

# Armed Services Technical Information Agency

Because of our limited supply, you are requested to return this copy WHEN IT HAS SERVED YOUR PURPOSE so that it may be made available to other requesters. Your cooperation will be appreciated.

**AD**

**28757**

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

Reproduced by  
**DOCUMENT SERVICE CENTER**  
KNOTT BUILDING, DAYTON, 2, OHIO

**UNCLASSIFIED**

AD No. **28757**

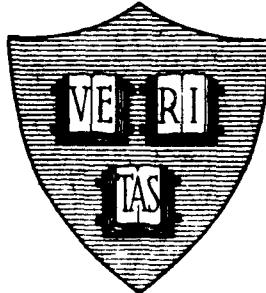
ASTIA FILE COPY

Office of Naval Research

---

Contract N50RI-76 • Task Order No.1 • NR-078-011

THE GAP PROBLEM IN ANTENNA THEORY



By

Ronold King

December 21, 1953

Technical Report No. 194

---

Cruft Laboratory  
Harvard University  
Cambridge, Massachusetts

Office of Naval Research

Contract N5ori-76

Task Order No. 1

NR-078-011

Technical Report

on

The Gap Problem in Antenna Theory

by

Ronold King

The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University, jointly by the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army, and the U. S. Air Force, under ONR Contract N5ori-76, T. O. 1.

Technical Report No. 194

Cruft Laboratory

Harvard University

Cambridge, Massachusetts

The Gap Problem in Antenna Theory

by

Ronold King

Cruft Laboratory, Harvard University

Cambridge 38, Massachusetts

Summary

The so-called gap problem in antenna theory is considered critically. It is shown that whereas there exist problems related to transmission-line end-effect and coupling between antenna and line, there is no gap, and hence no gap problem, when a physically realizable complete transmitting system is considered rather than an antenna with a fictitious mathematically convenient driving mechanism.

The essential parts of a complete and practical transmitting circuit include an antenna, a transmission line, and a coil in which an emf is induced by an alternating impressed magnetic field maintained by a generator. Typical practical circuits involving conventional open-wire and coaxial lines are in Figs. 1 and 2. Note that both circuits from one end of the antenna along one conductor of the transmission line around the coupling coil, back along the other conductor of the line, and out to the other extremity of the antenna (which may be the distant edge of a ground screen or the opposite pole of a great sphere) provide unbroken conducting paths. If, in the interest of pedagogical simplicity, the transmission line is reduced to zero and the coupling coil is contracted to a short section of the antenna itself in which an emf is induced by a varying magnetic field, an idealized system is obtained which, in the physically unavailable limit in which the distance along the antenna where the inducing field is active is reduced to zero while the field is increased to maintain a given emf, is equivalent to a so-called delta-function or slice generator. In this simple case the radiating circuit, consisting of a single straight conductor, is unbroken. Nowhere, either in the practical systems with long transmission lines and extended coupling coils or in its idealized contraction is there a gap. Yet, in spite

of the fact that in actual radiating systems there are no gaps, the so-called "problem of the gap" has been the subject of theoretical discussion.<sup>1</sup> Indeed, it has been asserted that the gap is "the essential part of the radiating system"; that it is "the only source of radiant energy."<sup>2</sup> Note that if this were true it would be necessary to conclude that none of the transmitters in practical operation could radiate! It is interesting and instructive to study and attempt to clarify the confusions and misunderstandings that must underlie statements that are as positive as they are untenable.

The origin of the "gap problem" in the study of antennas -- whether cylindrical, spheroidal, or spherical - is to be found in the attempt to analyze an antenna consisting, for example, of the two collinear conducting cylinders shown in Fig. 3 with adjacent flat ends separated a small distance  $2\delta$ , as if it constituted a complete transmitting system when a mathematically convenient electric field is postulated "across the gap." Actually, two essential components are missing. They are (1) the unbroken conducting path between the halves of the antenna, and (2), a localized induced emf along at least a part of the circuit. As soon as a transmitting system is completed in this manner, the gap and, with it, the gap problem disappears.

In the practical circuits in Figs. 1 and 2 where the conducting paths are conventional transmission lines and the emf is induced in a distant coil, the gap (together with an hypothetical, rotationally symmetrical exciting field  $E_z$  which is postulated for the antenna in Fig. 3) obviously is absent. (Note that Fig. 3 does not portray a physically meaningful complete transmitting system.) This is seen more clearly in Figs. 4 and 5 from which it is evident that the gap problem is replaced by precisely the transmission-line problem (including coupling and end-effects) which has been analyzed in the literature.<sup>3,4</sup> Another type of circuit in which the gap is apparently retained is in Fig. 6. The antenna consists of the same halves shown in Fig. 3 but the transmitting system is completed in a theoretically possible but practically unavailable manner. The gap in Fig. 3 is bridged at the center by a thin, perfectly conducting wire in which an alternating emf is induced. The presence of the wire and generator transforms the gap into a radial transmission line. Since the antenna is assumed to be perfectly conducting (or sufficiently highly conducting so that the skin depth is very

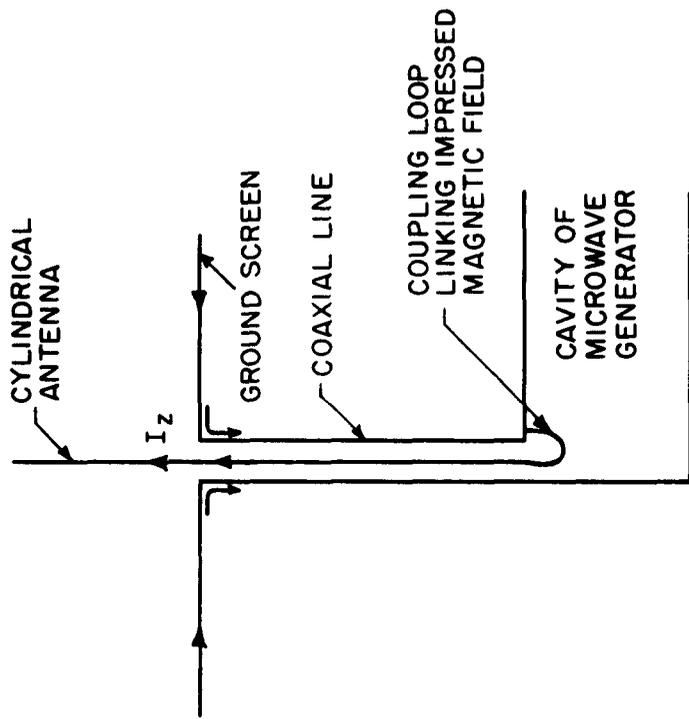


FIG. 2 TRANSMITTING SYSTEM WITH COAXIAL LINE

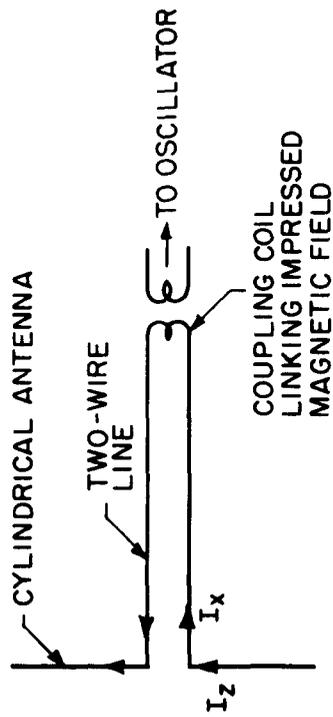


FIG. 1 TRANSMITTING SYSTEM WITH TWO-WIRE LINE

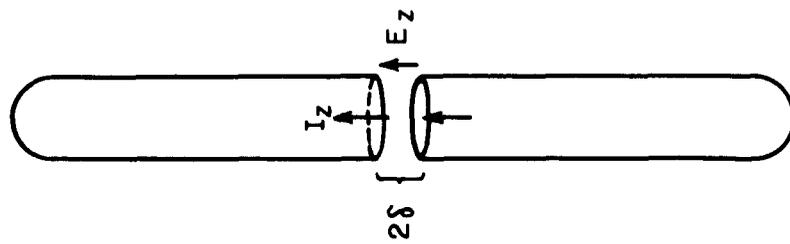


FIG. 3 CYLINDRICAL ANTENNA WITH GAP, ASSUMED FIELD, AND CURRENT

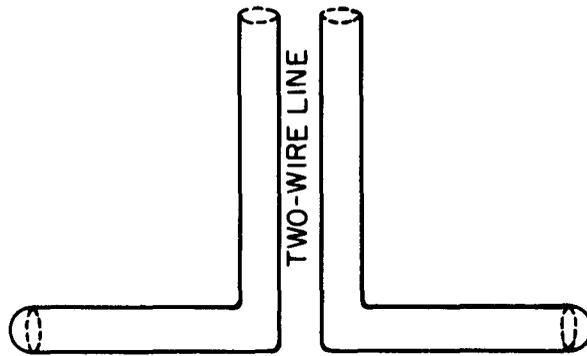


FIG. 4 TWO-WIRE DRIVE

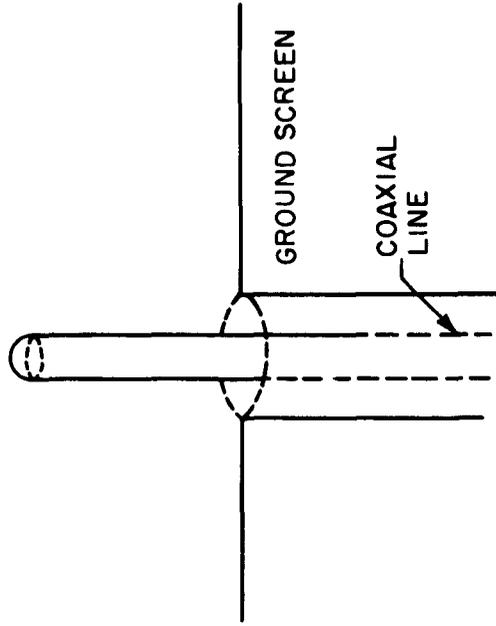


FIG. 5 COAXIAL DRIVE

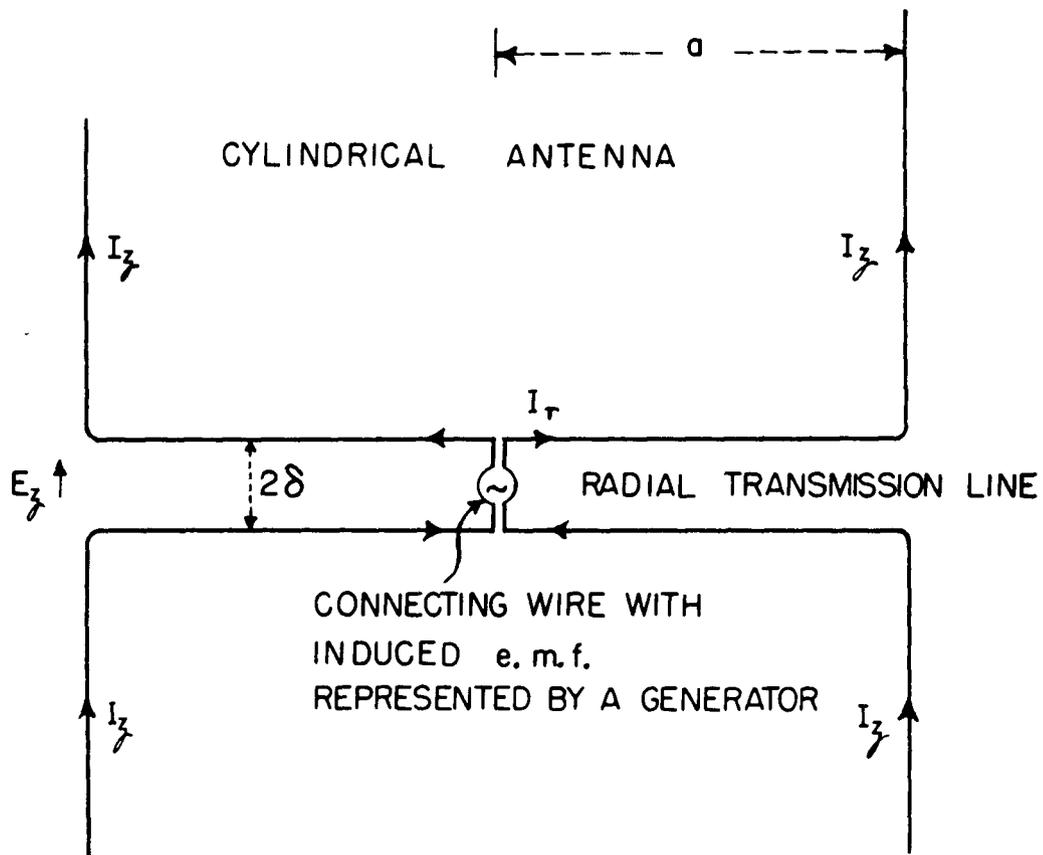


FIG. 6 TRANSMITTING SYSTEM WITH RADIAL TRANSMISSION LINE

small compared with the radius of the antenna) all currents are confined to a thin skin at the surface. Hence, radial sheets of surface current diverge from the center of the upper, converge to the center of the lower surface of the radial transmission line where it joins the vertical wire. Evidently, the determination of the electric field  $\underline{E}_z$  at the end  $r = a$  of the transmission line where it has the cylindrical antenna as its load, the definition of the voltage  $\underline{V}_\delta(r=a)$ , and the evaluation of the current  $\underline{I}_\delta(r=a)$  in terms of the emf  $V^e$  at  $r=0$  are parts of a transmission-line problem that includes radial end-effects near  $r = a$ . Note that Fig. 6 differs from a simple antenna with external generator of delta-function type only in having the continuous conducting path at the center of the antenna reduced from a radius equal to that of the antenna to a much smaller value. This complicates the problem on the one hand by inserting a radial transmission line between the antenna and the localized emf; it simplifies it, on the other hand, by permitting the use of a cross-sectionally dimensionless delta-function generator instead of one distributed as a belt around the antenna.

Instead of analyzing the circuit of Fig. 6, it is analytically more convenient and essentially equivalent to investigate the circuit shown in Fig. 7 where the radial transmission line is replaced by a biconical transmission line with a point-generator at its apex. This generator is mathematically attractive since it is equivalent to a singularity in the electric field. It may be approximated in practice by the arrangement in Fig. 8 where half of the cylindrical antenna is placed over a conducting ground screen and is driven from a conical transmission line. This, in turn, is connected at its apex to a coaxial transmission line of sufficiently small cross section so that the field at the end,  $r=a$ , of the conical transmission line is the same as when driven by a point generator. For the present study of the "gap problem" the simpler circuit of Fig. 7 is more convenient. In this case the driving voltage or emf of the delta-function generator is defined by

$$\underline{V}^e = \underline{V}(0) = \lim_{R \rightarrow 0} \int_{\pi-\theta_0}^{\theta_0} \underline{E}_\theta(R) R d\theta \quad (1)$$

The current in the generator at the apex is  $\underline{I}(0)$  and the impedance of the biconical line with its end load is

$$\underline{Z}_0 = \underline{V}(0)/\underline{I}(0) \quad (2)$$

As shown by Schelkunoff<sup>5</sup> the total radial current  $\underline{I}_R(R)$  in the upper cone of the transmission line is given by

$$\underline{I}_R(R) = \underline{I}_d(R) + \underline{I}_c(R) \quad (3)$$

where  $\underline{I}_d(R)$  is the dominant-mode current and  $\underline{I}_c(R)$  is the sum of the currents associated with the higher modes. These vanish identically at  $R=0$  for all angles  $\theta_0$  of the cone. The dominant-mode current is

$$\underline{I}_d(R) = \underline{I}(0) \cos \beta_0 R - jY_c \underline{V}(0) \sin \beta_0 R \quad (4)$$

where  $Y_c = 1/Z_c$  is the characteristic admittance and

$$Z_c = \frac{\xi_0}{\pi} \ln \cot \frac{\theta_0}{2} \quad (5)$$

is the characteristic impedance of the biconical transmission line. It can be shown that in so far as the current  $\underline{I}_d(0) = \underline{I}_R(0)$  in the generator is concerned, the effect of the complementary currents in cancelling all or part of the dominant-mode current may be simulated by ignoring the complementary currents and providing an apparent terminal admittance across the biconical line at  $R=l$  of such value that the part of the current  $\underline{I}_R(l)$  which enters this terminal admittance is equal to the part cancelled by the complementary currents. In the case of the thin biconical antenna with  $\theta_0$  very small, the total current  $\underline{I}_R(l)$  is essentially zero so that virtually the entire dominant-mode current  $\underline{I}_d(l)$  is cancelled by equal and opposite higher-mode currents since  $\underline{I}_d(l) \doteq -\underline{I}_c(l)$ . Hence, the complimentary currents could be ignored only by providing an apparent dominant-mode terminal admittance  $\underline{Y}_{la}$  that carried the entire current  $\underline{I}_d(l)$ . In the case at hand with  $\theta_0$  near  $\pi/2$  the situation is different since practically all of the dominant-mode current at  $R=l$  becomes the axially directed surface current  $\underline{I}_z(z=\delta)$ . When  $\theta_0$  is sufficiently near  $\pi/2$  the higher-mode current  $\underline{I}_c(l)$  is very small and cancels an insignificant part of the dominant-mode current  $\underline{I}_d(l)$ . Actually the higher-mode current is responsible for transmission-line end-effects. Just as in the case of the two-wire line<sup>3</sup> these

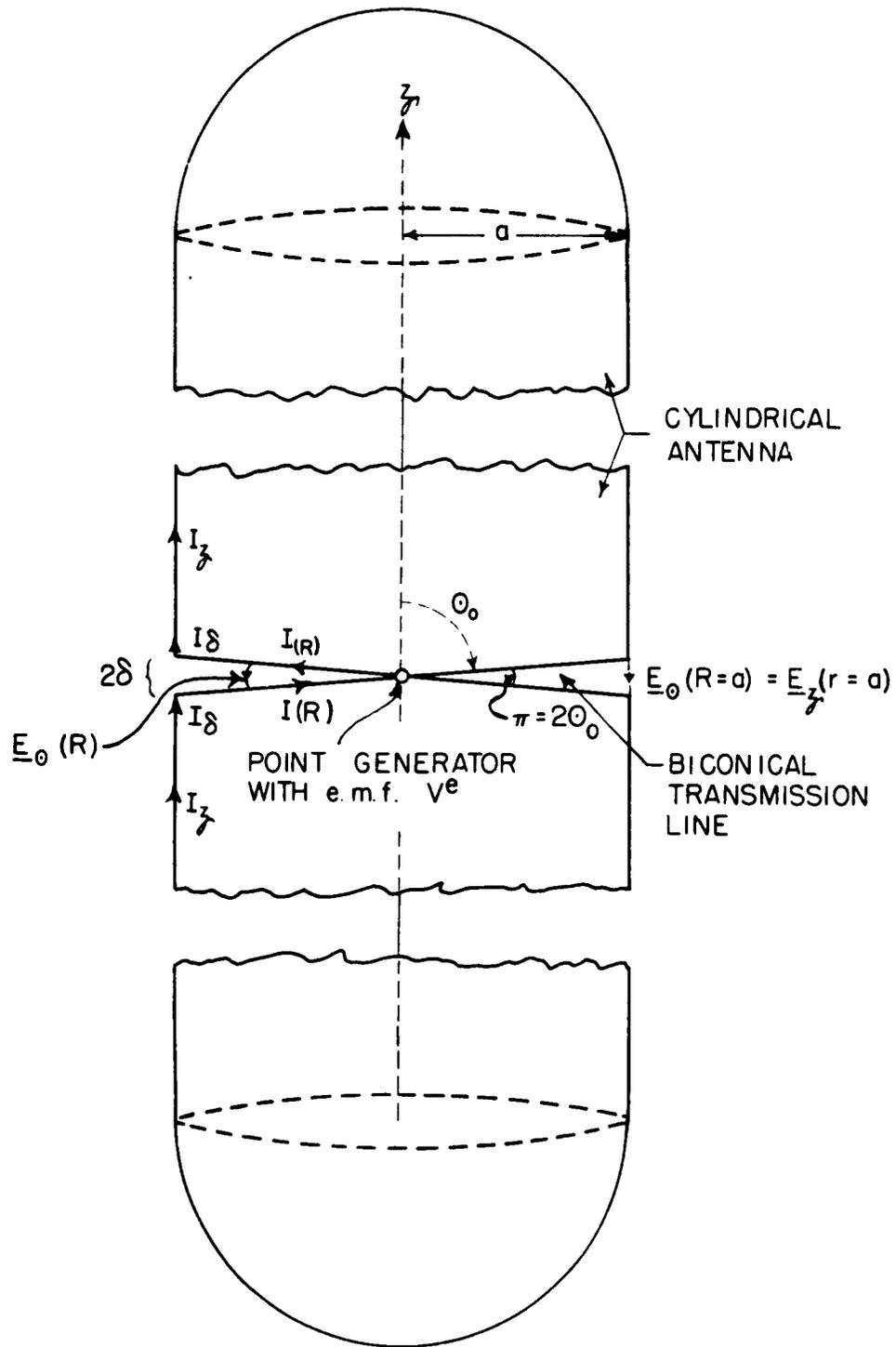


FIG. 7 TRANSMITTING SYSTEM WITH BICONICAL LINE AND POINT-GENERATOR

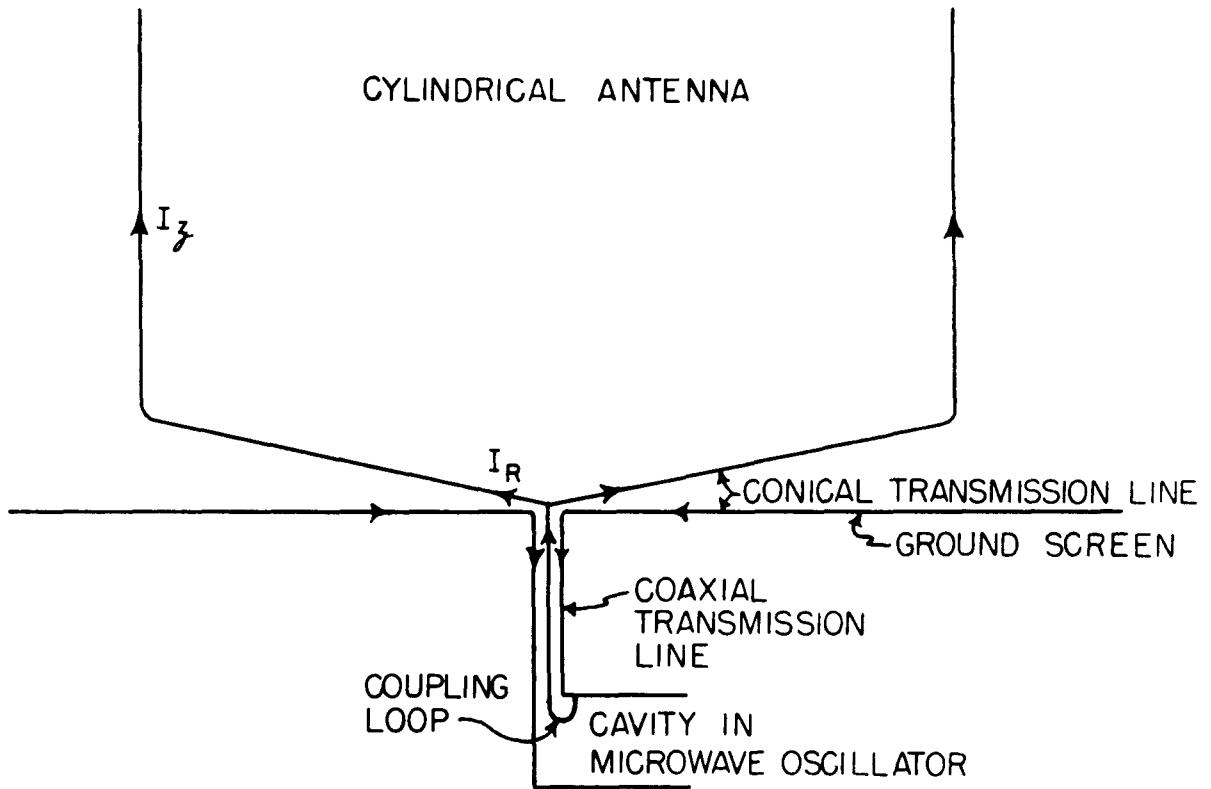


FIG. 8 TRANSMITTING SYSTEM WITH CONICAL AND COAXIAL LINES

may be ignored in their effect on the current far from the load if an appropriate terminal-zone admittance is provided. Thus, the total apparent terminal admittance is given by

$$\underline{Y}_{\text{ta}} = \underline{Y}_{\delta} + \underline{Y}_{\text{T}} \quad (6)$$

where  $\underline{Y}_{\delta}$  is the dominant-mode load impedance and  $\underline{Y}_{\text{T}}$  is the terminal-zone admittance that takes account of end-effect. With  $\theta_0$  near  $\pi/2$  it is essentially susceptive, so that

$$\underline{Y}_{\text{T}} \doteq j\omega C_{\text{T}} \quad (7)$$

The load impedance  $\underline{Z}_{\delta} = 1/\underline{Y}_{\delta}$  is defined by

$$\underline{Z}_{\delta} \doteq \frac{V(\ell)}{\underline{I}_{\delta}} = \frac{\int_{\theta_0}^{\pi-\theta_0} \underline{E}_{\text{d}\theta}(\ell) \ell d\theta}{\underline{I}_{\delta}} \quad (8)$$

where  $\underline{I}_{\delta}$  is the radial current leaving the end of the upper conductor of the biconical line at  $R = \ell \doteq a$  to become the axial surface current on the antenna. Since  $\underline{E}_{\text{d}\theta}(\ell)$  in (8) is the dominant-mode field in the biconical line where all currents are radial and, hence, perpendicular to the direction of  $\underline{E}_{\theta}$ , it follows from the definition of the scalar potential by  $-\text{grad } \phi = \underline{E} + j\omega \underline{A}$  that

$$\underline{E}_{\text{d}\theta} = -\frac{1}{R} \frac{\partial \phi}{\partial \theta} - j\omega \underline{A}_{\theta} \doteq -\frac{1}{R} \frac{\partial \phi}{\partial \theta} \quad (9)$$

where  $\phi$  is the scalar potential and  $\underline{A}_{\theta}$  a component of the vector potential. Substitution of (9) in (8) with  $R = \ell$ , gives

$$\underline{Z}_{\delta} = \frac{V(\ell)}{\underline{I}_{\delta}} = \frac{\int_{-\delta}^{\delta} d\phi}{\underline{I}_{\delta}} = \frac{\phi(\delta) - \phi(-\delta)}{\underline{I}_{\delta}} \quad (10)$$

If the exterior field is expressed in terms of the current on the cylindrical antenna and matched to the internal field at  $R = \ell \doteq a$ ,  $\pi - \theta_0 \leq \theta \leq \theta_0$  or  $-\delta \leq z \leq \delta$ , the impedance (10) may be determined. A part of the exterior field is required to match the interior complementary field. This represents the terminal-zone coupling between antenna

and biconical transmission line. The actual evaluation and matching of the interior and exterior fields is of no interest here.

The relation between the apparent load admittance  $\underline{Y}_{\ell a}$  of the biconical transmission line and the driving-point admittance  $\underline{Y}_0$  is obtained from the well-known transmission-line formulas for the dominant-mode voltage and current. They are

$$\underline{V}_d(\ell) = \underline{V}(0) \cos \beta_0 \ell - jZ_c \underline{I}(0) \sin \beta_0 \ell \quad (11a)$$

$$\underline{I}_d(\ell) = \underline{I}(0) \cos \beta_0 \ell - Y_c \underline{V}(0) \sin \beta_0 \ell \quad (11b)$$

It follows that

$$\underline{Y}_{\ell a} = \frac{\underline{I}_d(\ell)}{\underline{V}_d(\ell)} = \frac{\underline{Y}_0 \cos \beta_0 \ell - jY_c \sin \beta_0 \ell}{\cos \beta_0 \ell - jZ_c Y_0 \sin \beta_0 \ell} \quad (12)$$

where

$$\underline{Y}_0 \equiv \underline{I}(0)/\underline{V}(0) \quad (13)$$

is the driving-point admittance at the apex of the biconical line.

Up to the present no restrictions have been placed on the length  $\ell$  of the biconical transmission line. Since with  $\theta_0$  near  $\pi/2$ ,  $\ell \doteq a$ , where  $a$  is the radius of the cylindrical antenna which is assumed to satisfy

$$\beta_0 a \ll 1 \quad (14)$$

it follows that (12) reduces to

$$\underline{Y}_{\ell a} \doteq \frac{\underline{Y}_0 - jY_c \beta_0 a}{1 - jZ_c Y_0 \beta_0 a} \quad (15)$$

This formula may be simplified further by noting that with  $\theta_0$  near  $\pi/2$ ,  $Z_c = 120 \ln \cot(\theta_0/2)$  is very small. Hence,

$$\underline{Y}_{\ell a} \doteq \underline{Y}_0 - jY_c \beta_0 a = \underline{Y}_0 - j\omega C_g \quad (16)$$

where, with  $\beta_0 = \omega/v_0$ ,

$$C_g \equiv ac_0 = av_0 Z_c = \frac{a\pi\epsilon_0}{\ln \cot \frac{\theta_0}{2}} \quad (17)$$

In (17),  $c_o$  is the capacitance per unit radial length of the biconical line. It follows with (6) and (7) that the driving-point admittance at the apex of the biconical line is

$$\underline{Y}_o \doteq \underline{Y}_{\ell_a} + j\omega C_g = \underline{Y}_\delta + j\omega(C_T + C_g) \quad (18)$$

Note that (18) is a special form of (12) solved for  $Y_o$ , viz.,

$$\underline{Y}_o = \frac{\underline{Y}_{\ell_a} \cos \beta_o \ell + jY_c \sin \beta_o \ell}{\cos \beta_o \ell + jZ_c \underline{Y}_{\ell_a} \sin \beta_o \ell} \quad (19)$$

when both  $\beta_o \ell$  and  $Z_c$  are sufficiently small.

The confusion regarding the significance of the gap in antenna theory arises from a failure to recognize that (18) is not a general form but is a special case of (19).

The following conclusions may be drawn from (18). (1) The driving-point impedance  $\underline{Y}_o$  of a cylindrical antenna is equal to the impedance  $\underline{Z}_\delta = \underline{V}_\delta / \underline{I}_\delta$  (where  $\underline{V}_\delta$  is the voltage across the gap of length  $2\delta$  and  $\underline{I}_\delta$  is the current entering the antenna at the edge of the gap) in parallel with the effective "gap capacitance"  $C_g + C_T$  between the two adjacent end-surfaces of the antenna. (2) As the width  $2\delta$  of the gap is decreased by making  $\theta_o$  differ less and less from  $\pi/2$ , the essential part  $C_g$  of the gap capacitance increases without limit. This is seen from (17). (3) It follows that in the limit  $\delta \rightarrow 0$ , the driving-point susceptance  $B_o$  becomes infinite. Since the generator is short-circuited and completely enclosed by metal, no power can be radiated from the antenna. If it is now assumed that the interpretation of (18) is typical for all center-driven cylindrical antennas, the following conclusion is reached: (4) Only antennas with finite gaps can radiate. Finally a naive application of the Poynting vector theorem suggests that: (5) A radiation comes out of the gap.

In what way are these "conclusions" erroneous? This may be discovered readily by rephrasing them in a manner consistent with the general formula (19) instead of the special formula (18). (1) The driving-point admittance  $Y_o$  of a section of transmission line of length  $\ell$  with an apparent load admittance  $Y_{\ell_a}$  is given by (19). If  $\ell$  is sufficiently short and

the characteristic impedance  $\underline{Z}_c$  sufficiently small this reduces to (18). The apparent load admittance  $\underline{Y}_{\ell_a}$  is equal to the actual load admittance  $\underline{Y}_\delta = \underline{V}_\delta / \underline{I}_\delta$  in parallel with a lumped capacitance that takes account of terminal-zone effects. (2) As the distance between the two conductors of the transmission line is reduced, the characteristic impedance  $\underline{Z}_c$  decreases without limit. For the biconical line this is seen from (5). For coaxial or parallel wire lines  $\underline{Z}_c$  has the factor  $\ln(\frac{b}{a})$  or  $\cosh^{-1}(\frac{b}{2a})$  which reduces to zero when  $b$  approaches  $a$  or  $2a$ , respectively. (3) It follows from (19) that in the limit  $\delta \rightarrow 0$  when  $\underline{Z}_c \rightarrow 0$  and  $\underline{Y}_c \rightarrow \infty$  the input susceptance of the section of line is infinite since the two perfect conductors are everywhere in contact and do not constitute a transmission line. Since this leaves the antenna completely isolated from the generator, no currents are maintained in it and, therefore, no electromagnetic field is set up by them. That is, the antenna does not radiate. (4) A transmission line like that in Fig. 7 that happens to be short, biconical, and "inside" the antenna so that it looks like a simple gap instead of a section of line is characteristic only of certain special cases that actually are highly artificial and impractical. Every short section of transmission line, whether two-wire, coaxial, biconical, or radial, is not merely a lumped capacitance in parallel with the load. In all cases the entire current from the generator to the load traverses the two conductors—whether flat plates, cones, or wires—in opposite directions. They form a necessary series connection between generator and load. It is obvious that only transmission lines consisting of two conductors with finite spacing can transmit power to a load. This is as true of two-wire lines as of biconical or radial lines; it has nothing to do with gaps. Every antenna in which currents are induced radiates in the sense that these currents maintain a far-zone field. Practical antennas never have gaps. (5) It is readily verified that the Poynting-vector theorem is useful in locating the generator in a complete transmitting system, not the element that carries the currents that maintain the radiation field. In order to radiate, i.e., maintain far-zone fields, antennas must have currents maintained in them. This may be accomplished by direct connection to a transmission line from a distant generator or by coupling to a varying magnetic field maintained by a generator. A gap is not required. Thus, it may be concluded that there are transmission-line problems and coupling

problems in antenna theory but no gap problems.

References

1. See, for example, L. Infeld, "The Influence of the Width of the Gap in the Theory of Antennas," Quart. Appl. Math. 5, 113-132, (July, 1947).
2. E. Albert and J. L. Synge, "The General Problem of Antenna Radiation" Quart. Appl. Math. 6, 117-131 (July, 1948).
3. R. King, "Theory of Antennas Driven from a Two-Wire Line.I, J. Appl. Phys 20, 832 (Sept. 1949).
4. R. King and K. Tomiyasu, "Terminal Impedance and Generalized Two-Wire Line Theory," Proc. I.R.E., 37, 1134 (Oct. 1949).
5. S. A. Schelkunoff, "Antennas of Arbitrary Shape and Size," Proc. I.R.E., 29, 493 (1941).

## DISTRIBUTION LIST

### Technical Reports

2 Chief of Naval Research (427)  
Department of the Navy  
Washington 25, D. C.

1 Chief of Naval Research(460)  
Department of the Navy  
Washington 25, D. C.

1 Chief of Naval Research (421)  
Department of the Navy  
Washington 25, D. C.

6 Director (Code 2000)  
Naval Research Laboratory  
Washington 25, D. C.

2 Commanding Officer  
Office of Naval Research Branch Office  
150 Causeway Street  
Boston, Massachusetts

1 Commanding Officer  
Office of Naval Research Branch Office  
1000 Geary Street  
San Francisco 9, California

1 Commanding Officer  
Office of Naval Research Branch Office  
1030 E. Green Street  
Pasadena, California

1 Commanding Officer  
Office of Naval Research Branch Office  
The John Crerar Library Building  
86 East Randolph Street  
Chicago 1, Illinois

1 Commanding Officer  
Office of Naval Research Branch Office  
346 Broadway  
New York 13, New York

3 Officer-in-Charge  
Office of Naval Research  
Navy No. 100  
Fleet Post Office  
New York, N. Y.

1 Chief, Bureau of Ordnance (Re4)  
Navy Department  
Washington 25, D. C.

1 Chief, Bureau of Ordnance (AD-3)  
Navy Department  
Washington 25, D. C.

1 Chief, Bureau of Aeronautics (EL-1)  
Navy Department  
Washington 25, D. C.

2 Chief, Bureau of Ships (810)  
Navy Department  
Washington 25, D. C.

1 Chief of Naval Operations (Op-413)  
Navy Department  
Washington 25, D. C.

1 Chief of Naval Operations (Op-20)  
Navy Department  
Washington 25, D. C.

1 Chief of Naval Operations (Op-32)  
Navy Department  
Washington 25, D. C.

1 Director  
Naval Ordnance Laboratory  
White Oak, Maryland

2 Commander  
U. S. Naval Electronics Laboratory  
San Diego, California

1 Commander (AAEL)  
Naval Air Development Center  
Johnsville, Pennsylvania

1 Librarian  
U. S. Naval Post Graduate School  
Monterey, California

50 Director  
Signal Corps Engineering Laboratories  
Evans Signal Laboratory  
Supply Receiving Section  
Building No. 42  
Belmar, New Jersey

3           **Commanding General (RDRRP)**  
              **Air Research and Development Command**  
              **Post Office Box 1395**  
              **Baltimore 3, Maryland**

2           **Commanding General (RDDDE)**  
              **Air Research and Development Command**  
              **Post Office Box 1395**  
              **Baltimore 3, Maryland**

1           **Commanding General (WCRR)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

1           **Commanding General (WCRRH)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

1           **Commanding General (WCRE)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

2           **Commanding General (WCRET)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

1           **Commanding General (WCREO)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

2           **Commanding General (WCLR)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

1           **Commanding General (WCLRR)**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

2           **Technical Library**  
              **Commanding General**  
              **Wright Air Development Center**  
              **Wright-Patterson Air Force Base, Ohio**

1           **Commanding General (RCREC-4C)**  
              **Rome Air Development Center**  
              **Griffiss Air Force Base**  
              **Rome, New York**

1           **Commanding General (RCR)**  
              **Rome Air Development Center**  
              **Griffiss Air Force Base**  
              **Rome, New York**

- 2           **Commanding General (RCRW)**  
**Rome Air Development Center**  
**Griffiss Air Force Base**  
**Rome, New York**
- 6           **Commanding General (CRR)**  
**Air Force Cambridge Research Center**  
**230 Albany Street**  
**Cambridge 39, Massachusetts**
- 1           **Commanding General**  
**Technical Library**  
**Air Force Cambridge Research Center**  
**230 Albany Street**  
**Cambridge 39, Massachusetts**
- 2           **Director**  
**Air University Library**  
**Maxwell Air Force Base, Alabama**
- 1           **Commander**  
**Patrick Air Force Base**  
**Cocoa, Florida**
- 2           **Chief, Western Division**  
**Air Research and Development Command**  
**P. O. Box 2035**  
**Pasadena, California**
- 1           **Chief, European Office**  
**Air Research and Development Command**  
**Shell Building**  
**60 Rue Ravenstein**  
**Brussels, Belgium**
- 1           **U. S. Coast Guard (EEE)**  
**1300 E Street, N. W.**  
**Washington, D. C.**
- 1           **Assistant Secretary of Defense**  
**(Research and Development)**  
**Research and Development Board**  
**Department of Defense**  
**Washington 25, D. C.**
- 5           **Armed Services Technical Information Agency**  
**Document Service Center**  
**Knott Building**  
**Dayton 2, Ohio**

- 1           **Director**  
              **Division 14, Librarian**  
              **National Bureau of Standards**  
              **Connecticut Avenue and Van Ness St., N. W.**
- 1           **Director**  
              **Division 14, Librarian**  
              **National Bureau of Standards**  
              **Connecticut Avenue and Van Ness St., N. W.**
- 1           **Office of Technical Services**  
              **Department of Commerce**  
              **Washington 25, D. C.**
- 1           **Commanding Officer and Director**  
              **U. S. Underwater Sound Laboratory**  
              **New London, Connecticut**
- 1           **Federal Telecommunications Laboratories, Inc.**  
              **Technical Library**  
              **500 Washington Avenue**  
              **Nutley, New Jersey**
- 1           **Librarian**  
              **Radio Corporation of America**  
              **RCA Laboratories**  
              **Princeton, New Jersey**
- 1           **Sperry Gyroscope Company**  
              **Engineering Librarian**  
              **Great Neck, L. I., New York**
- 1           **Watson Laboratories**  
              **Library**  
              **Red Bank, New Jersey**
- 1           **Professor E. Weber**  
              **Polytechnic Institute of Brooklyn**  
              **99 Livingston Street**  
              **Brooklyn 2, New York**
- 1           **University of California**  
              **Department of Electrical Engineering**  
              **Berkeley, California**
- 1           **Dr. E. T. Booth**  
              **Hudson Laboratories**  
              **145 Palisade Street**  
              **Dobbs Ferry, New York**
- 1           **Cornell University**  
              **Department of Electrical Engineering**  
              **Ithaca, New York**

- 1 University of Illinois  
Department of Electrical Engineering  
Urbana, Illinois
- 1 Johns Hopkins University  
Applied Physics Laboratory  
Silver Spring, Maryland
- 1 Professor A. von Hippel  
Massachusetts Institute of Technology  
Research Laboratory for Insulation Research  
Cambridge, Massachusetts
- 1 Director  
Lincoln Laboratory  
Massachusetts Institute of Technology  
Cambridge 39, Massachusetts
- 1 Signal Corps Liaison Office  
Massachusetts Institute of Technology  
Cambridge 39, Massachusetts
- 1 Mr. Hewitt  
Massachusetts Institute of Technology  
Document Room  
Research Laboratory of Electronics  
Cambridge, Massachusetts
- 1 Stanford University  
Electronics Research Laboratory  
Stanford, California
- 1 Professor A. W. Straiton  
University of Texas  
Department of Electrical Engineering  
Austin 12, Texas
- 1 Yale University  
Department of Electrical Engineering  
New Haven, Connecticut
- 1 Mr. James F. Trosch, Administrative Aide  
Columbia Radiation Laboratory  
Columbia University  
538 West 120th Street  
New York 27, N. Y.
- 1 Dr. J. V. N. Granger  
Stanford Research Institute  
Stanford, California

Additional Reports Issued by Cruft Laboratory

(under Contract N5ori-76)

in the Field of Electromagnetic Radiation

- No.
- 2 D. D. King, "Measured Impedance of Cylindrical Dipoles," 1946. J. Appl. Phys., Oct. 1946.
- 6 D. D. King, "Impedance Measurements on Transmission Lines," 1946. Proc. I.R.E., May 1947.
- 8 B. C. Dunn, Jr. and R. W. P. King, "Currents Excited on a Conducting Plane. . .," 1947. Proc. I.R.E., Feb. 1948.
- 11 D. D. King et al, "Bolometer Amplifier for Minimum Signals," 1947. Electronics, Feb. 1948.
- 12 C. T. Tai, "Theory of Coupled Antennas," 1947. Part I Proc. I.R.E., April 1948; Part II, ibid, Nov. 1948.
- 16 Tung Chang, "Impedance Measurements of Antennas Involving Loop and Linear Elements," 1947.
- 18 C. T. Tai, "Propagation of Electromagnetic Waves from a Dissipative Medium to a Perfect Dielectric," 1947.
- 20 R. W. P. King, "Graphical Representation of the Characteristics of Cylindrical Antennas," 1947.
- 22 C. H. Papas and R. W. P. King, "Radiation Resistance of End-Fire Collinear Arrays," 1947. Proc. I.R.E., July 1948.
- 23 R. W. P. King, "Field of Dipole with Tuned Parasite at Constant Power," 1947. Proc. I.R.E., July 1948.
- 25 J. V. Granger, "Low-Frequency Aircraft Antennas," 1947.
- 27 C. H. Papas and R. W. P. King, "Surface Currents on a Conducting Plane. . .," 1948. J. Appl. Phys., Sept. 1948.
- 28 C. T. Tai, "Reflection and Refraction of a Plane Electromagnetic Wave. . .," 1948.
- 32 C. H. Papas and R. King, "Currents on the Surface of an Infinite Cylinder," 1948. Quart. Appl. Math., Jan. 1949.

- 35 P. Conley, "Impedance Measurements with Open-Wire Lines," 1948. J. Appl. Phys., Nov. 1949.
- 39 S. B. Cohn, "Theoretical and Experimental Study of a Waveguide Filter Structure," 1948.
- 40 C. T. Tai, "Reflection of Plane Electromagnetic Waves from Perfectly Conducting Grounded Half-Cylinder," 1948.
- 41 R. W. P. King, "Theory of Antennas Driven from a Two-Wire Line," 1948. J. Appl. Phys., Sept. 1949.
- 42 J. V. Granger, "Note on Broad-Band Impedance Characteristics of Folded Dipoles," 1948.
- 43 D. G. Wilson and R. King, "Measurement of Antenna Impedance Using Receiving Antenna," 1948.
- 44 E. Hallén, "Properties of Long Antennas," 1948. J. Appl. Phys., Dec. 1948.
- 46 E. Hallén, "Admittance Diagrams for Antennas. . .," 1948.
- 47 C. T. Tai, "On the Theory of Biconical Antennas," 1948. J. Appl. Phys., Dec. 1948.
- 48 K. Tomiyasu, "Problems of Measurement on Two-Wire Lines with Application to Antenna Impedance," 1948. Condensed version, J. Appl. Phys., Oct. 1949.
- 49 E. Hallén, "Traveling Waves and Unsymmetrically Fed Antenna," 1948.
- 50 D. D. King, "Measurement and Interpretation of Antenna Scattering," 1948.
- 52 C. H. Papas and R. King, "Input Impedance of Wide-Angle Conical Antennas," 1948. Proc. I.R.E., Nov. 1949.
- 53 D. K. Reynolds, "Surface-Current and Charge Measurements on Flat Metal Sheets," 1948.
- 55 C. T. Tai, "Study of the EMF Method," 1948. J. Appl. Phys., July 1949.
- 56 T. W. Winternitz, "The Cylindrical Antenna Center-Driven by a Two-wire Open Transmission Line," 1948. Quart. Appl. Math., 1949.
- 58 C. H. Papas, "On the Infinitely Long Cylindrical Antenna," 1948. J. Appl. Phys., May 1949.

- 61 C. H. Papas, "Radiation from a Transverse Slot in an Infinite Cylinder," 1948. J. Math. and Phys., Jan. 1950.
- 63 J. V. Granger and N. G. Altman, "Full-Scale Aircraft Antenna Measurements," 1949.
- 66 T. Morita, "Measurement of Current and Charge Distributions on Cylindrical Antennas," 1949. Proc. I.R.E., Aug. 1950.
- 67 T. Morita and C. E. Faflick, "Measurement of Current Distributions along Coupled Antennas. . .," 1949.
- 69 J. E. Storer and R. King, "Radiation Resistance of a Two-Wire Line," 1949.
- 70 J. V. Granger, "Shunt-Excited Flat-Plate Antennas. . .," 1949. Proc. I.R.E., March 1950.
- 71 }  
72 } B. C. Dunn, Jr., "Microwave Field Measurements," I (with  
73 } R. King), II and III, 1949.
- 74 R. King and K. Tomiyasu, "Terminal Impedance and Generalized Two-Wire Line Theory," 1949. Proc. I.R.E., Oct. 1949.
- 75 C. T. Tai, "Application of a Variational Principle to the Study of Biconical Antennas," 1949.
- 76 C. H. Papas, "Radiation from a Circular Diffraction Antenna," 1949.
- 77 C. T. Tai, "On Radiation and Radiating Systems in the Presence of a Dissipative Medium," 1949.
- 78 J. V. Granger and T. Morita, "Current Distribution on Aircraft," 1949.
- 81 K. Tomiyasu, "Loading and Coupling Effects of Standing-Wave Detectors," 1949. Proc. I.R.E., Dec. 1949.
- 83 C. H. Papas, "Diffraction by a Cylindrical Obstacle," 1949. J. Appl. Phys., April 1950.
- 84 R. King, "Theory of N Coupled Parallel Antennas," 1949. J. Appl. Phys., Feb. 1950.
- 86 K. Tomiyasu, "Unbalanced Terminations on a Shielded-Pair Line," 1949.
- 91 R. King, "Theory of Collinear Antennas," 1949.

- 92 C. H. Papas and R. King. "Radiation from Wide-Angle Conical Antennas. . .," 1949. Proc. I.R.E., Nov. 1949.
- 93 R. King, "Asymmetrically Driven Antennas and the Sleeve Dipole," 1949.
- 94 T. Morita, E. O. Hartig, and R. King, "Measurement of Antenna Impedance. . .," (Supplement to T. R. 43), 1949.
- 95 C. P. Hsu, "Theory of Helical Waveguides and Helical Radiators," 1950.
- 96 R. King, "Theory of V-Antennas," 1950.
- 98 D. J. Angelakos, "Current and Charge Distributions on Antennas and Open-Wire Lines," 1950.
- 100 H. Levine and C. H. Papas, "Theory of the Circular Diffraction Antennas," 1950.
- 101 J. E. Storer, "Variational Solution to the Problem of the Symmetrical Cylindrical Antenna," 1950.
- 104 G. Wheeler, "Coupled Slot Antennas," October 25, 1950.
- 105 R. D. Kodis, "An Experimental Investigation of Microwave Diffraction," 1950.
- 107 E. O. Hartig, "Circular Apertures and their Effects on Half-Dipole Impedances," 1950.
- 108 E. O. Hartig, "A Study of Coaxial-Line Discontinuities Using a Variational Method," 1950.
- 109 E. O. Hartig, "An Experimental and Theoretical Discussion of the Circular Diffraction Antenna," 1950.
- 118 R. King, "Self- and Mutual Impedances of Parallel Identical Antennas," 1950.
- 119 J. E. Storer, "The Impedance of an Antenna over a Large Circular Screen," 1950. J. Appl. Phys., August 1951.
- 121 R. King, "Theory of Collinear Antennas, II," 1950. J. Appl. Phys., December 1950.
- 122 J. Taylor and T. Morita, "Antenna Pattern-Measuring Range. 1951.
- 126 J. E. Storer, "The Radiation Pattern of an Antenna over a Circular Ground Screen," 1951.

- 128 J. Taylor, "The Sleeve Antenna," 1951.
- 129 T. E. Roberts, Jr., "Currents Induced on an Infinitely Long Wire by a Slice Generator," 1951.
- 130 R. King, "A Dipole with a Tuned Parasite: Theory and Experiment," 1951. J.I.E.E., January 1952.
- 132 R. King, "An Improved Theory of the Receiving Antenna," June 1951.
- 134 T. E. Roberts, Jr., "Properties of a Single-Wire Line," 1951.
- 138 C. Huang and R. D. Kodis, "Diffraction by Spheres and Edges at 1.25 Centimeters," 1951.
- 139 T. E. Roberts, Jr., "An Experimental Investigation of the Single-Wire Transmission Line," 1952.
- 141 R. King, "Theory of Electrically Short Transmitting and Receiving Antennas," 1952.
- 146 C. Moritz, "The Coupled Receiving Antenna, I.," 1952.
- 147 C. Moritz, "The Coupled Receiving Antenna, II.," 1952.
- 148 C. H. Papas and D. B. Brick, "Radiation of the Boss Antenna," 1952.
- 149 J. Sevick and J. E. Storer, "A General Theory of Plane-Wave Scattering from Finite Conducting Obstacles with Application to the Two-Antenna Problem," 1952.
- 150 J. Sevick, "Experimental and Theoretical Results on the Back-Scattering Cross Section of Coupled Antennas," 1952.
- 151 J. Sevick, "An Experimental Method of Measuring Back-Scattering Cross Sections of Coupled Antennas," 1952.
- 152 J. E. Storer, "Wave Propagation in a Two-Dimensional Periodic Medium," 1952.
- 153 R. V. Row, "Microwave Diffraction Measurements in a Parallel-Plate Region," 1952.
- 154 R. King, "An Alternative Method of Solving Hallén's Integral Equation and its Application to Antennas near Resonance," 1952.
- 155 P. A. Kennedy and R. King, "Experimental and Theoretical Impedances and Admittances of Center-Driven Antennas," April 1953.
- 159 L. S. Sheingold, "The Susceptance of a Circular Obstacle to an Incident Dominant Circular-Electric Wave," 1952.

- 160 J. E. Storer, L. S. Sheingold, and S. Stein, "A Simple Graphical Analysis of Waveguide Junctions," 1952.
- 161 R. D. Turner, "Scattering of Plane Electromagnetic Radiation by an Infinite Cylindrical Mirror," May 15, 1953.
- 162 T. Morita and L. S. Sheingold, "A Coaxial Magic-T," 1952.
- 163 C. Huang, "On the Diffraction of Electromagnetic Waves by Annular, Elliptical and Rectangular Apertures," May 1953.
- 170 R. V. Row, "Electromagnetic Scattering from Two Parallel Conducting Circular Cylinders," May 1, 1953.
- 172 D. B. Brick, "The Radiation of a Hertzian Dipole over a Coated Conductor," May 10, 1950.
- 173 R. Turner and A. F. Downey, "A Tabulation of the Fresnel Integrals." March 15, 1953.
- 174 R. King, "End-Correction for Coaxial Line When Driving an Antenna over a Ground Screen." June 15, 1953.
- 180 J. E. Storer, "Modification of Standard Network Synthesis Techniques to Use Lossy Elements," June 20, 1953.