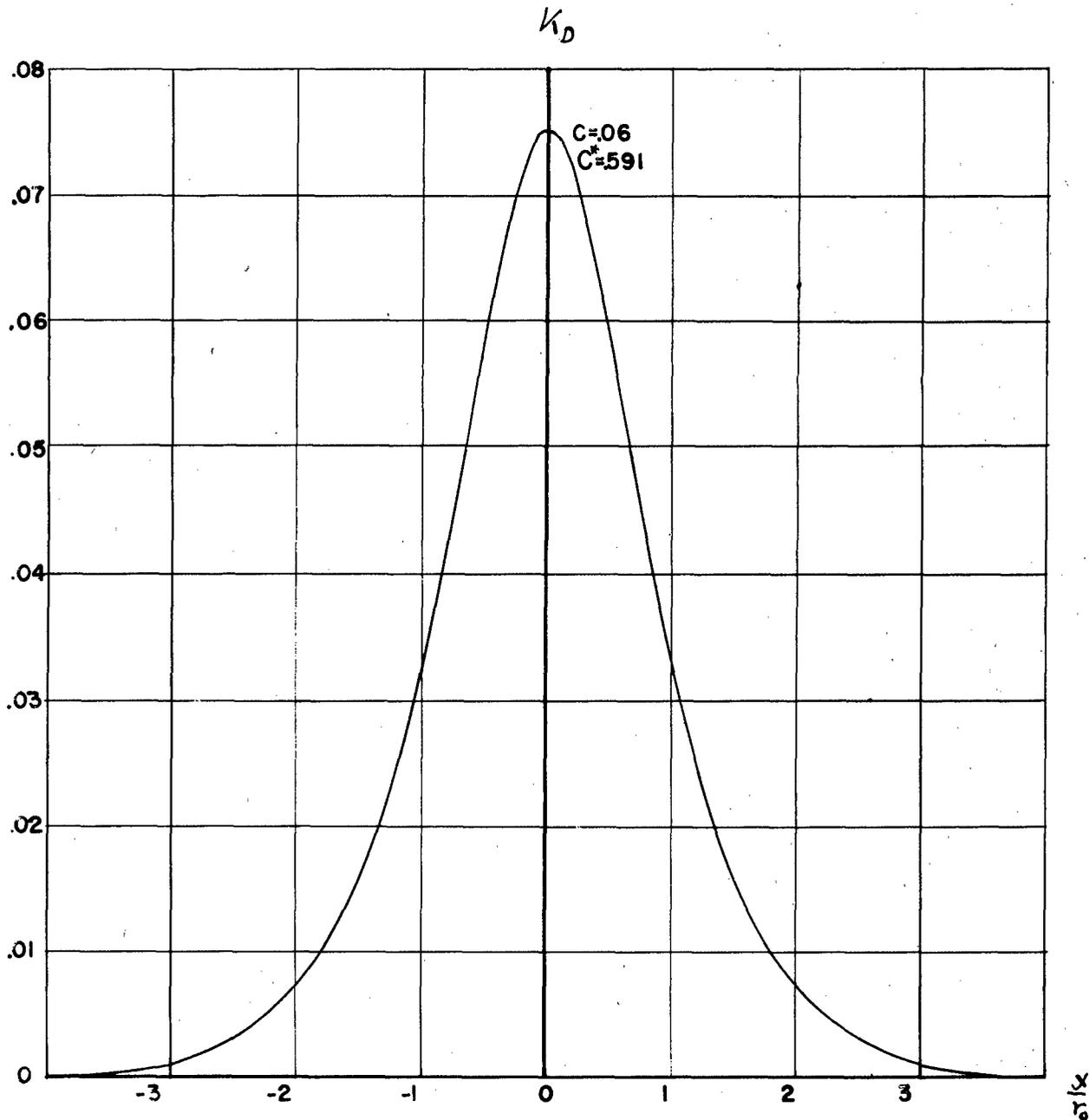


Figure 8: Penetration of the Jet into the Slots for Doublet (Function  $K_D$  Versus  $x/r_0$  for the Values of  $c$  or  $C^*$  of Giure 7)

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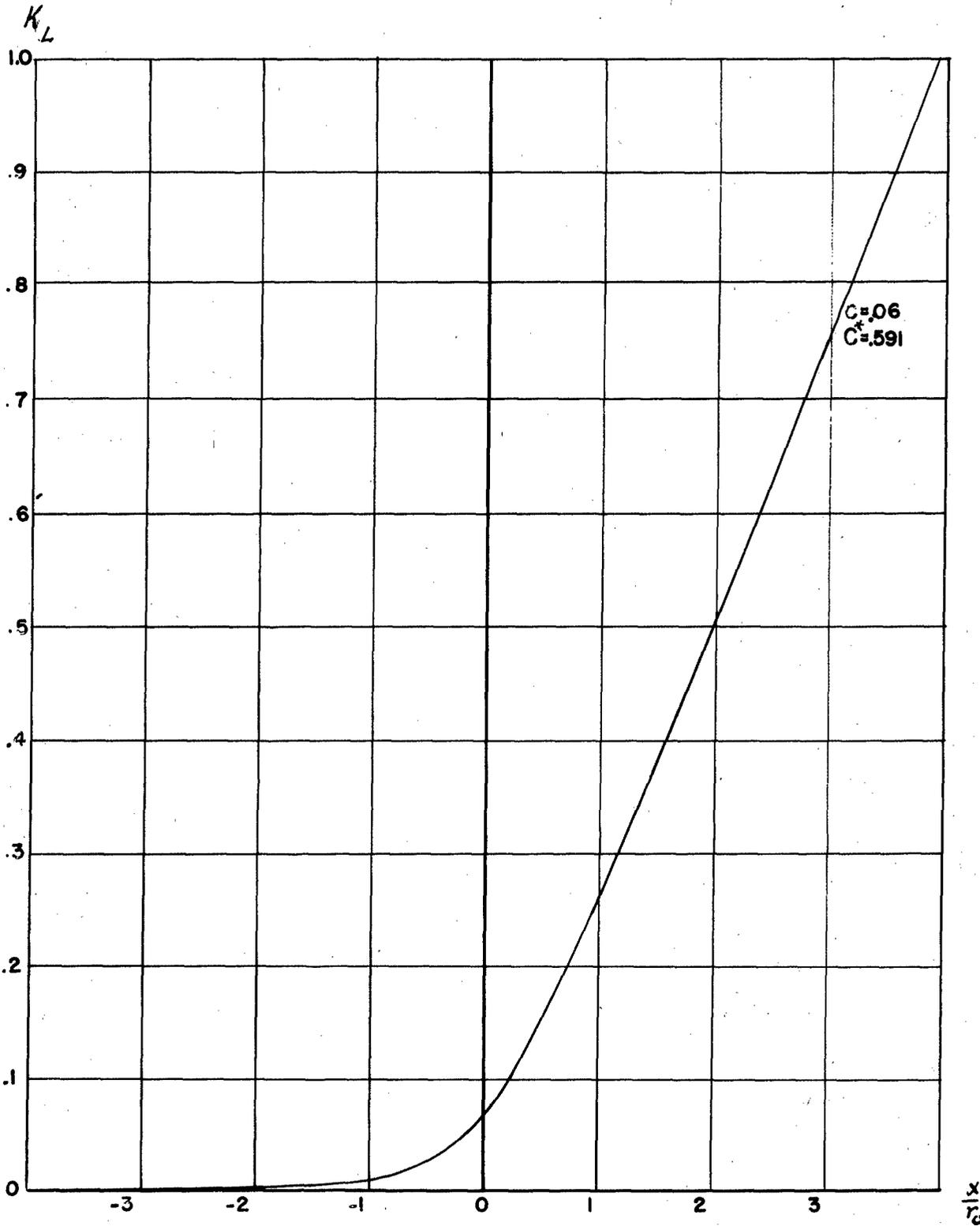


Figure 9: Penetration of the Jet into the Slots for Lift (Function  $K_L$  Versus  $x/r_0$  for the Values of  $c$  or  $C^*$  of Figure 7)

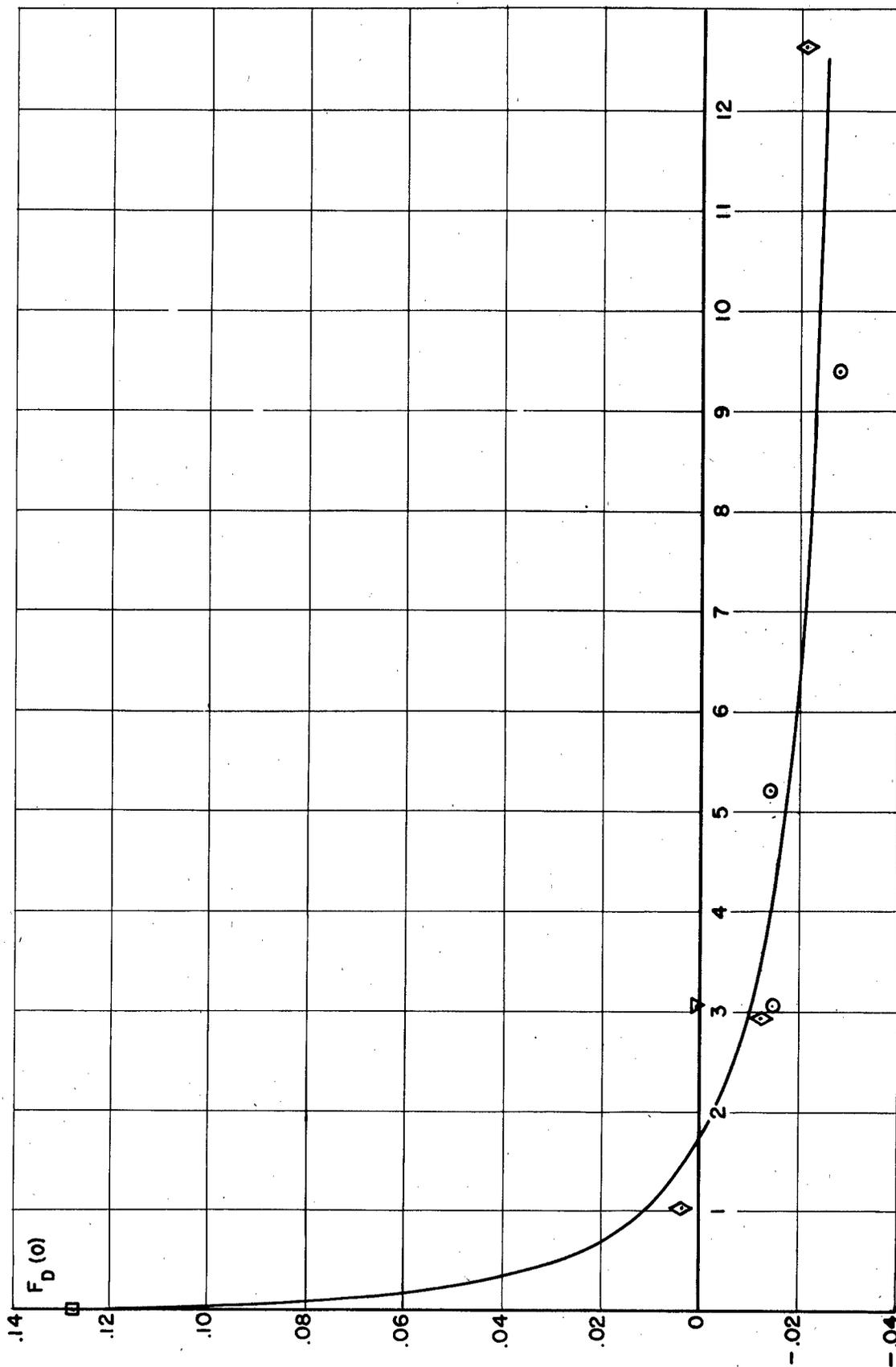


Figure 10: Correction Velocities at the Wind Tunnel Axis for  $x = 0$  Versus  $C^{*-1}$  (Function  $F_D(0)$  Versus  $C^{*-1}$ )  
(The scattered points correspond to the results of the computation in Reference 3.)

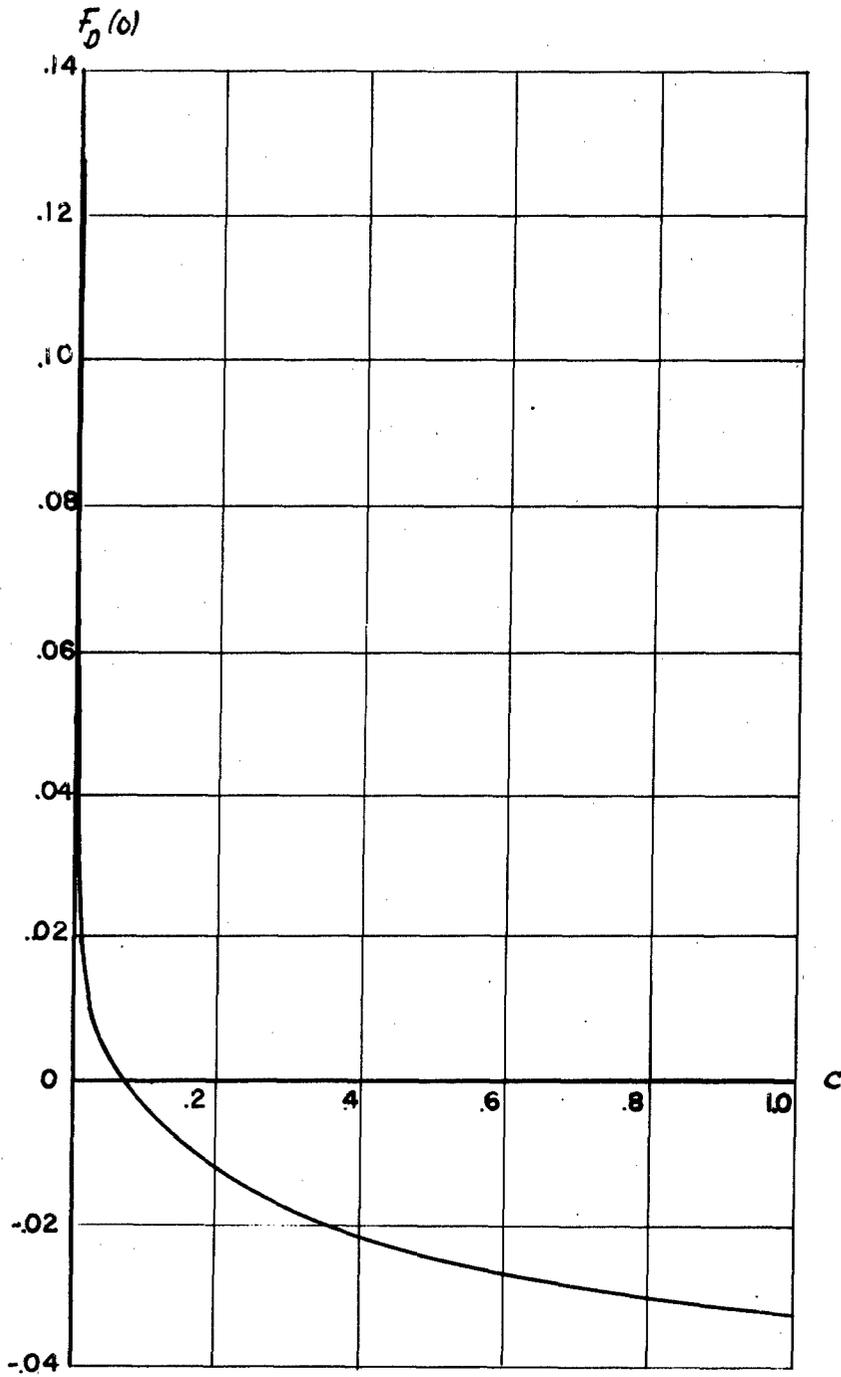


Figure 11: Correction Velocities as in Figure 10 Versus Opening Ratio  $c$  for Wind Tunnel with Eight Slots

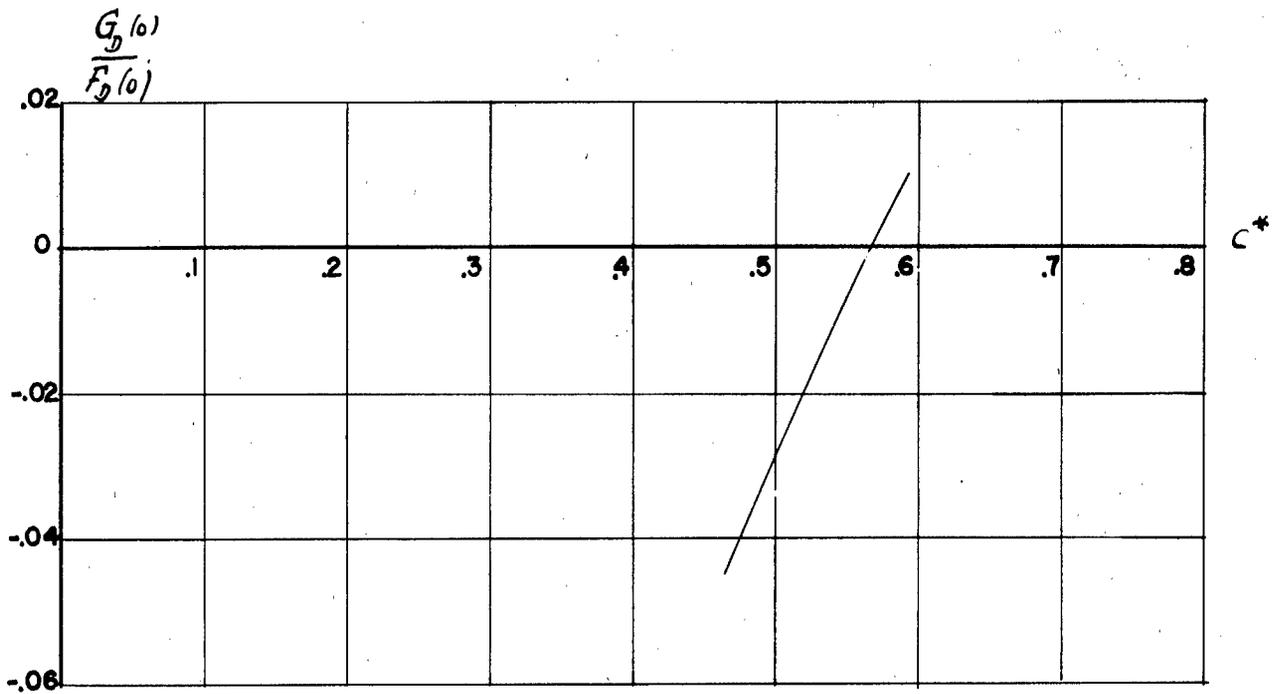


Figure 12: Ratio of Average Wall Velocity Deviation to the Velocity Perturbations Due to the Wall at the Wind Tunnel Axis Versus  $C^*$

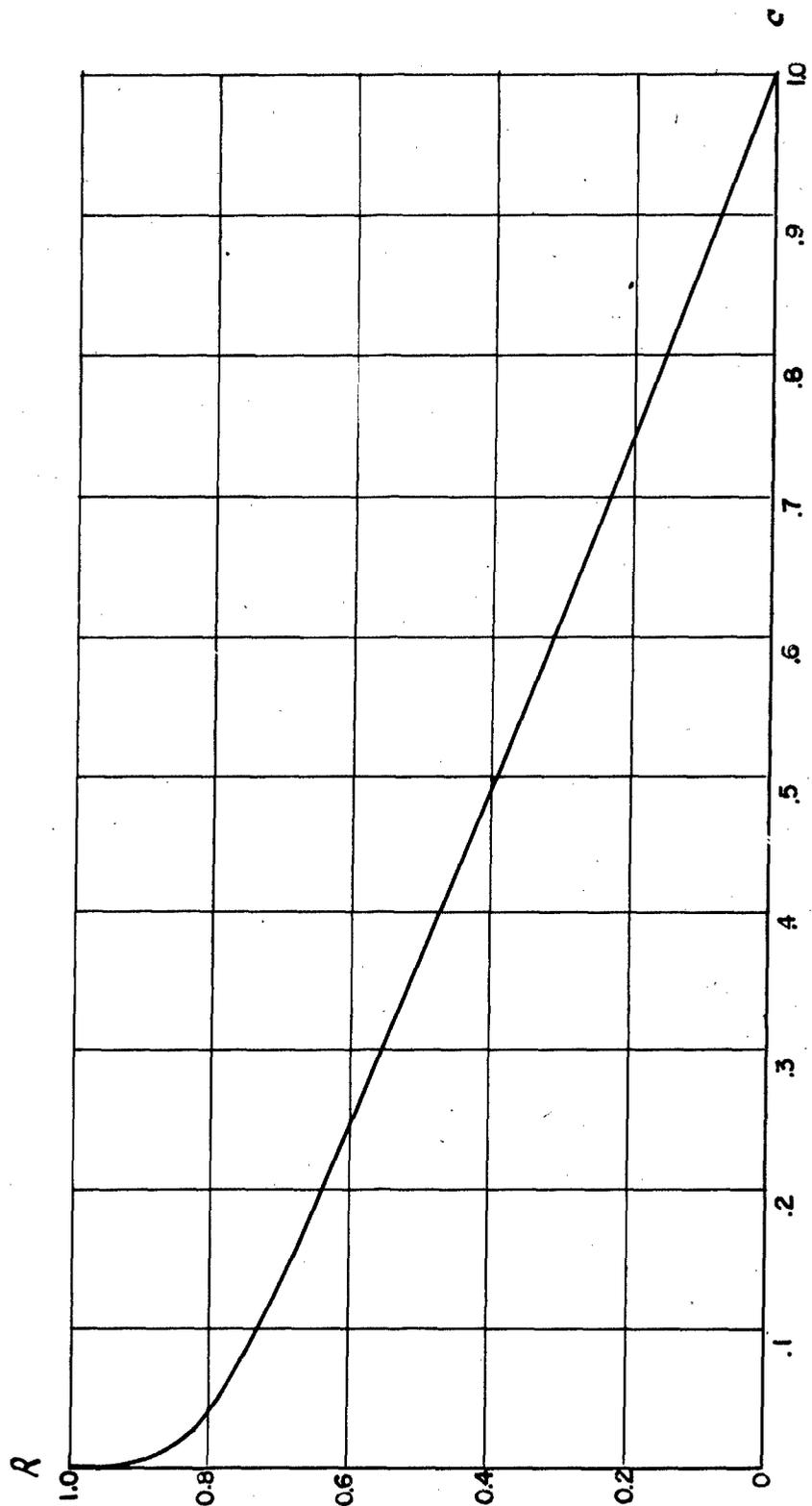


Figure 13: Ratio of Maximum to Average Wall Velocity Deviation (R Versus c)

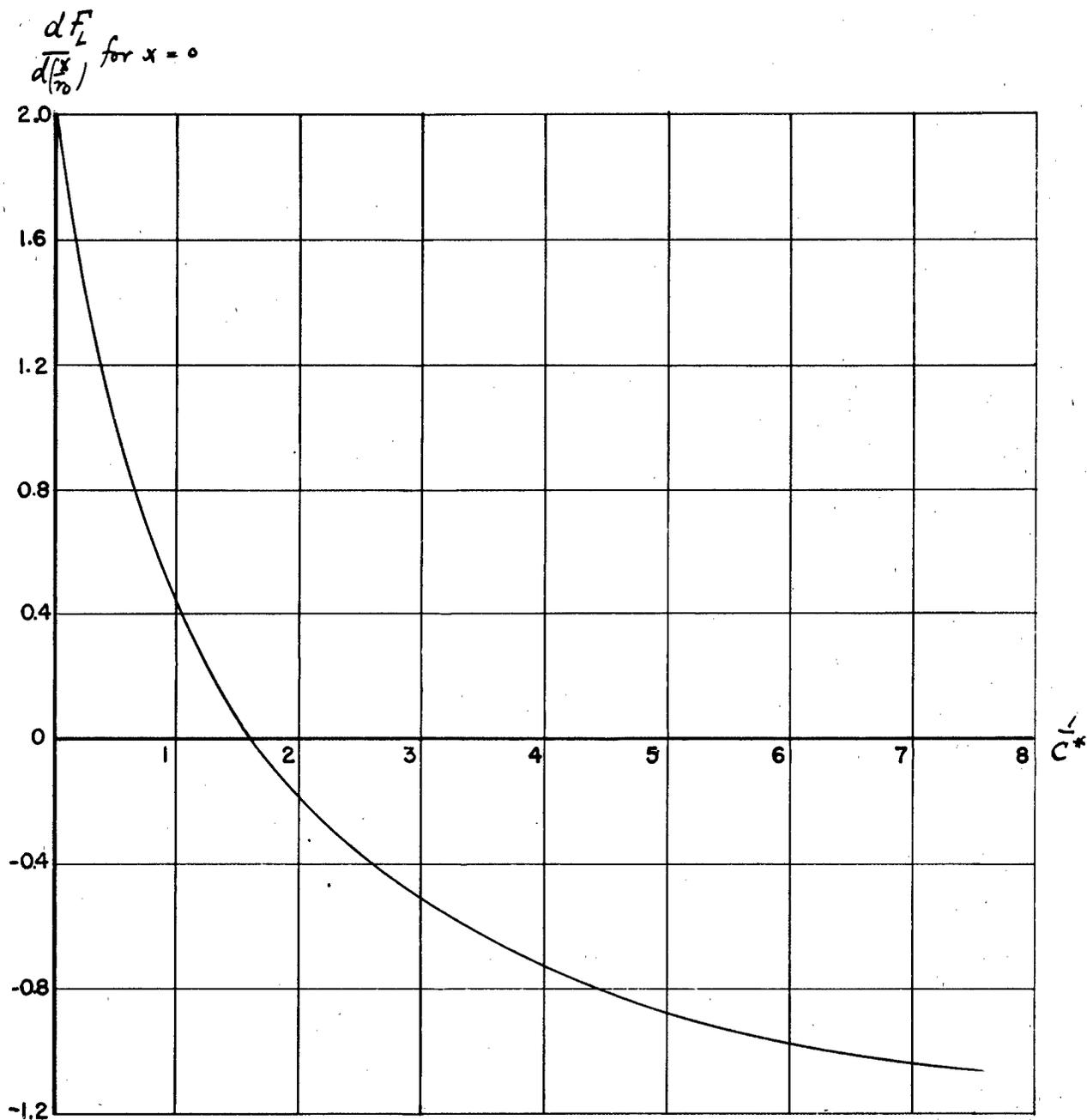


Figure 14: Curve for the Determination of the Camber Correction

$(\frac{dF_L}{d(\frac{x}{r_0})}$  for  $x=0$ ) Versus  $C^*^{-1}$

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A P P E N D I X

THE BEHAVIOR OF A CERTAIN INTEGRAL FOR LARGE VALUES OF  $C^*$

To study the integral

$$\int_0^{\infty} \frac{i H_0'''(i\lambda) - C^* \lambda (-H_1'''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda^2 d\lambda$$

we first form the difference between its value for an arbitrary value of  $C^*$  and its limiting value for  $C^* \rightarrow \infty$ . One obtains

$$\begin{aligned} & \int_0^{\infty} \frac{i H_0'''(i\lambda) - C^* \lambda (-H_1'''(i\lambda))}{J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))} \lambda^2 d\lambda + \int_0^{\infty} \frac{(-H_1'''(i\lambda)) \lambda^2 d\lambda}{(-i J_1(i\lambda))} \\ &= \int_0^{\infty} \frac{i H_0'''(i\lambda) (-i J_1(i\lambda)) + H_1'''(i\lambda) J_0(i\lambda)}{(J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))) (-i J_1(i\lambda))} \lambda^2 d\lambda \\ &= \frac{2}{\pi} \int_0^{\infty} \frac{\lambda d\lambda}{(J_0(i\lambda) + C^* \lambda (-i J_1(i\lambda))) (-i J_1(i\lambda))} \end{aligned}$$

Now we introduce

$$C = \frac{1}{j^2}$$

Then the above expression is given by

$$\frac{2}{\pi} \int_0^{\infty} \frac{j^2 \lambda d\lambda}{(j^2 J_0(i\lambda) + \lambda (-i J_1(i\lambda))) (-i J_1(i\lambda))}$$

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Its derivative with respect to  $y$  is

$$\frac{2}{\pi} \int_0^{\infty} \frac{2y \lambda d\lambda}{(y^2 J_0(i\lambda) + \lambda(-iJ_1(i\lambda)))(-iJ_1(i\lambda))} - \frac{2}{\pi} \int_0^{\infty} \frac{2y^3 J_0(i\lambda) \lambda d\lambda}{\{(y^2 J_0(i\lambda) + \lambda(-iJ_1(i\lambda)))^2(-iJ_1(i\lambda))\}}$$

To evaluate the expression for  $y = 0$  is not possible immediately, since then the integrals will not converge. Therefore, we split the

integrals up into two parts:  $\int_0^{\infty} = \int_0^{\epsilon} + \int_{\epsilon}^{\infty}$ . In the integrals

which extend from  $\epsilon$  to infinity it is allowed to put  $y = 0$ . In the other integral the Bessel functions can be developed with respect to  $\lambda$  and one obtains

$$\begin{aligned} & \frac{2}{\pi} \int_0^{\epsilon} \frac{2y \lambda d\lambda}{(y^2 + \frac{1}{2}\lambda^2)\frac{1}{2}\lambda} - \frac{2}{\pi} \int_0^{\epsilon} \frac{2y^3 \lambda d\lambda}{\{y^2 + \frac{1}{2}\lambda^2\}^2 \frac{1}{2}\lambda} \\ &= \frac{8}{\pi} \sqrt{2} \left\{ \int_0^{\lambda=\epsilon} \frac{\frac{d\lambda}{\sqrt{2} \cdot y}}{(1 + \frac{1}{2} \frac{\lambda^2}{y^2})} - \int_0^{\epsilon} \frac{\frac{d\lambda}{\sqrt{2} \cdot y}}{(1 + \frac{\lambda^2}{2y^2})^2} \right\} \end{aligned}$$

If  $y \rightarrow 0$  then the upper limit of the integrals in terms of  $\frac{\lambda}{\sqrt{2} \cdot y}$

as variable of integration tends to infinity. The first integral gives  $\pi/2$ , the second one  $\pi/4$ . Thus the derivative of the above expression with respect to  $y$  is  $2 \cdot \sqrt{2}$ .

Accordingly, the curve of Figure 10 behaves at  $1/C^* \rightarrow 0$  as constant + constant  $C^{*-1/2}$ . The curve in Figure 11 behaves for  $c \rightarrow 0$  as constant + constant  $(\log c)^{-1/2}$ .