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THE APPLICATION OF SOME NONLINEAR TECHNIQUES  
FOR THE IMPROVEMENT OF AIRCRAFT BEAM FOLLOWING

David L. Markusen

John S. Norton

Orville P. Pomeroy

Minneapolis-Honeywell Regulator Company

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February 1953

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The Application of Some Nonlinear Techniques  
for the Improvement of Aircraft Beam Following

David L. Markusen  
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February 1953

Aeronautical Research Laboratory  
Contract No. AF33(038)-22893  
RDO. No. 461-2

Wright Air Development Center  
Air Research and Development Command  
United States Air Force  
Wright-Patterson Air Force Base, Ohio

## FOREWORD

This report was prepared by the Research Department, Aeronautical Division, Minneapolis-Honeywell Regulator Company, Mr. D. L. Markusen acting as project engineer, under USAF Contract No. AF33 (038)- 22893, RDO No. 461-2 (Basic Research in Nonlinear Mechanics), and Supplemental Agreement No. S-2 (52-321). The contract was initiated under the research project identified by Expenditure Order No. 460-42-4BR1 and extended under Expenditure Order No. R461-2BR-1. It is administered under the direction of the Mechanics Research Branch, Flight Research Laboratory, Dr. C. L. Morrison acting as project engineer. This report will be followed by a final report at a later date.

## ABSTRACT

Preliminary results in a study of nonlinear techniques as applied to aircraft beam following are presented. The use of nonlinear displacement or rate gain and the use of second order predicting networks are considered as beam following aids. In addition, beam noise reducing schemes such as a simplified acceleration limited filter, velocity limiting with variable limits and limiting in the coupler lead network are discussed.

## PUBLICATION REVIEW

Manuscript Copy of this report has been reviewed and found satisfactory for publication.

For MINNEAPOLIS-HONEYWELL  
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## TABLE OF CONTENTS

Section I	Aid to Bracketing	Page 1
Section II	Reduction of Response to Noise	Page 15

## LIST OF ILLUSTRATIONS

Figure		Page
1	REAC Results for Linear 90° Approach at +16 miles	2
2	REAC Results for Linear 45° Approach at +16 miles	2
3	REAC Results for Linear Parallel Approach at +16 miles	3
4	REAC Results for Non-Linear 90° Approach at +16 miles	3
5	REAC Results for Non-Linear 45+ Approach at +16 miles	4
6	REAC Results for Non-Linear Parallel Approach at +16 miles	4
7	Scheduling of Lead Network Time Constant as Function of Distance	5
8	REAC Results for Approach Begun at 16 miles and Heading of 90°	6
9	REAC Results for Approach Begun at 16 miles -Parallel to Beam	7
10	REAC Results for Approach Begun at 8 miles	8
11	REAC Results for Approach Begun at 4 miles	9
12	Lateral Distance from Course Center at which Displacement and Rate are Equal	11
13	REAC Diagram of Beam Position Sensor	13
14	Block Diagram of Original Perception Filter with Fixed Limits	15
15	Block Diagram of Second Order Filter with Undetermined Limiting	16
16	Block Diagram of Second Order Filter with Fixed Acceleration Limits	17
17	REAC Diagram of Second Order Filter with Fixed Acceleration Limits	18
18	REAC Diagram for Variable Limit Amplifier	19
19	REAC Diagram of Second Order Filter with Fixed and Variable Limits on Second Derivative of Output	20
20	Amplitude of Output vs. Frequency of Input for Ratios of $A/\omega$	21
21	Phase of Output vs. Frequency of Input for Ratios of $A/\omega$	22
22	Frequency Response of Saturating Amplifier Servo	24
23	Phase Shift of Saturating Amplifier Servo	25

Figure		Page
24	Output vs Noise Amplitude	26
25	Effect of Noise Frequency on Output vs Noise Amplitude	27
26	Output vs Noise Amplitude	28
27	Output vs Noise Amplitude	29
28	Comparison of Standard Network with Non-Linear Network	30
29	Vector Diagram	31

## INTRODUCTION

This report summarizes the work done on the beam following phase of Non-Linear Project AD5042 under USAF Contract No. AF33(038)-22893 from 1 September 1952 to 15 February 1953. The entire program has three distinct fields of endeavor: (1) Beam Following, (2) Jet Engine Control, (3) Interceptor Fire Control. The beam following is divided into the two subclasses of improvement of bracketing and noise reduction. Non-linear gains, rate gain scheduling, and second order lead (prediction) networks have been investigated in order to improve the bracketing and damping. Of these the rate gain scheduling gives the greatest improvement although it will not be possible to implement this type of system until DME becomes generally available at localizer sites. Noise reduction has been tackled by reducing the complexity of the acceleration limited servo, adding variable limits to the velocity limited filter and by limiting in the coupler lead network.

The purpose of the investigation being conducted on the beam following system is to determine whether the system operation may be improved by the introduction of non-linearities in the system. In its present form the system is essentially linear or at least the non-linearities are of such a magnitude as to permit a linear analysis with good accuracy. It would appear that the introduction non-linearities might be of benefit in improving the system bracketing performance and in reducing system response to noise. Previous work was concerned primarily with reducing system response to noise and resulted in the perception filter which was demonstrated in flight tests last year. Present work has been concerned with investigating non-linearities that may be an aid to bracketing performance and with simplification of the means of obtaining the functions of the perception filter.

## SECTION I AIDS TO BRACKETING

One of the non-linear systems investigated was one in which the system gain was made a non-linear function of coupler output. The system was simulated on the REAC for the investigation and the non-linearity was so arranged to give linear operation for a relative narrow range of coupler inputs either side of beam center and increased system gain for coupler inputs beyond this range. For very large error or coupler input, the gain approaches an upper limit asymptotically.

Attached are typical solutions obtained on the REAC. Figure 1, 2, and 3 are for a linear system and approaches at 16 miles and headings of  $0^\circ$ ,  $45^\circ$ , and  $90^\circ$ . Figures 4, 5, and 6 are for the same conditions except the system is non-linear. It is evident from these that the non-linear system is not significantly better than the linear system insofar as time required to bracket the beam is concerned. The solutions do indicate that the non-linear system has effectively reduced the overshoot. Similar results were obtained for approaches at 8 and 4 miles.

It was found that a linear range of about 25 MV either side of beam center before entering non-linear operation produced good results. A linear range much less than 25 MV resulted in instability at distances close to the transmitter and linear ranges much larger gave results similar to a linear system.

When the linear range of operation was made a function of the displacement and displacement rate so as to increase the linear range for large rate and decrease it for large displacement, the results were very much the same as those obtained with a fixed linear range of 25 MV.

Another non-linear system investigated was one in which the time constant of the lead network was made a function of the distance from the localizer transmitter. A REAC simulation was set up in which the lead network time constant varied so that at 16 miles the time constant was 50 seconds and at 1 mile it was 10 seconds. Between these two values, the time constant varied as indicated in Figure 7. The steady state attenuation of the lead network was 20 and the coupler gain was set to give  $10^\circ$  of bank for a  $2\text{-}1/2^\circ$  course error.

Figures 8, 9, 10, and 11 are REAC solutions indicating the system performance for brackets made at 4, 8, and 16 miles with headings of  $90^\circ$ ,  $45^\circ$ , and parallel to course center. The REAC solutions also indicate the system response to sudden displacements of the beam center. With one exception the system appeared to be critically damped and responded without overshoot. The exception was for a sudden beam displacement of  $2\text{-}1/2^\circ$  at 10,000 feet from the transmitter and in this case a small amplitude oscillation of about two cycles duration resulted. A  $2\text{-}1/2^\circ$  beam displacement is actually a very severe disturbance and would not be encountered in flight.

At the present time the system discussed here cannot be realized for general use since distance measuring equipment is still in the development stage and not in general use, but it was thought that a performance

Figure 1 REAC RESULTS FOR LINEAR  
90° APPROACH AT 16 MILES

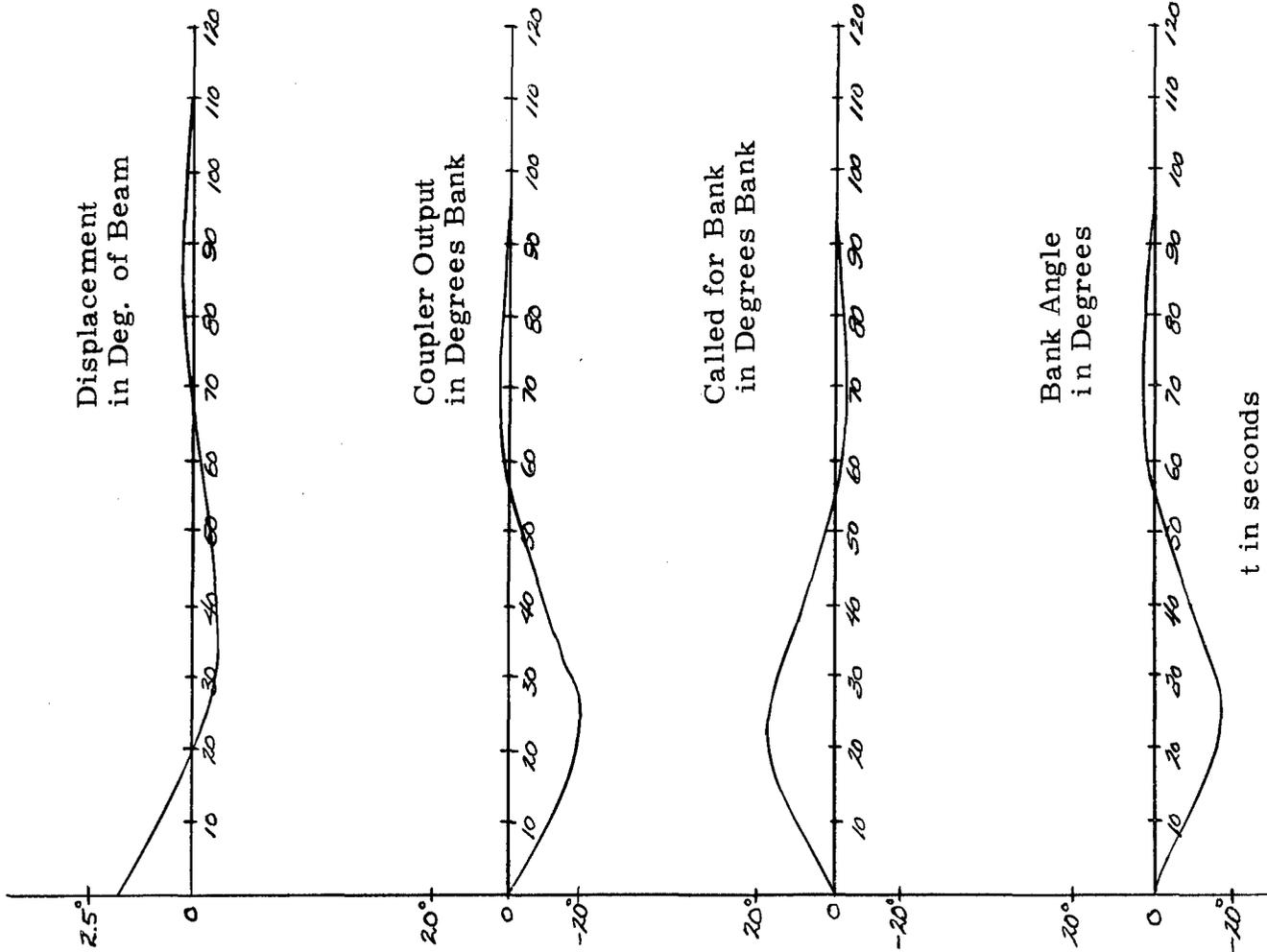


Figure 2 REAC RESULTS FOR LINEAR  
45° APPROACH AT 16 MILES

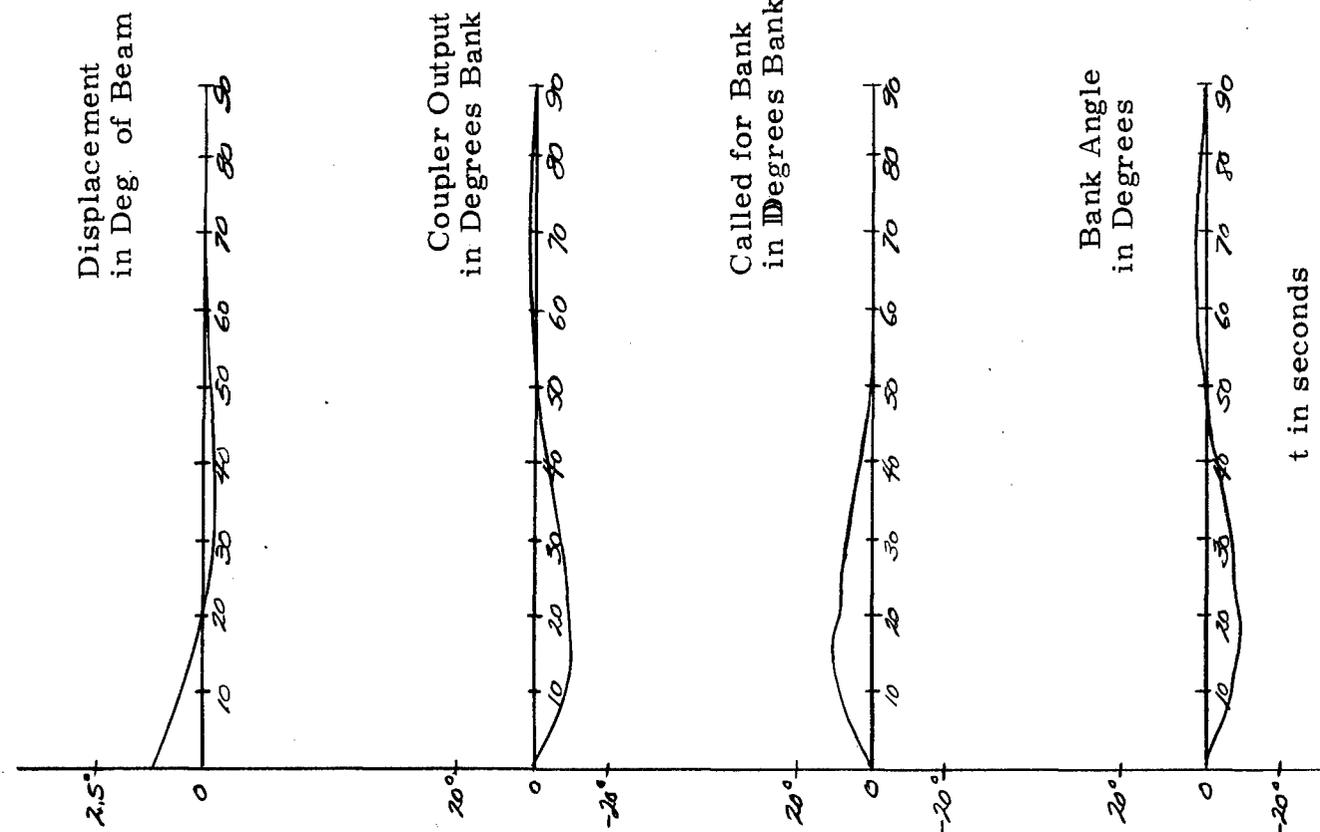


Figure 3. REAC RESULTS FOR LINEAR PARALLEL APPROACH AT 16 MILES

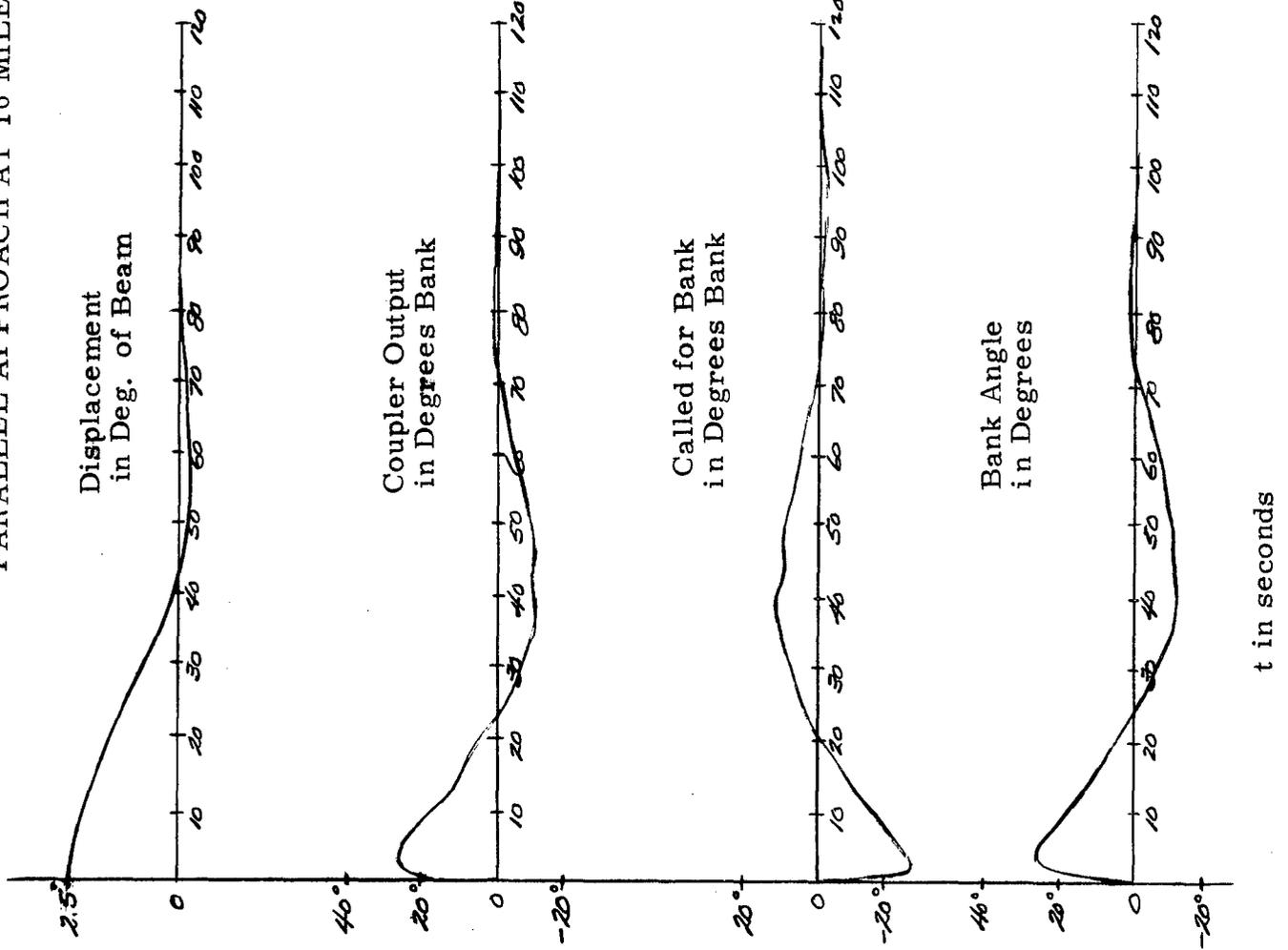


Figure 4. REAC RESULTS FOR NON-LINEAR 90° APPROACH AT 16 MILES

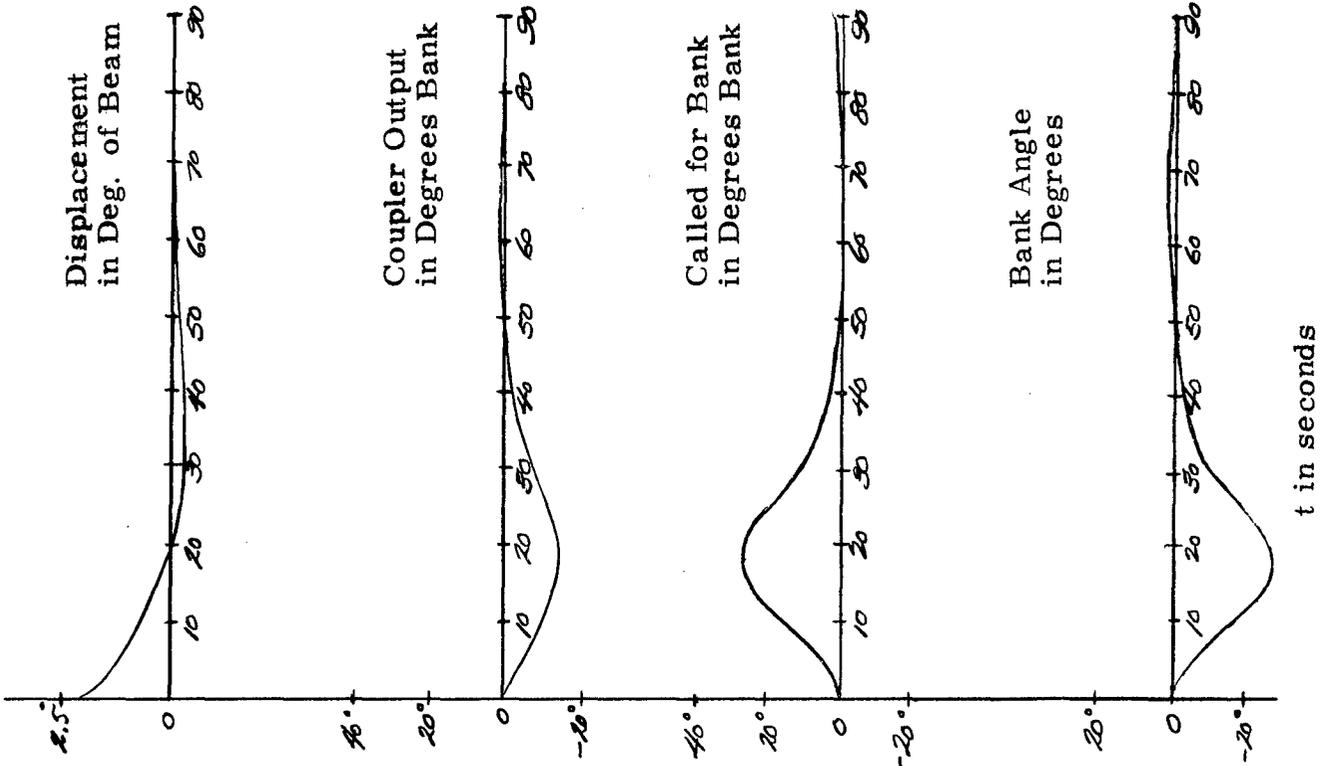


Figure 5. REAC RESULTS FOR NON-LINEAR  
45° APPROACH AT 16 MILES

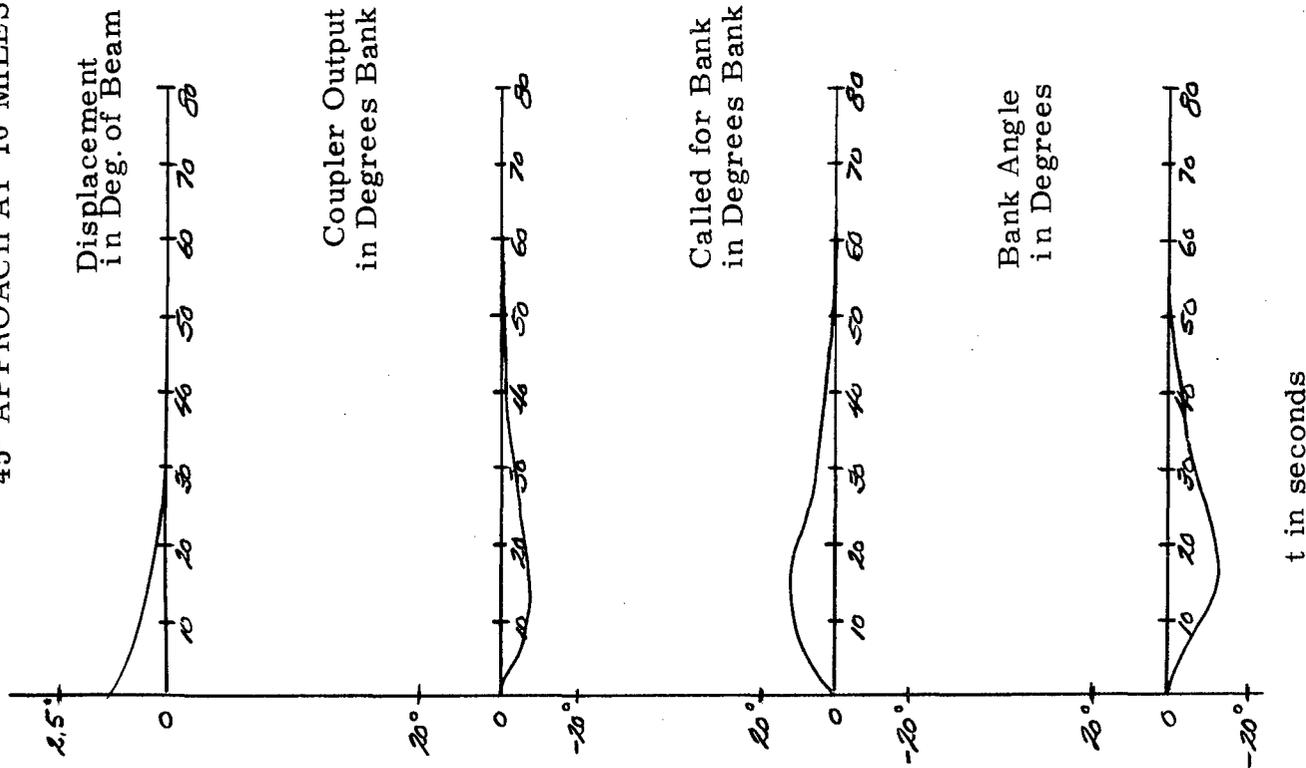


Figure 6. REAC RESULTS FOR NON-LINEAR  
PARALLEL APPROACH AT 16 MILES

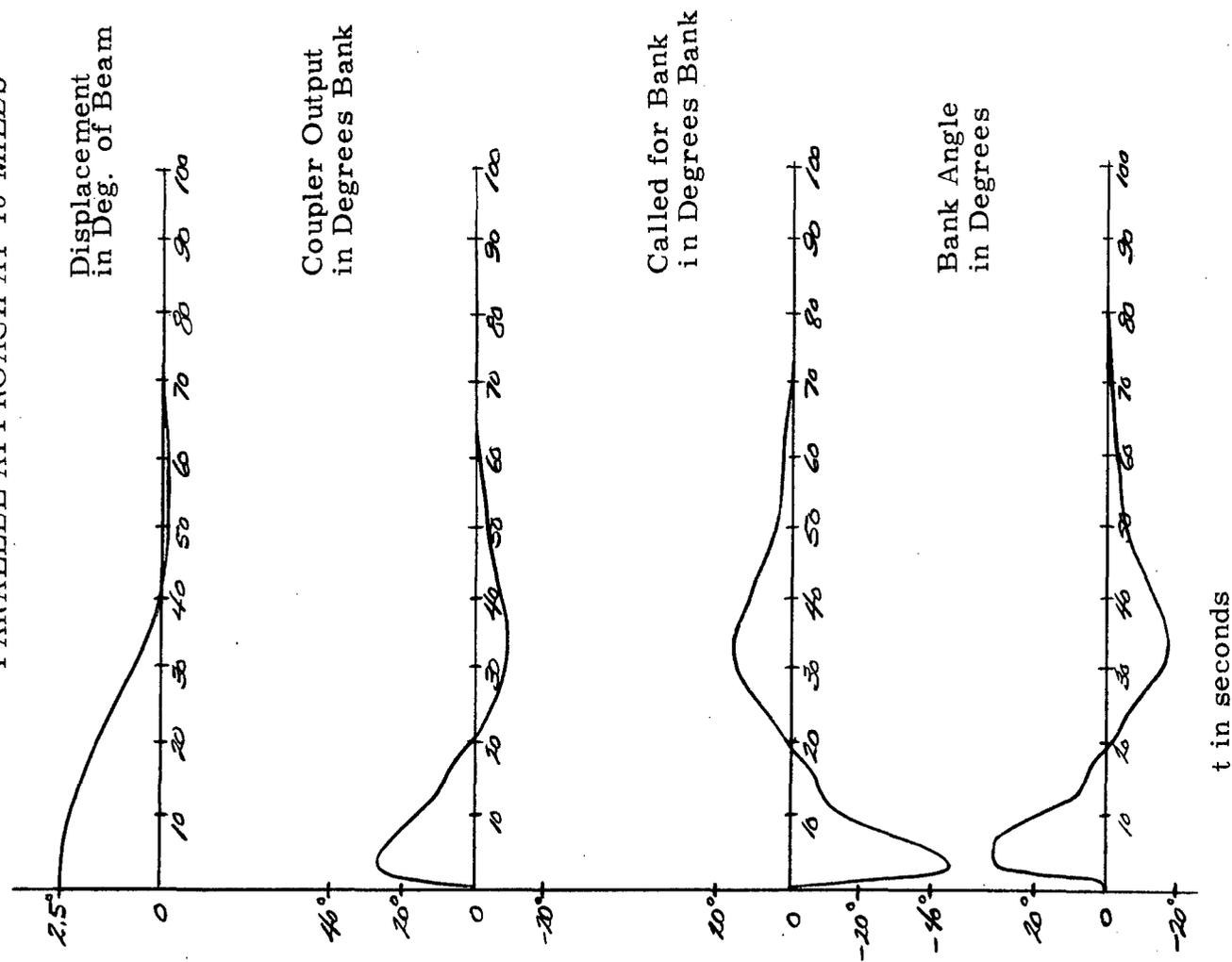


Figure 7. SCHEDULING OF LEAD NETWORK TIME CONSTANT AS  
FUNCTION OF DISTANCE

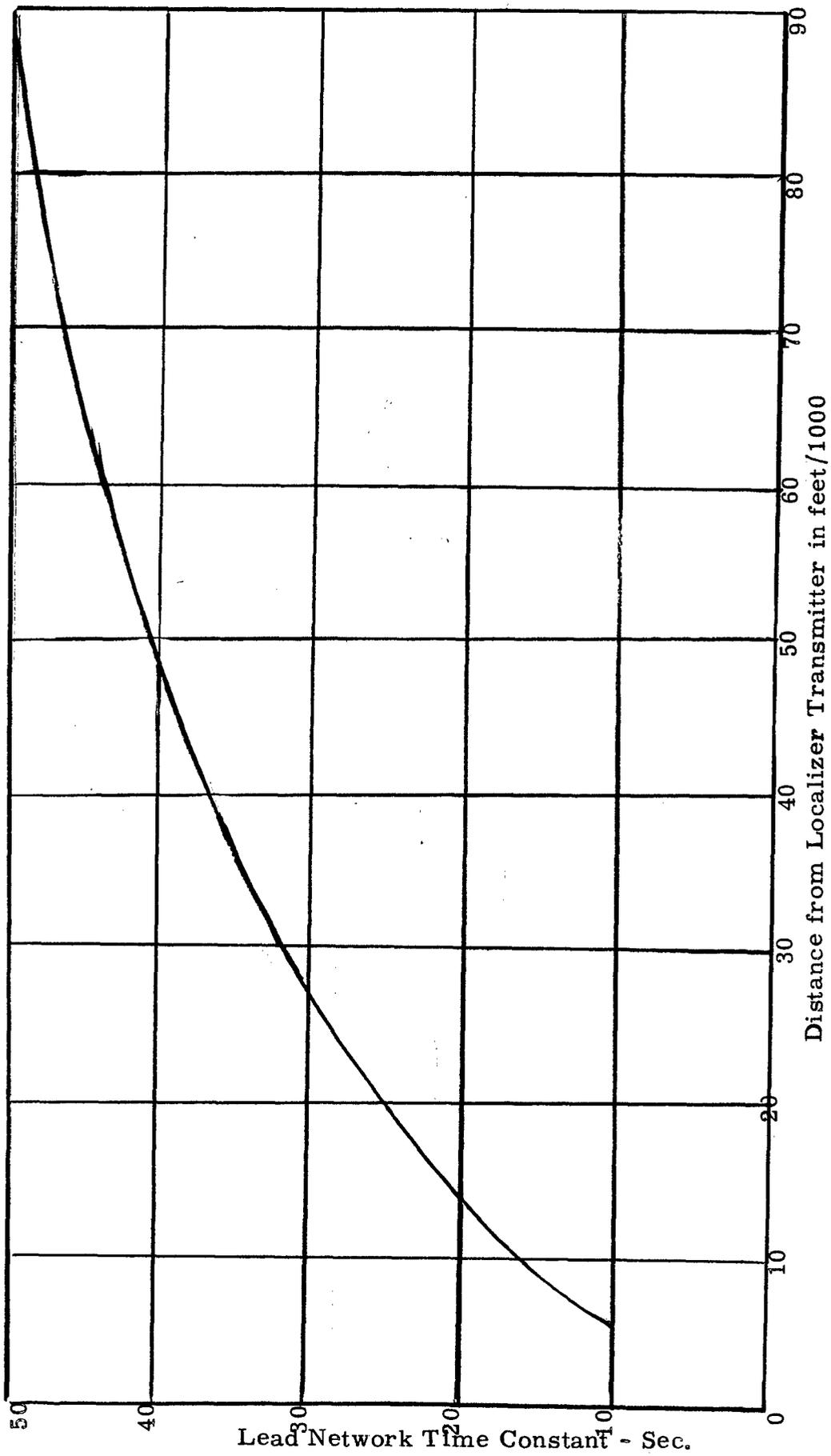


Figure 8. REAC RESULTS FOR APPROACH BEGUN AT 16 MILES AND HEADING OF 90°

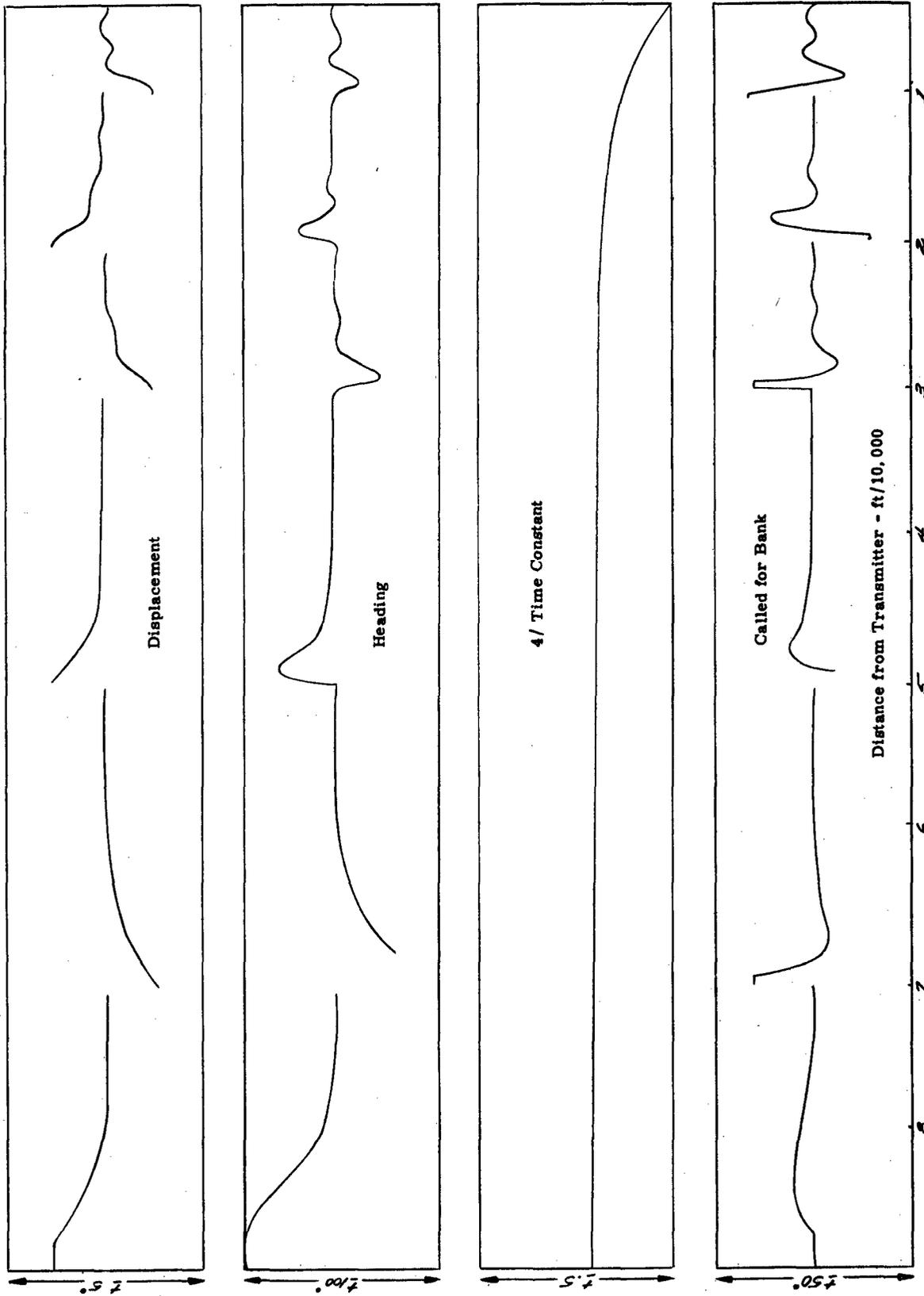


Figure 9. REAC RESULTS FOR APPROACH BEGUN AT 16 MILES & PARALLEL TO BEAM

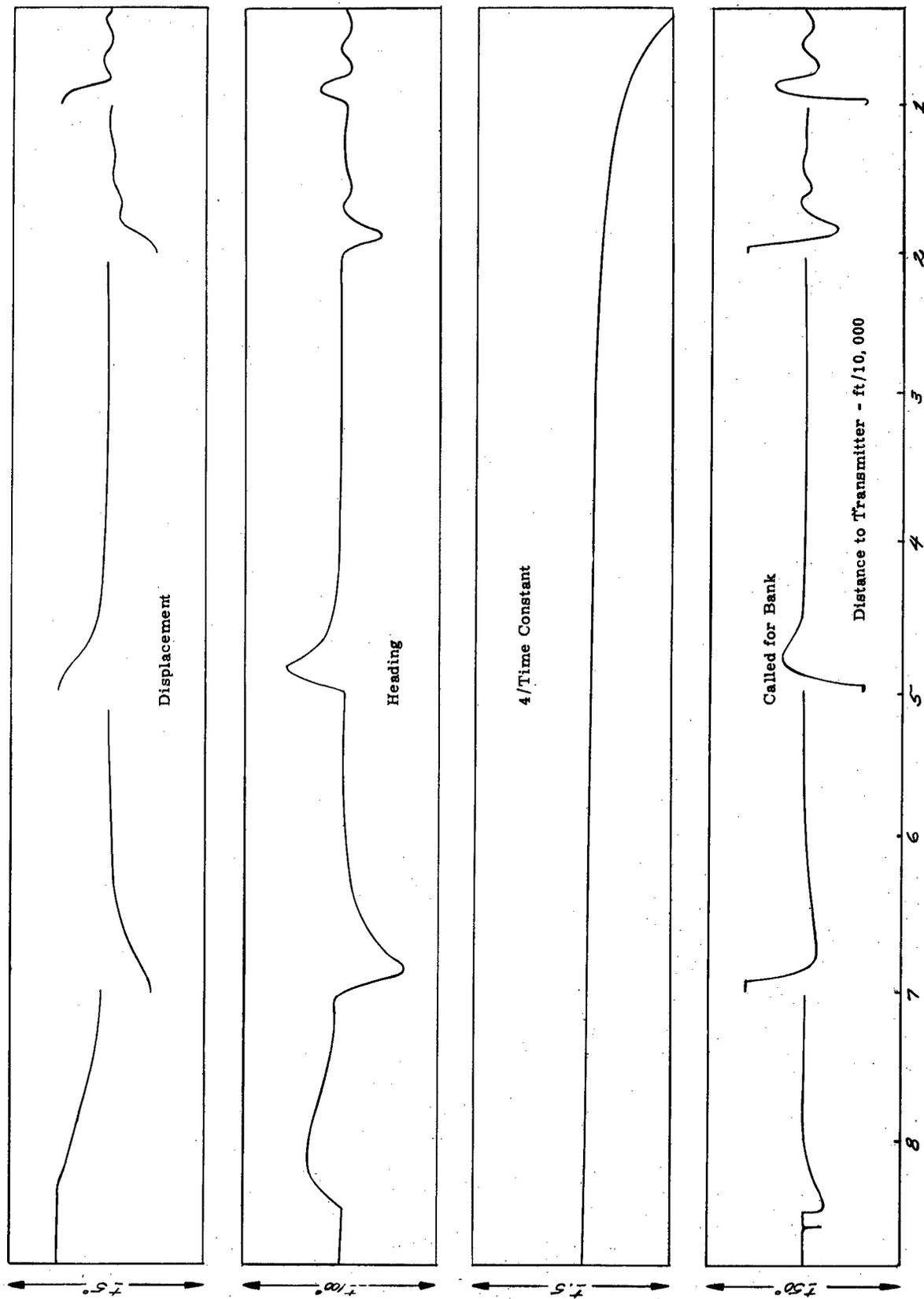


Figure 10. REAC RESULTS FOR APPROACH BEGUN AT 8 MILES

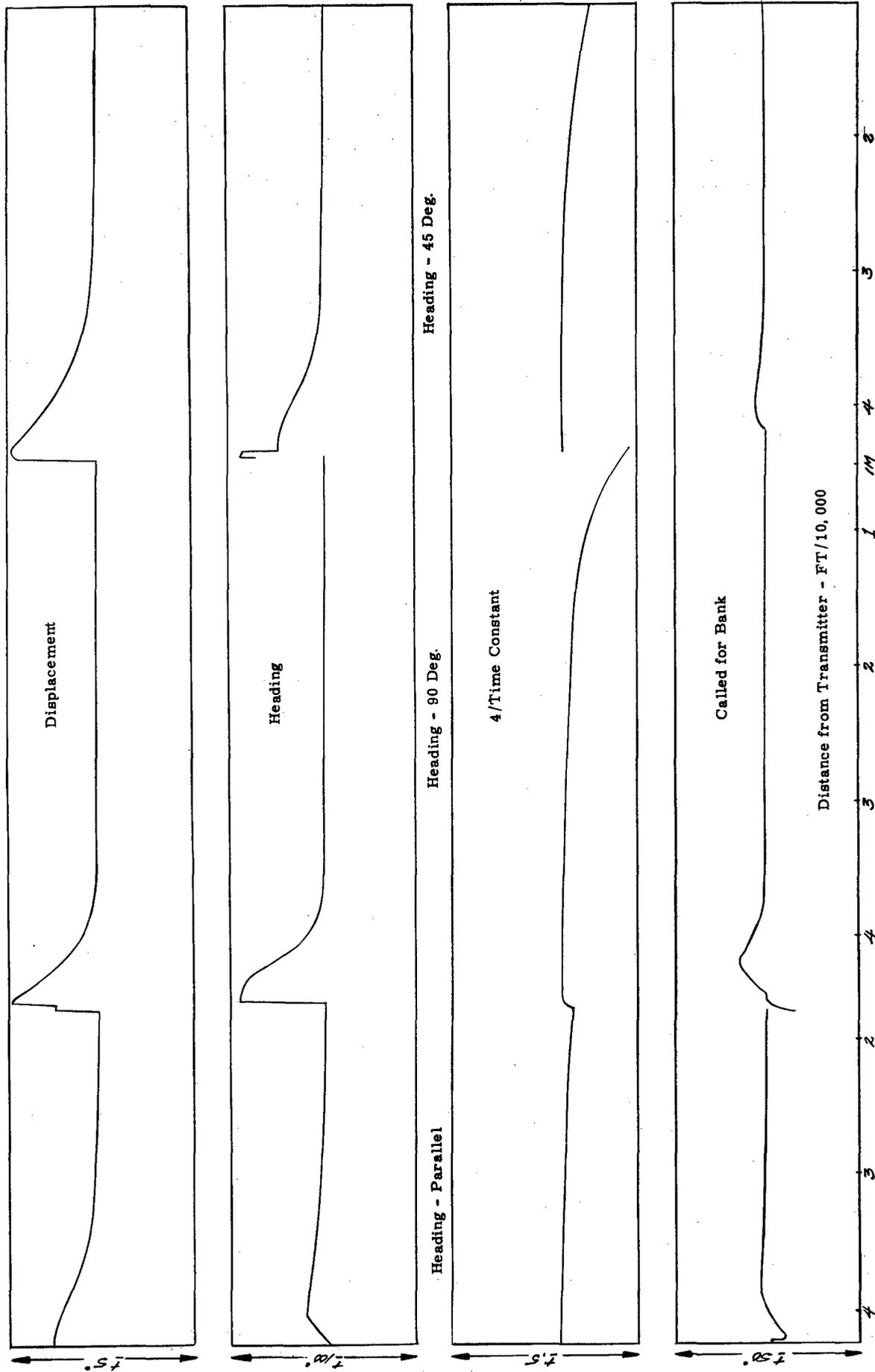
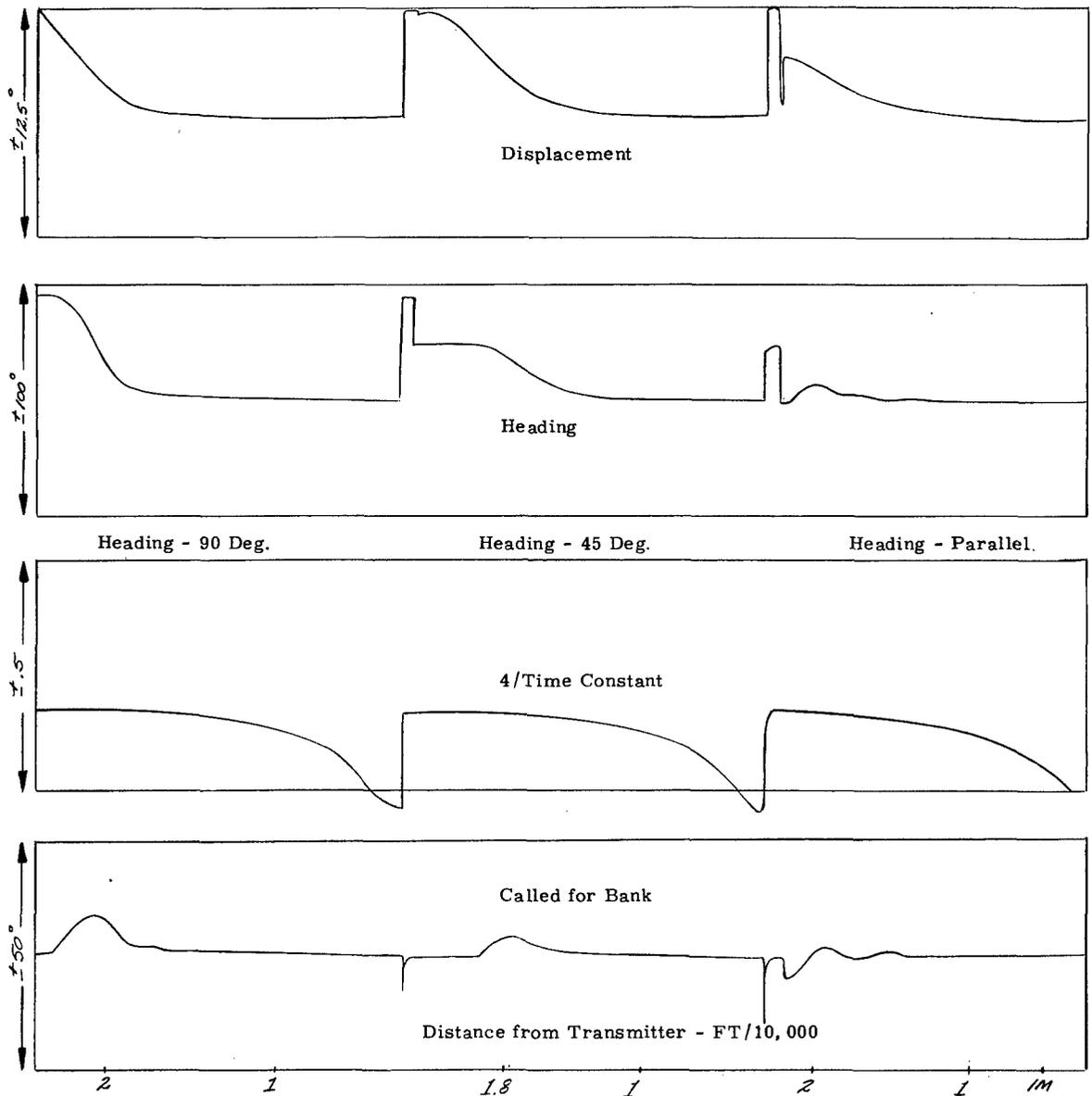


Figure 11. REAC RESULTS FOR APPROACH BEGUN AT 4 MILES



approximating this might be obtained by switching time constant at a known distance such as a middle marker. In such a case the part of the approach before the middle marker would be made with a relatively longer time constant than that used from the middle marker to touch down.

Further investigation aimed at producing the compromised system mentioned above has revealed that the switching at the middle marker becomes somewhat superfluous when the system is made fail-safe. That is to say, if the system is required to complete the approach safely in the event automatic switching at the middle marker fails, there is nothing gained by switching at the middle mark at any time since the system that would be used on the final portion of the approach must be engaged manually at the outer marker. The alternate would be manual switching at both the outer and middle marker. However, this increases the burden on the pilot and is not likely to meet with favor.

Also in connection with the investigation of the compromise system, various second order lead networks were considered as means of increasing the range over which the system was critically damped and thus obtain a response similar to that obtained with rate gain scheduling between bracketing and outer marker or from outer marker to touch down. Several of the networks gave a slight improvement, but it was not considered significant with respect to the additional complication the lead network.

Another investigation with the aim of improving bracketing involved a beam position sensor. A beam position sensor is a device which automatically engages the localizer coupler upon approach to the beam. By engaging the coupler at the earliest possible moment, the overshoot in bracketing is reduced to a minimum and the maximum achieved bank angle is low. The coupler should be engaged at the moment rate and displacement signals are equal and opposite. This will cause the aircraft to turn towards the proper heading at the earliest time.

Definition of the desired characteristics appears to be the greatest problem of the beam position sensor. Since rate and displacement may be equal outside of the normal  $2\text{-}1/2^\circ$  beam or, on the other hand, they may never be equal because of an approach at a constant small angle such as  $2\text{-}1/2^\circ$ , the problem becomes one of defining the various conditions under which the system may be engaged. Engagement of the system under the conditions of equal rate and displacement signals occurs at a constant distance in feet from beam center for a given course intersection angle. For example, a  $45^\circ$  intersection angle and 20 second lead network time constant will cause rate and displacement signals to be equal at 2100 feet from beam center providing the beam is linear out to 2100 feet. The nominally  $\pm 2\text{-}1/2^\circ$  localizer beam will be  $\pm 2100$  feet wide at 9 miles. Since the linear beam width is generally greater than  $2\text{-}1/2^\circ$ , it would be possible to engage before the cross pointer meter leaves the stop at distances less than 9 miles.

Figure 13 is a REAC diagram of a system which operated satisfactorily. The system will automatically engage at the moment the rate signal becomes larger than the displacement signal. In case the approach path is approximately parallel to course center the system will engage if the displacement is less than  $2.5^\circ$ . Should it happen that an angular approach path was continued until the aircraft neared or even passed beam center without the pilot throwing the master engage switch, the combination of

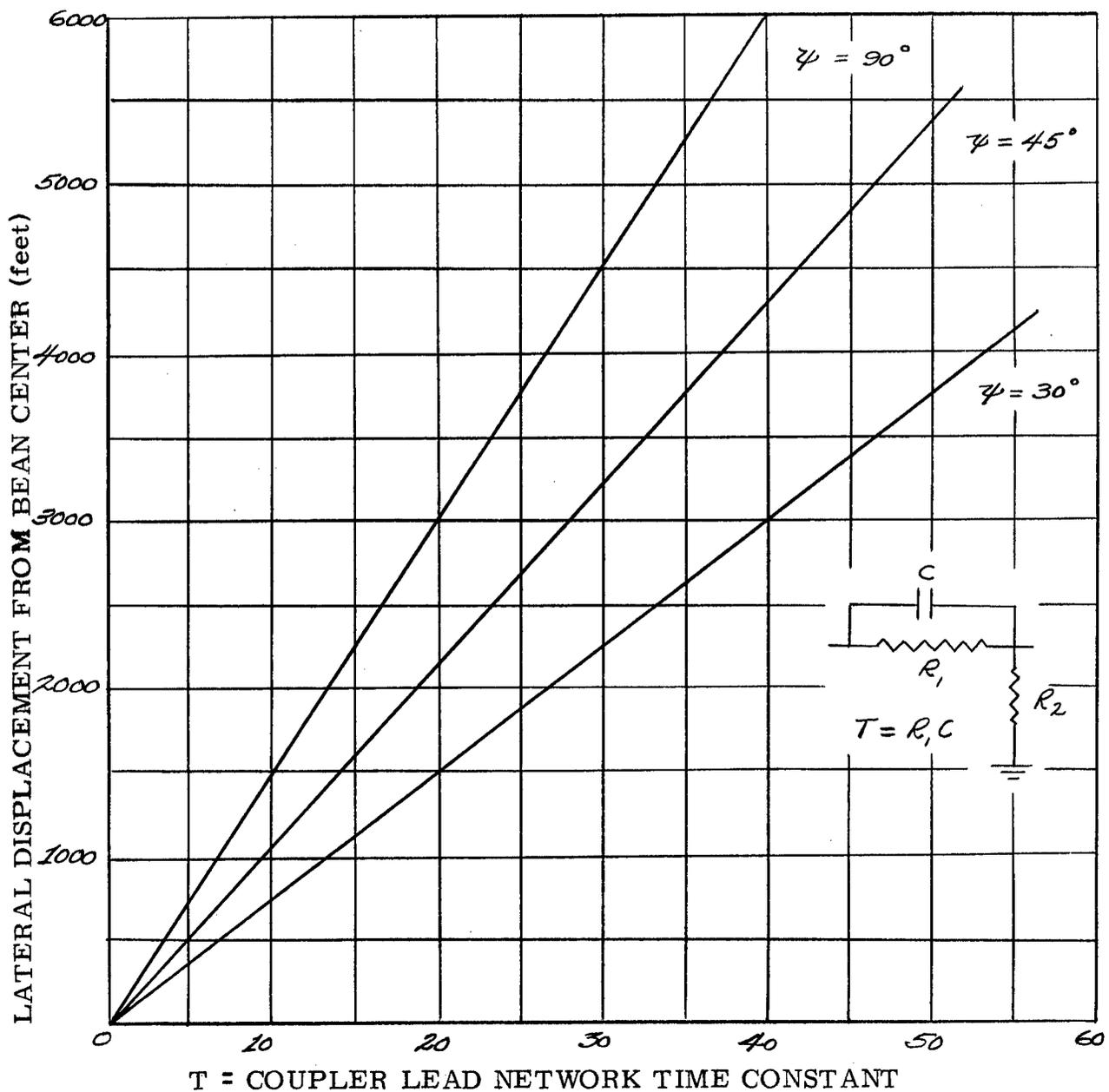
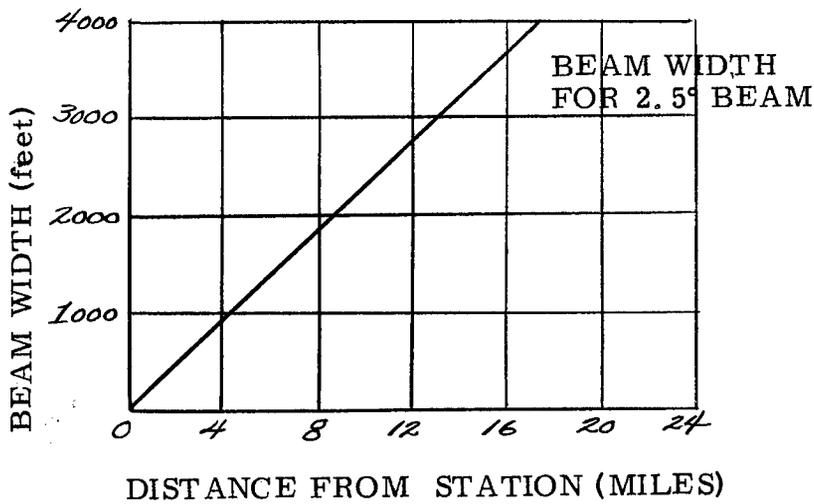


Figure 12. LATERAL DISTANCE FROM COURSE CENTER AT WHICH DISPLACEMENT AND RATE ARE EQUAL

rate and displacement would determine whether the system would be automatically engaged upon closing the master. For example, a 90° approach at 8 miles could only be engaged between the limits  $y = \hat{y}$  and beam center. At 4 miles, engagement could only occur between  $y = \hat{y}$  and 2.1°. On the other hand, the same 90° approach at 16 miles could be engaged anywhere from  $y = \hat{y}$  to approximately 1° on the other side of the beam. The lower the angle of intersection and the farther out on the beam the more nearly the range of permissible engagement approaches  $\pm 2.5^\circ$ . Late switching of the system at close distances and high approach angles is prevented because of the difficulty of obtaining a satisfactory bracketing at a reasonable distance from the runway and in order to prevent the aircraft path from getting too far outside the linear beam limits. The above limiting conditions were arbitrarily chosen and may not represent the best limits for a practical system.

A brief discussion of the operation of the system of Figure 13 follows. The coupler lead network was split into two parts in order to obtain the rate and displacement components separately. It was further necessary to full-wave rectify (take absolute value) these signals in order to enable the system to operate independently of a left or right approach to the beam. The open position of a differential relay allowed the system to be engaged providing the pilot's localizer master switch was closed. The sum of negative rate and positive displacement was taken in amplifier 10, and passed through a variable limiter to a differential relay. The polarity was such that whenever the magnitude of the rate exceeded the displacement the relay would open and permit system engagement. Amplifier 3 operated the variable limit from the sum of positive rate plus positive displacement,  $|R| + |D|$ , minus the sum of rate and displacement,  $-|R + D|$ . Whenever rate and displacement were of different signs (plane approaching beam center)  $|R| + |D| > |R + D|$ , the output of amp 3 was negative allowing the limiter to pass signals from amp 10 to open the relay whenever rate exceeded displacement. However, if rate was zero or if rate and displacement were of the same sign  $|R| + |D| = |R + D|$ , the output of amp 3 was zero which prevented the limiter from passing signals to close the relay. Limiter 18 prevented engagement whenever the displacement was over 2.5° on a parallel approach by maintaining an output of amp 3. Amplifier 14 was a safety device to prevent opening the relay whenever the value of rate plus displacement became substantially greater than an equivalent 2.5° of displacement. It is this amplifier which prevents the system from a late engaging on a high angle close-in approach.

Further study of the beam position sensor has been made by the Flight Operations Group. They have concluded that the improvement in beam following would not be sufficient to warrant the additional equipment of the beam sensor. In addition, there are objections from a safety standpoint. The following comments were made by T. U. Grove, Manager of Flight Operations.

"There is considerable question regarding the practical value of a beam sniffer for ILS approach application. There are a number of considerations that indicate such a device would not appreciably improve an ILS approach system from the operational standpoint. Several individuals expressed the opinion that an overshoot during bracketing is not objectionable. In fact, there is some advantage to be gained from a moderate overshoot, in that the pilot has a positive indication of system operation that would not be evident in some cases of an asymptotic approach.

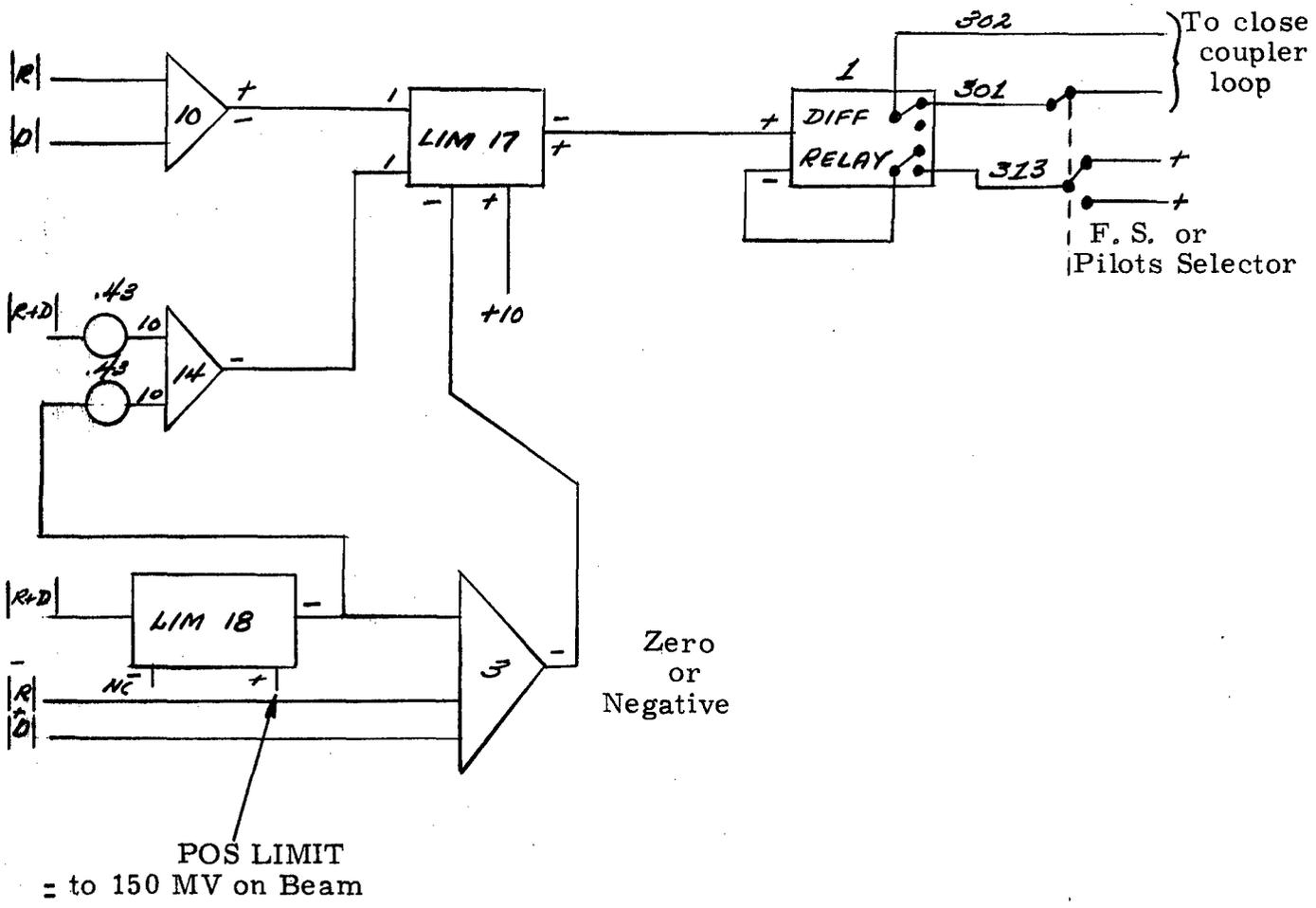
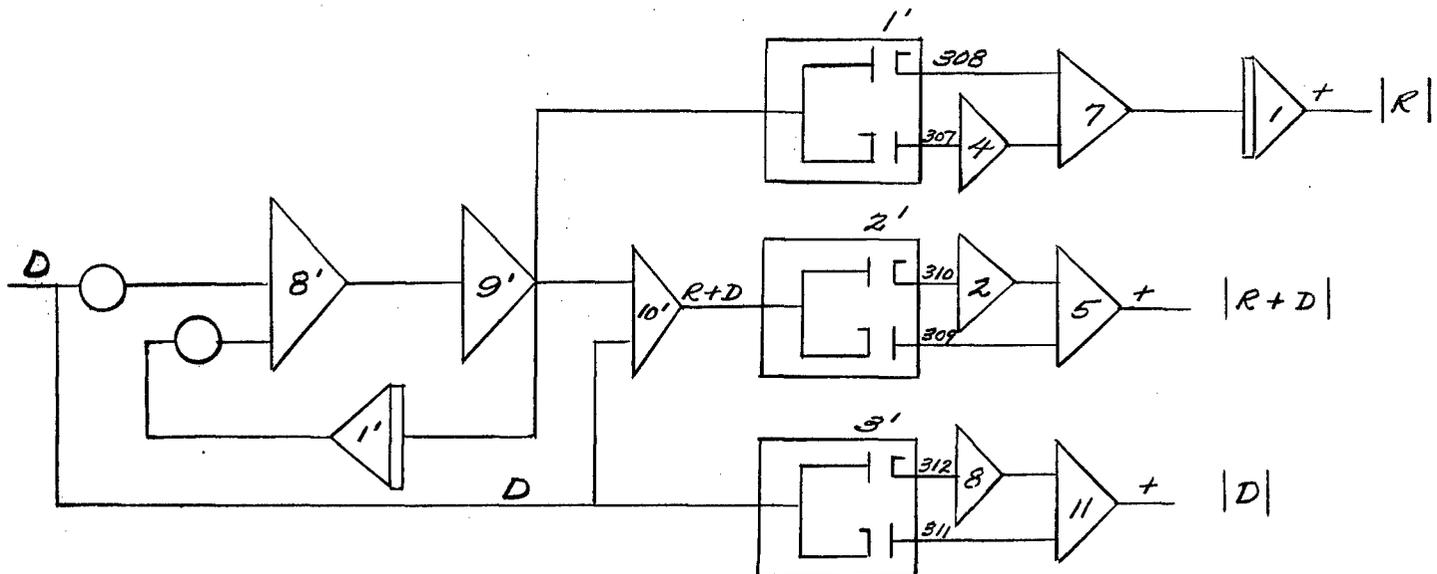


Figure 13  
REAC Diagram of Beam Position Sensor

"The trend in approach procedures has been in the direction of less standardization during the initial phase of the ILS approach procedure. The primary objective of the recent trend in instrument approach procedure has been to expedite the flow of traffic with the minimum of spacing and consequently maximum traffic density. This has resulted in a greater flexibility of procedure in the initial approach zone. That is, aircraft #1 may approach straight in from 50 miles out and require no bracketing at all. Airplane #2 may approach the terminal area on a reciprocal heading and thus require a standard procedure turn at 10 miles from touchdown. Airplane #3 may approach from a 90° position and find it advantageous to intersect 15 miles from touchdown.

"The primary factor making it practical to use such a variety of initial approach procedures is the use of radar vectoring for bringing aircraft into the final approach zone. This system is now in use at Minneapolis as well as a half-dozen other major centers. With this procedure, it is convenient for the pilot to introduce command signals to the autopilot through the turn controller as he complies with GCA instructions for vectoring, holding and initial intersection of the localizer. No standard procedure for intersect angle or distance used for intersecting the inbound localizer course is followed. Use of the beam sniffer would require considerable indoctrination before pilots would gain sufficient confidence in a new procedure such as this. It is necessary for the pilot to know his position relative to localizer course prior to making bracketing commitment. With the present system he has the means of gaining this information.

"There now exists a satisfactory degree of flexibility utilizing autopilot turn controller; therefore, an automatic engage system would appear to be of questionable advantage."

## SECTION II REDUCTION OF RESPONSE TO NOISE

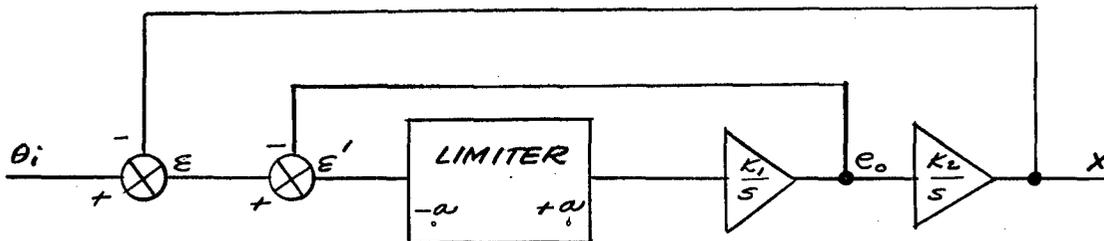
The perception filter was developed for the non-linear program to be used for minimizing the effects of beam noise on the ILS-A/P-A/C control system. Flight tests with this system were encouraging since the filter made the control system relatively immune to the detrimental effects of beam noise. Electrically, the device can be characterized as a low-pass filter with a variable cut-off frequency and a controlled limit on the second-derivative of the output.

The original filter implementation was achieved by using two electronic integrators in the forward path of a feedback system to represent the second-order low-pass filter. Diode limiters were used to control the magnitude of the second derivative of the output. The cut-off frequency was varied by motor driven potentiometers controlling suitable gains. Some difficulty was experienced using the synchronous chopper type of integrator and it seemed desirable to try to eliminate this device.

The problem with which this section is concerned is the use of passive (RC) integrators in a second-order low-pass filter having limiting on the second derivative of the output. The filter configuration is determined by making its transfer functions similar to those of the original

circuit. The data taken on the REAC is presented as evidence that the performance of this filter is similar to a corresponding portion of the perception filter.

In order to understand the type of limiting involved, it is helpful to describe the original system. This is done by specifying the open loop transfer function. For the present, the variable limit feature will be neglected and only the fixed limits will be considered. The following diagram shows the original filter configuration.



Block Diagram of Original Perception Filter with Fixed Limits

Figure 14

The attack is first to describe the minor loop and second to specify the major loop. The minor loop equation is

$$(1) -\alpha < \epsilon' < +\alpha \quad \frac{1}{K_1} \dot{e}_o + e_o = \epsilon \quad \text{where } \dot{e}_o = \frac{de_o}{dt}$$

$$(2) \epsilon' > +\alpha \quad \frac{1}{K_1} \dot{e}_o = +\alpha$$

$\pm\alpha$  is the magnitude of the fixed limits. It is unnecessary to write the equation for the case  $\epsilon' < -\alpha$  since it is similar to (2) with the exception of the signs preceding  $\alpha$ . Equation (2) indicates that the output rate is identical (in steady state) whether the loop is open or closed, if the limiter is operating.

The open loop equation for the major loop can be written as

$$(3) -\alpha < \epsilon' < +\alpha \quad \frac{1}{K_1 K_2} \ddot{x} + \frac{1}{K_2} \dot{x} = \epsilon$$

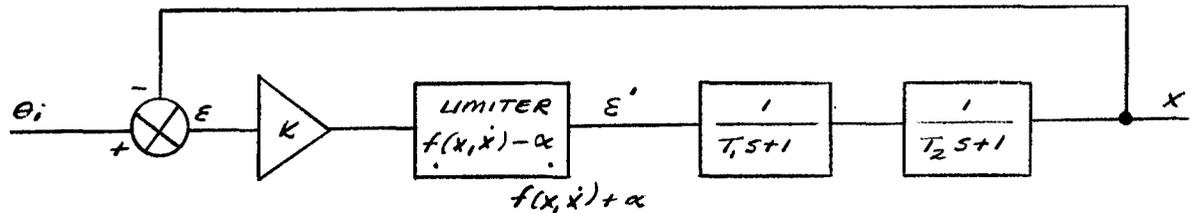
$$(4) \epsilon' < +\alpha \quad \frac{1}{K_1 K_2} \ddot{x} = +\alpha$$

The last equation shows that during the limiting action the second derivative of  $x$  is constant. Equation (4) is descriptive of the type of system desired during limiting. Equation (3) will be modified to show the closed loop equations. The system to be synthesized must have the following characteristics:

$$(5) \text{ No limiting} \quad \frac{1}{K_1 K_2} \ddot{x} + \frac{1}{K_2} \dot{x} + x = \theta_i$$

$$(6) \text{ Positive limiting} \quad \frac{1}{K_1 K_2} \ddot{x} = +\alpha$$

As a start on the filter, consider the following diagram which contains two RC integrators ( $T_1$  and  $T_2$ ) and a limiter. The limits will be taken as functions  $x$  and  $\dot{x}$ . The problem is then to determine the form of the limit function subject to the conditions that the equations of the filter satisfy conditions (5) and (6).



Block Diagram of Second Order Filter with Undetermined Limiting

Figure 15

This system is described by the following equations:

$$(7) \quad f(x, \dot{x}) - \alpha < K\epsilon < f(x, \dot{x}) + \alpha \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + (1 + K)x = K\theta_i$$

$$(8) \quad K\epsilon > f(x, \dot{x}) + \alpha \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + x = f(x, \dot{x}) + \alpha$$

If (8) is to have the same form as (6), then

$$(9) \quad f(x, \dot{x}) = (T_1 + T_2) \dot{x} + x$$

If the problem is reduced to that of applying a limiter voltage of the form

$$(10) \quad a\ddot{x} + x \pm \alpha \quad \text{where } a = T_1 + T_2$$

If this can be accomplished then the equations for Figure 15 will be

$$(11) \quad \text{No limiting} \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + (1 + K)x = K\theta_i$$

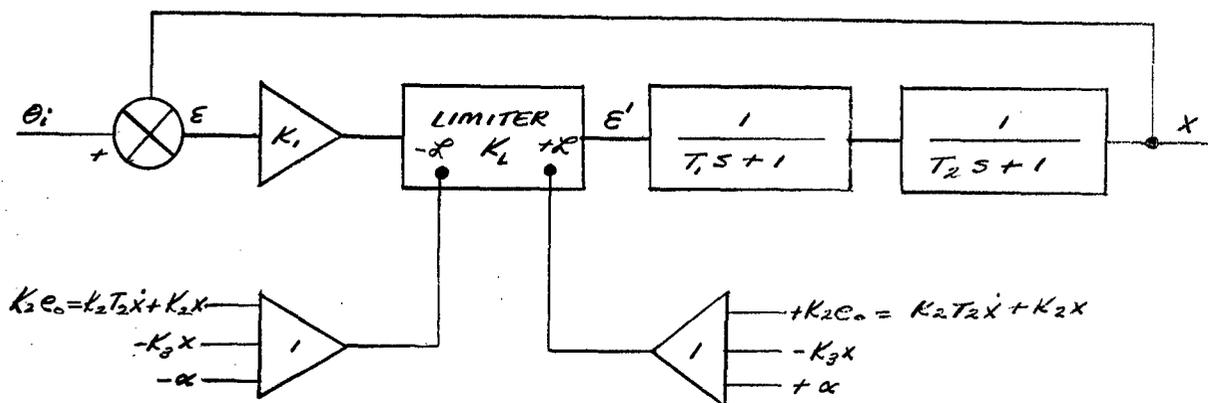
$$(12) \quad \text{Limiting} \quad T_1 T_2 \ddot{x} = +\alpha$$

These are similar to the desired form of (5) and (6).

A system has been devised that has the desired limit function. The success of the scheme depends on calibrated values which is often undesirable. This shortcoming must be compared with some of the less desirable features of d-c integrators. The block diagram is shown in Figure 16.

In Figure 16, the limits are  $\pm \alpha$ . The amplifiers are not the inverting type and their gains are shown inside the blocks. This system without limiting action, is described by

$$(13) \quad -\alpha < K_1 E < +\alpha \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + (1 + K_1 K_L) x = K_1 K_L \theta_i$$



Block Diagram of Second Order Filter with Fixed Acceleration Limits

Figure 16

When positive limiting occurs,

$$(14) \quad K_1 E > +\alpha \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + x = E' = [K_2 T_2 \dot{x} + (K_2 - K_3) x + \alpha] K_L$$

This reduces to the desired form (equation 12) when

$$K_L K_2 T_2 = T_1 + T_2 \quad \text{and} \quad K_2 - K_3 = \frac{1}{K_L}$$

Then, the calibrations must make

$$(15) \quad K_2 = \left(1 + \frac{T_1}{T_2}\right) \frac{1}{K_L} \quad \text{and} \quad K_3 = \frac{T_1}{T_2} \frac{1}{K_L}$$

By satisfying these conditions, the filter of Figure 16 has the relations

$$(16) \quad \text{No limiting} \quad T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + (1 + K_1 K_L) x = K_1 K_L \theta_i$$

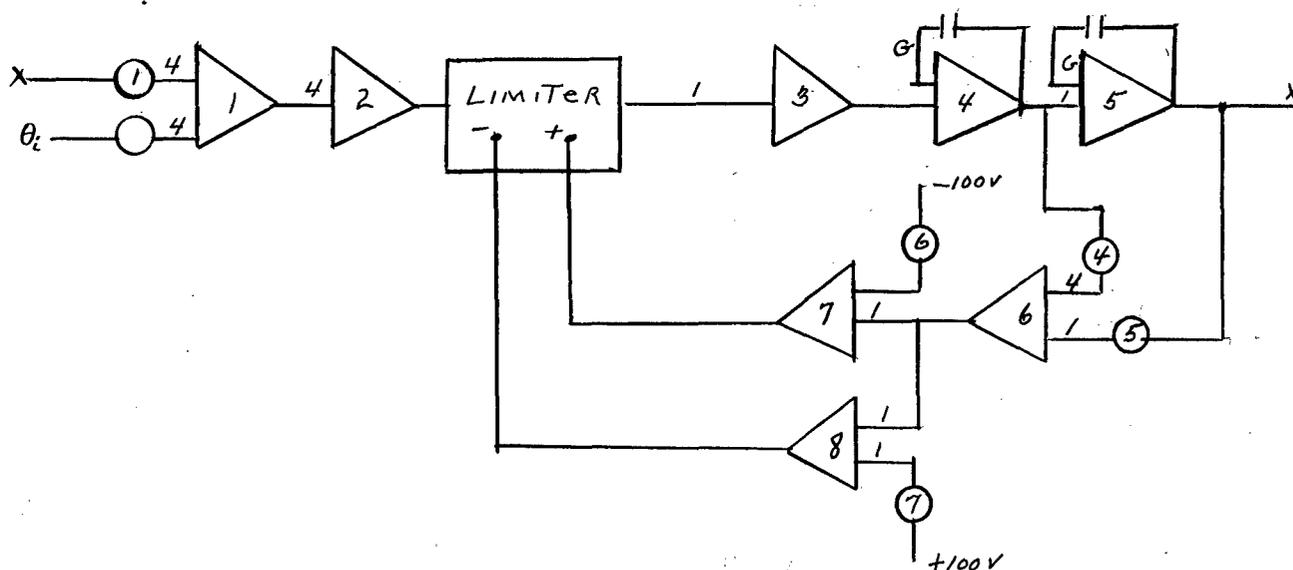
$$(17) \quad \text{Limiting} \quad T_1 T_2 \ddot{x} = +\alpha K_L$$

These are similar to the desired form in equations (5) and (6).

Figure 16 has been reduced to a REAC diagram to allow testing of the acceleration limited filter. The coefficients in equations (15, 16, and 17) are represented in Figure 17 in the following manner

$$\begin{aligned} K_1 &= 16 P_1 & K_2 &= 4 P_3 & K_3 &= P_4 \\ &= P_5 = P_6 & T_1 &= C_1 & T_2 &= C_2 \end{aligned}$$

$P_1$  is setting of potentiometer No. 1 and varies from 0 - 1.



REAC Diagram of Second Order Filter with Fixed Acceleration Limits

Figure 17

The perception filter made use of a variable limit feature. This assumes that the acceleration signal (in the presence of large amounts of noise at the input) is predominantly noise signal. The limits are opened up when the acceleration signal is large. The effect is to reduce the phase shift of the signal and to keep a constant amount of noise in the output as the signal to noise ratio decreases at the input.

The variable limit control is developed in the following manner. The desired limit function is of the form

$$(18) \quad \alpha = \alpha_0 + m/\ddot{x}$$

where  $\alpha$ , and  $m$  are constants:  $\ddot{x}$  in (18) is rectified and filtered. The computation of  $\ddot{x}$  remains to be accomplished.

In Figure 17 without limiting, the output of amplifier 2 is

$$e_2 = \frac{1}{K_L} \left[ T_1 T_2 \ddot{x} + (T_1 + T_2) \dot{x} + x \right]$$

and the output of amplifier 6 is

$$e_6 = \left[ K_2 T_2 \dot{x} + (K_2 - K_3) x \right]$$

Subtracting these equations gives

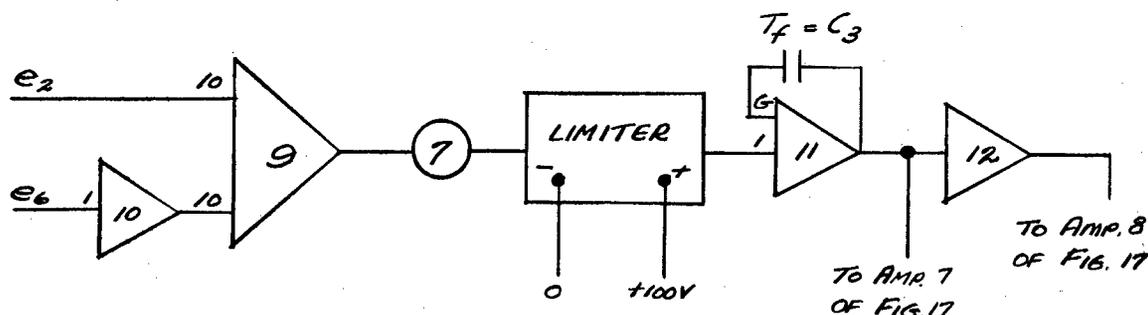
$$(19) \quad e_2 - e_6 = \frac{T_1 T_2}{K_L} \ddot{x} + \left( \frac{T_1 + T_2}{K_L} - K_2 T_2 \right) \dot{x} + \left( \frac{1}{K_L} - K_2 + K_3 \right) x$$

If the conditions of equation (16) are satisfied (19) becomes

$$(20) \quad e_2 - e_6 = \frac{T_1 T_2}{K_L} \ddot{x}$$

This signal is rectified, filtered and amplified. Adding this to the limit voltages of Figure 17 gives the limit control similar to equation (18).

For the variable limit case, the following diagram indicates the process of obtaining a limit voltage that increases as the noise increases.



REAC Diagram for Variable Limit Amplifier

Figure 18

The quantities  $e_2$  and  $e_6$  refer to output voltages of amplifiers 2 and 6 in Figure 17. Potentiometer 7 provides control of  $m$  in equation (18). Figure 17 and Figure 18 comprise the REAC diagram of the variable limit filter.

The non-linear filter was simulated on the REAC to verify experimentally the predictions of the analysis. The type of data taken from the REAC simulation was similar to that used (Figures 14, 15, 23, 25) to demonstrate characteristics of the perception filter.

The curves determined were as follows:

1. Amplitude and phase of output vs. frequency for various ratios of input to limit magnitudes.
2. Effect of noise frequency on signal output amplitude vs. noise amplitude
3. Effect of limit magnitude on signal output amplitude vs. noise amplitude.

The REAC configuration used is shown in Figure 19. Under linear conditions the natural frequency was 6 radians per second and the damping ratio was 1/4 critical. The original work utilized a damping ratio of 1/2 critical.

The frequency response curves are shown in Figures 20 and 21. For comparison similar curves are included for the perception filter, Figures 22 and 23. The parameter for both sets of curves is the ratio of input amplitude  $A$ , to the limit voltage  $\alpha$ . To conform to the curves of Figures 22 and 23, this ratio is multiplied by the open loop gain between the input and the limiter. The characteristics peculiar to acceleration limiting are the sharp peaks of amplitude and the large slope of phase for large values of  $A/\alpha$ . Both sets of curves have these properties.

The effect of noise frequency on output signal versus noise amplitude is shown in Figure 24. Figure 25 is for original filter and is included for purposes of comparison. In these curves, the increase in signal amplitude and phase with noise frequency is shown. It is this feature that contributes to a noise induced control system instability. The effects of noise frequency were similar in each case.

The effect of limit magnitude on output amplitude vs. noise amplitude is shown in Figures 24 and 25. As the limit magnitude was decreased both curves show an increase in the signal amplitude and phase. In the case of the modified filter, however, the sensitivity to change of the limit magnitude was less than in the original filter. This effect has not been explained.

One limitation on the use of a configuration similar to Figure 19 should be mentioned. It cannot be used to produce an output greater than the value of the constant limit. When this was attempted the output became unstable and diverged to a value determined by the maximum output of the amplifiers. Second order systems with lower damping ratios seemed slightly less susceptible to this divergence when subjected to step inputs of magnitudes near the critical value. For this reason the 1/4 critical damping ratio was preferred rather than the 1/2 critical used in previous work. This applies only to the configuration for constant limits. When the circuit in Figure 18 was added, to increase the limits with  $x$  amplitude, this divergence did not exist.

The various curves presented show that the configuration in Figure 19 has characteristics quite similar to those given for the perception filter. It appears then that the modified circuit exhibits an acceleration or second derivative form of limiting. This has been accomplished without the use of integrators - as normally defined - which was the aim of the investigation. Present plans include some time for devising circuits to implement this device. The complexity of the filter after this stage of development will determine whether a real simplification has been achieved.



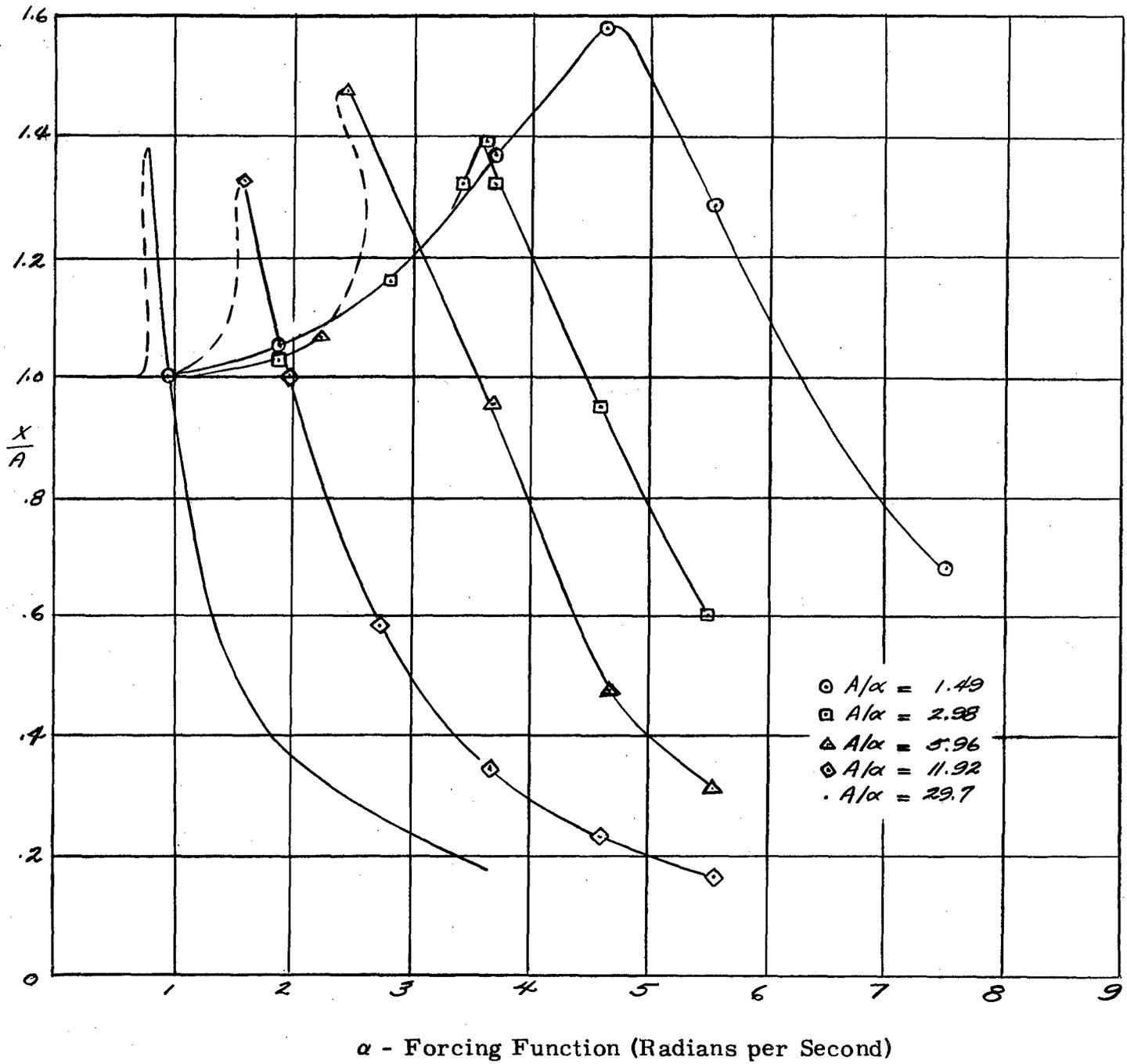


Figure 20  
 AMPLITUDE OF OUTPUT VS  
 FREQUENCY OF INPUT FOR RATIOS OF  $A/\alpha$

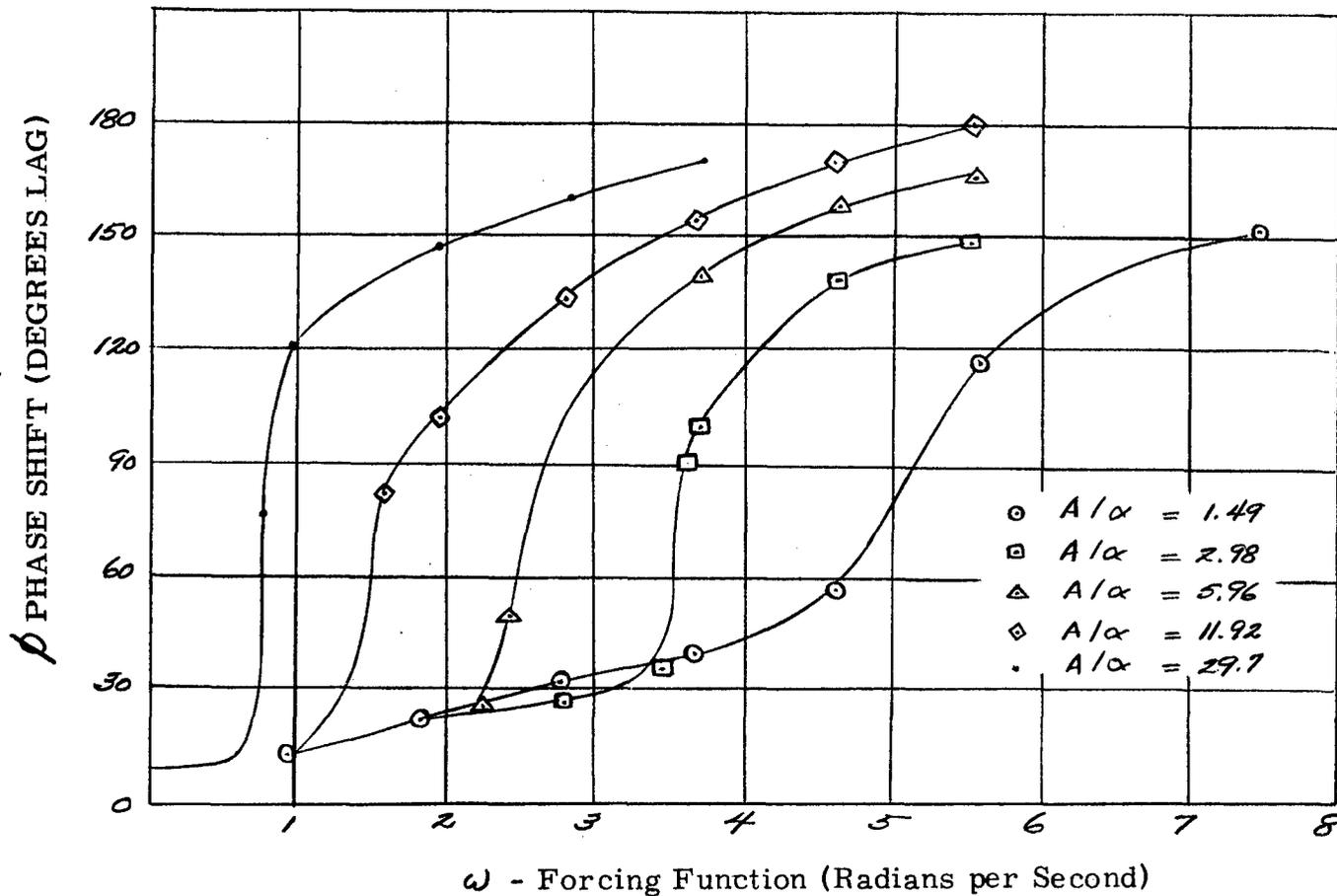


Figure 21  
 PHASE OF OUTPUT VS FREQUENCY OF INPUT FOR RATIOS OF  $A/\alpha$

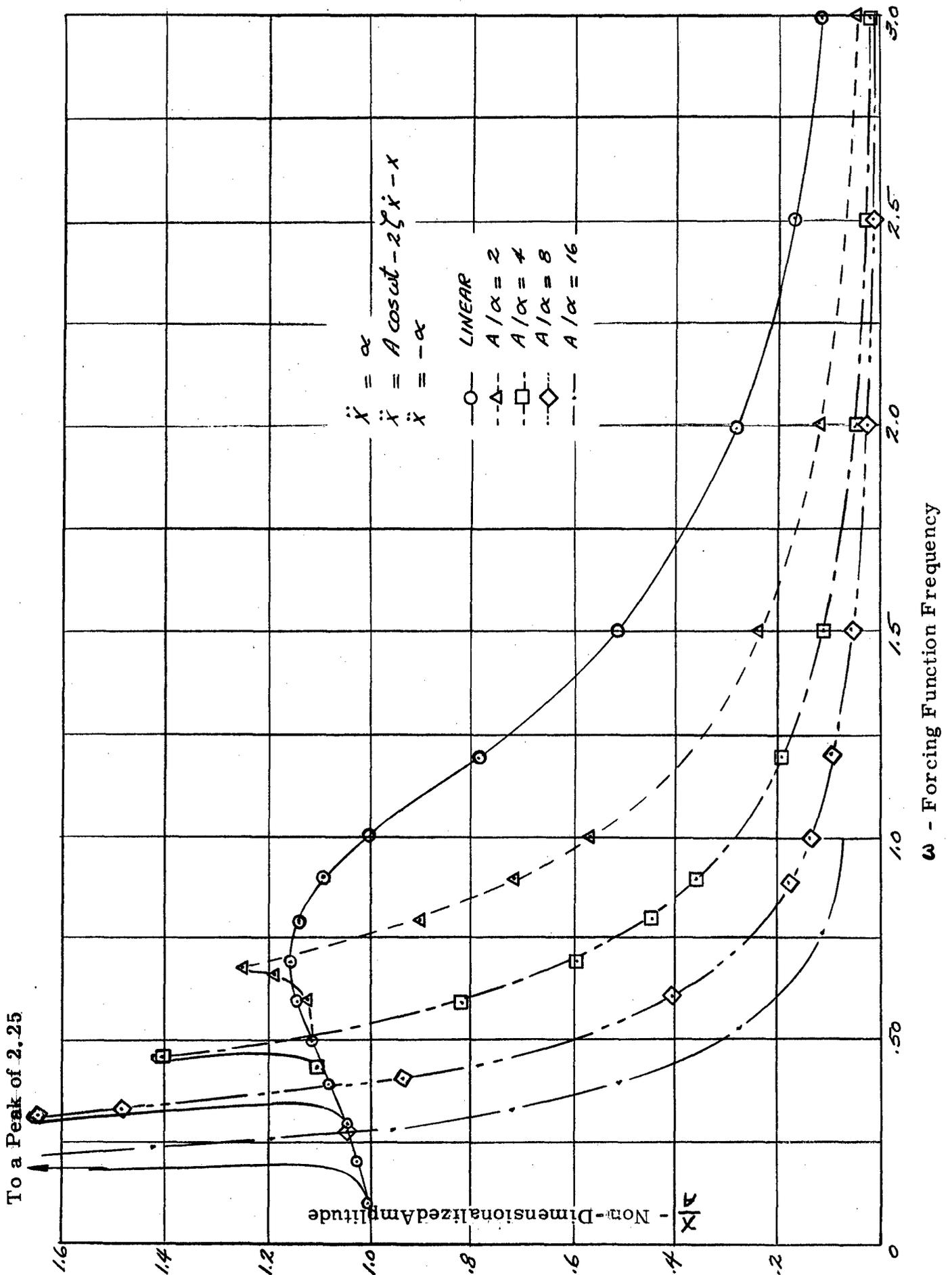


Figure 22. FREQUENCY RESPONSE OF SATURATING AMPLIFIER SERVO

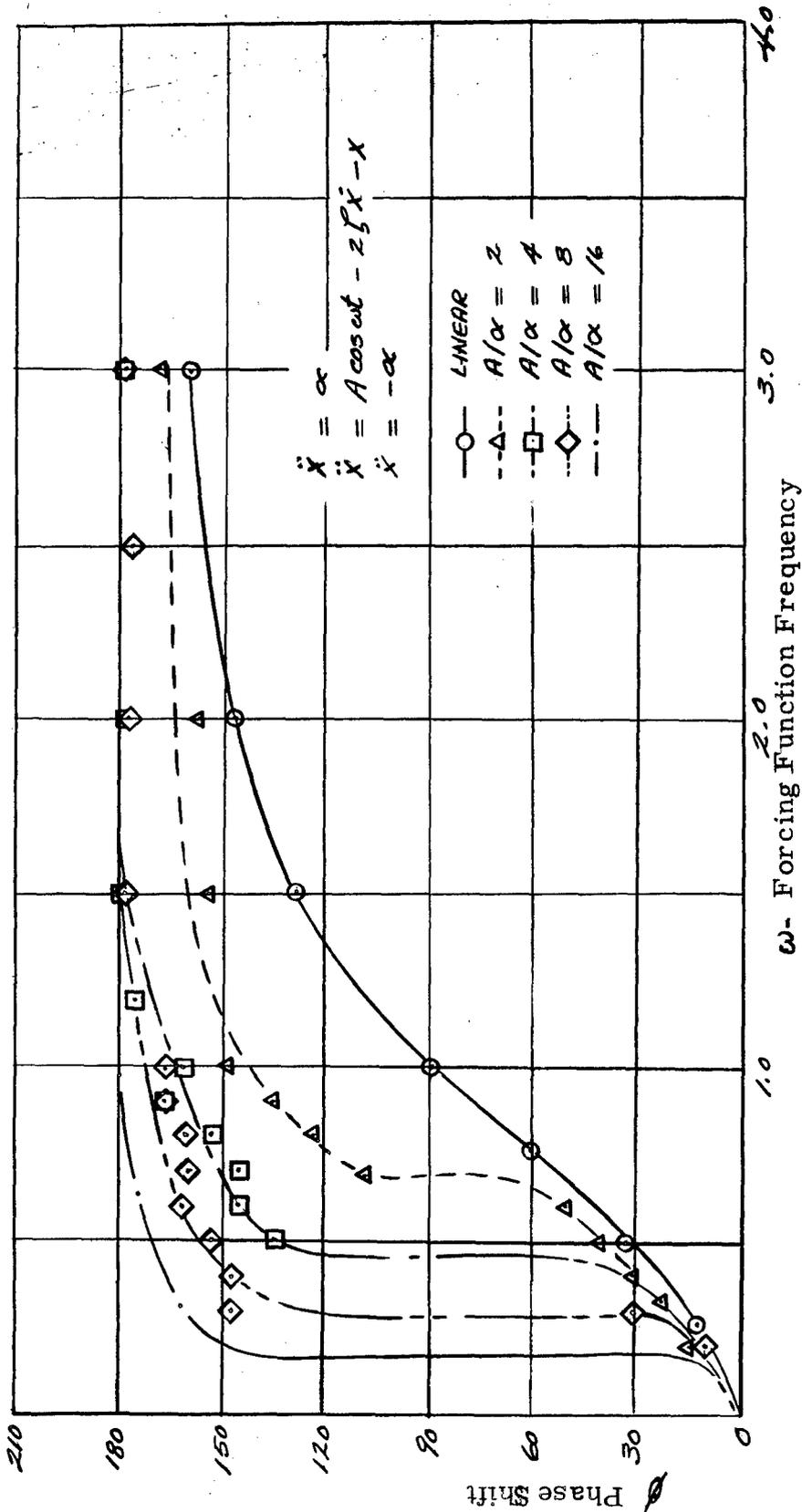


Figure 23. PHASE SHIFT OF SATURATING AMPLIFIER SERVO,  $\zeta = 1/2$

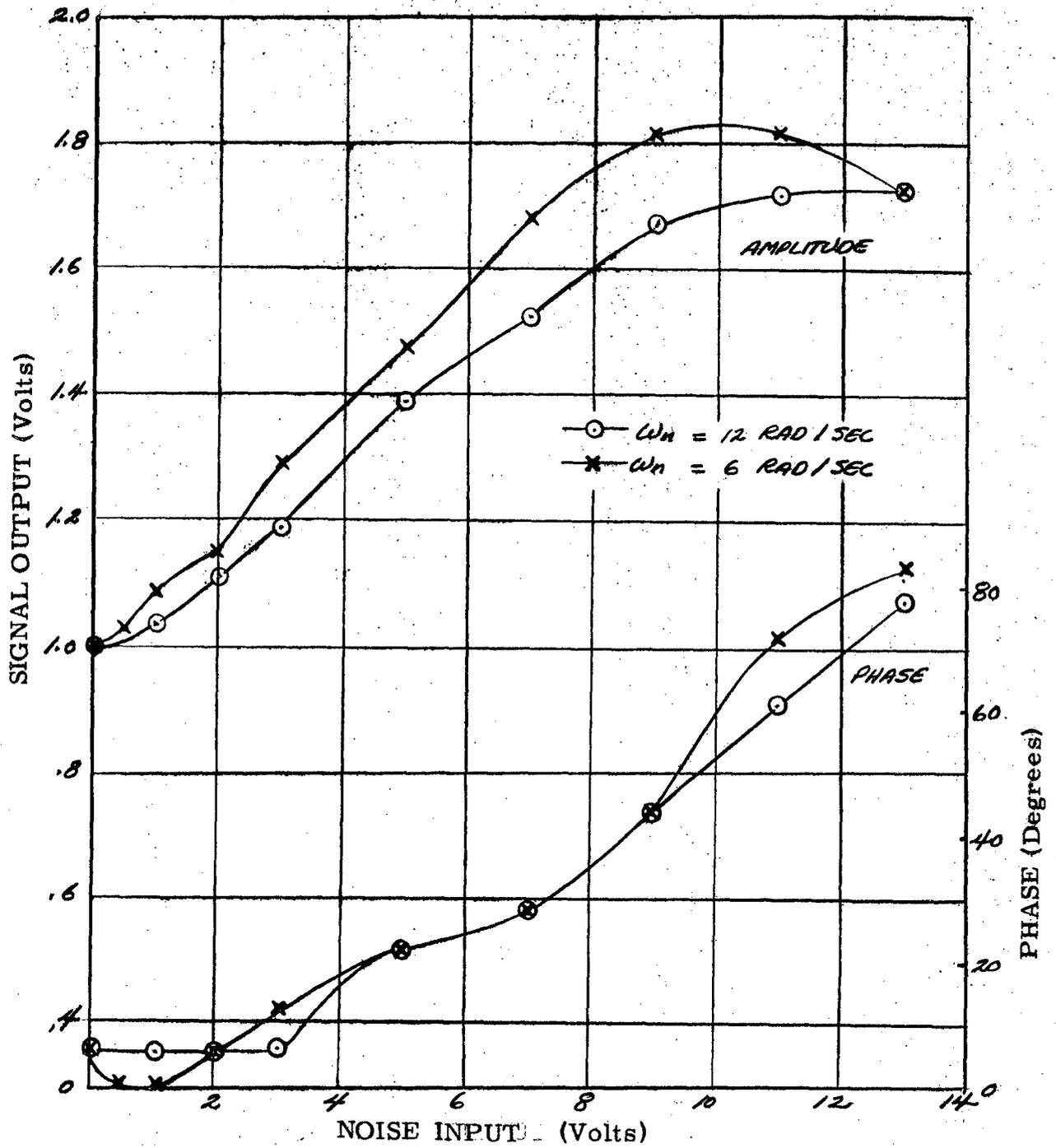


Figure 24.  
OUTPUT VS NOISE AMPLITUDE

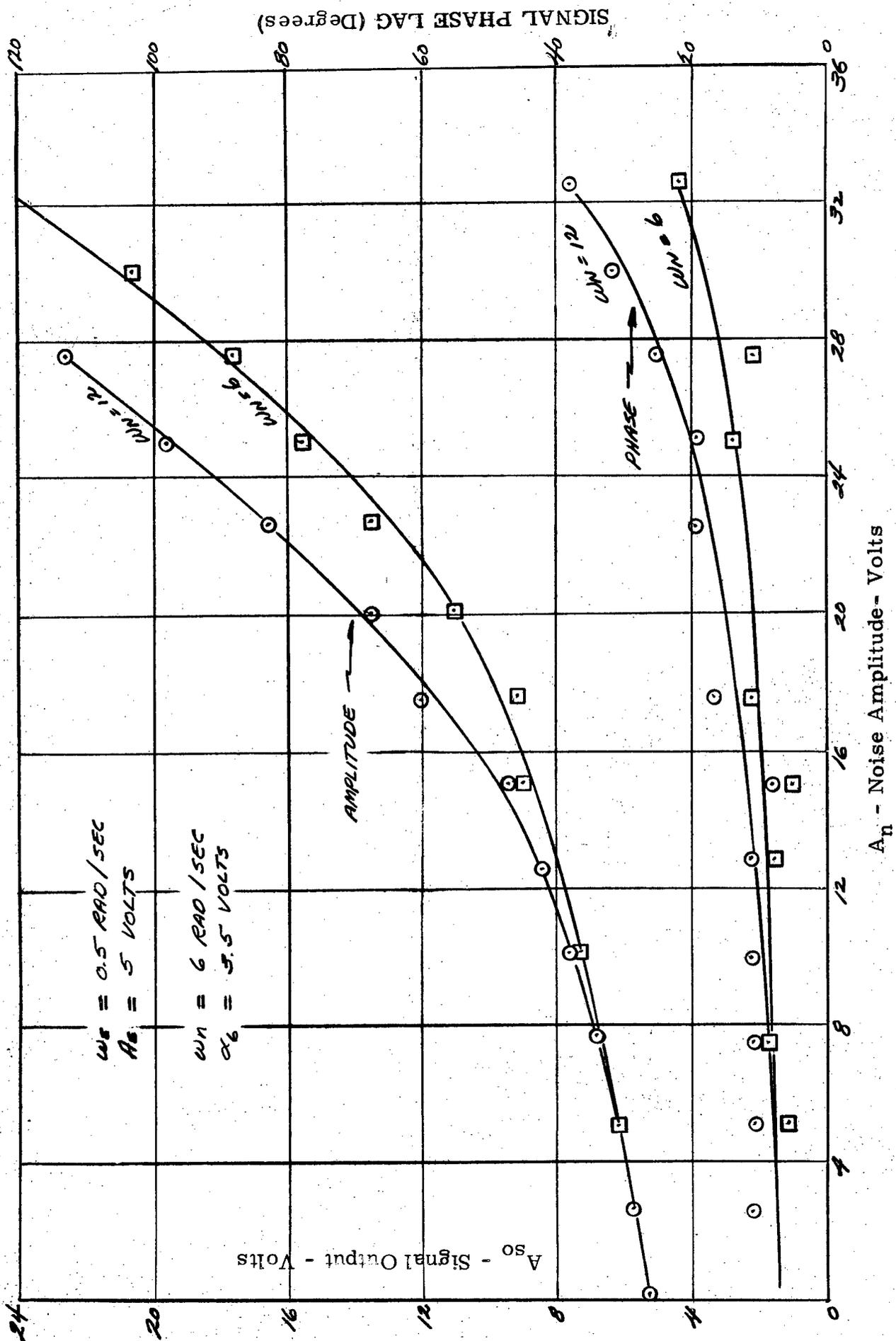


Figure 25. EFFECT OF NOISE FREQUENCY ON OUTPUT VS NOISE AMPLITUDE

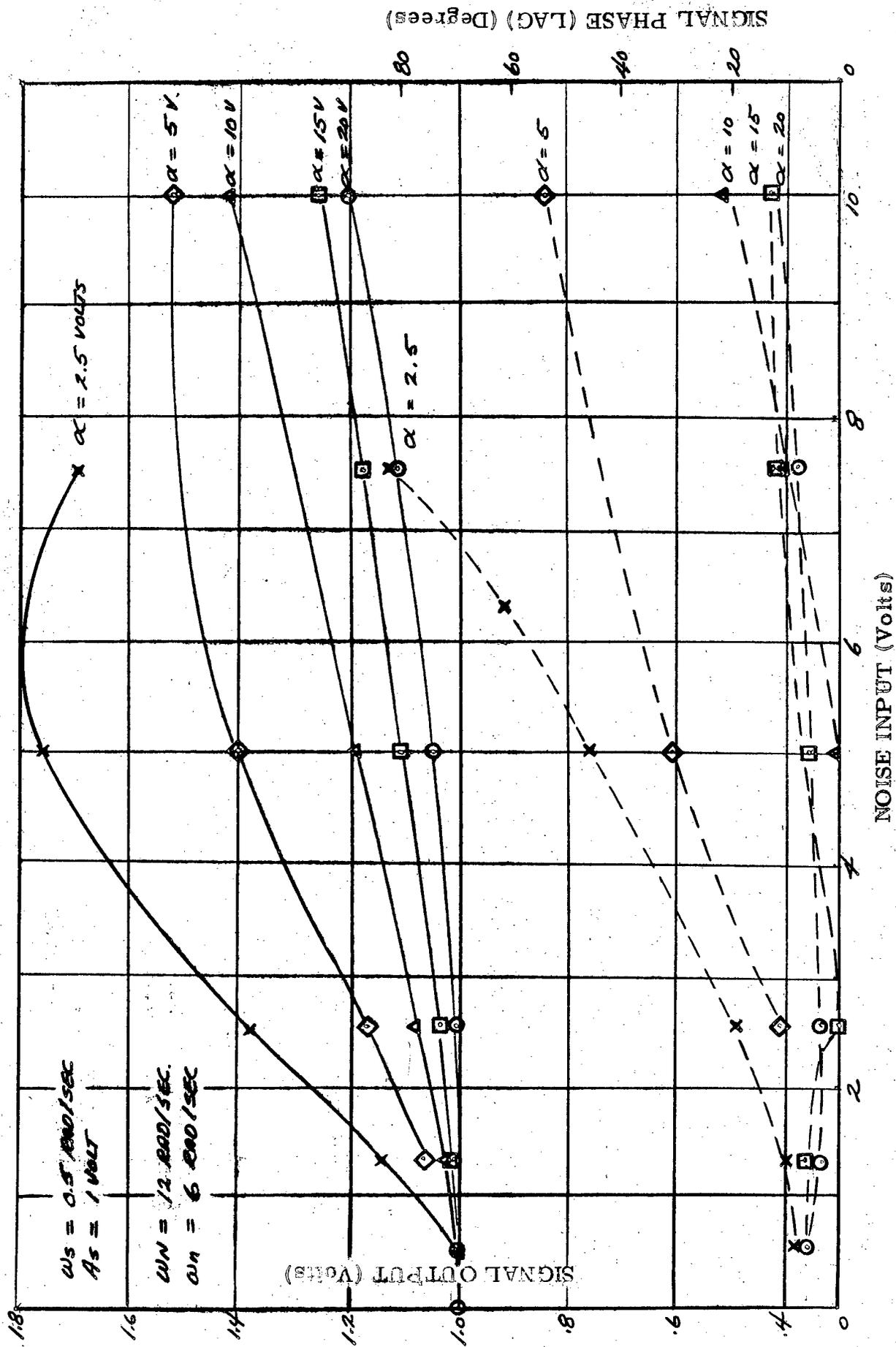


Figure 26. OUTPUT VS NOISE AMPLITUDE

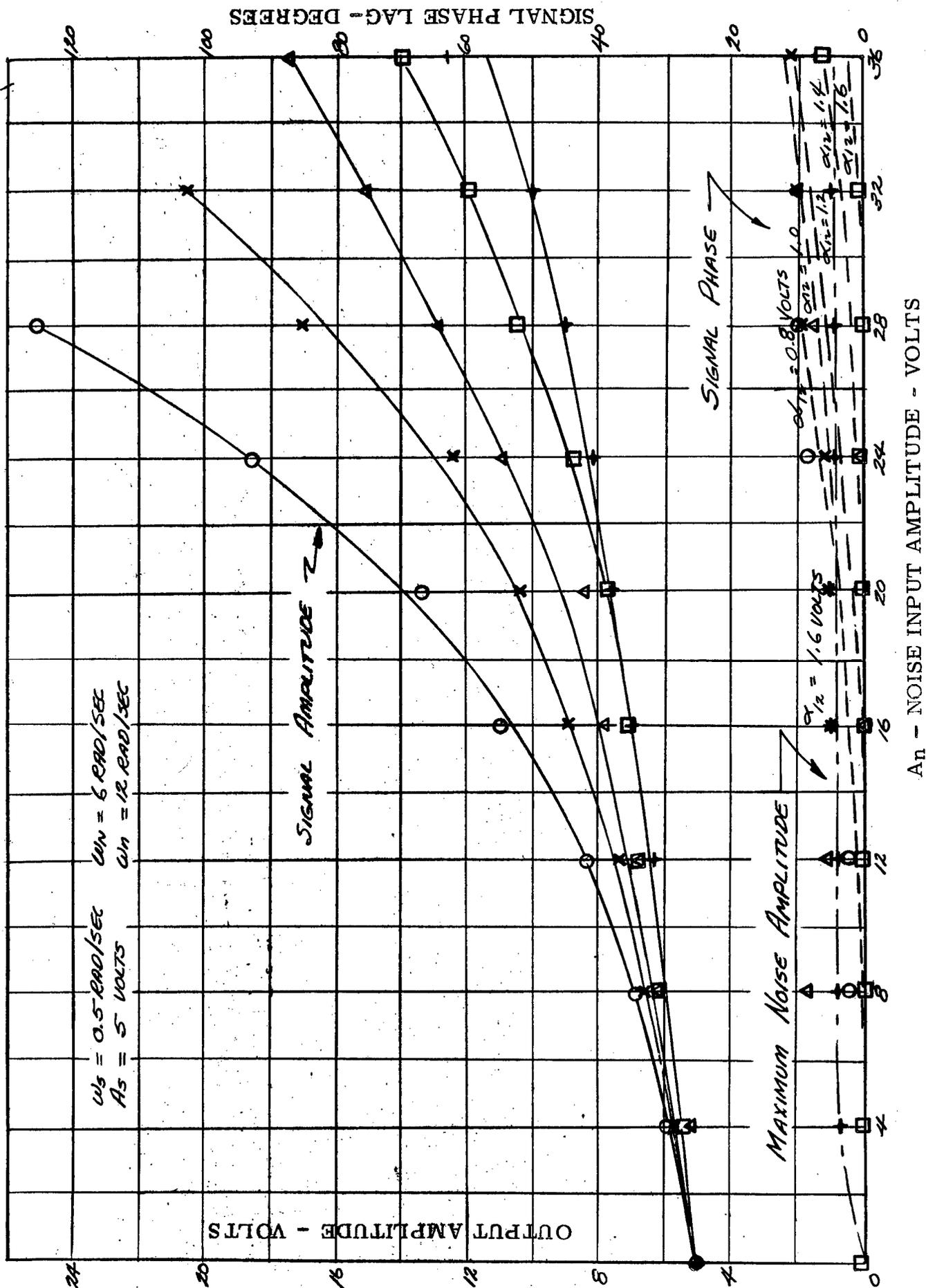


Figure 27. OUTPUT VS NOISE AMPLITUDE

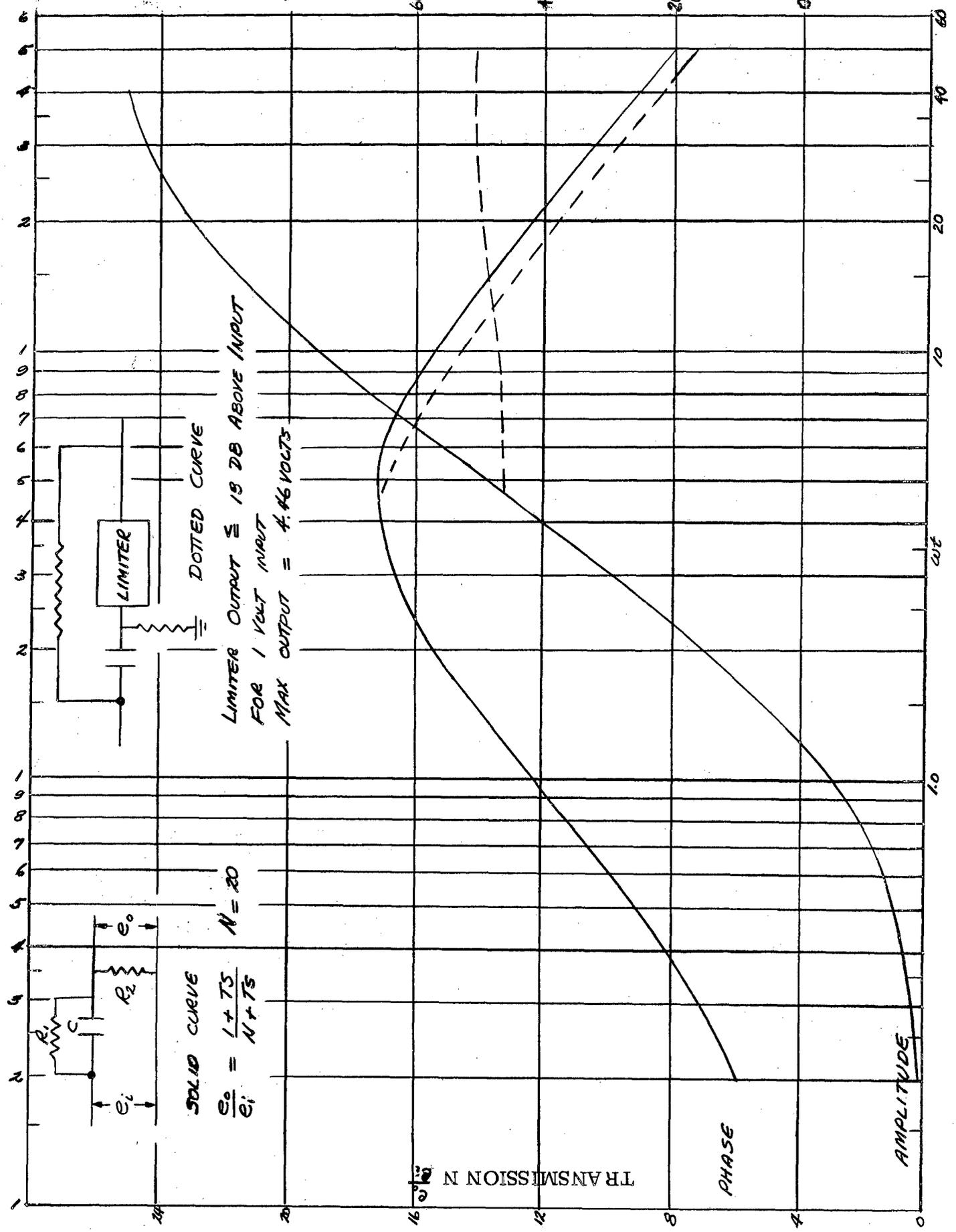


Figure 28. COMPARISON OF STANDARD NETWORK WITH NON-LINEAR NETWORK

A REAC investigation was also made of a velocity limited filter in which the limit on the rate of output was controlled by the rate of the input. For large input rate the limit on the output is raised and for smaller input rate the limits are lowered which is similar to the manner of controlling acceleration limits on the perception filter. The filter was included as a part of a simulation of the ILS system on the REAC and data obtained for bracketing at 12 miles out with a heading of  $90^\circ$  to the beam center. The data indicated the system stability in the presence of noise to be essentially the same as in the absence of noise. Further investigation of this method of filtering will be necessary before definite conclusions can be drawn as to its general utility.

Another possibility of reducing the effect of noise is limiting in the lead network itself. Although this type of limiting is more nearly displacement than acceleration limiting, there may be possibilities for system improvement.

A plot of attenuation and phase of the non-linear network is compared to that of a linear network in Figure 28. The non-linear curve is valid only for one amplitude; at smaller amplitudes of forcing function, the amplitude break would come at higher frequencies. The general characteristics of this curve would be desirable since it would attenuate the objectionable large amplitude noise. However, in common with other non-linear circuits such as velocity and acceleration limiting, the non-linear lead network will not maintain the proper phase and amplitude of a low frequency in the presence of a large amount of high frequency. Therefore, it is necessary to use variable limits on a practical system unless the noise amplitudes are nearly constant.

A vector diagram illustration of the behavior of the non-linear network may be made by considering the output as the sum of a proportional signal and the limited output of an R-C network.

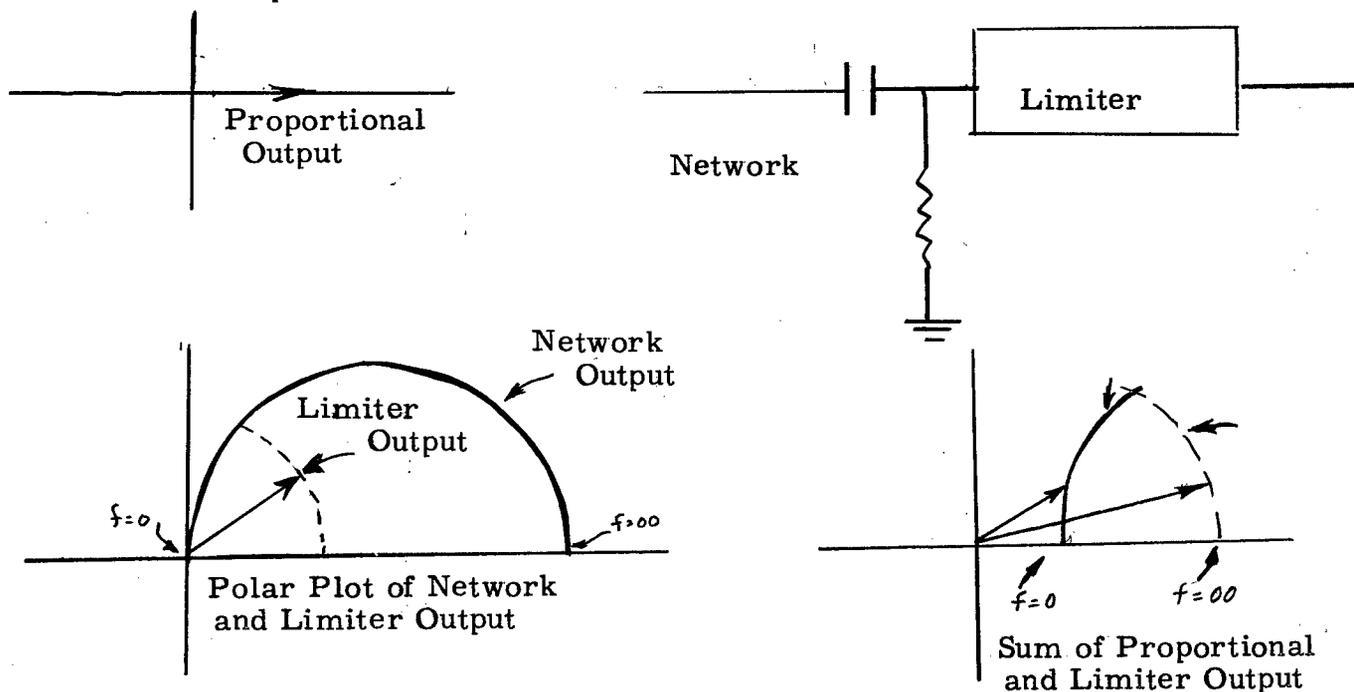


Figure 29.  
VECTOR DIAGRAM