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TECHNICAL REPORT

Some considerations on normal monthly temperatures

By Capt Robert W. Lennert, USAF, and Walter A. Baum



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SOME CONSIDERATIONS ON NORMAL MONTHLY TEMPERATURES

By Capt. Robert W. Lenhard, USAF, and Werner A. Baum

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Werner A. Baum
Principal Investigator

ABSTRACT

The concept of a satisfactory normal monthly temperature is examined. January and July mean temperature records at seven stations in the United States are examined. It is found that, except in the western United States in summer, such temperature records may be accepted as constituting a random sample. It is also found that, in general, mean monthly temperatures are not normally distributed.

The method of confidence limits is applied to determination of a satisfactory normal temperature. Seasonal and geographical variation in reliability of normal temperatures is observed. Consideration is given to the adequacy with which the normal characterizes the temperature record, and to the influence of trends and cyclic fluctuations on this adequacy. It is suggested that a normal of desired reliability be computed from the most recent portion of the record to be most representative.

Normals computed by different methods are compared and show a statistically significant difference in July only, a difference that may be of no practical importance.

1. Introduction

Because normal monthly temperatures are a standard climatological item, there is a need for them to be reliable. Most standard reference sources indicate that 30 to 40 years of record should be used to establish a satisfactory normal. The reasons for choice of this length of record are not clear; one suspects that belief in the Bruckner cycle is a major reason.

In 1935 the then existing International Meteorological Organization [1] recommended a period of 30 years as appropriate for establishment of normal temperature conditions, and suggested 1901-1930 as a universal period for calculation of normals. During the discussions leading to the recommendation, the dependence of the reliability of means on the variability of climate was brought out.

In 1941 the U. S. Weather Bureau [2] adopted the 40-year period 1899-1938 for the preparation of maps of normal temperatures.

Kendrew [3] notes the varying length of record necessary to establish normals in different latitudes, as does Landsberg [4], who also notes the seasonal change in variability of mean temperature at a middle-latitude station.

In general, a satisfactory normal temperature is incompletely specified but is considered to be one that is stable and representative of the record. At times, statistical measures of reliability have been used to establish such a satisfactory normal. Hann's [5] formula and computations have been most widely repeated and quoted. In this formula, the probable error of the mean is expressed in terms of the number of items n and the mean of the deviations (departures) of these

items from their average value, disregarding algebraic sign:

$$\text{Probable error} = 1.1955(2n - 1)^{-\frac{1}{2}} \times \text{mean departure.}$$

This formula is credited by Hann to Fechner, without restriction as to its applicability and without specific reference. Study of some of Fechner's [6] work indicates that the formula was developed to show the probable error of the means of a series of physiological and psychological measurements. Attempts to convert this formula, into one currently accepted in statistical theory, fail.

The simplest modern statistical procedure in dealing with random distributions is to use the standard deviation of the mean to specify confidence limits; this procedure will be examined here. The primary objective of the study is to provide information and criteria for use in selection of a procedure for computing a normal that will be appropriate to the application that is to be made of it.

The assumption is made that instrumental errors do not bias the data.

2. Definitions and data

Terminology that will differentiate between mean temperatures is needed. Daily mean temperatures are averaged to form a mean temperature for an individual month, designated the monthly mean temperature. The latter are averaged over a period of record to form another mean, usually spoken of as the normal temperature for the month. This designation has some misleading connotations, but a clear differentiation of the two means is necessary when both are being discussed. When symbols are used, T will designate monthly mean temperature and \bar{T} normal monthly temperature. When the discussion is solely of statistical technique, the word "mean" will be used in a general sense.

For working data, January and July have been selected, to provide the maximum seasonal difference. To provide a variety of physical conditions, the following stations have been selected: Portland, Oregon; San Diego, California; Salt Lake City, Utah; Bismark, North Dakota; Cairo, Illinois; Blue Hill, Massachusetts; and Jacksonville, Florida.

As an example, fig. 1 presents a summary of the record at Jacksonville. Running curves for estimated normal January and July temperatures, and for corresponding standard deviations of the normals, are shown. The abscissa gives the number of items that entered into the computation, and the effect on the normal and its standard deviation of increasing sample size may be observed directly. The samples were accumulated chronologically moving backward in time, beginning with the year 1952. Results for less than 10 years of record are not shown, due to the extreme fluctuations that occur.

3. Randomness of sample

For the application of confidence limits, and for many other statistical techniques, it must be assumed that a succession of monthly mean temperatures constitutes a random sample. The specification of random sampling is not satisfied a priori in climatological data. Because of the sequential nature of the data and the known interdependence of weather conditions in time, the data must be tested for randomness.

The temperature records at the seven selected stations were tested for randomness of sequence. A test for randomness is given by Wald and Wolfowitz [7]. If X_1, X_2, \dots, X_n is the sequence

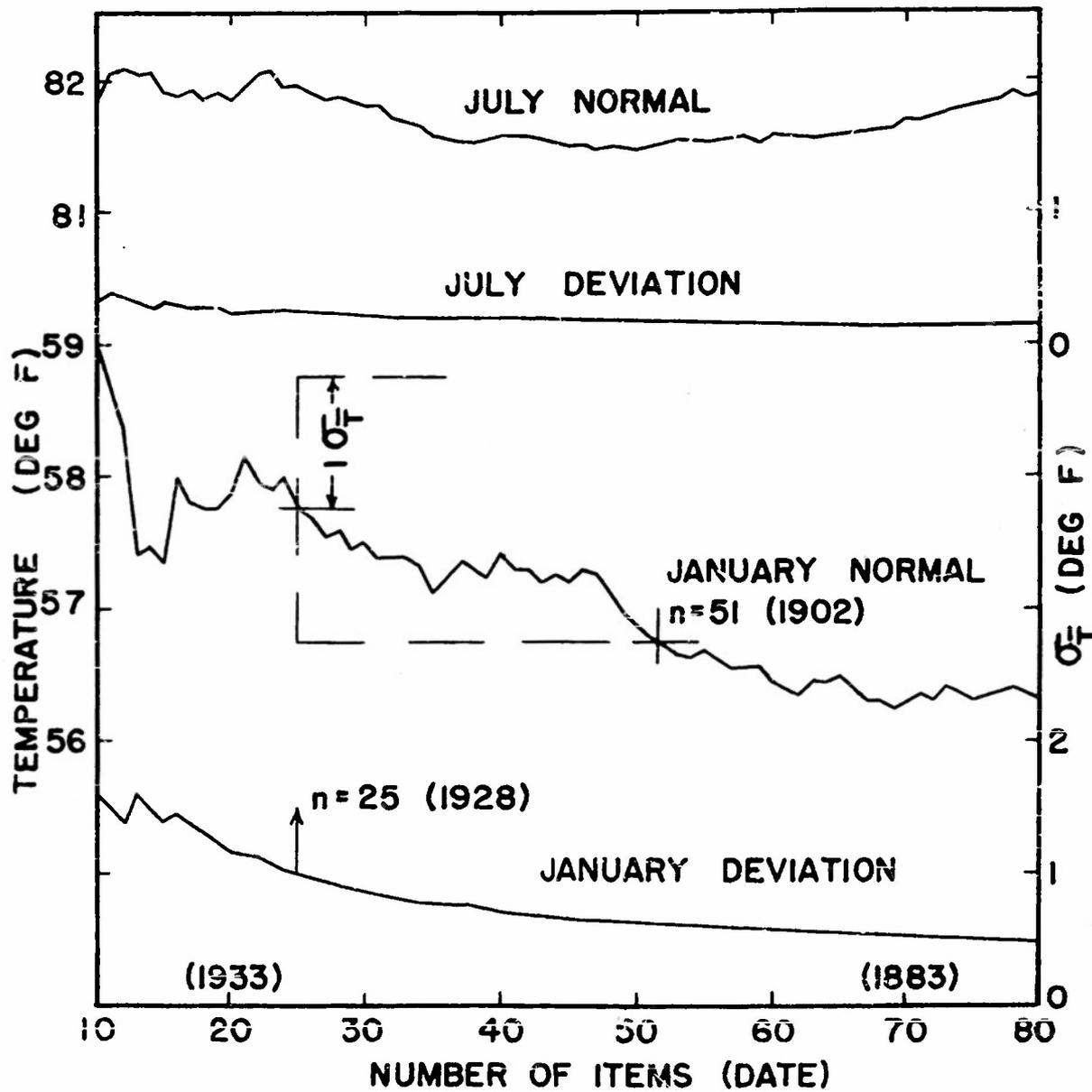


FIGURE 1. NORMAL TEMPERATURE AND DEVIATION, JANUARY AND JULY, JACKSONVILLE, FLORIDA.

to be tested, the statistic

$$R = \sum_{i=1}^{n-1} X_i X_{i+1} + X_n X_1,$$

is considered for all possible $n!$ permutations of the sequence. If all possible permutations of the sequence are treated as equally likely to occur, the statistic R is approximately normally distributed for large values of n . It may therefore be used to test the hypothesis of zero serial correlation. The only quantities necessary are the mean and variance of R , given by

$$M_R = \frac{S_1^2 - S_2}{n-1},$$

and

$$\sigma_R^2 = \frac{S_2^2 - S_4}{n-1} + \frac{S_1^4 - 4S_1^2 S_2 + 4S_1 S_3 + S_2^2 - 2S_4}{(n-1)(n-2)} - M_R^2,$$

where

$$S_k = \sum_{i=1}^n X_i^k.$$

The analysis for randomness was performed at points 10 years apart. Each point at which the analysis is made, then, is a sample that is larger by ten items than the preceding sample. Table 1 shows the probability of obtaining a value of $|M_R - R|$, as large or larger than that computed, from a sample drawn from a population of zero serial correlation.

In January, no probability is less than 0.12 and most are considerably greater. Within the climatic regions represented, it appears valid to consider January mean temperatures as a randomly

distributed variable.

In July, the probabilities are generally smaller. Nevertheless, in the eastern and midwestern United States, it appears valid to treat July mean temperature as a randomly distributed variable. In the west, however, particularly in the regions represented by Salt Lake City and Portland, one would be inclined to reject the hypothesis that July mean temperature is a randomly distributed variable. This statistical result poses a meteorological problem that warrants further investigation. It should be pointed out that selected periods from the records at Salt Lake City and Portland do test out as random. At Salt Lake City, for example, the period from 1952 to 1933 may be accepted as such.

4. Applicability of Gaussian Law

In many problems, the normality of the frequency distribution of mean monthly temperatures is an important consideration. Croxton and Cowden [8] present several tests for the normality of a frequency distribution. The most useful and practicable one for the present problem is Fisher's test, wherein two statistics are computed: g_1 , a measure of skewness, and g_2 , a measure of kurtosis. For samples drawn from normal populations, these statistics are normally distributed with mean values of zero and variances depending solely on the size of the sample. They may therefore be used to test the hypothesis of zero skewness and normal kurtosis.

As in the test for randomness, the analysis was performed at points 10 years apart. Table 2 shows the probability of obtaining a value of g_1 , as large or larger than that computed, from a population of zero skewness. It is quite clear that the hypothesis of zero skewness in the population cannot be accepted generally.

Sufficient evidence is already provided that January and July mean temperatures at some of the selected stations may not be treated as forming a normal distribution. However, for the sake of completeness, table 3 shows the probability of obtaining a value of g_2 , as large or larger than that computed, from a population with kurtosis equal to that of a Gaussian frequency distribution.

The general lack of normality in the frequency distributions of T precludes the possibility of using normal probability tables for determination of the likelihood of a specific T from a knowledge of \bar{T} and σ_T , the standard deviation of the mean monthly temperatures. However, even though the original population is not normal, \bar{T} will be normally distributed with standard deviation $\sigma_{\bar{T}} = \sigma_T/n^{1/2}$, if the population is random and homogeneous.

5. Homogeneity of data

The importance of computing a mean from a portion of the record that is homogeneous is emphasized by Conrad and Pollak [9]. Homogeneity of weather records is questionable on the grounds that the locations of instruments are known to have been changed and that, in many cases, the local environment is known to have been changed radically over a period of time as a result of man's activities.

Dixon and Massey [10] present a number of tests for homogeneity. Application of these tests is complicated by the restrictions on the conditions under which they may be used. For example, the well-known analysis-of-variance test requires that the observations are from normally distributed populations and that the variance of each group is the same.

Insofar as the limitations on the tests made it possible, the temperature records were broken down into samples by locations and samples by decades, and tested by one or more methods. With very few exceptions, the records were found to be statistically acceptable as homogeneous.

An interesting anomaly was provided by Jacksonville. The entire January record was found to be statistically acceptable as homogeneous. But when the July record was tested for homogeneity by locations, it proved acceptable only when the period 1903-1932 was dropped from the record; the years which remain are all those in which locations of the instruments were less than 100 ft above the surface.

6. Confidence limits

The above considerations lead to the conclusion that, in general but not always, satisfactory temperature normals can be obtained by the use of confidence limits of reliability.

To illustrate, a sample of the temperature record consisting of the last 25 years at Jacksonville in January gives $\bar{T} = 57.8F$ with $\sigma_{\bar{T}} = 1.0F$, or a normal temperature of $57.8 \pm 1.0F$ with a confidence of 0.68. That is to say that if 100 randomly selected samples of 25 years of data each were taken, on the average 68 of them would include the true mean in the range $\bar{T} \pm \sigma_{\bar{T}}$. Obviously this does not specify which of the 100 samples do include the true mean; a single sample is more likely to be one of 68 than one of 32, but it can belong to either group and there is no way of telling to which group it belongs.

The limitation of single sampling is discussed by Shewhart [11], and may be illustrated with the January record at Jacksonville. When the sample is accumulated backward in time from 1952, an accuracy of 1F with confidence of 0.68 is reached with 1928, at $n = 25$; the normal

at that point is 57.8F. The minimum expected value for the normal temperature would be 56.8F. If the population is the entire record, the normal temperature is found to be 56.2F; the normal, computed as the sample accumulated in size, dropped below 56.8F at $n = 52$. Another sample may be obtained by accumulating backward in time from 1941. The two samples are shown in fig. 2. In the latter sample, an accuracy of 1F with confidence of 0.68 is reached with 1922, at $n = 20$, when the normal temperature is 56.7F. If the sample is extended back to the beginning of the record, the normal changes but remains always within the range $56.7 \pm 1.0F$. Thus, if the population is the entire record, the sample 1922-1941 would be satisfactory whereas the 1928-1952 sample would not be. That this is not an unusual occurrence is demonstrable in two ways: first, the example was not specifically selected but was obtained at the first attempt to find an illustration; 1941 was the starting year resulting from choosing a sample whose possible final size was 70. Secondly, the mean of the 1928-1952 sample is but $1.4\sigma_{\bar{T}}$ away from the mean of the entire record; deviations as large or larger could be expected in 17 percent of random samples of 25 from this population. Finally, it is clear that the choice of one standard deviation, corresponding to a confidence of 0.68, is an arbitrary base for discussion.

In acceptance of $\bar{T} \pm \sigma_{\bar{T}}$ as including the true normal with a confidence of 0.68, a risk of 0.32 is taken of rejecting such a statement when it is true, of making an error of type 1. Since the estimated normal temperature is invariably accepted, this statement of risk is unrealistic. The confidence expressed is also unrealistic, since

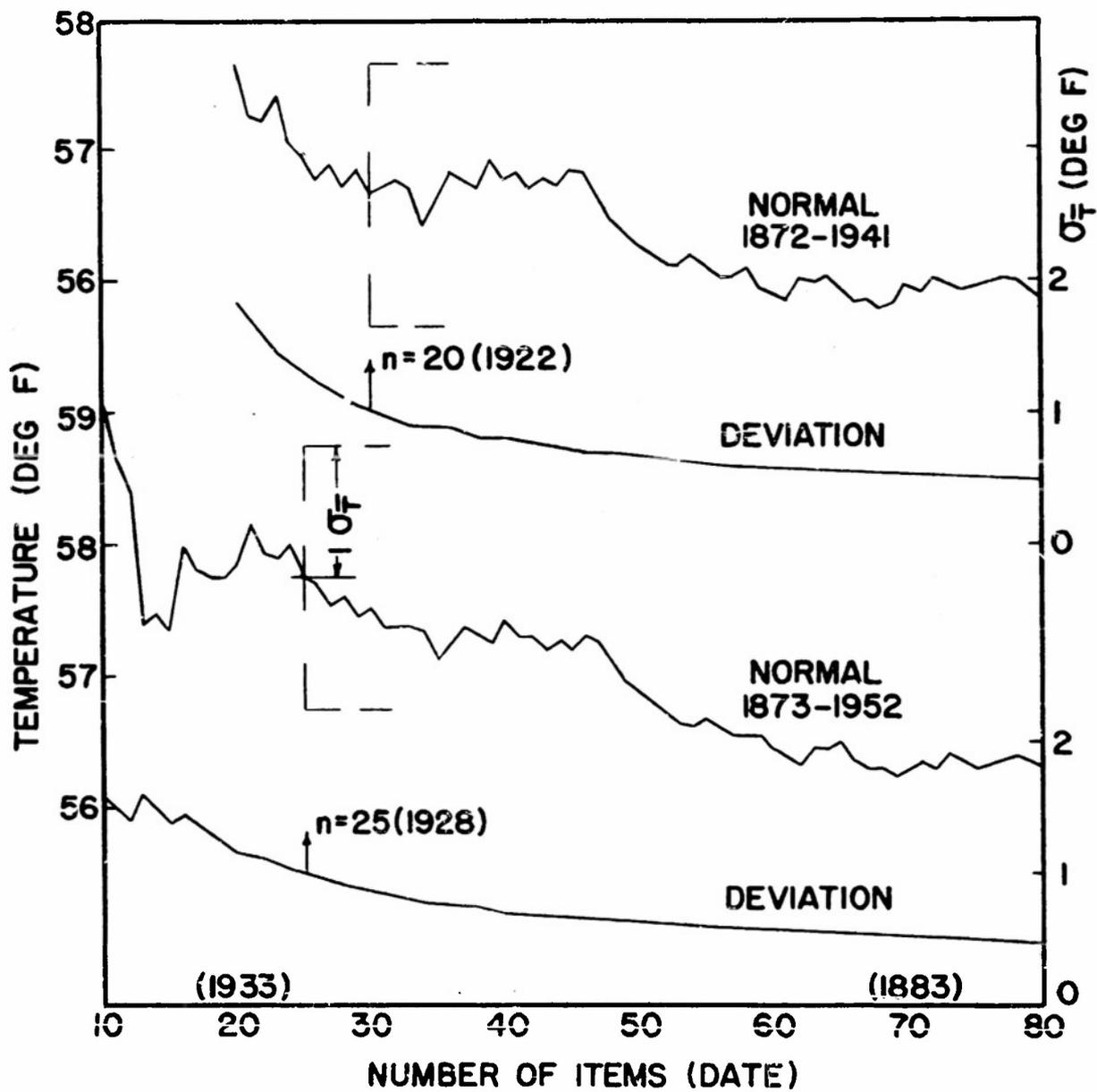


FIGURE 2. COMPARISON OF SAMPLES OF JANUARY NORMAL TEMPERATURE AND DEVIATION AT JACKSONVILLE, FLORIDA .

there will not be 99 more samples taken to demonstrate the confidence.

If the acceptance range of $\bar{T} \pm k\sigma_{\bar{T}}$ does not include the true normal, a false statement is accepted as true, and an error of type 2 has been made. The probability of this type of error is designated by β , and Ferris et al [12] show that the magnitude of β is the probability that the difference between true and sample means will be between specified limits:

$$\beta = P \left[(\mu - a - k\sigma_{\bar{T}}) < (\bar{T} - \mu) < (\mu - a + k\sigma_{\bar{T}}) \right], \quad (1)$$

where k is a factor governing the confidence in the acceptance limits, μ is the true mean, and a is an arbitrary reference constant about which the range $\pm k\sigma_{\bar{T}}$ is centered. If $a = \mu$, (1) reduces to the expression for confidence limits,

$$P \left[(\mu - k\sigma_{\bar{T}}) < \bar{T} < (\mu + k\sigma_{\bar{T}}) \right] = 1 - \mathcal{L},$$

where \mathcal{L} is the probability of an error of type 1.

When $|\mu - a| < k\sigma_{\bar{T}}$, μ will be included in the range $\bar{T} \pm k\sigma_{\bar{T}}$ and no type 2 error is made. As $|\mu - a|$ exceeds $k\sigma_{\bar{T}}$, the probability of a type 2 error is expressed by the area under one tail of the normal distribution curve of sample values of μ . This probability is at a maximum when $|\mu - a| = k\sigma_{\bar{T}}$ and decreases as $|\mu - a|$ increases. At this maximum value,

$$\beta = P \left[0 < (\bar{T} - \mu) < 2k\sigma_{\bar{T}} \right] \text{ or } P \left[2k\sigma_{\bar{T}} < (\bar{T} - \mu) < 0 \right].$$

The two expressions are equivalent in magnitude, and either one, but not both, may occur.

A statement of the following form may be made: If the confidence is $1 - \mathcal{L}$ that $\mu = \bar{T} \pm k\sigma_{\bar{T}}$, the probability is β that this estimate is in error by as much as $\pm |\mu - a|$, or that $\mu \geq \bar{T} \pm (k\sigma_{\bar{T}} + |\mu - a|)$.

Specification of either β or $|\mu - a|$ will determine the value of the other variable. A probability statement of this form can be of direct applicability in planning.

Equation (1) has been applied to the January temperature record at Jacksonville, and the results are presented in table 4.

When confidence is increased by increasing the acceptance range, β increases. If the January normal temperature at Jacksonville is stated as $57.8 \pm 1.0F$ with a confidence of 0.68, there is but 2 per cent chance of this estimate being in error by as much as 3.0F; if the normal is stated as $57.8 \pm 2.0F$ with a confidence of 0.95, the chance of an error in estimate as great as 3.0F is 16 per cent.

When confidence is increased, holding the acceptance range constant, β decreases. If the normal is stated as $56.3 \pm 1.0F$ with a confidence of 0.95, which occurs with a sample of 75 items, there is but 2 per cent chance of the estimate being in error by as much as 2.0F.

7. Comparison of normal temperatures

Table 5 presents the normal January and July temperatures, with their standard deviations, for four periods: 1901-1930, the period recommended by the International Meteorological Organization; 1901-1952, the entire period of record; and the minimum record required to obtain a normal of specified reliability. This reliability is 1F with confidence of 68 per cent for January normals. Where this reliability was achieved with less than ten items in the sample, the normal and its standard deviation for a sample of ten years was used; slightly larger samples may actually be necessary for statistical techniques to be applicable. In the computation of July normals, a reliability of

0.45F with a confidence of 68 percent was used, so that as few stations as possible would have a record of but ten items.

To a limited extent, table 5 may be used to examine the seasonal and geographic variation of reliability of normal January and July temperatures in the United States. In general, July standard deviations of normal temperatures are about one-half as large as January values. The reliability in summer, then, will be greater than in winter. This increased reliability in summer could appear either as a narrower tolerance range or as greater confidence in the same range as for winter.

The table indicates, as would be expected, that there is a northward decrease of reliability in middle latitudes, and that maritime control of climate leads to greater reliability than does continental control of climate.

In table 5, the averages of the January and July normals of the seven stations are shown for each period on which the normals are based. The range of these averages is 0.5F in January and 1.2F in July. The analysis-of-variance test was applied to the normals in the table to determine if any significant difference exists in the methods for obtaining normal temperatures. At the 5 per cent level of significance, the test ratio has the value $F = 3.16$. The ratios obtained are: January, $F = 1.2$; July, $F = 22.5$. Although the validity of applying the test in this case was not investigated, the results may at least be considered indicative. It seems that, statistically, it makes no difference which of the four periods is used to compute the normal temperature in January; the difference may be considered considerable

in July. The practical significance of the conclusion that periods are not equivalent for July is dependent upon the application to be made of the normal temperature. In view of the high reliability of July normals, even with short periods of record, any differences arising out of different periods for computation of the normal would probably be negligible. Since July records are generally less likely to be homogeneous than those in January, a July normal based on a selected homogeneous portion is probably the most accurate.

8. Representativeness of the normal temperatures

The extent to which the normal temperature represents the entire record of mean monthly temperatures is dependent on the variation of the mean temperature of a month from year to year. There are at least two classes of time variations that may limit the representativeness of the normal temperature, namely trends and cyclical fluctuations. To examine these variations, the monthly mean temperature records were plotted as exemplified by Jacksonville in fig. 3. The solid lines connect normal temperatures for decades, thus providing a smoothed trace of the time variation of mean monthly temperature. The dashed lines connect the normal temperatures of the half-records and show the trend of mean monthly temperature. Other, more accurate methods of measuring trend are available; this method of semi-averages is of sufficient accuracy for illustrative purposes. The dotted lines are drawn at the levels of the normal temperatures as computed from the entire records. Quasi-cyclical fluctuations of the decade normals may be observed either about this long-term normal line or about the trend line.

Trend. The effect of trend on the representativeness of the normal can be illustrated with the January record at Jacksonville. To

judge from the changing normal shown in fig. 1, the normal temperature of $57.8 \pm 1.0F$, established at $n = 25$, adequately characterizes the record for the period 1902-1952, but is not adequate for describing the entire record. Beyond 1902, the long-term normal continues to fall as it has throughout the record. This tendency to fall is consistent with the trend shown in fig. 3. In general, a normal determined from a sample at one end of the record will not be representative of the entire record when a trend is present.

To be considered satisfactory, a normal temperature must be not only reliable but representative. If trend exists, some compensation for that trend must be made in determining the normal. The proper method of determination will depend on the use to be made of the mean. If an analysis of the past record is the objective, the entire record should be used as fully as the analysis requires. The normal then obtained will be the true one for the population under consideration, but will be inadequate without specification of the trend if the model resulting from the analysis is to symbolize the record.

If extrapolation of the record is desired, inclusion of those portions of the record remote from the point of extrapolation will tend to bias the normal. If the attempt were to be made to extrapolate Jacksonville January temperatures, assuming for the moment that the trend will continue undisturbed, the normal at $n = 25$ would be more useful than that obtained from the entire record.

Projection of a trend is a dangerous process, unless the physical causes of the trend can be determined and the continuance of their effects can be assured. Consider the two alternatives for the

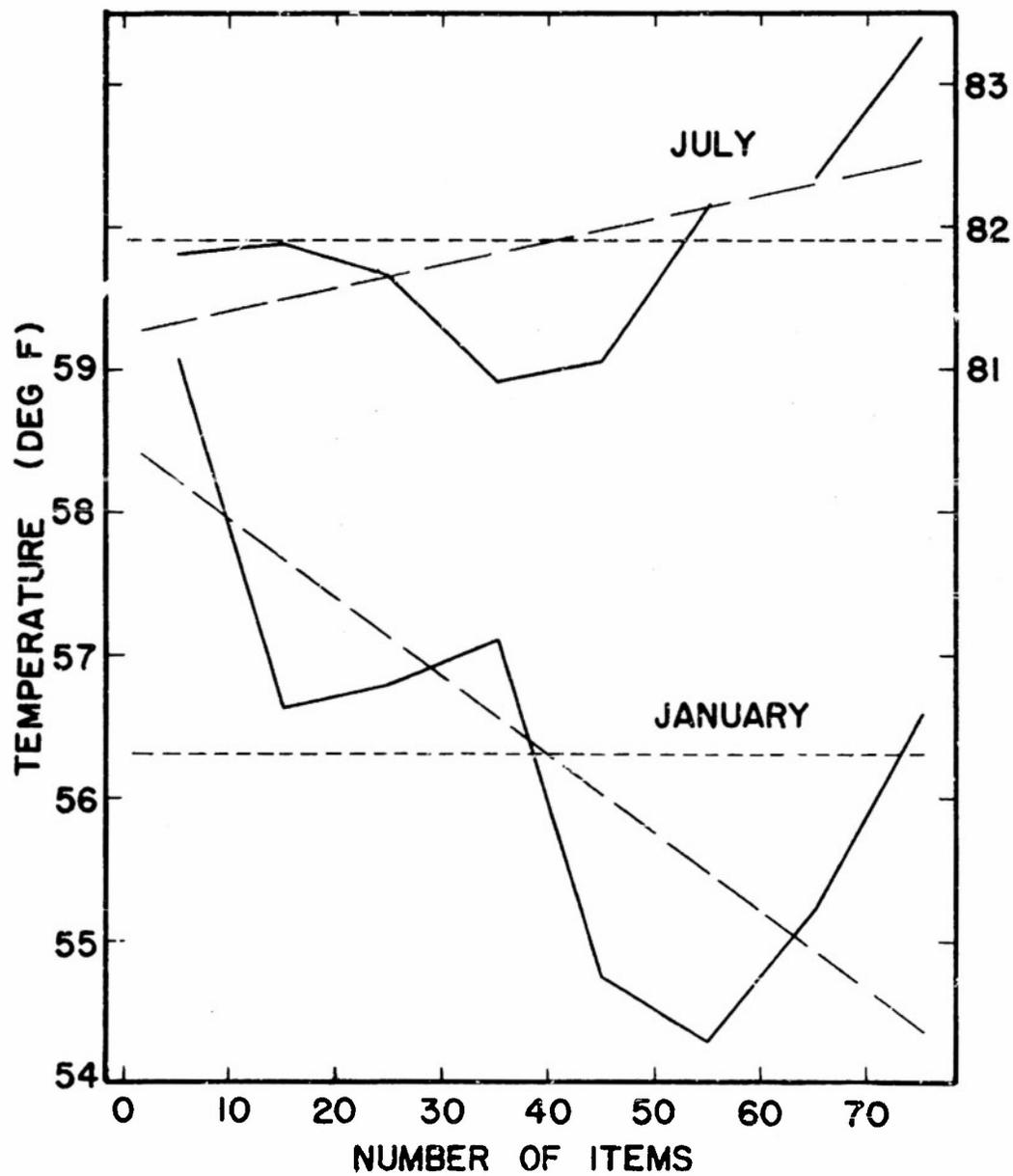


FIGURE 3. DECADE NORMAL TEMPERATURE JANUARY AND JULY, JACKSONVILLE, FLORIDA.

Jacksonville example: (1) the warming may be due to the growth of the city and increasing industrialization; (2) the warming may be due to atmospheric causes. If the warming is due to the first alternative, its future behavior is as unpredictable as are the growth rate of the city and the effects of industrial activity. If the warming is due to atmospheric causes, it might be expected to continue; or the apparent trend may just be a portion of a cycle having a long period. Which eventuality is more likely is largely a matter of speculation.

Most likely, both causes would be operating. Whatever the facts, extrapolation of the record is of dubious validity. Yet such extrapolation is implicit in almost all activities. Whenever a normal temperature is used as a design temperature, the assumption is made that the normal temperature used will be characteristic of times to come. Probably the best forecast that can be made is to use the normal from the part of the record nearest to the point of extrapolation, and for this normal to be established over the shortest period that will give a useful reliability. It was with this end in mind that samples were accumulated backward in time in this study.

Cyclical fluctuations.- With respect to the decade normals, if judgement as to representative ability of the Jacksonville January normal of $57.8 \pm 1.0F$ established at $n = 25$ is based on fig. 3, it is adequate only for the decade 1913-1922. The range fails to include either of the first two decade normals (proceeding backward in time), barely includes the third decade normal, and includes the fourth but no more decade normals. These additional normals would be eliminated by the trend. The quasi-cyclical fluctuation brings the most remote decade normal, that for 1873-1882, almost up to inclusion within the

reliability range of the normal temperature for $n = 25$. Whether or not a reliable normal temperature will be representative when quasi-cyclical fluctuations are present depends on the magnitude of the fluctuations and the portion of the cycle considered in computing the normal.

In any attempt to compensate for cyclical fluctuations, they should be presented directly if the goal is analysis of the past record. In the event that such cycles are to be analyzed further and are desired as deviations from the normal temperature, care must be taken that the normal is computed over a number of years equal to a full cycle or multiple thereof. If the purpose is extrapolation, the cyclical variations rather than the normal should be extrapolated if they can be established as physically real and if there is an adequate length of record. Such a length of record will rarely be encountered.

Generally the evidence in support of cycles is questionable, so there will frequently be hesitancy in extrapolating them. The mean is then the best prognostic parameter. Ideally it should be computed over a period such that equal positive and negative deviations of the cyclical variations from the trend line are included. To meet the ideal condition, it is necessary to examine a longer record than that necessary to establish a reliable normal. Inaccuracies arising from a normal obtained from a period covering an unbalanced portion of a cycle will, of course, depend on the amplitude of the cycle.

9. Conclusions

Satisfactory temperature normals can be obtained by the use of confidence limits of reliability in many instances. The assumption of homogeneity of the temperature record must be validated, especially when the record involves a change of location.

The conclusions that monthly mean temperatures do not form normal frequency distributions and that the sequential record constitutes an acceptably random sample would seem to be suitable approximations. Since these conclusions are based on a "most usual" basis, they are not valid in every case.

The presence of secular and quasi-cyclical variations of temperature weakens the representativeness of the normal. This weakening can be minimized by computation of the normal over the shortest possible period adjacent to the point of extrapolation.

There exist wide ranges of reliability and representativeness of normal temperatures obtained from synchronous records. An alternative to synchronous records for the comparison of normals between stations is the use of normals of equal reliability obtained from records with synchronous origins in time.

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Table 1. Probability of obtaining value of $|M_R - R|$, as large or larger than computed, from population with no serial correlation.

Station	Years of record						
	20	30	40	50	60	70	80
January							
Portland	0.43	0.48	0.57	0.77	0.77	0.94	0.80
San Diego	0.52	0.65	0.47	0.81	0.84	0.62	0.42
Salt Lake City	0.24	0.45	0.15	0.20	0.22	0.48	0.52*
Bismark	0.34	0.45	0.75	0.96	0.78	0.56	0.76*
Cairo	0.85	0.68	0.49	0.33	0.12	0.80	0.42
Blue Hill	0.28	0.25	0.24	0.78	0.96	0.72**	—
Jacksonville	0.94	0.87	0.81	0.75	0.81	0.89	0.85
July							
Portland	0.03	0.15	0.12	0.10	0.02	0.02	0.01
San Diego	0.82	0.87	0.85	0.28	0.05	0.06	0.05
Salt Lake City	0.77	0.90	0.10	0.01	0.01	0.01	0.01*
Bismark	0.83	0.74	0.94	0.80	0.72	0.71	0.67*
Cairo	0.71	0.95	0.37	0.52	0.29	0.12	0.31
Blue Hill	0.94	0.93	0.71	0.83	0.92	0.77**	—
Jacksonville	0.98	0.91	0.67	0.69	0.64	0.55	0.51

*78 years of record; **67 years of record.

Table 2. Probability of obtaining value of g_1 , as large or larger than computed, from population with no skewness.

Station	Years of record						
	20	30	40	50	60	70	80
	January						
Portland	0.08	0.03	0.03	0.01	0.01	0.01	0.01
San Diego	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Salt Lake City	0.01	0.03	0.04	0.01	0.01	0.02	0.02*
Bismark	0.25	0.21	0.25	0.47	0.40	0.90	0.74*
Cairo	0.35	0.60	0.01	0.01	0.01	0.04	0.27
Blue Hill	0.67	0.55	0.87	0.94	0.75	0.92**	—
Jacksonville	0.90	0.54	0.52	0.35	0.17	0.10	0.08
	July						
Portland	0.03	0.13	0.54	0.94	0.70	0.70	0.94
San Diego	0.20	0.28	0.27	0.12	0.07	0.14	0.25
Salt Lake City	0.01	0.01	0.01	0.01	0.12	0.24	0.39*
Bismark	0.13	0.64	0.62	0.02	0.01	0.01	0.01*
Cairo	0.38	0.31	0.60	0.84	0.92	0.92	0.94
Blue Hill	0.02	0.01	0.40	0.23	0.45	0.66**	—
Jacksonville	0.04	0.20	0.60	0.70	0.40	0.14	0.02

*78 years of record; **67 years of record.

Table 3. Probability of obtaining value of g_2 , as large or larger than computed, from population with normal kurtosis.

Station	Years of record						
	20	30	40	50	60	70	80
	January						
Portland	0.97	0.89	0.84	0.83	0.56	0.62	0.45
San Diego	0.11	0.01	0.41	0.15	0.29	0.47	0.92
Salt Lake City	0.16	0.42	0.86	0.31	0.29	0.67	0.44*
Bismark	0.65	0.23	0.29	0.24	0.30	0.11	0.05*
Cairo	0.98	0.51	0.05	0.09	0.11	0.83	0.69
Blue Hill	0.31	0.34	0.43	0.61	0.89	0.79**	—
Jacksonville	0.32	0.22	0.34	0.48	0.39	0.42	0.34
	July						
Portland	0.51	0.52	0.17	0.12	0.48	0.76	0.83
San Diego	0.69	0.09	0.10	0.08	0.12	0.05	0.03
Salt Lake City	0.76	0.79	0.77	0.34	0.28	0.27	0.29*
Bismark	0.01	0.01	0.01	0.01	0.01	0.01	0.01*
Cairo	0.54	0.84	0.13	0.28	0.27	0.38	0.32
Blue Hill	0.12	0.01	0.01	0.01	0.01	0.05**	—
Jacksonville	0.01	0.63	0.75	0.62	0.84	0.90	0.74

*78 years of record; **67 years of record.

Table 4. Probability of accepting a false statement under varying forms of statement of normal January temperature at Jacksonville.

$1 - \alpha$	n	\bar{T}	$\sigma_{\bar{T}}$	$\mu - a$	k	$k\sigma_{\bar{T}}$	μ	β
0.68	25	57.8F	1.0F	1.0F	1	1.0F	$57.8 \pm 2.0F$	0.48
0.68	25	57.8	1.0	2.0	1	1.0	57.8 ± 3.0	0.16
0.68	25	57.8	1.0	3.0	1	1.0	57.8 ± 4.0	0.02
0.95	25	57.8	1.0	2.0	2	2.0	57.8 ± 4.0	0.50
0.95	25	57.8	1.0	3.0	2	2.0	57.8 ± 5.0	0.16
0.68	80	56.3	0.5	1.0	1	0.5	56.3 ± 1.5	0.16
0.68	80	56.3	0.5	1.5	1	0.5	56.3 ± 2.0	0.02
0.95	80	56.3	0.5	2.0	2	1.0	56.3 ± 3.0	0.02
0.68	80	?	0.33	0.66	1	0.33	$? \pm 1.0$	0.16
0.68	30	55.9	0.66	1.33	1	0.66	55.9 ± 2.0	0.16

*Period 1901-1930.

Table 5. Normal temperatures and their standard deviations.

Station	1901-1930		1901-1952		Entire record		Minimum record		
	\bar{T}		\bar{T}		\bar{T}		\bar{T}		n
January									
Portland	39.3	0.79	39.5	0.63	39.3	0.47	39.1	1.00	26
San Diego	55.0	0.38	55.1	0.32	54.8	0.26	54.4	0.87	10
Salt Lake City	30.1	0.97	29.6	0.62	29.1	0.52	28.8	0.98	28
Bismark	9.0	1.40	9.4	1.18	8.2	0.97	8.3	1.00	73
Cairo	36.1	0.86	37.1	0.68	36.5	0.60	37.7	1.00	18
Blue Hill	25.4	0.85	25.9	0.67	25.7	0.58	27.1	1.00	25
Jacksonville	55.9	0.67	56.7	0.62	56.3	0.48	57.8	1.00	25
Average	35.8		36.2		35.7		36.2		
July									
Portland	67.6	0.38	68.1	0.27	67.6	0.22	69.0	0.44	14
San Diego	67.3	0.26	67.9	0.24	67.7	0.18	68.9	0.38	10
Salt Lake City	76.5	0.39	77.3	0.30	76.7	0.25	77.6	0.38	10
Bismark	70.0	0.57	71.3	0.53	70.7	0.40	71.0	0.45	65
Cairo	79.5	0.38	80.0	0.28	79.6	0.23	80.5	0.43	18
Blue Hill	68.6	0.34	69.1	0.27	68.8	0.24	69.8	0.44	19
Jacksonville	81.1	0.19	81.5	0.17	81.9	0.15	81.8	0.33	10
Average	72.9		73.6		73.3		74.1		